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Relationships among depolarizations in scattering of polarized protons from ^{3}He at intermediate energies

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We derive relation formulas for proton depolarizations in proton-^{3}He (p-^{3}He) elastic scattering in a form that makes it easy to identify contributions of spin-dependent interactions. The formulas explain approximate relations found in the depolarizations measured at $E_p = 800$ MeV when the magnitudes of scattering amplitudes due to p-^{3}He spin–spin and tensor interactions are small. This nature of the interactions is investigated from the viewpoint of folding models. It is shown that the spin–spin and tensor interactions are significantly diminished owing to characteristics of nucleon densities of ^{3}He, which are calculated from a solution of the Faddeev equation. A folding model calculation with the densities and a simple nuclear potential shows that the p-^{3}He spin–spin interaction is much weaker than the spin-independent central interaction.

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Recently a complete set of proton depolarizations $D_i^j(\theta)$ for p-^{3}He elastic scattering at $E_p = 800$ MeV were measured together with differential cross sections and analyzing powers [1]. We first point out that the measured depolarizations satisfy the following relations, as shown in Fig. 1,

$$D_x^x(\theta) \approx D_z^z(\theta), \quad D_z^x(\theta) \approx -D_x^z(\theta), \quad D_y^y(\theta) \approx 1, \tag{1}$$

which were not discussed in Ref. [1]. Here, the coordinate axes are chosen as $\boldsymbol{z} \| \boldsymbol{k}_{\rm i}$ and $\boldsymbol{y} \| \boldsymbol{k}_{\rm i} \times \boldsymbol{k}_{\rm f}$, where $\boldsymbol{k}_{\rm i}$ ($\boldsymbol{k}_{\rm f}$) is the p-^{3}He relative momentum in the initial (final) state. These depolarizations are exactly related to each other as $D_x^x(\theta) = D_z^z(\theta)$, $D_z^x(\theta) = -D_x^z(\theta)$, and $D_y^y(\theta) = 1$ in proton scattering from spinless nuclei. Thus, the observed deviations from the exact relations will describe contributions of the ^{3}He spin in the p-^{3}He interaction.

In the present paper, we will theoretically derive relation formulas for the depolarizations. The formulas include characteristics of the p-^{3}He spin-dependent interactions and then provide the approximate relations, Eq. (1), when the magnitudes of scattering amplitudes due to p-^{3}He spin–spin and tensor interactions are small, implying the interactions to be weak. The weakness of these interactions is the reflection of the spin structure of the ^{3}He nucleus within the framework of folding models. To verify such a prediction, spin densities of ^{3}He are calculated from a wave function obtained by solving the Faddeev equation, and the spin–spin and tensor interactions are shown to be significantly diminished owing to the characteristics of the densities. In fact, a folding-model calculation is performed with the densities to give a very weak spin–spin interaction.

In theoretical developments we will follow those for hyperon-nucleon scattering [2], since the spins of related particles are $\frac{1}{2}$ in the both cases. We will calculate the depolarizations in a way that makes it easy to identify the contribution of each spin-dependent interaction. For that purpose, we expand the T matrix $\boldsymbol{M}$ into spin-space tensors $\boldsymbol{S}_\kappa^{(K)}$, where $K(\kappa)$ is the rank (z component) of the tensor,

$$\boldsymbol{M} = \sum_{K\kappa} (-)^\kappa \boldsymbol{S}_{-\kappa}^{(K)} \boldsymbol{R}_\kappa^{(K)}, \tag{2}$$

where $\boldsymbol{R}_\kappa^{(K)}$ is the counterpart, the coordinate space tensor. The tensors $\boldsymbol{S}_{-\kappa}^{(K)}$ and $\boldsymbol{R}_\kappa^{(K)}$ depend on the magnitudes of spins of related particles. In the following, this dependence is designated by the channel spins, $s_{\rm i}$ and $s_{\rm f}$, where $s_{\rm i}$ and $s_{\rm f}$ are 0 or 1. Denoting the z component of spin s by ν, Eq. (2) gives the scattering amplitude

$$\langle s_{\rm f}\nu_{\rm f}; \boldsymbol{k}_{\rm f} | \boldsymbol{M} | s_{\rm i}\nu_{\rm i}; \boldsymbol{k}_{\rm i}\rangle = \sum_{K=0}^{2} (-)^{s_{\rm f}-\nu_{\rm f}} (s_{\rm i} s_{\rm f} \nu_{\rm i} - \nu_{\rm f} | K\kappa) \times M_\kappa^{(K)}(s_{\rm i}s_{\rm f}; \boldsymbol{k}_{\rm i}, \boldsymbol{k}_{\rm f}), \tag{3}$$

with

$$M_\kappa^{(K)}(s_{\rm i}s_{\rm f}; \boldsymbol{k}_{\rm i}, \boldsymbol{k}_{\rm f}) = \frac{(-)^{s_{\rm i}-s_{\rm f}}}{\sqrt{2K+1}} (s_{\rm f} \| \boldsymbol{S}^{(K)} \| s_{\rm i}) \times \langle \boldsymbol{k}_{\rm f} | \boldsymbol{R}_\kappa^{(K)}(s_{\rm i}s_{\rm f}) | \boldsymbol{k}_{\rm i}\rangle. \tag{4}$$

The amplitudes with $K = 0$, $M_0^{(0)}(s_{\rm i}s_{\rm f}; \boldsymbol{k}_{\rm i}, \boldsymbol{k}_{\rm f})$, describe scattering by central interactions, those with $K = 1$, $M_\kappa^{(1)}(s_{\rm i}s_{\rm f}; \boldsymbol{k}_{\rm i}, \boldsymbol{k}_{\rm f})$, scattering by spin-orbit (SO) ones, and so on. Higher-order contributions of interactions are also included in these amplitudes according to their tensorial properties in the spin space.

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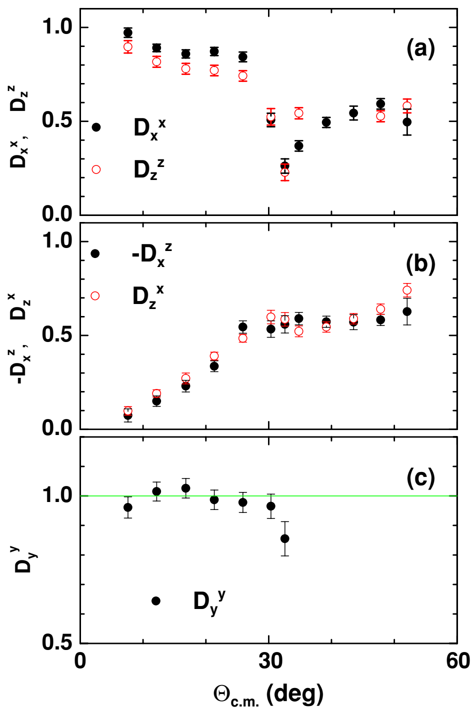

FIG. 1. (Color online) Measured depolarizations for p-^{3}He elastic scattering at $E_p = 800$ MeV [1]. (a) $D_x^x(\theta)$ (solid circles) and $D_z^z(\theta)$ (open circles). (b) $-D_x^z(\theta)$ (solid circles) and $D_z^x(\theta)$ (open circles). (c) $D_y^y(\theta)$ (solid circles).

In the present reference frame, the parity conservation gives [3]

$$M_{-\kappa}^{(K)}(s_i s_f; \boldsymbol{k}_i, \boldsymbol{k}_f) = (-)^{K-\kappa} M_{\kappa}^{(K)}(s_i s_f; \boldsymbol{k}_i, \boldsymbol{k}_f). \tag{5}$$

Further, in elastic scattering, the time reversal theorem gives [2]

$$M_{\kappa}^{(1)}(10; \boldsymbol{k}_i, \boldsymbol{k}_f) = -M_{\kappa}^{(1)}(01; \boldsymbol{k}_i, \boldsymbol{k}_f), \tag{6}$$

and

$$\sqrt{\tfrac{3}{2}} M_0^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) - M_2^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) = -2\cot\theta M_1^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f). \tag{7}$$

In total, six amplitudes, namely, two scalar, two vector, and two tensor amplitudes, are the independent ones. Expressions of polarization observables with these amplitudes are given in the appendix of Ref. [2].

To identify the contribution of spin components of the p-^{3}He interaction, we introduce a model interaction referring to the general form of the p-^{3}He scattering amplitude [4],

$$\begin{aligned}\langle \boldsymbol{k}_f | \boldsymbol{M} | \boldsymbol{k}_i \rangle = U_\alpha &+ (\boldsymbol{s}_p \cdot \boldsymbol{s}_h) U_\beta \\ &+ (\boldsymbol{s}_p \cdot \boldsymbol{n}) S_p + (\boldsymbol{s}_h \cdot \boldsymbol{n}) S_h \\ &+ \hat{S}_T(\boldsymbol{n}) T_n + (\hat{S}_T(\boldsymbol{\ell}) - \hat{S}_T(\boldsymbol{m})) T_{\ell m}. \end{aligned} \tag{8}$$

Here, $\boldsymbol{\ell}$ and $\boldsymbol{n}$ are the unit vectors parallel to $\boldsymbol{k}_i + \boldsymbol{k}_f$ and $\boldsymbol{k}_i \times \boldsymbol{k}_f$, respectively, and $\boldsymbol{m} = \boldsymbol{n} \times \boldsymbol{\ell}$; $\hat{S}_T(\boldsymbol{a})$ is the tensor operator with respect to a unit vector $\boldsymbol{a}$ given as

$$\hat{S}_T(\boldsymbol{a}) = 12(\boldsymbol{s}_p \cdot \boldsymbol{a})(\boldsymbol{s}_h \cdot \boldsymbol{a}) - 4(\boldsymbol{s}_p \cdot \boldsymbol{s}_h). \tag{9}$$

In Eq. (8), U_α and U_β describe the scattering by the spin-independent and spin-spin central interactions, S_p and S_h the scattering by the proton SO and the ^{3}He SO interactions, and T_n and $T_{\ell m}$ the scattering by the tensor interactions. Here, a $\hat{S}_T(\boldsymbol{\ell}) + \hat{S}_T(\boldsymbol{m})$ term that possibly arises is included in the $\hat{S}_T(\boldsymbol{n})$ term by the use of the symmetry relation $\hat{S}_T(\boldsymbol{n}) + \hat{S}_T(\boldsymbol{\ell}) + \hat{S}_T(\boldsymbol{m}) = 0$. From interaction (8), Eq. (4) gives

$$M_0^{(0)}(00; \boldsymbol{k}_i, \boldsymbol{k}_f) = U_\alpha - \frac{3}{4} U_\beta, \tag{10}$$

$$M_0^{(0)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) = \sqrt{3} U_\alpha + \frac{\sqrt{3}}{4} U_\beta, \tag{11}$$

$$M_{+1}^{(1)}(01; \boldsymbol{k}_i, \boldsymbol{k}_f) = \frac{i}{2\sqrt{2}}(S_p - S_h), \tag{12}$$

$$M_{+1}^{(1)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) = -\frac{i}{2}(S_p + S_h), \tag{13}$$

$$M_0^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) = 3\sqrt{6}\cos\theta T_{\ell m} - \sqrt{6} T_n, \tag{14}$$

$$M_{+1}^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) = -6\sin\theta T_{\ell m}, \tag{15}$$

$$M_{+2}^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) = -3\cos\theta T_{\ell m} - 3T_n. \tag{16}$$

For later convenience, we define two tensor amplitudes, T_α and T_β, as

$$\begin{aligned} T_\alpha &= \frac{1}{\sqrt{6}} M_0^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) + M_2^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) \\ &= -4T_n, \end{aligned} \tag{17}$$

$$\begin{aligned} T_\beta &= -\frac{1}{\sin\theta} M_1^{(2)}(11; \boldsymbol{k}_i, \boldsymbol{k}_f) \\ &= 6T_{\ell m}. \end{aligned} \tag{18}$$

We will calculate the depolarizations in terms of $U_\alpha, U_\beta, S_p, S_h, T_\alpha$, and T_β by using

$$D_i^j(\theta) = \frac{1}{4\sigma(\theta)} \mathrm{Tr}(\boldsymbol{M}\sigma_i \boldsymbol{M}^\dagger \sigma_j), \tag{19}$$

where $\sigma(\theta)$ is the cross section and $\boldsymbol{\sigma}$ is the Pauli spin matrix of the proton. Then we get the following formulas for relations between the depolarizations:

$$D_x^x(\theta) - D_z^z(\theta) = -\frac{\cos\theta}{\sigma(\theta)} \mathrm{Re}\{(U_\beta + T_\alpha)^* T_\beta\}, \tag{20}$$

$$D_x^z(\theta) + D_z^x(\theta) = \frac{\sin\theta}{\sigma(\theta)} \mathrm{Re}\{(U_\beta + T_\alpha)^* T_\beta\}, \tag{21}$$

$$1 - D_y^y(\theta) = \frac{1}{\sigma(\theta)} \left\{ \frac{1}{4}|U_\beta + T_\alpha|^2 + |T_\beta|^2 \right\}, \tag{22}$$

with

$$\begin{aligned}\sigma(\theta) = |U_\alpha|^2 &+ \tfrac{3}{16}|U_\beta|^2 + \tfrac{1}{4}(|S_p|^2 + |S_h|^2) \\ &+ \tfrac{3}{8}|T_\alpha|^2 + \tfrac{1}{2}|T_\beta|^2. \end{aligned} \tag{23}$$

The numerators on the right-hand sides of Eqs. (20), (21), and (22) are fully governed by the spin–spin central amplitude U_β and the tensor amplitudes, T_α and T_β. When the magnitudes of these amplitudes are small, Eqs. (20), (21), and (22) lead to Eq. (1). That is, the experimentally observed relations should

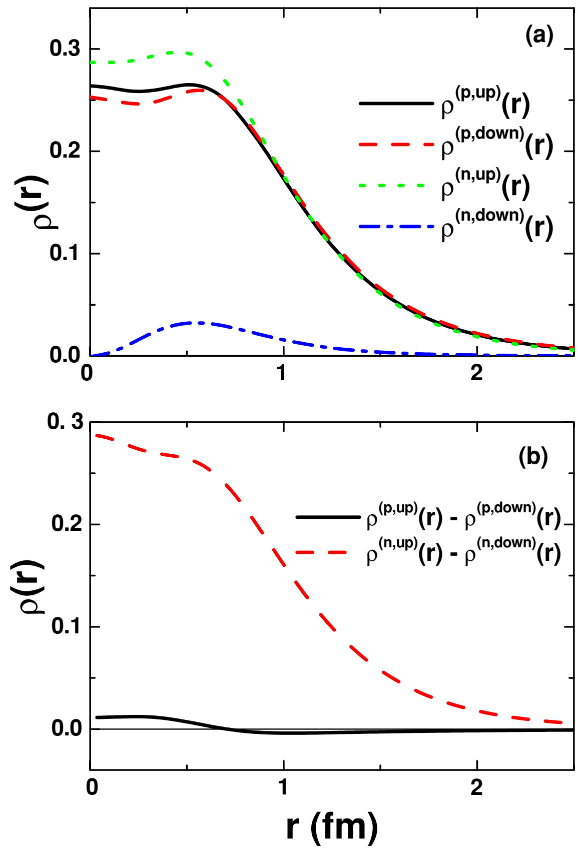

FIG. 2. (Color online) Nucleon densities in ^{3}He as functions of the distance from the center of the ^{3}He nucleus, r. (a) The solid and dashed curves depict the proton densities for spin-up and spin-down, respectively. The dotted and dashed-dotted curves show the neutron densities for spin-up and spin-down, respectively. (b) The solid and dashed curves depict the up–down differences for the proton and the neutron, respectively.

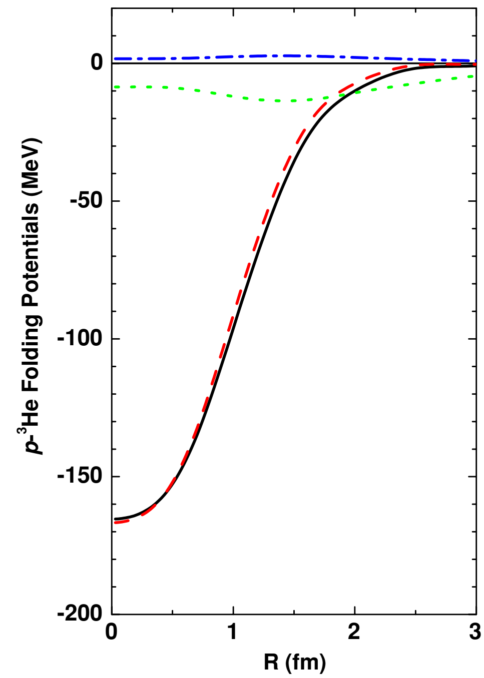

FIG. 3. (Color online) Proton-^{3}He central potentials calculated by a folding model. The real and imaginary parts of the spin-independent potentials $V_0^{(p-h)}(R)$ are shown as the solid and dashed curves, respectively, and the corresponding spin–spin potential $V_\sigma^{(p-h)}(R)$ by the dotted and dashed-dotted curves, respectively. The abscissa R is the distance between the proton and ^{3}He.

be a sign of the characteristic feature of the p-^{3}He interaction, the weakness of the spin–spin and tensor interactions. In detail, however, one sees small but finite differences between the measured depolarizations as shown in Fig. 1. Such differences are due to the contribution of the amplitudes, $U_\beta + T_\alpha$ and T_β, and thus provide measures of the spin–spin and tensor interactions. Eliminating $\mathrm{Re}\{(U_\beta + T_\alpha)^* T_\beta\}$ from Eqs. (20) and (21), one can derive the following relation formula:

$$\frac{D_x^z(\theta) + D_z^x(\theta)}{D_z^z(\theta) - D_x^x(\theta)} = \tan\theta, \tag{24}$$

which agrees with the formula required by the time reversal theorem in nucleon–nucleon scattering [5].

The features of the p-^{3}He spin–spin and tensor scattering amplitudes should originate from those of the respective components of the p-^{3}He interaction. From this viewpoint, we will examine if folding-model interactions have the above features. The spin structure of the ^{3}He nucleus is one ingredient to help explain the p-^{3}He interaction in the folding model. Figure 2 shows the nucleon densities in the ^{3}He nucleus, whose wave function is calculated by the Faddeev method [6,7]. The calculation includes the proton–proton Coulomb interaction and the Brazil-model three-nucleon force [8] in addition to the Argonne V_{18} two-nucleon force [9]. In Fig. 2(a), $\rho^{(x,\mathrm{up})}(r)$ and $\rho^{(x,\mathrm{down})}(r)$ denote the density distributions of a nucleon x, where x is the proton (p) or the neutron (n), for its spin direction up and down, respectively, with the choice of the ^{3}He spin to be up. Here r is the distance from the center of the ^{3}He. The up–down differences $\rho^{(x,\mathrm{up})}(r) - \rho^{(x,\mathrm{down})}(r)$ are shown in Fig. 2(b). From these figures, it is easily seen that the density of the spin-up proton is very close to that of the spin-down proton except for small r, indicating the total of the proton spins is effectively almost zero at most places in the nucleus. On the other hand, the density of the spin-up neutron is dominant over that of the spin-down neutron. Such characteristics of the densities are shown in the form of their volume integrals in Table I. These indicate that the protons in ^{3}He scarcely contribute to the p-^{3}He spin–spin and tensor interactions and that only the neutron significantly contributes to the

TABLE I. Volume integrals of the proton and neutron densities in the ^{3}He nucleus.

x	$N^{x,\mathrm{up}}$	$N^{x,\mathrm{down}}$	$N^{x,\mathrm{up}} + N^{x,\mathrm{down}}$	$N^{x,\mathrm{up}} - N^{x,\mathrm{down}}$
Proton	0.973	1.027	2.000	-0.054
Neutron	0.935	0.065	1.000	0.870

interactions. The strengths of these interactions therefore will be much reduced.

With the above nucleon densities, we will fold a p-x central potential, which consists of the spin-independent component $v_0^{(p\text{-}x)}(r_{px})$ and the spin–spin $v_\sigma^{(p\text{-}x)}(r_{px})(\boldsymbol{s}_p \cdot \boldsymbol{s}_x)$, to obtain the p-^{3}He potential. The resultant potential has also a spin-independent central component $V_0^{(p\text{-}h)}(R)$ and the spin–spin component $V_\sigma^{(p\text{-}h)}(R)$, where R is the distance between the proton and ^{3}He. They are described as

$$V_{(0/\sigma)}^{(p\text{-}h)}(R) = \sum_{x=p,n} \int d\boldsymbol{r}\, v_{(0/\sigma)}^{(p\text{-}x)}(|\boldsymbol{R} - \boldsymbol{r}|) \times [\rho^{(x,\mathrm{up})}(r) \pm \rho^{(x,\mathrm{down})}(r)], \tag{25}$$

where the plus sign is for $V_0^{(p\text{-}h)}(R)$ and the minus for $V_\sigma^{(p\text{-}h)}(R)$.

Since our present purpose is to examine the suppression of $V_\sigma^{(p\text{-}h)}$ in comparison with $V_0^{(p\text{-}h)}$, we will estimate such an effect in a simple way, adopting a three-range Gaussian form for $v_{(0/\sigma)}^{(p\text{-}x)}$. Range and strength parameters are taken from Ref. [10] (G3RS); these parameters are determined to reproduce low-energy nucleon–nucleon (*NN*) phase shifts. The use of the G3RS at intermediate energies such as $E_p = 800$ MeV is less valid because of the neglect of inelastic channel contributions. To compensate for such defects, we extend the G3RS to be complex by multiplying by complex factors so that the volume integrals of $v_{(0/\sigma)}^{(p\text{-}x)}$ are equivalent to those by the Franey–Love effective *NN* interaction at $E_p = 800$ MeV [11]. In Fig. 3, we show the resultant folding potentials. As expected from Fig. 2, the final magnitude of the calculated $V_\sigma^{(p\text{-}h)}$ is quite small compared with that of the calculated $V_0^{(p\text{-}h)}$ except for large R, where the magnitudes of potentials are small.

In Ref. [1], the depolarizations calculated by a folding model with sophisticated nuclear interactions are presented; there the difference between $D_x^x(\theta)$ and $D_z^z(\theta)$ and that between $D_x^z(\theta)$ and $-D_z^x(\theta)$ are hardly identified, though the calculated $D_y^y(\theta)$ deviates from one by small amounts. This means that their spin–spin central and tensor interactions are weak, as we predicted. Since the weakness of the spin–spin and tensor interactions is a reflection of the nuclear structure of ^{3}He, it will be worthwhile to explore, through measurements of the depolarizations, whether such features of the interactions are observed at other energies. These measurements will also provide information of other p-^{3}He scattering amplitudes, the spin-independent central one and the spin–orbit ones [2], which will complement the analysis of cross sections and analyzing powers [4].

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