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Analysis of a Polarization Splitter With a Multilayer Filter Using a Padé-Operator-Based Power-Conserving Fourth-Order Accurate Beam-Propagation Method

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***Abstract*—A (1,1) Padé-operator-based fourth-order accurate finite-difference beam-propagation method is described for the analysis of the transverse-magnetic (TM) wave, with an emphasis on power conservation. As an application, a polarization splitter with a multilayer filter is analyzed. Reflectivities of more than 98% for the transverse-electric wave and transmissivities of more than 94% for the TM wave are obtained over a wide wavelength range of 1.1–1.65 μm.**

***Index Terms*—Beam-propagation method (BPM), finite-difference time-domain (FDTD) method, multilayer filter, optical waveguide, polarization splitter.**

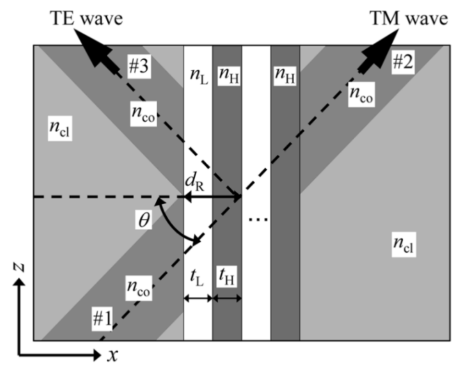

Fig. 1. Configuration of a polarization splitter ($n_{co} = 1.536$, $n_{cl} = 1.532$, $n_H = 2.25(TiO_2)$, $n_L = 1.45(SiO_2)$, $\theta = 52.5°$).

I. Introduction

THE beam-propagation method (BPM) has widely been used to simulate propagating beams in longitudinally varying optical waveguides. It is known that the power calculated on the basis of the paraxial equation no longer remains constant [1], [2]. Schmidt [1] attempted to control the reference refractive index so as to minimize the power deviation. Meanwhile, Vassallo [2] introduced modified unknown fields to keep the power constant for the transverse-electric (TE) wave. Vassallo [3] also indicated that the inclusion of the first drivative of the refractive index with respect to the propagation direction $\partial n^{-2}/\partial z$, which is often neglected in previous works [4], is neccessary for the transverse-magnetic (TM) wave. The authors [5] clearly demonstrated the effects of including the $\partial n^{-2}/\partial z$ term, but the formulation was limited to the paraxial equation. Subsequently, Ho and Lu [6] introduced a single scatter approximation for the wide-angle TM wave.

Recently, the authors [7], [8] developed a (1,1) Padé-operator-based finite-difference (FD)-BPM in which the $\partial n^{-2}/\partial z$ term is included and the fourth-order FD scheme [9] is adopted. This FD-BPM was succesfully employed to assess the performance of a demultiplxer with a multilayer filter. However, brief discussion was only provided regarding the FD-BPM because of limited space.

In this letter, we explicitly describe the (1,1) Padé-operator-based FD-BPM with an emphasis on power conservation. We calculate the power, taking into account the effect of the first derivative of the field with respect to the propagating beam direction [7], [8]. Although the present method is simple and the direct extension of the well-known Padé-operator-based BPM, reasonable results are obtained by our simple algorithm.

As an application, we treat a polarization splitter with a multilayer filter (see Fig. 1) [10]. Note that the reflectivity of the multilayer filter can be evaluated by the BPM, provided the multilayer interface is placed parallel to the propagating beam direction [7], [8], [11]. The use of the BPM leads to high computational efficiency as compared with the finite-difference time-domain (FDTD) method [12]. In this letter, we discuss wideband characteristics of the present polarization splitter, confirming the validity of the results through comparison with those obtained from the FDTD method.

II. Numerical Method

To obtain the propagating fields of the TE and TM modes, we employ a (1,1) Padé-operator-based BPM. Since the Padé-operator-based BPM is well-known for the TE wave [13], [14], we will only describe the formulation for the TM wave.

By eliminating the electric field $\mathcal{E}$ from Maxwell's equations, we obtain the vector wave equation

$$\nabla^2 \boldsymbol{\mathcal{H}} + \frac{\nabla n^2}{n} \times (\nabla \times \boldsymbol{\mathcal{H}}) + n^2 k_0^2 \boldsymbol{\mathcal{H}} = 0 \tag{1}$$

where k_0 is the free-space wave number and n is the refractive index. For a planar geometry, (1) reduces to the following exact wave equation for the TM wave:

$$n^2 \frac{\partial}{\partial z}\left(\frac{1}{n^2}\frac{\partial \mathcal{H}_y}{\partial z}\right) + n^2 \frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial \mathcal{H}_y}{\partial x}\right) + k_0^2 n^2 \mathcal{H}_y = 0. \tag{2}$$

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After expressing the field as $\mathcal{H}_y = H(x,z)\exp(-jk_0n_0z)$, where n_0 is the reference refractive index, we obtain

$$n^2\frac{\partial}{\partial z}\left[\frac{1}{n^2}\left(jk_0n_0H-\frac{\partial H}{\partial z}\right)\right]+jk_0n_0\frac{\partial H}{\partial z} = n^2\frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial H}{\partial x}\right)+k_0^2\left(n^2-n_0^2\right)H. \quad (3)$$

Equation (3) can be transformed into

$$\left[-\frac{\partial}{\partial z}-n^2\left(\frac{\partial}{\partial z}\frac{1}{n^2}\right)+2jk_0n_0\right]\frac{\partial H}{\partial z}=PH \quad (4)$$

where

$$P=n^2\frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\right)+k_0^2\left(n^2-n_0^2\right)-jk_0n_0n^2\left(\frac{\partial}{\partial z}\frac{1}{n^2}\right). \quad (5)$$

By analogy with the derivation of the Padé-operator-based BPM for the TE wave, we finally obtain the following (1,1) Padé expression for the TM wave:

$$\frac{\partial H}{\partial z}=-\frac{\frac{jP}{2k_0n_0}}{1+\frac{P}{4k_0^2n_0^2}}H. \quad (6)$$

Note that the last term in (5) has often been neglected [4]. In this letter, this term is evaluated using [5, eq. (11)]. Auxiliary calculation shows that the inclusion of the second term in the left-hand side of (4) does not necessarily lead to reasonable results.

To evaluate the second derivative with respect to the transverse direction, we employ the fourth-order FD scheme [9], which takes into account the boundary condition at the dielectric interface.

The reference refractive index is updated as the field propagates [1]

$$n_0^2=\frac{\int\left(k_0^2|H|^2-\frac{1}{n}^2\left|\frac{\partial H}{\partial x}\right|^2\right)dx}{k_0^2\int\left(\frac{|H|^2}{n^2}\right)dx}. \quad (7)$$

Based on the Poynting vector, the exact power can be expressed as [7]

$$P_{\mathrm{TM}}=\frac{1}{2\omega\epsilon_0}\mathrm{Re}\int\left(k_0n_0\frac{|H|^2}{n^2}+j\frac{H^*}{n^2}\frac{\partial H}{\partial z}\right)dx \quad (8)$$

in which ϵ_0 and ω are the free-space permittivity and the angular frequency, respectively. For the TE wave, we also use the equations corresponding to (7) [1] and (8) [7].

It is interesting to note that the second term in (8) can be neglected, provided the slowly varying envelope approximation is maintained. This fact suggests that the second term should be included for the case where the variation of the field cannot be negligible in the propagation direction, such as in the filter region.

III. Analysis of Polarization Splitter

Fig. 1 shows the configuration to be considered here. We excite the field of either TE or TM mode from Waveguide #1, and intend to extract the TM wave in Waveguide #2 and the TE wave in Waveguide #3 over a wide range of wavelengths. The refractive indexes of the core and cladding are taken to be $n_{\mathrm{co}} = 1.536$ and $n_{\mathrm{cl}} = 1.532$, respectively. The normalized frequency of the waveguide is almost equal to $V = 1.5$ at 1.31 μm. The number of layers is taken to be 12. The thicknesses of the layers are determined by $t_H(\text{or } t_L) = \lambda_S/(4n_H(\text{or } n_L)\cos\theta)$, where n_H and n_L are the refractive indexes in the high- and low-index layers, respectively. In this analysis λ_S is chosen to be 1.11 μm. The transverse and longitudinal sampling widths are taken to be $\Delta x = t_H/9$ and $\Delta z = \Delta x\tan\theta$, respectively.

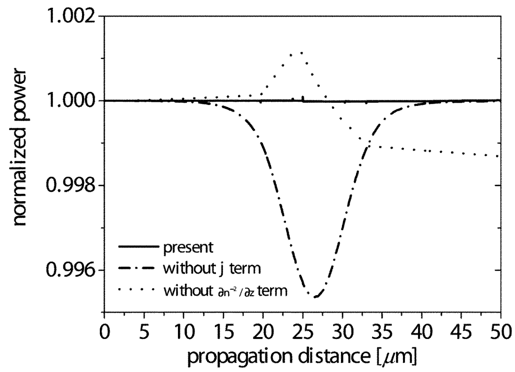

Fig. 2. Normalized power of the TM wave as a function of propagation distance (n_0 is updated using (7), $\lambda = 1.55$ μm).

Assuming that the incident field is regarded as a plane wave, the Brewster condition is satisfied when the angle of incidence is such that [10]

$$\sin^2\theta=\frac{n_H^2n_L^2}{n_{\mathrm{co}}^2\left(n_H^2+n_L^2\right)}. \quad (9)$$

In the present case ($n_H = 2.25$ and $n_L = 1.45$), θ is calculated to be 52.5°. It should be noted that the appropriate angle of incidence is not sensitive to wavelength change, leading to wideband characteristics of filtering properties.

The effective reflection distance designated as d_R should be chosen to maximize the reflectivity for the TE wave. Preliminary calculation at 1.31 μm shows that the maximum reflectivity is obtained at $d_R = t_H + t_L$.

Before investigating polarization splitting behavior in detail, we first assess the effectiveness of the present method for evaluating the power. We calculate the normalized power as a function of propagation distance, in which n_0 is updated using (7). The results for the TM wave are shown in Fig. 2, in which the data obtained without the $\partial n^{-2}/\partial z$ term in (5) are shown for comparison. It is observed that the inclusion of the $\partial n^{-2}/\partial z$ term contributes to power conservation. Fig. 2 also shows that negligence of the second (j) term in (8), corresponding to the simple squared norm, results in variation of the normalized power in the filter region. Further examination shows that the inclusion of the second (j) term leads to insenstivity of the power to the change of n_0 (see Fig. 3). Note also that the power with or without the second (j) term approaches unity at $z = 50$ μm once n_0 is updated using (7), as shown in Fig. 2.

Fig. 4 shows the reflectivity and transmissivity as a function of wavelength. To validate the use of the FD-BPM, we also present the results obtained with the FDTD method. Good agreement is found to exist between both results (since the FDTD method is based on the second-order FD scheme, Δx

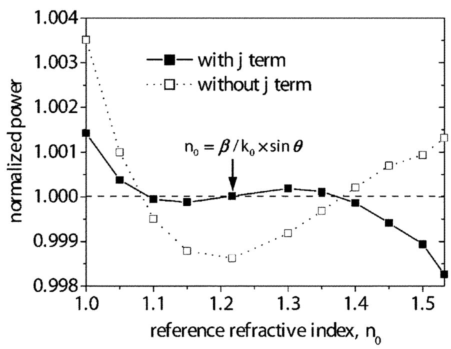

Fig. 3. Normalized power of the TM wave observed at $z = 50$ μm as a function of n_0 (with $\partial n^{-2}/\partial z$ term, $\lambda = 1.55$ μm).

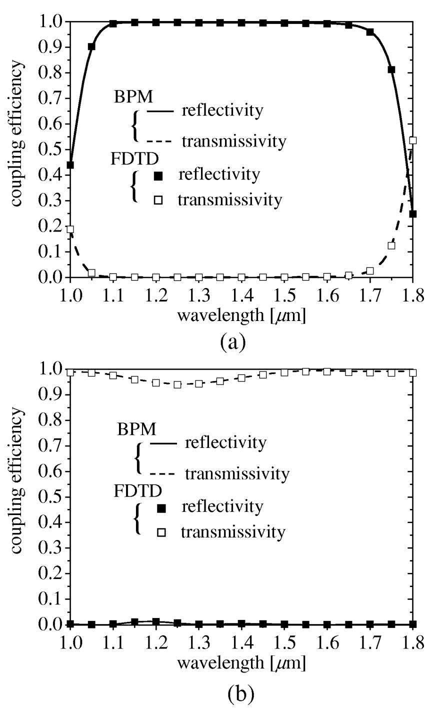

Fig. 4. Wavelength responses. (a) TE wave. (b) TM wave.

is chosen to be half that used in the FD-BPM to generate the comparable accuracy).

It is worth mentioning in Fig. 4 that a TM-pass/TE-reflection property is achieved over a wide range of wavelengths, which is not obtainable in the polarization splitters based on interference properties [15]. Reflectivities of more than 98% for the TE wave and transmissivities of more than 94% for the TM wave are obtained over a wavelength range of 1.1–1.65 μm.

Fig. 5 shows the extinction ratios observed at the outputs of Waveguides #2 and #3. Over a wavelength range of 1.1–1.65 μm, the extinction ratios are evaluated to be more than 21 dB in Waveguide #2 and 18 dB in Waveguide #3 (the use of Fresnel-operator-based power-conserving BPM causes a maximum error of about 3 dB).

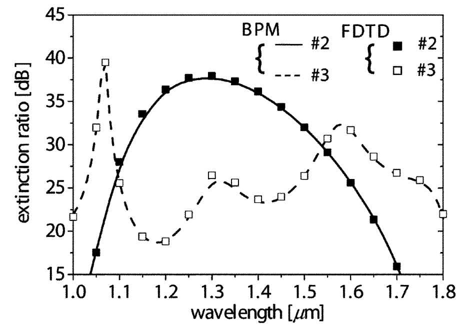

Fig. 5. Extinction ratios as a function of wavelength.

IV. Conclusion

A (1,1) Padé-operator-based FD-BPM with a power conserving property has been used to analyze a polarization splitter with a multilayer filter. As a result, reflectivities of more than 98% for the TE wave and transmissivities of more than 94% for the TM wave are obtained over a wide wavelength range of 1.1–1.65 μm.

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