The Theory of the Harmonic Waves of the *m-n* Symmetrical Machines

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ABSTRACT

This paper describes the new theory of the higher harmonics waves due to the magnetomotive force distribution of the *n*-phase squirrel cage induction machines. The analytic method of this paper is the tensor analysis, using the *m*- and *n*-phase symmetrical coordinate matrices. The mathematical model of this paper is described in the following way that the stator circuits have the *m*-phase symmetrical winding that is connected with the star connection and the rotor circuits have the *n*-phase symmetrical winding that consist of the bars. The impedance matrix of this model is transformed into the symmetrical coordinate by the *m*- and *n*-phase symmetrical coordinate matrices and its results are classified into the four cases by the relation of the number of rotor bars or phase and the pair of poles.

INTRODUCTION

The study about the abnormal phenomena due to the space harmonics waves produced by the MMF distribution and the slot permeance variation of the machine's air gap are described by several other papers. Kron pointed out the slot combinations, using the permeance wave and the electromagnetic noise due to the space harmonics waves that is very important problem and is not solved perfectly is studied by Alger, Jordan and Erdelyi, (1)(2)(3)(4)

This paper is an attempt to clarify the new nature of the MMF harmonics waves of the *m*-phase squirrel cage induction machines which is the characteristic of the mutual action of the MMF space harmonics waves that plays the important role of the synchronous torque and the high frequency current of the stator circuit related with the number of the rotor bars and pair of poles.

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SYMBOLS AND NOTATIONS

m	: number of the stator phase
n	: number of the rotor phase
p	: number of the pair of poles
g, g', h, h'	: positive integer including zero
$\nu=2mh\pm1$	: order of the stator MMF harmonics waves
μ	: order of the rotor MMF harmonics waves, in this case positive integer
γ	: order of the MMF harmonics waves smaller than "n"
f	: stator impressed source frequency
s	: slip
ω	: $2\pi f$
θ	: $(1-s)\omega t$, angle between the stator and rotor circuit transformed into the electrical angle
$H_1(\nu)$	: magnetic field due to the stator MMF
$H_2(\mu)$	: magnetic field due to the rotor MMF
L	: kinetic energy term of the Lagrange function, in this case magnetic field energy
F	: Rayleigh dissipative function, in this case electrical resistance power loss
$f_i(t)$	: i-th coordinate external forced function, in this case stator impressed voltage
x_i	: variable of the i-th coordinate, in this case electrical charge
$[Z]$	: total impedance matrix
$[Z_{11}]$	: stator impedance matrix
$[Z_{22}]$	: rotor impedance matrix
$[M_{12}], [M_{21}]$	: mutual inductance matrix
R_1, R_2	: stator and rotor circuit resistance respectively
$L\nu, L\mu$	: self-inductance of the ν-th and μ-th harmonic wave respectively
l_1, l_2	: leakage inductance of the stator and rotor circuit respectively
$Mp\nu$	: mutual inductance of the ν-th harmonic wave between the stator and rotor circuit
$*$	: denotes the conjugate quantity
$'$	: denotes the transformed quantity in the symmetrical coordinate
$\cdot$	: denotes the differential respective to the time
δ	: 2 or m
α	: $j\,2\pi/m$, element of the m-phase symmetrical coordinate matrix
β	: $j\,2\pi/n$, element of the n-phase symmetrical coordinate matrix
$[m], [n]$	: m- and n-phase symmetrical coordinate matrix respectively
$[\ \ ]_t$	: transpose of the matrix
$[\ \ ]^{-1}$	: denotes the inverse matrix

THEORY

Fundamental Impedance Matrix : The magnetic field due to the stepwise MMF distribution of the m-phase and $2p$ poles stator circuit is written in the following equation.

$$H_1=\sum_{\nu} H_1(\nu) \cdots\cdots (1)$$

The magnetic field due to the saw wave MMF distribution of the n-phase or bars of the rotor circuit (Fig. 1.) is written in the following equation.

$$H_2=\sum_{\mu} H_2(\mu) \cdots\cdots (2)$$

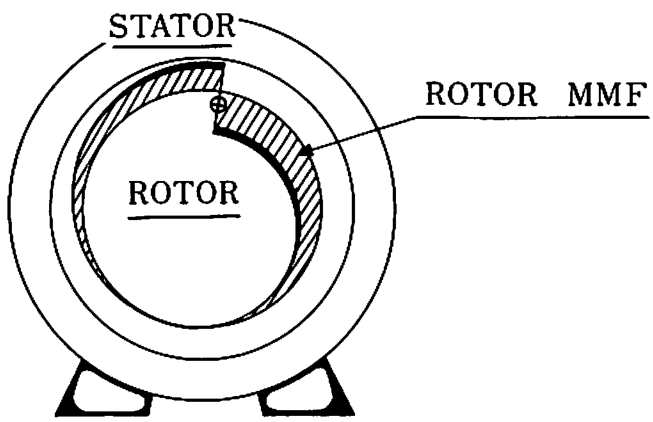


Fig. 1　Rotor MMF distribution of the one bar.

The kinetic energy term of Lagrange equation is calculated from the Eq. (1) and Eq. (2) and Rayleigh's dissipative function can be written as the electrical resistance power loss in each coordinate. Therefore, Euler-Lagrange equation is written in the following equation.

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right)-\frac{\partial L}{\partial x_i}+\frac{\partial F}{\partial \dot{x}_i}=f_i(t) \cdots\cdots (3)$$

The fundamental equation is derived by the Eq. (3) and the fundamental impedance matrix is written by the following equation

$$[Z]=\left[\begin{array}{c|c}[Z_{11}] & [M_{12}] \\ \hline [M_{21}] & [Z_{22}]\end{array}\right] \cdots\cdots (4)$$

and each impedance matrix of Eq. (4) is shown in the following equations. (Note: $p\nu$ denotes the common harmonic wave's order of the stator and rotor MMF harmonics waves)

$[Z_{11}]=$

$R_1+\frac{d}{dt}(\sum_v L_v+l_1)$	$\frac{d}{dt}\sum_v L_v\cos\frac{2\pi}{m}v$	·	$\frac{d}{dt}\sum_v L_v\cos\frac{m-1}{m}2\pi v$
$\frac{d}{dt}\sum_v L_v\cos\frac{2\pi}{m}v$	$R_1+\frac{d}{dt}(\sum_v L_v+l_1)$	·	$\frac{d}{dt}\sum_v L_v\cos\frac{m-2}{m}2\pi v$
· · ·	· · ·	·	· · ·
$\frac{d}{dt}\sum_v L_v\cos\frac{m-1}{m}2\pi v$	$\frac{d}{dt}\sum_v L_v\cos\frac{m-2}{m}2\pi v$	·	$R_1+\frac{d}{dt}(\sum_v L_v+l_1)$

$[M_{12}]=[M_{21}]_t=\frac{d}{dt}\sum_v M_{pv}\times$

$\cos \nu\theta$	$\cos \nu(\theta+\frac{1}{n}2\pi p)$	·	$\cos \nu(\theta+\frac{n-1}{n}2\pi p)$
$\cos \nu(\theta-\frac{1}{m}2\pi)$	$\cos \nu(\theta-\frac{2\pi}{m}+\frac{1}{n}2\pi p)$	·	$\cos \nu(\theta-\frac{2\pi}{m}+\frac{n-1}{n}2\pi p)$
· · ·	· · ·	·	· · ·
$\cos \nu(\theta-\frac{m-1}{m}2\pi)$	$\cos \nu(\theta-\frac{m-1}{m}2\pi+\frac{1}{n}2\pi p)$		$\cos\nu(\theta-\frac{m-1}{m}2\pi+\frac{n-1}{n}2\pi p)$

$[Z_{22}]=$

$R_2+\frac{d}{dt}(\sum_\mu L_\mu+l_2)$	$\frac{d}{dt}\sum_\mu L_\mu\cos\frac{1}{n}2\pi\mu$	·	$\frac{d}{dt}\sum_\mu L_\mu\cos\frac{n-1}{n}2\pi\mu$
$\frac{d}{dt}\sum_\mu L_\mu\cos\frac{1}{n}2\pi\mu$	$R_2+\frac{d}{dt}(\sum_\mu L_\mu+l_2)$	·	$\frac{d}{dt}\sum_\mu L_\mu\cos\frac{n-2}{n}2\pi\mu$
· · ·	· · ·	·	· · ·
$\frac{d}{dt}\sum_\mu L_\mu\cos\frac{n-1}{n}2\pi\mu$	$\frac{d}{dt}\sum_\mu L_\mu\cos\frac{n-2}{n}2\pi\mu$	·	$R_2+\frac{d}{dt}(\sum_\mu L_\mu+l_2)$

Coordinate Transformation: The coordinate transformation is done by the *m*- and *n*-phase symmetrical coordinate matrices. The stator and the rotor impedance matrix is transformed by the *m*- and *n*-phase symmetrical coordinate matrices respectively and the mutual inductance matrix is transformed by the *m*- and *n*-phase symmetrical coordinate matrices.

The following relations between the space harmonics waves produced by the stator or rotor MMF distribution and *m*- or *n*-phase symmetrical coordinate matrix operator are established.

$$\alpha^{\nu}=\alpha^{2mh\pm1}=\alpha^{\pm1} \quad \cdots\cdots (5)$$

$$\beta^{\mu}=\beta^{gn\pm\gamma}=\beta^{\pm\gamma} \quad \cdots\cdots (6)$$

The coordinate transformation is carried out by the following equation, using the relations of Eq. (5) and Eq. (6).

$$[Z']=\left[\begin{array}{c|c}[m][Z_{11}][m]^{-1} & [m][M_{12}][n]^{-1} \\ \hline [n][M_{21}][m]^{-1} & [n][Z_{22}][n]^{-1}\end{array}\right] \quad \cdots\cdots (7)$$

The term $[Z'_{11}]=[m][Z_{11}][m]^{-1}$ consists of the elements (2,2) and (m, m)

$$R_1+\frac{d}{dt}\left[\frac{m}{2}\sum_{\nu}L_{\nu}+l_1\right]$$

and the other elements on the diagonal line of the matrix $[Z']$ are the

$$R_1+\frac{d}{dt}l_1$$

that are zero phase impedance of the stator circuits and the other elements of the matrix $[Z'_{11}]$ are zero.

The term $[Z'_{22}]=[n][Z_{22}][n]^{-1}$ consists of the elements

$$R_2+\frac{d}{dt}(\sum_{g}L_{gn}+l_2)$$

that are zero phase impedance of the rotor circuits and the elements $(\gamma+1, \gamma+1)$ and $(n-\gamma+1, n-\gamma+1)$ of the matrix $[Z'_{22}]$ are the

$$R_2+\frac{d}{dt}\left(\frac{n}{2}\sum_{g}L_{gn+\gamma}+l_2\right)$$

and the other elements of the matrix $[Z'_{22}]$ are zero.

The term $[M'_{12}]=[m][M_{12}][n]^{-1}$ consists of the elements $(\delta, 1)$ of the matrix $[M'_{12}]$

$$\frac{\sqrt{mn}}{2}\sum_{g}M_{gn}\exp\left(\pm j\frac{gn\theta}{p}\right)$$

that are the zero phase mutual inductances of the space harmonics waves of rotor circuit and the elements $(2,\gamma+1)$ of the matrix $[M'_{12}]$ are the

$$\frac{\sqrt{mn}}{2}\sum_{g}M_{gn+\gamma}\exp\left\{\pm j(gn+\gamma)\frac{\theta}{p}\right\}$$

and their conjugate quantities are in the position $(m, n+1-\gamma)$ of the matrix $[M'_{12}]$.

The term $[M'_{21}]=[n][M_{21}][m]^{-1}$ is derived from the following equation.

$$[M'_{21}]=[M'_{12}]^{*}_{t}$$

The transformed mutual inductance matrix $[M'_{12}]$ or $[M'_{21}]$ is classified into the four cases by the relation of the number of rotor phase and pole pairs.

Each case of the transformed impedance matrix $[Z']$ is expressed by the following equations without zero phase components of the stator and rotor circuit.

(A) If $n(1+g+g')$ is equal to $2pm(h+h')$, the $p(2mh\pm1)-th$ and $p(2mh'\mp1)-th$ harmonics wave's mutual inductances are in the same column in the matrix $[M'_{12}]$ and this case transformed impedance matrix $[Z']$ is written by the Eq. (8).

(B) If $n(1+g+g')$ is equal to $2p[m(h+h')+1]$, the $p(2mh+1)-th$ and $p(2mh'+1)-th$ harmonics wave's mutual inductances are in the same column in the matrix $[M'_{12}]$ and this case transformed impedance matrix $[Z']$ is written by the Eq. (9).

$R_1+\frac{d}{dt}\left\{\frac{m}{2}\sum_{v} L_v+l_1\right\}$		·	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g} Me_{gn+\gamma}^{\pm j(gn+\gamma)\frac{\theta}{p}}$	·	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} Me_{1\mp g'n-\gamma}^{\pm j(1\mp g'n-\gamma)\frac{\theta}{p}}$	·
	$R_1+\frac{d}{dt}\left\{\frac{m}{2}\sum_{v} L_v+l_1\right\}$	·	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} Me_{1\mp g'n-\gamma}^{\mp j(1\mp g'n-\gamma)\frac{\theta}{p}}$	·	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g} Me_{gn+\gamma}^{\mp j(gn+\gamma)\frac{\theta}{p}}$	·
· · ·	· · ·	·				
$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g} Me_{gn+\gamma}^{\mp j(gn+\gamma)\frac{\theta}{p}}$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} Me_{1\mp g'n-\gamma}^{\pm j(1\mp g'n-\gamma)\frac{\theta}{p}}$		$R_2+\frac{d}{dt}\left\{\frac{n}{2}\sum_{g} L_{gn+\gamma}+l_2\right\}$			
· · ·	· · ·			·		
$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} Me_{1\mp g'n-\gamma}^{\mp j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g} Me_{gn+\gamma}^{\pm j(gn+\gamma)\frac{\theta}{p}}$				$R_2+\frac{d}{dt}\left\{\frac{n}{2}\sum_{g} L_{gn+\gamma}+l_2\right\}$	
· · ·	· · ·					·

··············(8)

$R_1+\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{m}{2}\sum_{v}L_v+l_1\right\}$		$\cdot$	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g}Me_{gn+\gamma}^{j(gn+\gamma)\frac{\theta}{p}}$	$\cdot$	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g'}Me_{1\mp g'n-\gamma}^{j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\cdot$
	$R_1+\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{m}{2}\sum_{v}L_v+l_1\right\}$	$\cdot$	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g'}Me_{1\mp g'n-\gamma}^{-j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\cdot$	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g}Me_{gn+\gamma}^{-j(gn+\gamma)\frac{\theta}{p}}$	$\cdot$
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$	$\cdot$				
$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g}Me_{gn+\gamma}^{-j(gn+\gamma)\frac{\theta}{p}}$	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g'}Me_{1\mp g'n-\gamma}^{j(1\mp g'n-\gamma)\frac{\theta}{p}}.$		$R_2+\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{n}{2}\sum_{g}L_{gn+\gamma}+l_2\right\}$			
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$			$\cdot$		
$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g'}Me_{1\mp g'n-\gamma}^{-j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g}Me_{gn+\gamma}^{j(gn+\gamma)\frac{\theta}{p}}$				$R_2+\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{n}{2}\sum_{g}L_{gn+\gamma}+l_2\right\}$	$\cdot$
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$					$\cdot$

$$\cdots\cdots\cdots\cdots\cdots(9)$$

$R_1+\frac{d}{dt}\left\{\frac{m}{2}\sum_v L_v+l_1\right\}$		$\cdot$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_g M_{gn+\gamma} e^{-j(gn+\gamma)\frac{\theta}{p}}$	$\cdot$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} M_{1\mp g'n-\gamma} e^{-j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\cdot$
	$R_1+\frac{d}{dt}\left\{\frac{m}{2}\sum_v L_v+l_1\right\}$	$\cdot$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} M_{1\mp g'n-\gamma} e^{j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\cdot$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_g M_{gn+\gamma} e^{j(gn+\gamma)\frac{\theta}{p}}$	$\cdot$
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$	$\cdot$				
$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_g M_{gn+\gamma} e^{j(gn+\gamma)\frac{\theta}{p}}$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} M_{1\mp g'n-\gamma} e^{-j(1\mp g'n-\gamma)\frac{\theta}{p}}$		$R_2+\frac{d}{dt}\left\{\frac{n}{2}\sum_g L_{gn+\gamma}+l_2\right\}$			
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$			$\cdot$		
$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g'} M_{1\mp g'n-\gamma} e^{j(1\mp g'n-\gamma)\frac{\theta}{p}}$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_g M_{gn+\gamma} e^{-j(gn+\gamma)\frac{\theta}{p}}$				$R_2+\frac{d}{dt}\left\{\frac{n}{2}\sum_g L_{gn+\gamma}+l_2\right\}$	
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$					$\cdot$

$\cdots\cdots\cdots\cdots\cdot(10)$

$R_1+\frac{d}{dt}\left\{\frac{m}{2}\sum_{v}L_v+l_1\right\}$		$\cdot$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g}M_{gn+\gamma}e^{j(gn+\gamma)\frac{\theta}{p}}$	$\cdot$		$\cdot$
	$R_1+\frac{d}{dt}\left\{\frac{m}{2}\sum_{v}L_v+l_1\right\}$	$\cdot$		$\cdot$	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g}M_{gn+\gamma}e^{-j(gn+\gamma)\frac{\theta}{p}}$	$\cdot$
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$	$\cdot$				
$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g}M_{gn+\gamma}e^{-j(gn+\gamma)\frac{\theta}{p}}$			$R_2+\frac{d}{dt}\left\{\frac{n}{2}\sum_{g}L_{gn+\gamma}+l_2\right\}$			
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$			$\cdot$		
	$\frac{d}{dt}\frac{\sqrt{mn}}{2}\sum_{g}M_{gn+\gamma}e^{j(gn+\gamma)\frac{\theta}{p}}$				$R_2+\frac{d}{dt}\left\{\frac{n}{2}\sum_{g}L_{gn+\gamma}+l_2\right\}$	
$\cdot\ \cdot\ \cdot$	$\cdot\ \cdot\ \cdot$					$\cdot$

$\cdots\cdots\cdots\cdots\cdots(11)$

(C) If $n(1+g+g')$ is equal to $2p[m(h+h')-1]$, the $p(2mh-1)-th$ and $p(2mh'-1)-th$ harmonics wave's mutual inductances are in the same column in the matrix $[M_{12}]$ and this case transformed impedance matrix $[Z']$ is written by the Eq. (10).

(D) If $n(1+g+g')$ is not equal to $2p[m(h+h')\pm(^{1}_{0})]$, each harmonic wave's mutual inductance exists independently in the matrix $[M_{12}]$ and this case transformed impedance matrix $[Z']$ is written in the Eq. (11).

Current and Torque : If the stator impressed m-phase voltage are perfectly balanced, the transformed voltage matrix by the m-phase symmetrical coordinate matrix consists of the second row element $V'\exp(\mathrm{j}\omega t)$ and its conjugate value of the m-th row. The currents are calculated by the iteration method and the example of the current matrix derived from the first iterative approximation of the case (B) is written by the following equation.

$$[I']=\begin{bmatrix} i_1 e^{\mathrm{j}\omega t}+\sum_{g}\sum_{g'} i_1'^{*} e^{-\mathrm{j}\omega t+\mathrm{j}(1+g+g')\frac{n\theta}{p}} \\ i_1^{*} e^{-\mathrm{j}\omega t}+\sum_{g}\sum_{g'} i_1' e^{\mathrm{j}\omega t-\mathrm{j}(1+g+g')\frac{n\theta}{p}} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \sum_{g} i_{gn+\gamma} e^{\mathrm{j}\omega t-\mathrm{j}(gn+\gamma)\frac{\theta}{p}}+\sum_{g'} i^{*}_{1\mp g'n-\gamma} e^{-\mathrm{j}\omega t+\mathrm{j}(1\mp g'n-\gamma)\frac{\theta}{p}} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ \sum_{g} i^{*}_{gn+\gamma} e^{-\mathrm{j}\omega t+\mathrm{j}(gn+\gamma)\frac{\theta}{p}}+\sum_{g'} i_{1\mp g'n-\gamma} e^{\mathrm{j}\omega t-\mathrm{j}(1\mp g'n-\gamma)\frac{\theta}{p}} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \end{bmatrix} \quad \cdots\cdots (12)$$

The torque is calculated from the following equation

$$T=\frac{p}{4}\left\{[I^{*}]\,t\left(\frac{\partial}{\partial\theta}[Z']\right)[I']\right\} \cdots\cdots (13)$$

and the example of the torque calculation, using the Eq. (9) and the Eq. (12), results in the following torques equations.

$$T_1=\sum_{g}\mathrm{j}(gn+\gamma)\frac{\sqrt{mn}}{4}M_{gn+\gamma}\,i_1^{*}\cdot i_{gn+\gamma} \cdots\cdots (14)$$

$$T_1=\sum_{g'}\mathrm{j}(1\mp g'n-\gamma)\frac{\sqrt{mn}}{4}M_{1\mp g'n-\gamma}\,i_1^{*}\cdot i_{1\mp g'n-\gamma} \cdots\cdots (15)$$

$$T_2=\sum_{g}\sum_{g'}\mathrm{j}(gn+\gamma)\frac{\sqrt{mn}}{4}M_{gn+\gamma}\,i_1'\cdot i^{*}_{1\mp g'n-\gamma} \cdots\cdots (16)$$

$$T_2=\sum_{g}\sum_{g'}\mathrm{j}(1\mp g'n-\gamma)\frac{\sqrt{mn}}{4}M_{1\mp g'n-\gamma}\,i_1'\cdot i^{*}_{gn+\gamma} \cdots\cdots (17)$$

$$T_3=\sum_{g}\sum_{g'} \mathrm{j}(gn+\gamma)\frac{\sqrt{mn}}{4}M_{gn+\gamma}(i_1^{*}\cdot i_{1\mp g'n-\gamma}^{*}-i_1'^{*}\cdot i_{gn+\gamma}^{*})\mathrm{e}^{-\mathrm{j}2\omega t+\mathrm{j}(1+g+g')\frac{n\theta}{p}} \quad \ldots\ldots\ldots (18)$$

$$T_3=\sum_{g}\sum_{g'} \mathrm{j}(1\mp g'n-\gamma)\frac{\sqrt{mn}}{4}M_{1\mp g'n-\gamma}(i_1'\cdot i_{1\mp g'n-\gamma}-i_1\cdot i_{gn+\gamma})\mathrm{e}^{\mathrm{j}2\omega t-\mathrm{j}(1+g+g')\frac{n\theta}{p}} \quad \ldots\ldots\ldots (19)$$

and the total torque is written by the following equation.

$$T=T_1+T_1^*+T_2+T_2^*+T_3+T_3^*$$

The examination of the current matrix Eq. (12) gives the following relations that the rotor circuit currents consist of the term due to the $(gn+\gamma)$ harmonic wave's mutual inductance and the term due to the $[(1+g')n-\gamma]$ harmonic wave's mutual inductance, and the stator circuit currents consist of the stator impressed frequency term and the high frequency term due to the mutual action between the $(gn+\gamma)$ and the $[(1+g')n-\gamma]-th$ harmonics waves.

The examination of the torques terms results in the following relations that the Eq. (14) and Eq. (15) are the asynchronous torques produced by the stator impressed frequency current and the rotor harmonics currents, and the Eq. (16) and Eq. (17) are the asynchronous torques produced by the stator high frequency currents and the rotor harmonics currents, and the Eq. (18) and Eq. (19) are the synchronous torques effected at a particular slip

$$S=1-\frac{2p}{n(1+g+g')}$$

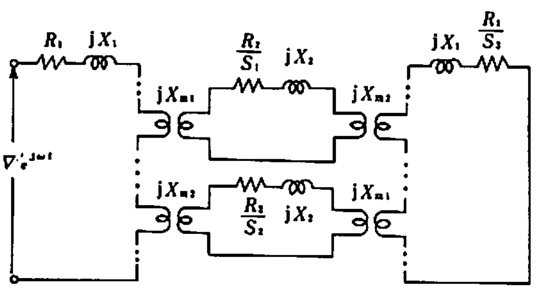
that consist of the term due to the stator impressed frequency current and the rotor harmonics waves currents and the term due to the stator high frequency currents and the rotor harmonics waves currents.

The current matrix and the torques of the other case are calculated in the same way as the previous example and its results are described as following : the current matrix of the case (A) consists of the several different high frequency stator currents and the rotor harmonics waves currents, and its case torques include the locking torques, and the current matrix of the case (C) is the same form as the case (B), but the stator high frequency currents have a different frequency from the case (B), therefore, its case torques include the synchronous torque in the braking region, and the current matrix of the case (D) consists of the stator impressed frequency current and the rotor harmonics waves currents, and the torques in this case consist only of asynchronous torques.

Commutation Matrix : If the harmonics wave's mutual inductances in the same column and same row in the transformed mutual inductance matrix exist independently, the commutation matrix can be established except the case (A), and the example of the commutation matrix for case (B) is shown by the Eq. (20) and the result of the commutation of the Eq. (9), using the Eq. (20), is expressed by the Eq. (21), and the equivalent circuit derived from the Eq. (21) for the stator transformed voltage $V'\exp(j\omega t)$ is shown in the Fig. 2, and the numerical example of the torque-slip curve calculated by the equivalent circuit of the Fig. 2 is shown in the Fig. 3, and its circuit constants are assumed in the table 1.

TABLE 1

Phase	Stator	$m=3$
	Rotor	$n=40$
Poles		4 poles
Voltage (Transformed value)		$V'=200$ (V)
Harmonics waves order (g=g'=0)		$gn+\gamma=p$
		$(1+g')n-\gamma=19p$
Resistance (Constant value assumed)		$R_1=R_2=5$ (Ω)
Mutual reactance	p-th harmonic wave	$X_{m1}=95$ (Ω)
	$19p$-th harmonic wave	$X_{m2}=95/(19\times19)$ (Ω)
Total reactance (Leakage reactance included)		$X_1=X_2=100$ (Ω)



$$X_1=\omega\left\{\frac{m}{2}\sum_{\nu}L_{\nu}+l_1\right\},\qquad X_2=\omega\left\{\frac{n}{2}\sum_{g}L_{gn+\gamma}+l_2\right\}$$

$$X_{m1}=\omega\frac{\sqrt{mn}}{2}M_{gn+\gamma},\qquad X_{m2}=\omega\frac{\sqrt{mn}}{2}M_{(1+g')n-\gamma}$$

$$S_1=\frac{\omega-(gn+\gamma)\dfrac{(1-s)\omega}{p}}{\omega},\qquad S_2=\frac{\omega-((1+g')n-\gamma)\dfrac{(1-s)}{p}}{\omega}$$

$$S_3=\frac{\omega-(1+g+g')\dfrac{n}{p}(1-s)\omega}{\omega}$$

Fig. 2 Steady state equivalent circuit of the case (B).

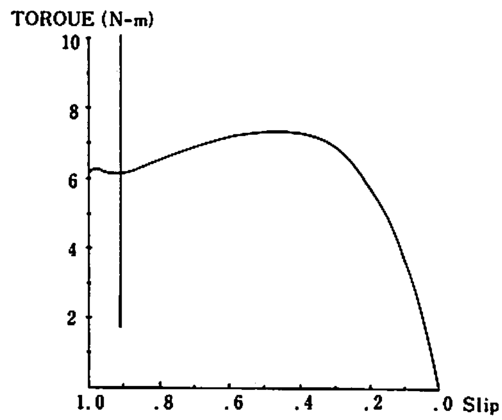


Fig. 3 Torque-slip curve of the case (B).

$[K]=$

1						
	$\sum_{g}\sum_{g'} e^{j(1+g+g')\frac{\theta}{p}n}$					
		·				
			$\sum_{g} e^{j(gn+\gamma)\frac{\theta}{p}}$			
				·		
					$\sum_{g'} e^{j(1\mp g'n-\gamma)\frac{\theta}{p}}$	
						·

$\cdots\cdots\cdots(20)$

$[K][Z'][K^*]t=$

$R_1+\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{m}{2}\sum_v L_v+l_1\right\}$		•	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_g M_{gn+\gamma}$	•	$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\sqrt{mn}}{2}\sum_{g'} M_{1\mp g'n-\gamma}$	•
	$R_1+\sum_g\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(1+g+g')\frac{n}{p}(1-s)\omega\right\}\cdot\left\{\frac{m}{2}\sum_v L_v+l_1\right\}$	•	$\sum_g\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(1+g+g')\frac{n}{p}(1-s)\omega\right\}\cdot\frac{\sqrt{mn}}{2}M_{1\mp g'n-\gamma}$	•	$\sum_g\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(1+g+g')\frac{n}{p}(1-s)\omega\right\}\cdot\frac{\sqrt{m}}{2n}M_{gn+\gamma}$	•
• • •	• • •	•				
$\sum_g\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(gn+\gamma)\frac{(1-s)}{p}\omega\right\}\cdot\frac{\sqrt{mn}}{2}M_{gn+\gamma}$	$\sum_g\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(gn+\gamma)\frac{(1-s)}{p}\omega\right\}\cdot\frac{\sqrt{mn}}{2}M_{1\mp g'n-\gamma}$		$R_2+\sum_g\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(gn+\gamma)\frac{(1-s)}{p}\omega\right\}\cdot\left\{\frac{n}{2}L_{gn+\gamma}+l_2\right\}$			
• • •	• • •			•		
$\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(1\mp g'n-\gamma)\frac{(1-s)}{p}\omega\right\}\cdot\frac{\sqrt{mn}}{2}M_{1\mp g'n-\gamma}$	$\sum_g\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(1\mp g'n-\gamma)\frac{(1-s)}{p}\omega\right\}\cdot\frac{\sqrt{mn}}{2}M_{gn+\gamma}$				$R_2+\sum_g\sum_{g'}\left\{\frac{\mathrm{d}}{\mathrm{d}t}-\mathrm{j}(1\mp g'-\gamma)\frac{n}{p}(1-s)\omega\right\}\cdot\left\{\frac{n}{2}L_{gn+\gamma}+l_2\right\}$	
• • •	• • •					•

$\cdots\cdots\cdots\cdots(21)$

CONCLUSION

As shown above, I have tried to elucidate the new nature of the MMF space harmonics waves of the *m*-phase squirrel cage induction machines modeled by the *m-n* symmetrical machines.

The particular pair of the space harmonics waves has the mutual action in the particular sets of the number of pole pairs and the rotor phase or bars, therefore, the particular pair of the space harmonics waves must be calculated simultaneously for the rigorous calculation of the phenomena due to the MMF space harmonics waves of the *m*-phase squirrel cage induction machines.*

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