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A Three-Dimensional Horizontally Wide-Angle Noniterative Beam-Propagation Method Based on the Alternating-Direction Implicit Scheme

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***Abstract*—A three-dimensional horizontally wide-angle beam-propagation method is proposed on the basis of the alternating-direction implicit scheme, in which the Padé approximant is applied only to the horizontal direction. The present formulation reduces the splitting error to the first order without an iteration procedure. The effectiveness is demonstrated through the wide-angle propagating beam analysis of a tilted optical waveguide.**

***Index Terms*—Alternating-direction implicit (ADI) scheme, beam-propagation method (BPM), optical waveguide, Padé approximant, planar lightwave circuit.**

I. Introduction

To efficiently analyze the field propagation in three-dimensional (3-D) optical waveguides, the beam-propagation method (BPM) based on the alternating-direction implicit (ADI) scheme has widely been employed [1]–[3]. Note, however, that the ADI scheme has not been extended to the Padé-based wide-angle equation [4], [5], since the straightforward extension gives rise to a zeroth-order splitting error term. Chui and Lu [5] reformulated the wide-angle equation as in the ADI iteration scheme for elliptic problems. While this approach successfully solves the full-vectorial wide-angle equation, it requires several iterations resulting in a decrease of the efficiency of the ADI scheme.

Here we pay attention to the fact that a circuit pattern in most planar waveguiding structures is confined to the horizontal plane. This means that wide-angle beam propagation may occur only in the horizontal direction. Therefore, the application of the Padé approximant [6]–[9] only to the horizontal direction is expected to be sufficient for most practical waveguide problems.

In this letter, we propose a 3-D semivectorial horizontally wide-angle BPM based on the ADI scheme, in which the Padé approximant is applied only to the horizontal direction. The present formulation reduces the splitting error to the first order, without resorting to an iteration procedure. It is demonstrated through the analysis of a tilted optical waveguide that the present BPM maintains high accuracy for the analysis of the wide-angle beam propagation regardless of the choice of the reference refractive index.

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II. Formulation

The 3-D semivectorial equation with the (1, 1) Padé approximant operator using recurrence relation [9] is expressed as

$$\frac{\partial \phi}{\partial z} = \frac{\frac{-j}{2kn_0}(D_{xx} + D_{yy} + \nu)}{1 + \frac{1}{4k^2 n_0^2}(D_{xx} + D_{yy} + \nu)}\phi \tag{1}$$

where

$$\begin{aligned} \phi &= H_y \\ D_{xx} H_y &= n^2 \frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial H_y}{\partial x}\right) \\ D_{yy} H_y &= \frac{\partial^2 H_y}{\partial y^2} \end{aligned}$$

for the quasi-TE mode

$$\begin{aligned} \phi &= E_y \\ D_{xx} E_y &= \frac{\partial^2 E_y}{\partial x^2} \\ D_{yy} E_y &= \frac{\partial}{\partial y}\left[\frac{1}{n^2}\left(\frac{\partial}{\partial y}(n^2 E_y)\right)\right] \end{aligned}$$

for the quasi-TM mode and $\nu = k^2(n^2 - n_0^2)$, in which k is the free-space wavenumber, n the refractive index, and n_0 the reference refractive index to be appropriately chosen. The second term in the denominator of the right-hand side of (1) results from the Padé approximant. If we ignore this term, i.e., a value of the denominator is chosen to be one, the equation reduces to the Fresnel (paraxial) equation, which can be efficiently solved by the ADI scheme.

It is known that the direct application of the ADI scheme to (1) gives rise to the zeroth-order splitting error term. The coefficient of the error term is derived as

$$\left(\frac{1}{4k^2 n_0^2} \pm \frac{j\Delta z}{4kn_0}\right)^2.$$

Note that this error does not converge to zero, even when Δz approaches zero.

Here we apply the Padé approximant only to the horizontal (x) direction, considering the fact that a circuit pattern is gener-

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ally confined to the horizontal plane for most waveguiding structures, such as planar lightwave circuits. Doing so leads to

$$\frac{\partial \phi}{\partial z} = \frac{\frac{-j}{2kn_0}(D_{xx} + D_{yy} + \nu)}{1 + \frac{1}{4k^2n_0^2}(D_{xx} + \nu)}\phi. \tag{2}$$

We discritize (2) by the Crank–Nicolson scheme, so that

$$\left[1 + \left(\frac{1}{4k^2n_0^2} + \frac{j\Delta z}{4kn_0}\right)(D_{xx} + \nu) + \frac{j\Delta z}{4kn_0}D_{yy}\right]\phi^{l+1} = \left[1 + \left(\frac{1}{4k^2n_0^2} - \frac{j\Delta z}{4kn_0}\right)(D_{xx} + \nu) - \frac{j\Delta z}{4kn_0}D_{yy}\right]\phi^{l}. \tag{3}$$

To solve (3) by the ADI scheme, we divide it as follows:

$$\left[1 + \left(\frac{1}{4k^2n_0^2} + \frac{j\Delta z}{4kn_0}\right)(D_{xx} + \nu)\right]\left[1 + \frac{j\Delta z}{4kn_0}D_{yy}\right]\phi^{l+1} = \left[1 + \left(\frac{1}{4k^2n_0^2} - \frac{j\Delta z}{4kn_0}\right)(D_{xx} + \nu)\right] \times \left[1 - \frac{j\Delta z}{4kn_0}D_{yy}\right]\phi^{l}. \tag{4}$$

Finally, we obtain the following two-step algorithm:

$$\left[1 + \frac{j\Delta z}{4kn_0}D_{yy}\right]\phi^{l+1/2} = \left[1 + \left(\frac{1}{4k^2n_0^2} - \frac{j\Delta z}{4kn_0}\right)(D_{xx} + \nu)\right]\phi^{l} \tag{5}$$

$$\left[1 + \left(\frac{1}{4k^2n_0^2} + \frac{j\Delta z}{4kn_0}\right)(D_{xx} + \nu)\right]\phi^{l+1} = \left[1 - \frac{j\Delta z}{4kn_0}D_{yy}\right]\phi^{l+1/2}. \tag{6}$$

The second derivatives D_{xx} and D_{yy} are approximated using the modified finite-difference formulae for the E [10] and H fields [11]. Finally, we obtain a tridiagonal matrix, which can be efficiently solved by the Thomas algorithm, leading to almost the same computational speed as that of the conventional ADI-BPM based on the Fresnel equation.

We alternatively obtain the horizontally wide-angle BPM using the exponential operators [6]–[8]. The relation between ϕ^{l+1} and ϕ^{l} is formally expressed as [7]

$$\phi^{l+1} = \exp\left[\delta\left(\sqrt{1 + \frac{D_{xx} + D_{yy} + \nu}{k^2n_0^2}} - 1\right)\right]\phi^{l} \tag{7}$$

where $\delta = -jkn_0\Delta z$. We here adopt the second-order expansion in D_{xx} and ν, and the first-order in D_{yy}, so that the exponential operator in (7) can be approximated as

$$\exp\left[\frac{\delta}{2}\left(\frac{D_{xx} + \nu}{k^2n_0^2} - \frac{(D_{xx} + \nu)^2}{4k^4n_0^4}\right)\right]\exp\left[\frac{\delta}{2}\frac{D_{yy}}{k^2n_0^2}\right]. \tag{8}$$

Note that the term ν regarding the phase variation should be included in the first operator. Using the second-order [7] and first-order approximations to the first and second operators, respectively, in (8), we finally derive the same equation as (4).

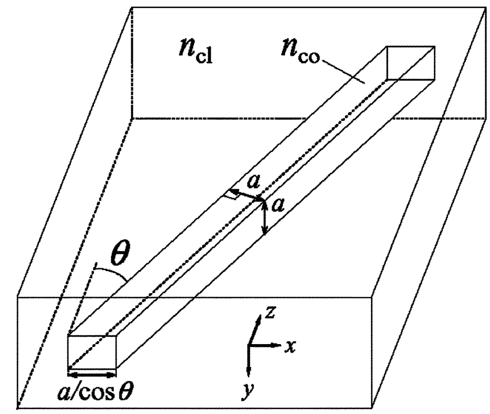

Fig. 1. Tilted optical waveguide.

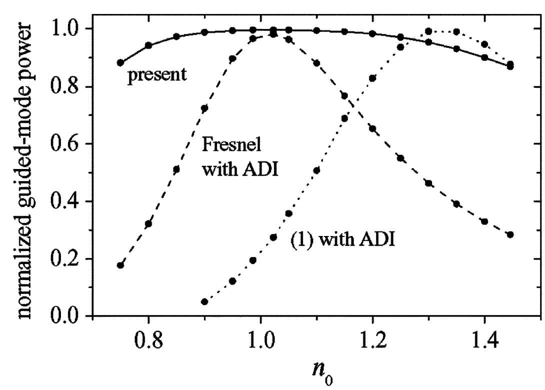

Fig. 2. Normalized guided-mode power for $\theta = 45°$, as a function of reference refractive index n_0. Each result is obtained using the conventional noniterative ADI scheme.

Expanding (4), we obtain the splitting error term whose coefficient is

$$\frac{j\Delta z}{4kn_0}\left(\frac{1}{4k^2n_0^2} \pm \frac{j\Delta z}{4kn_0}\right).$$

It should be emphasized that this error is the first order with respect to Δz. Therefore, the effect of the error can be suppressed with a reasonably small Δz.

III. Numerical Results

We analyze the fundamental mode propagation in a tilted waveguide to investigate the accuracy for the wide-angle beam propagation. The tilted waveguide to be analyzed is illustrated in Fig. 1, where the refractive indexes of the core and cladding are $n_{\text{co}} = 1.450$ and $n_{\text{cl}} = 1.446$, respectively. The square core ($a = 5$ μm) is used and the tilt angle is set to be $\theta = 45°$. The wavelength is chosen to be $\lambda = 1.55$ μm. The spatial sampling widths are as follows: $\Delta x = \Delta y = \Delta z = 0.05$ μm.

Fig. 2 shows the guided-mode power of the E_y field observed at $z = 40$ μm, normalized to the input power of the fundamental eigenmode, as a function of reference refractive index n_0. The results obtained from the present method are indicated by the solid line. For comparison, also included are the results obtained from the Fresnel equation with the ADI and from (1) with the ADI, which are denoted by the broken and dotted lines,

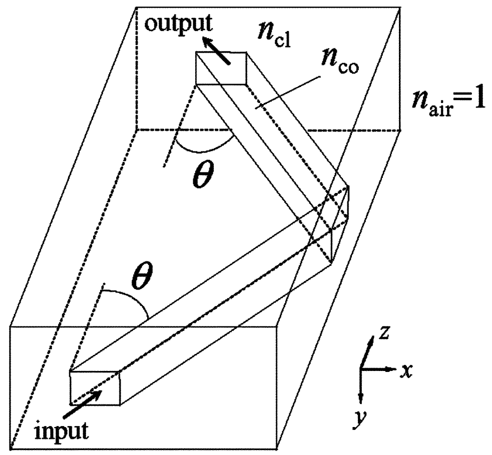

Fig. 3. V-shaped waveguide with a facet.

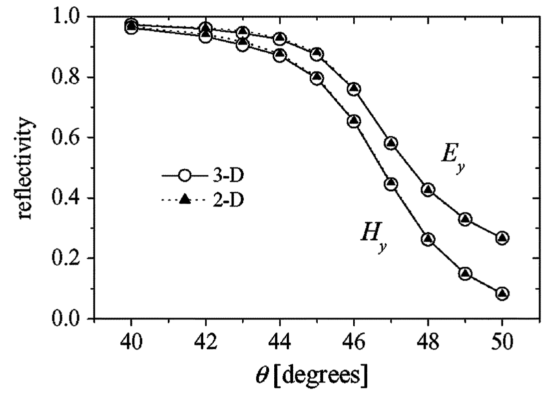

Fig. 4. Reflectivity as a function of tilt angle θ.

respectively (the latter corresponds to the case where the Padé approximant is applied to both x and y directions). When the eigenmode field propagates without deformation, the normalized guided-mode power remains unity.

It is found in Fig. 2 that the present method attains high accuracy over a wide range of n_0. Note that the results obtained from the Fresnel equation shows a mode power close to unity, only when n_0 is taken to be $n_e \times \cos 45°(\simeq 1.02)$, where n_e is the effective index of the eigenmode. For the use of (1), the mode power is sensitive to n_0 as in the case of the Fresnel equation. This result is caused by the incorrect noniterative ADI scheme with the zeroth-order splitting error. Although the maximum power observed probably stems from the reduction in the splitting error at $n_0 \simeq 1.3$, an optimum value of n_0 may not be predicted in advance from the waveguide parameters. As a result, the present method offers more accurate results for the wide-angle beam propagation in the planar waveguiding structures, regardless of the choice of n_0. This also leads to the advantage that problems involving radiation modes can be treated more accurately, when compared with the conventional ADI-BPM based on the Fresnel equation.

As an application, we analyze the V-shaped waveguide with a facet, shown in Fig. 3. The perfectly matched layer [12] is employed to absorb the outgoing waves. The waveguide parameters are the same as those used in Fig. 1. We excite the field of the fundamental mode from the input port and extract the reflected field in the output port. Since this waveguide is a weakly guiding structure, it is expected that a close correspondence be found between the numerical results of the 3-D analysis and those of the two-dimensional (2-D) analysis. Therefore, we also perform the analysis using the 2-D wide-angle BPM with the equivalent index method.

Fig. 4 depicts the power reflectivity observed at the output port, as a function of tilt angle. It can be seen that the reflectivity decreases with an increase in the tilt angle, where the polarization dependence is found. In addition, the 3-D results are in excellent agreement with the 2-D ones. This again demonstrates the high accuracy of the present method for the analysis of the 3-D horizontally wide-angle beam propagation.

IV. Conclusion

We have proposed a 3-D semivectorial horizontally wide-angle BPM based on the ADI scheme without an iteration procedure. The effectiveness of the present method is investigated through the analysis of the tilted optical waveguide. It is shown that the present method attains high accuracy for the analysis of the wide-angle beam propagation, regardless of the choice of the reference refractive index. The application to the full-vectorial BPM is now under consideration.

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