Reliability and Responsibility: A Theory of Endogenous Commitment*

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Abstract

A common assumption in Political Science literature is policy commitment: candidates maintain their electoral promises. We study its validity and we prove that is an costless electoral is an effective way of transmitting information to voters. We investigate the responsiveness of policies to electoral promises depending on politicians' motivations. The results are robust to relevant equilibrium refinements.

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1 Introduction

It is commonplace to say that electoral promises cannot be taken at their face value. However, parties and candidates invest a considerable amount of effort and resources in producing electoral messages. While earlier studies of the "Columbia School" (Lazarsfeld, Berelson and Gaudet 1948 and Berelson, Lazarsfeld and McPhee 1954) assigned only marginal effect to electoral campaigns, there is now a large consensus among scholars that they do matter (see Alvarez 1998 and Brady, Johnston and Sides 2006).

But if campaigns were a mere act of promising why should they influence citizens? A widely employed explanation is that politicians and elected officials seek reelection. Electoral promises affect voters' expectations about the policies that the elected officials will choose. They provide a benchmark linking promises, policies and reelection, because a credible threat to reelection is imposed (see Barro 1973, Ferejohn 1986 and Austen-Smith and Banks 1989).

The disciplining role of electoral competition is only one face of the coin. Electoral promises provide also a solution to the informational asymmetries between candidates and politicians. Campaigns convey information useful to predict future policies and future policies should be predictable from present ones. The intuition dates back at least to Downs (1957), who underlies the relationship between preelection statements and post election behavior.

> Now we try to prove that a party's ideology must be consistent with either (1) its actions in prior election periods, or (2) its statements in the preceding campaign (including its ideology), or (3) both... A party is **reliable** if its policy statements at the beginning of an election period-including those in its preëlection campaign-can be used to make accurate predictions of its behavior... A party is **responsible** if its policies in one period are consistent with its ac-

> tions (or statements in the preceding period)... We conclude that reliability is a logical necessity in any rational election system, and that responsibility-though not logically necessary-is strongly implied by rationality as we define it ...(pp. 103-107).

However, most of the formal models of electoral competitions assume that politicians commit to their electoral announcements. The questions about the credibility of campaign promises are left unanswered.

The objective of this paper is to provide an explanation based on the interaction of informational asymmetries and reelection concerns. We provide a model where they concur in shaping implemented policies

Each one of the arguments alone is not able to provide a satisfactory solution to the puzzle. Under complete information, politicians cannot credibly commit to policies different from their favorite ones unless elections are infinitely repeated (Alesina 1988). In this case, the use of trigger strategies and retrospective voting raise doubts about the credibility of the electoral threats.[1] If information is incomplete and elections are not repeated electoral campaign cannot be informative (Harrington 1992a) unless one drops the assumption of full policy enforceability (Harrington 1992b).

We introduce a model where politicians' and voters' policy preferences are private informations. The type space is continuous and beliefs are derived from a common prior. Politicians care both about the policy and holding the office. Elections are held twice. Prior to the first election the candidates make a costless policy announcement. The elected candidate implements a policy, not necessarily the announced one and runs for reelection against a randomly chosen opponent. The elected politician will finally implement a policy that will coincide with her favorite one, because there are not further elections.

[1] Aragonés and al. (2006) attempt to characterize to which policies candidates can credibly commit with an infinite horizon.

The analysis focuses on symmetric and monotonic equilibria in which centrist politicians are elected with higher probabilities and implement more centrist policies. Monotonic equilibria exclude unlikely behaviors where extremists present themselves as centrist, while moderates implement extremist policies.[2] Out-of-equilibrium beliefs with regard to completely unexpected policies, are refined adapting D1 Criterion (Cho and Kreps 1987), to monotonic environments,, using a refinement first introduced by Bernheim and Severinov (2003).[3] The set of the equilibria that survive are completely characterized.

The game does not have fully separating equilibrium. Reelection pressures and policy motivations interact making electoral promises relevant, when candidates' policy preferences are intense enough. At the campaign stage the candidates endogenously differentiate in two pools: centrist candidates and extremists. The degree in which incumbents are made accountable of their announcements different and vary with their ideal policies. Electoral pressure is more effective on centrist candidates who, in the first term, stick to the announced platform, the one that guarantees the highest chances of reelection. They actuate as pure office seekers, but they have to implement policy that is faraway from their favorite one. Extremist politicians suffer a stronger tension between pleasing their constituencies and seeking the reelection so they separate in policy. Still, they are forced to support more moderate policies than their favorite ones and see their reelection chances reduced.[4] This prevent policy convergence. The result accounts for partial but relevant responsiveness of politicians to their electoral statements, consistently with empirical studies (see Harrington 1992b).

We present an example of electoral equilibrium where extremists completely separate in campaign and in policy. This equilibrium eventually converges to a

[2] We prove that in every non-monotonic equilibrium electoral campaign must provide some valuable information.

[3] See also Kartik (2005).

[4] This centripetal tendency is consistent with the results obtained in different setups by Calvert (1985), Alesina and Spear (1988) and Harrington (1992c).

completely separating one, when candidates degree of policy concern approaches infinity. It is not robust to the equilibrium refinement introduced but is nevertheless interesting, because it suggests the possibility of an even more informative role for electoral campaign.

The paper that is closest to our approach is Harrington (1993). He presents a model of finitely (twice) repeated elections under bilateral asymmetric information, with a finite type space (two politicians' types and two electors' types). Beliefs are not consistent with the common prior assumption. Candidates care about their income, politicians are lexicographic, first they care about reelection then about their favorite policy. An exogenous form of uncertainty about the effect of the policy is present. The author proves there exist equilibria in which each candidate truthfully announces and implements her favorite policy. This result is driven by the effect of reelection concerns and by the residual uncertainty on the policy outcome, that prevents the equilibrium to unravel. Policy preferences play only a tie-breaking role. The model present also equilibria in which politicians implement their least favorite policy, because there is no interaction between reelection concerns and policy preferences.

Related to our paper are also the works by Banks (1990) and Callander and Wilkie (2007). They study a model of a one-shot electoral competition with incomplete information where there is no commitment to policy so that every candidate will implement his ideal position if elected. However, candidates suffer an exogenous cost of lying that increases with the magnitude of their lies and makes possible an informative electoral campaign. In our model endogenous lying costs appear for the concurring effects of electoral concerns and policy preferences. An interesting contribution of Callander and Wilkie (2007) is introducing a portion of politicians who do not suffer any costs of lying.[5]

[5] Kartik and McAfee (2007) present a complementary approach where some candidates

The structure of the paper is the following. Section 2 presents the model of electoral competition. In Section 4 I present some preliminary results that clarify our choices and I prove the impossibility of fully honest behavior. Section 5 introduces the equilibrium refinement. The main results are presented in Section 6. Section 7 discusses the results. Finally, Section 8 draws the conclusion and suggests possible directions of future research. The Proofs are in the appendix.

2 The Model

The models builds on a standard Downs one dimensional location game. Elections are repeated twice.

The policy space is denoted by $X = [-D, D]$, where $D > 0$. There is a continuum of voters having quadratic preferences over the policy space, and who care about the policy implementing in the following term. Each voter is identified by her ideal point so a voter of type $\alpha \in [-D, D]$ derives utility $V(x, \alpha) = -(\alpha - x)^2$ from the implementation of policy x. Median voter's ideal point is drawn from a symmetric distribution G on $[-D, D]$, with continuous density, $g(\cdot) = G'(\cdot)$.[6]

There are two candidates: R(ight) and L(eft). Departing from Downs, we assume that they care both about being elected and the policy they will implement once in office. Like voters they are be identified by their favorite policy. A positive real number $y > 0$ represents the benefit a candidate gets from holding the office, and a factor $k > 0$ measures the degree in which they care about the policy. Their intertemporal factor discount is $\delta \in (0, 1]$. A politician of type α

present an infinity disutility from lying, in a perfect commitment setup.

[6] The assumption of uncertainty on voters' preferences is necessary for the equilibrium not to unravel at the second election election.

has utility function $U(x, \alpha)$ where x is the implemented policy and

$$\begin{aligned} U(x,\alpha) &= y - k(\alpha - x)^2 \text{ if she wins the elections} \\ U(x,\alpha) &= 0 \text{ otherwise.} \end{aligned}$$

Candidates R and L have their ideal policies in $X_R = [0, D]$ and $X_L = [-D, 0]$, respectively. Due to party discipline, they are bounded to implement policies within their respective policy spaces. Candidates' ideal points are private knowledge. Candidate R type, is drawn from the cdf $F(\cdot)$ on $[0, D]$, with continuous density, $f(\cdot) = F'(\cdot)$, where $f(\alpha) > 0$ on $[0, D]$.

The timing of the game is as follows:

1. **Campaign.** Candidates R and L announce a policy $m_R \in X_R$ and $m_L \in X_L$, respectively.
2. **First Election**. Each citizen, taking in account the electoral campaign, casts a vote for one of the two candidates.
3. **Policy implementation.** The elected politician implements a policy from her policy space.[7]
4. **Second Election.** Each citizen observes the policy implemented by the incumbent and casts a vote for one of the two candidates.
5. **Policy implementation.** The elected politician implements a policy from her policy space.

The losing candidate is replaced by a candidate drawn from her original distribution. At the second election the median voter's ideal point is drawn from the original distribution G.

[7] The introduction of electoral campaign in this stage would not alter the analysis.

If an incumbent is confirmed in the office, she will implement her favorite policy, so her second period utility level will be simply y.

Let μ be voters' beliefs about candidates' policy preferences. and let $s_R(\cdot)$, $s_L(\cdot)$, respectively, the policy she expects from a candidate R and from a candidate R if elected. Let m_v be the median voter's ideal policy.

She votes for candidate R if and only if $E\left[(m_v - s_R(\cdot))^2 \mid \mu\right] < E\left[(m_v - s_L(\cdot))^2 \mid \mu\right]$ which is if and only if $m_v > e(\mu)$ where

$$e(\mu) = \frac{1}{2}\frac{E\left[s_R^2(\cdot) \mid \mu\right] - E\left[s_L^2(\cdot) \mid \mu\right]}{E\left[s_R(\cdot) \mid \mu\right] - E\left[s_L(\cdot) \mid \mu\right]}$$

The voter with ideal policy $e(\mu)$ will be called the decisive voter. Every voter with ideal point on the right of $e(\mu)$ will vote for R and every voter with ideal point on the left will vote for L. Politician L is elected with probability $\pi(\mu) = 1 - G(e(\mu))$.

The next example presents the values of decisive voters for two different specifications of the beliefs.

Example 1 *In this example we compute the decisive voter in different informative situation that are of relevance for the papers*

1. If voters believe that candidate R is of type $\alpha > 0$, $s_R(\alpha) = \alpha$ and candidate L is randomly drawn from her original distribution with density $f(-x)$ on $[-D, 0]$ then the decisive voter is:

$$e(\alpha, f(\cdot)) = \frac{1}{2}\frac{\alpha^2 - \int_0^D \beta^2 f(\beta)\, d\beta}{\alpha + \int_0^D \beta f(\beta)\, d\beta}$$

   In this case we denote by $\pi((\alpha, f(\cdot)))$ R's probability of election.

2. Instead, if voters believe that candidate R's type belongs to interval (α_1, α_2), and it is drawn from the distribution F, while candidate L is randomly

drawn from her original distribution then the decisive voter is

$$e\left([\alpha_1,\alpha_2],f(\cdot)\right) = \frac{1}{2}\frac{\int_{\alpha_1}^{\alpha_2}\beta^2 f(\beta)\,d\beta-(F(\alpha_2)-F(\alpha_1)\int_0^D\beta^2 f(\beta)\,d\beta}{\int_{\alpha_1}^{\alpha_2}\beta f(\beta)\,d\beta+(F(\alpha_2)-F(\alpha_1)\int_0^D\beta f(\beta)\,d\beta}$$
$$\frac{1}{2}\frac{\int_{\alpha_1}^{\alpha_2}\beta^2 f(\beta)\,d\beta-(F(\alpha_2)-F(\alpha_1)\int_0^D\beta^2 f(\beta)\,d\beta}{\int_{\alpha_1}^{\alpha_2}\beta f(\beta)\,d\beta+(F(\alpha_2)-F(\alpha_1)\int_0^D\beta f(\beta)\,d\beta}.$$

We denote by $\pi\left((\alpha_1,\alpha_2,f(\cdot)\right))$ R's probability of election.

3. Assume that voters believe that candidate R and L are of type $\alpha>0$ and $\beta<0$ respectively, then the decisive voter is:

$$e\left(\alpha,\beta,s_R,s_L\right)=\frac{1}{2}\frac{s_L^2(\alpha)-s_R^2(\beta)}{s_L(\alpha)-s_R(\beta)}=\frac{s_L(\alpha)+s_R(\beta)}{2}$$

From elementary Real Analysis follows that:

(i) $e\left(\alpha_3,f(\cdot)\right)>e\left([\alpha_1,\alpha_2],f(\cdot)\right)>e\left(\alpha_0,f(\cdot)\right)$ if $\alpha_3>\alpha_2>\alpha_1>\alpha_0>0$.

(ii) $e\left(\alpha,f(\cdot)\right)$ is strictly increasing in α. $e\left([\alpha_1,\alpha_2],f(\cdot)\right)$ is strictly decreasing in α_1,α_2 (separately).

(iii) $\lim_{\alpha_1\to\alpha_2^-} e\left([\alpha_1,\alpha_2],f(\cdot)\right)=e\left(\alpha_2,f(\cdot)\right)$.

(iv) $\lim_{\alpha_2\to\alpha_1^+} e\left([\alpha_1,\alpha_2],f(\cdot)\right)=e\left(\alpha_1,f(\cdot)\right)$.

(v) If s_R (resp s_L) is strictly increasing (resp decreasing) then $e\left(\alpha,\beta,s_R,s_L\right)$ is strictly increasing (resp decreasing) in α (resp β).

3 Electoral Equilibrium

Strategies

Candidates' campaign strategies are simply costless announcements of a policy. Formally the **campaign strategies** of candidates R and L are functions $m_R:P_R\to P_R$ and $m_L:P_L\to P_L$, respectively.

Once observed electoral campaign citizens have to casts their vote for one of the candidate, depending on their own ideal policies. A **first election voting strategy** is a function $r_{1R} : P_R \times P_L \times P \to \left\{0, \frac{1}{2}, 1\right\}$, $r_{1R}(m_{R,}m_L, \gamma)$ represents the probability a median voter of type $\gamma \in P$ once candidates R and candidate L have announced policies m_R and m_L, respectively. The median voter votes for candidate L with probability $1 - r_{1R}$.

The elected politician implements a policy from her policy space. As, at the second elections he will be opposed to a randomly drawn candidate, there is no loss of generality in considering strategies that depend only on her past electoral announcement and on her type.

A **policy strategy** for an R incumbent is simply a function $s_R : P_R \times P_R \to P_R$, $s_R(m_R, \alpha)$ denotes the strategy implemented by a politician of ideal policy α, who has announced policy m_R. A policy strategy for an L incumbent is defined analogously.

After policy implementation citizens have decide if to confirm the incumbent or to fire her. There is no loss of generality in assuming that her decision will depend only on the incumbent electoral campaign and on her performance. If the incumbent is R the **second election voting strategy** is a function $r_{2R} : P_R \times P_R \times P \to \left\{0, \frac{1}{2}, 1\right\}$, $r_{2R}(m_R, s_R, \gamma)$ represents the probability a median voter of type γ confirms the incumbent after has announced policy m_R and implemented policy s_R. The second election voting strategy when the incumbent is L is defined analogously and denoted by r_{2L}.

Beliefs

Voters' form their beliefs, at each information set depending on politicians' announcements and performances.

A **belief** at the first election about candidates is a function μ_1 from the Cartesian product of campaign messages $P_L \times P_R$ to the set of joint probability distributions on P^2. At the second election we assume that voters' beliefs depend only on the announcements and the policy of the incumbent, like voting strategies. A belief at the second election is a function μ_2 from the Cartesian product of campaign messages, first stage voting outcomes, and policy outcomes to the set of joint probability distributions on P^2.

An equilibrium is given by strategies and beliefs such that agents' strategies are optimal, given the beliefs.

Definition 1 *An* ***electoral equilibrium*** *consists of strategies* $(m_R, m_L, s_R, s_L, r_{1R}, r_{2R}, r_{2L})$ *and beliefs* (μ_1, μ_2) *such that*
(1) For all $\alpha \in P_R$*,* $m_R(\alpha)$ *maximizes in* m

$$\int_{-D}^{D}\int_{-D}^{0} r_{1R}(m, m_L(\beta), \gamma) \left[y - k(\alpha - s_R(\alpha, m)^2\right] f(\beta)g(\gamma)d\beta d\gamma +$$
$$\int_{-D}^{D}\int_{-D}^{0} r_{1R}(m, m_L(\beta), \gamma_1) r_{2R}(m, s_R(\alpha, m), \gamma_2)\delta y f(\beta)g(\gamma_1)g(\gamma_2)d\beta d\gamma_1 d\gamma_2.$$

(2) For all $(\alpha, m) \in [0, D] \times [0, D]$*,* $s_R(\alpha, m)$ *maximizes in* $s \in [0, D]$*:*

$$-k(\alpha - s)^2 + \int_{-D}^{D} r_{2R}(m_R(\alpha), s_j, \gamma))\delta y g(\gamma)d\gamma.$$

Analogous requirement are imposed on candidate L*'s strategies*

(3) For all $(m_R, m_L, \gamma) \in [0, D] \times [-D, 0] \times [-D, D]$*:*
$r_{1R}(m_R, m_L, \gamma) = 1 \, \text{\textit{if}} \, E\left[(\gamma - s_R(\cdot))^2 \mid \mu_1(m_R, m_L)\right] < E\left[(\gamma - s_L(\cdot))^2 \mid \mu_1(m_R, m_L)\right].$
$r_{1R}(m_R, m_L, \gamma) = \frac{1}{2} \, \text{\textit{if}} \, E\left[(\gamma - s_R(\cdot))^2 \mid \mu_1(m_R, m_L)\right] = E\left[(\gamma - s_L(\cdot))^2 \mid \mu_1(m_R, m_L)\right].$
$r_{1R}(m_R, m_L, \gamma) = 0 \, \text{\textit{if}} \, E\left[(\gamma - s_R(\cdot))^2 \mid \mu_1(m_R, m_L)\right] > E\left[(\gamma - s_L(\cdot))^2 \mid \mu_1(m_R, m_L)\right].$

.(4) For all $(m, s, \gamma) \in [0, D] \times [0, D] \times [-D, D]$*:*

$r_{2R}(m,s,\gamma) = 1$ *if* $E\left[(\gamma-\alpha_R)^2 \mid \mu_2(m_R,s)\right] < E\left[(\gamma-\alpha_L)^2 \mid \mu_2(m_R,s)\right]$.

$r_{2R}(m,s,\gamma) = \frac{1}{2}$ *if* $E\left[(\gamma-\alpha_R)^2 \mid \mu_2(m_R,s)\right] = E\left[(\gamma-\alpha_L)^2 \mid \mu_2(m_R,s)\right]$.

$r_{2R}(m,s,\gamma) = 0$ *if* $E\left[(\gamma-\alpha_R)^2 \mid \mu_2(m_R,s)\right] > E\left[(\gamma-\alpha_L)^2 \mid \mu_2(m_R,s)\right]$.

(5) Beliefs are computed using Bayes' rule whenever possible.

Conditions (1) and (2) state that each candidate's electoral and policy strategies are sequentially optimal given her opponent's strategies and voters' decision. Conditions (3) and (4) state that voters' decisions are optimal at each election, given their beliefs.

All along the paper we will devote our attention to symmetric equilibrium, that have great intuitive appeal.

Definition 2 *An electoral equilibrium is* ***symmetric*** *if* $m_R(\alpha) = -m_L(-\alpha)$ *and* $s_R(m_R(\alpha),(\alpha)) = -(s_L(-\alpha), m_L(-\alpha))$ *for all* $\alpha \in [0,D]$.

We next introduce notation for politicians expected probability of elections given their strategy.

$$\pi_{1R}(m_R, m_L(\cdot)) = \int_{-D}^{D}\int_{-D}^{0} r_{1R}(m_R, m_L(\beta),\gamma) f(\beta) g(\gamma) d\beta d\gamma.$$

$$\pi_{2R}(m_R, s) = \int_{-D}^{D} r_{2R}(m,s,\gamma) g(\gamma) d\gamma.$$

π_{1R} and π_{2R} are candidate R's probabilities of winning the first and the second election, respectively. Define analogous quantities for candidate L. Taking in account that, that, if reelected a politician R of type $\alpha \in X_R$ will implement her ideal policy, her expected utility at equilibrium will be

$$\pi_{1R}\left[y - k(\alpha - x_R)^2 + \pi_{2R}\delta y\right]$$

where arguments are omitted for simplicity.

We consider only equilibrium in which more centrists politicians implement more centrist policies and are elected with higher probability.

Definition 3 *An electoral equilibrium is* ***monotonic*** *if:*

(R) $\pi_{1R}(m_R(\alpha), m_L(\cdot))$ *and* $\pi_{2R}(m_R(\alpha), s_R(\alpha))$ *are decreasing on* $[0, D]$*, and* $s_R(\alpha, m_R(\alpha))$ *is increasing on* $[0, D]$[8].

(L) $\pi_{1L}(m_R(\cdot), m_L(\alpha))$ *and* $\pi_{2L}(m_L(\alpha), s_L(\alpha))$ *are increasing on* $[-D, 0]$ *and* $s_L(\alpha, m_L(\alpha))$ *is decreasing on* $[-D, 0]$.

Apart of the intuitive appeal of the notion of monotonic equilibrium there is a further reason to consider them. We will prove that in every equilibrium that is non monotonic (if any exists) there must be at least two candidates with different ideal policy making different electoral announcements.

Given an electoral equilibrium $\{(m_j, s_j), (r_{1j}, r_{2j}), (\mu_1, \mu_2)\}_{j=R,L}$ an electoral pool is a set of candidates of different ideal policies making the same electoral announcements. For every $x \in X$, $\Omega(x) = \{\alpha : m_R(\alpha) = x\}$ is an **electoral pool** if it has positive measure.

A policy pool instead, is a set of incumbents of different ideal policies implementing the same policy in their first term. Let $x, z \in [0, D]$ and set $\Omega(x, z) = \{\alpha : s_R(\alpha, x) = z\}$. $\Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha)))$ is called a **policy pool** if it has positive measure.

Let $x, \alpha, \alpha' \in [0, D]$, $\alpha \neq \alpha'$. If $\Omega(m_R(\alpha)) = \{\alpha\}$ or $\Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha))) = \{\alpha\}$ we say that α **separates in campaign** or **in policy**, respectively. Otherwise, we say that α **pool**. If $\{\alpha, \alpha'\} \subset \Omega(m_R(\alpha))$ or if $\{\alpha, \alpha'\} \subset \Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha)))$ we say that α and α' **pool** (together) in campaign or in policy, respectively. If $\Omega(m_R(\alpha)) = \{\alpha\}$ and $\Omega(m_R(\alpha), s_R(\alpha, m_R(\alpha))) = \{\alpha\}$ for all $\alpha \in [0, D]$, then

[8] Unless otherwise stated, decreasing and increasing will stay for weakly decreasing and weakly increasing, respectively.

the equilibrium is **fully separating**. If only one of the two conditions holds, then the equilibrium will be said **separating in campaign** and **in policy**, respectively.
Analogous definitions hold for candidate L.

We devote our attention to symmetric monotonic equilibria. Thus, it is sufficient to consider only the strategies of one of the two candidates. We will analyze R's strategies omitting the subscript R, when there is no risk of ambiguity. Furthermore we use $s(\alpha)$ for $s(m(\alpha), \alpha)$, $\pi_1(\alpha)$ for $\pi_1(m_R(\alpha), m_L(\cdot))$ and $\pi_2(\alpha)$ for $\pi_2(m(\alpha), s(\alpha))$, on the equilibrium path.

Monotonic equilibria have interesting connectivity properties: if candidates of different types $\alpha_1 < \alpha_2$ announce the same policy then every candidate α, with $\alpha_1 < \alpha < \alpha_2$ will make the same campaign and will implement the same policy after the first electoral term. Furthermore they will be elected with the same probability at both elections. The easy proof is omitted.

Remark 1 *Let* $\{(m_j, s_j), (r_{1j}, r_{2j}), (\mu_1, \mu_2)\}_{j=R,L}$ *be a monotonic equilibrium.*
(1) For each $\alpha \in [0, D]$, $\Omega(m(\alpha), s(\alpha))$ *is connected, hence it is an interval.*

(2) If $s(\alpha) = s(\alpha')$ *then* $\pi_1(\alpha) = \pi_1(\alpha')$ *and* $\pi_2(\alpha) = \pi_2(\alpha')$.

There is no loss of generality in assuming that candidates having the same probability of election at the first stage use the same electoral campaign (we assume candidates only use pure strategies). Under this assumption, we can state (2) as.
(3) $s(\alpha) = s(\alpha') \Rightarrow m(\alpha) = m(\alpha')$.

Then monotonicity gets rid of unlikely equilibria where centrists and moderates present different electoral platforms, but extremists pool with centrists.

We say that the electoral campaign is significative at equilibrium if there exist politicians of different ideal policies inducing different beliefs through electoral campaign. Formally:

Definition 4 *The electoral campaign is **informative** at equilibrium if there are two electoral messages* $m = m_R(\alpha)$, $m' = m_R(\alpha')$, $m'' = m_L(\beta)$ *for some* $\alpha, \alpha' \in [0, D]$ *and* $\beta \in [-D, 0]$ *such that* $\mu_1(\cdot \mid m, m'') \neq \mu_1(\cdot \mid m', m'')$

4 Preliminary results

The first result provides an additional reason that makes monotonic equilibrium a reasonable choice in this environment. Any electoral equilibrium is, locally, monotonic. In all electoral pools, equilibrium policies are monotonic and second stage election probabilities are decreasing.

Lemma 1 *Let* $x \in [0, D]$ *be a campaign message. In all symmetric equilibria:*

(i) $s(\alpha, x)$*, is increasing in* α *on* $\Omega(x)$.

(ii) $\pi_2(x, s(\alpha, x))$ *is decreasing in* α *on* $\Omega(x)$[9].

Symmetric claims hold for candidate L.

Both claims are emptily true if the type sending message x is separating. Otherwise the first claim is easily proved using incentive compatibility. In order to prove the second one we use (i) of Example 1

From Lemma 1, it follows:

Corollary 1 *In any non monotonic equilibrium the electoral campaign is significative.*

We next prove that in any monotonic equilibrium, if the types (α_1, α_2) belong to the same policy pool, then there is a set of unused policies. This result will

[9] Property (i) holds in any electoral equilibrium, either symmetric or asymmetric.

be frequently used. It implies that the policy function has a discontinuity, at the end of any policy pool.

Lemma 2 *Let $x \in [0, D]$ be a policy and let $\alpha_1 < \alpha_2$. Assume that $s(\alpha) = x < D$ for all $\alpha \in (\alpha_1, \alpha_2)$, but $s(\alpha) \neq x$ for all $\alpha > \alpha_2$ then there exists $h > 0$ such that policies in $(x, x + h)$ are not used or $s(\alpha) = x$ on $(\alpha_2, D]$.*

If the incumbents with ideal policies in (α_1, α_2) are implementing x, while agents with $\alpha > \alpha_2$ are implementing a different policy then there must be a discontinuity in voters' beliefs. A discontinuity in the policy function is needed to prevent an agent of type $\alpha > \alpha_2$ from mimicking a more centrist politician.

The next result shows that the threat of reelection is effective on the incumbent. In order not to decrease her chances of reelection, she will implement a policy which is more centrist than her favorite one.

Lemma 3 *In a monotonic equilibrium, if $s(\alpha)$ is separating on $[\alpha_1, \alpha_2)$ then $s(\alpha) < \alpha$ on $[\alpha_1, \alpha_2)$.*

The intuition for this result is simple. If $s(\alpha) > \alpha$ then an agent of type $s(\alpha)$ would profit from imitating α because she would increase her election perspectives and she would implement her favorite policy. The result does not preclude that $s(D) = D$.

It follows that in any monotonic equilibrium some candidates' types are pooling in order to increase the probability of winning the elections. This fact implies that a full separating equilibrium does not exist.

Proposition 2 *There exists no policy separating monotonic equilibrium. Hence there is no full separating equilibrium.*

Proof. Otherwise, from Lemma 3, $s(0) < 0$. Any full separating equilibrium is equivalent to a monotonic equilibrium so the second claim follows from the first one. ■

We have assumed that right wing politician can only implement right wing policies. The result holds also we allow right wing candidates to implement left wing policies.

If $s(\cdot)$ is increasing then it has at most a countable set of discontinuity points and it is differentiable almost everywhere (see Royden 1988). There is no loss of generality in assuming that the electoral campaign is monotonic increasing and that $m(\alpha) = \alpha$ when type α is separating, and that agents having the same probability of being elected at the first election make the same announcement.

Denote by $\alpha_1, \alpha_2, ...\alpha_k, ...$ where $0 \leq \alpha_1 < \alpha_2 < ... < \alpha_k < ... \leq D$ the discontinuity points of s. The intervals (α_i, α_{i+1}) $i \geq 1$ will be called continuity regions of s.

Not surprisingly, if the politicians belonging to the same continuity region implement the same policy then also their electoral announcement is the same. The result will be repeatedly used in providing a characterization of SMD1 equilibria. The result holds for any monotonic equilibrium, independently on any refinement.

Lemma 4 *Assume that, for some $i = 1, 2, ..., k, ...$, the types in (α_i, α_{i+1}) belong the same policy pool. Then they belong to the same campaign pool.*

The proof of the results is by contradiction. The intuition is simple: if it was not the case extremists of the continuity region (α_i, α_{i+1}) would find profitable to mimicking more centrists candidates.

5 The SMD1 refinement

The refinement used in this‘paper builds on the one introduced by Bernheim and Severinov (2003) and studied also in Kartik (2005). It develops the same basic intuition that led Cho and Kreps (1987) to introduce the D1 criterion.

The receiver should not attribute the deviation to a particular type if there is some different type that is willing to deviate for a larger set of responses. The MD1 Criterion strengthens the D1 criterion and applies the test to beliefs about policies that are never used in equilibrium, taking in account the monotonicity of equilibrium messages and the sequential release of information.

The lowest and the highest response an incumbent can expect after implementing a policy x are, in this setup, the highest and lowest probability of election following policy x. We denote them by $\pi_{lR}(x)$ and $\pi_{hR}(x)$ respectively. For all $x \in [0, D]$ set

$$\begin{aligned}
\pi_{lR}(x) &= \sup_{s_R(\alpha)>x} \pi_{2R}(\alpha) \text{ if } s_R(\alpha) > x \text{ for some } \alpha \in [0, D] \\
\pi_{lR}(x) &= \pi_{2R}(D, f(\cdot)) \text{ otherwise}
\end{aligned}$$

and

$$\begin{aligned}
\pi_{hR}(x) &= \inf_{s_R(\alpha)<x} \pi_{2R}(\alpha), \text{ if } s_R(\alpha) < x \text{ for some } \alpha \in [0, D] \\
\pi_{hR}(x) &= \pi(0, f(\cdot)) \text{ otherwise}
\end{aligned}$$

Analogous bounds are symmetrically defined for candidate L. Imagine that an incumbent implements an out of equilibrium policy x, $\pi_{lR}(x)$ is the supremum of the probabilities of elections of candidates that, at equilibrium, use policies, that are more extremist than x. Similarly, $\pi_{hR}(x)$ is the infimum of the probabilities of elections of candidates that, at equilibrium, use more extremist policies. Lemma 2 implies that $\pi_{lR}(x) < \pi_{hR}(x)$ for all out of equilibrium policies x with $x < s(D)$. For all $x \in [s(D), D]$, $\pi_{lR}(x) = \pi_{hR}(x) = \pi_2(D)$.[10]

[10] If the D1 criterion was used $[\pi_{lR}(x), \pi_{hR}(x)] = [\pi(D, f(\cdot)), \pi(0, f(\cdot))]$.

Definition 5 *An electoral equilibrium satisfies the* ***sequential monotonic D1 (SMD1) criterion*** *if*

(1) It is monotonic

(2) Let $m^* = (m_R(\beta_R^*), m_L(\beta_L^*))$ *for some* $(\beta_R^*, \beta_L^*) \in P_R \times P_L$*. Let* $x \in [0, D]$ *with* $\mu(x \mid \beta_R, \beta_L) = 0$ *for all* $(\beta_R, \beta_L) \in P_R \times P_L$*. If there exist some* $\alpha' \in [0, D]$ *such that, if* $\pi \in [\pi_{lR}(x), \pi_{hR}(x)]$ *and* $\alpha \in [0, D]$*:*

$$\pi_{1R}(\beta)\left(y - k(x-\alpha)^2 + \pi\delta y\right) \;\geq\; \pi_{1R}(\alpha)\left(y - k(s_R(\alpha)-\alpha)^2 + \pi_2(\alpha)\delta y\right) \implies$$
$$\pi_{1R}(\beta)(y - k(x-\alpha')^2 + \pi\delta y) > \pi_{1R}(\alpha')\left(y - k(s_R(\alpha')-\alpha')^2 + \pi_2(\alpha')\delta y\right)$$

then $\mu(\alpha', \cdot \mid m, x) = 1$*.*

Analogous requirement is symmetrically imposed on candidate L*.*

Requirement (2) extends the monotonicity requirements to out of equilibrium beliefs. If an elected official implements out of equilibrium policy x, she should expect of being reelected with probability between $\pi_{lR}(x)$ and $\pi_{hR}(x)$. The refinement assign positive probability only to those types who benefit most from this deviation. Differently from the monotonic D1 criterion, in assessing her beliefs, the receiver takes also in account the information provided by the campaign, which can have an influence on the probability of winning the first electoral competition. In this electoral game the politician in deviating will take in account both the effect on the first election, and on the second one.

For all α, β set

$$T(\alpha,\beta,x,\pi_2) = \pi_1(\beta)\left(y - k(x-\alpha)^2 + \pi_2\delta y\right) - \pi_1(\alpha)\left(y - k(s(\alpha)-\alpha)^2 + \pi_2(\alpha)\delta y\right).$$

$T(\alpha,\beta,x,\pi_2)$ is the gain, for type α in mimicking type β campaign and implementing policy x.

Condition (2) of Definition 5 can be written in this case as

Let $m^* = (m_R(\beta_R^*), m_L(\beta_L^*))$ and let $x \in [0, D]$ with $\mu(x \mid \beta_R, \beta_L) = 0$ for all

$(\beta_R, \beta_L) \in P_R \times P_L$. If there exists a non-empty set of types $\Omega \subset [0, D]$ such that, for each $\alpha \notin \Omega$, if there exists some $\alpha' \in \Omega$ such that, for all $\pi \in [\pi_l(x), \pi_h(x)]$

$$T(\alpha, \beta, x, \pi_2) \geq 0 \Longrightarrow T(\alpha', \beta, x, \pi_2) > 0$$

then $\mu(\cdot, \cdot \mid m_R^*, x) = \mu_R(\cdot) f(\cdot)$, where $supp \mu_R(\cdot \mid m_R^*, x)) \subset \Omega$.

In particular if α is the unique maximizer of $T(\cdot, \beta, x, \pi_h(x))$ then voters must assign probability 1 to type α when they observe an incumbent who has been elected presenting an electoral platform $m_R(\beta)$ and implements policy x.

6 Equilibrium characterization and existence

Every SMD1 equilibrium is characterized by a cut-off type such all politician with ideal point to the left of it implement the most centrist policy, while the others separate in policy. In this equilibria centrists act as pure office seekers. More extremists candidates, despite the electoral concerns cannot bear to implement a policy that is too faraway from the ideal one.

Theorem 3 *Any SMD1 is essentially equivalent*[11] *to an equilibrium in which, for all i, there exists $\alpha^* \in (0, D]$ such that*
(i) $s_R(\alpha) = 0$ on $[0, \alpha^*]$
(ii) If $\alpha^* < D$ then $s_R(\alpha)$ is separating on $(\alpha^*, D]$ and $s_R(D) = D$.

Let's provide an intuition for the role played by the SMD1 criterion in the proof. Let x_i be the most extremist policy implemented by an incumbent with ideal policy in (α_i, α_{i+1}) then and if x_{i+1} is the most centrist policy implemented by an incumbent with ideal policy in $(\alpha_{i+1}, \alpha_{i+2})$. From Lemma 2 $x_i < x_{i+1}$.

[11] Essentially equivalent means that it is equal, excepted, at most a zero measure set of types.

The SMD1 criterion implies that, if voters observe a policy in (x_i, x_{i+1}) they must deduce that the incumbent politician has ideal point α_{i+1}. We use this result to proof that if agents in (α_i, α_{i+1}) pool then politicians in $(\alpha_{i+1}, \alpha_{i+2})$ separate and that if politicians in (α_i, α_{i+1}) separate then $\alpha_{i+1} = D$.

Theorem 3 and Lemma 4 together restrict all SMD1 equilibria to only four categories:

(i) **Babbling:** equilibria in which all types pool in campaign and first-term policy.

Insert Figure 1 here

(ii) **Campaign irrelevant but policy significative** equilibria, in which all types send the same electoral message, but the more extremist types separate in first-term policy.

Insert Figure 2 here

(iii) **Polarized campaign** equilibria in which centrists and extremists belong different campaign pools centrists implement the same policy and extremists separate in first-term policy

Insert Figure 3 here

(iv) **Partially separating campaign** equilibria where centrists pool on the same electoral promise and on the same policy and extremists separate both in campaign and in first-term policy.

Insert Figure 4 here

We finally prove the existence of SMD1 equilibria and we refine the characterization. The larger is the degree in which candidates cares about the policy they implement, the larger are the possibilities of relevant electoral campaign. We prove that partially separating campaign equilibrium do not exist.

Theorem 4 *A SMD1 equilibrium exists. There exist $k_0 < k_1 < k_2$ and there exists strictly decreasing functions $\alpha_1(k), \alpha_2(k)$ with*
$\lim_{k\to\infty} \alpha_i(k) = 0$ for $i = 1, 2, 3$, such that

(i) For $k \leq k_0$ all MD1 equilibria are fully pooling, which is $m(\alpha) = m(0)$ and $s(\alpha) = 0$ for all $\alpha \in [0, D]$. If $k > k_0$ such equilibria are not SMD1.

(ii) For $k \geq k_0$ there exists an SMD1 equilibria such that $m(\alpha) = m(0)$ for all $\alpha \in [0, D]$, $s(\alpha) = 0$ for all $\alpha \in [0, \alpha_1(k)]$, $s(\alpha)$ is separating on $(\alpha(k), D]$.

(iii) For $k \geq k_1$ there exists an SMD1 equilibrium in which $m(\alpha) = m(0)$ for all $\alpha \in [0, \alpha_2(k))$ and $m(\alpha) = m(\alpha_2(k)) \neq m(0)$ for all $\alpha \in [\alpha_2(k), D]$, $s(\alpha) = 0$ for all $\alpha \in [0, \alpha_2(k)]$ and $s(\alpha)$ is separating on $[\alpha_2(k), D]$.

Any SMD1 equilibrium is essentially equivalent to one of the equilibria described above.

It is interesting to observe that if an high degree of policy concern makes campaign relevant, it makes more difficult for politicians to stick to their promises.. When $k \to \infty$, $\alpha_2(k) \to 0$ so a larger proportion of politician separate in policy.

The babbling equilibrium is always an equilibrium. When candidates care enough about the policy they implement if elected extremists candidate are willing to reduce their chances of elections in order to implement their favorite policy, so the babbling equilibrium does no longer satisfies the SMD1 criterion. The rest of the proof is constructive.

Observe that $\alpha = \arg\max_{\alpha'} \pi_1(\alpha')) \left[y - k\left(s(\alpha') - \alpha\right)^2 + \pi_2\left(\alpha'\right)\delta y \right]$. So, almost everywhere:

$$\pi_1'(\alpha)\left[y - k\left(s(\alpha) - \alpha\right)^2 + \pi_2\left(\alpha\right)\delta y \right] + \pi_1(\alpha))\left[-2ks'(\alpha)\right)\left(s(\alpha) - \alpha\right) + \pi_2'\left(\alpha\right)\delta y] = 0 \tag{1}$$

In particular, if all agents in (α', α'') pool on the same campaign but separate in policy, $(\pi_1(\alpha)))' = 0$ and

$$[-2ks'(\alpha))\left(s(\alpha) - \alpha\right) + \pi_2'\left(\alpha\right)\delta y] = 0 \text{ on } \left(\alpha', \alpha''\right) \tag{2}$$

In a SMD1 equilibrium $S(D) = D$ so the problem defined by 2 and $S\left(D\right) = D$ has not in general a solution, because does not satisfies the Lipschitz condition in a neighborhood of D, and nothing guarantees the unicity of the solution. In the appendix we proof that the problem has a unique increasing solution which satisfies $S(D) = D$ and $s\left(x\right) < x$ for $x < D$. Actually it could be proved that

the problem has a unique decreasing solution which satisfies $S(D) = D$ and $s\left(x\right) > x$ for $x < D$.

If some candidates separated in campaign then Equation 1 would be an integro-differential equation of the form

$$\frac{ds}{d\gamma}_{|\gamma=\alpha} = \frac{-\widehat{U}_3\left(s\left(\alpha\right), \alpha, \alpha, s\left(\cdot\right)\right)}{\widehat{U}_1\left(s\left(\alpha\right), \alpha, \alpha, s\left(\cdot\right)\right)}$$

with final condition $s\left(D\right) = D$. In the Appendix we prove that $\widehat{U}_3\left(s\left(\alpha\right), \alpha, \alpha, s\left(\cdot\right)\right) < 0$ and that $\widehat{U}_1\left(s\left(D\right), D, D, s\left(\cdot\right)\right) < 0$ if $s\left(D\right) = D$. No solution s to the the problem can then be increasing.

6.1 Asymptotically separating equilibria

Theorem 4 proves that there is no equilibria in which an interval of politicians separate in campaign that are robust to the SMD1 criterion. Nevertheless monotonic equilibria where some politicians separate in campaign in general exist. In the Appendix we describe a family of such equilbria, that are very similar to the ones described in Theorem 4, but with $s(D) < D$. Asymptotically, for $k \to \infty$ such equilibria converge to a fully revealing equilibrium.

Insert Figure 5 here

Proposition 5 *Let f and g be C^1. There exists k^* such that for all $k \geq k^*$ a monotonic equilibrium with partially separating campaign exists. There exists $\alpha(k) \in (0.D)$ $m(\alpha) = m(0)$ for all $\alpha \in [0, \alpha(k))$ and $m(\alpha) = \alpha$ for all $\alpha \in [\alpha_2(k), D]$, $s(\alpha) = 0$ for all $\alpha \in [0, \alpha_2(k)]$ and $s(\alpha)$ is separating on $[\alpha(k), D]$. Such equilibria converge to a completely separating equilibrium:* $\lim_{k\to\infty} \alpha(k) = 0$.

7 Discussion

Theorem 4 relies on the credible threat imposed on incumbent reelection perspectives. In the real world electoral disappointment does have an effect on electors. The model we presents does not capture this aspect because the idiosyncratic shocks defining median voter exact position is independent across periods and uncorrelated to actions. Electoral disappointment can be introduced as a shift of voters distribution, correlated with the degree of electoral fulfillment. To make things simple as possible assume that median voter distribution is shifted to left in the case of an R incumbent, or to the right in the case of an L incumbent of a fix factor $x > 0$, if the elected officer deviates from the expected policy(ies)[12]. The reader can easily verify that the claim of Theorem 4 holds even if we impose voters' beliefs about the two candidates to be independent. For $x \geq D - \frac{D^2 - \int_0^D \beta f(\beta) d\beta}{D + \int_0^D \beta f(\beta) d\beta}$ the proof of the result would not change at all. Otherwise the value of k_0, k_1, k_2 needed would be larger as it would harder to induce extremists not to pool in campaign. Similar result can be obtained also through a shock which continuously depends on the distance between expected policy and implemented one.

The SMD1 refinement applies only to zero probabilities policies. It is strong enough to shrink dramatically the set of possible equilibria.

The claim of Proposition 2 relies on the boundedness of the type space. Allowing for an unbounded type space can lead to full separation in sender-receiver games with both costly messages and cheap talk (see Kartik 2005). It is not the case here. We would obtain full separation in policy, but total pooling in campaign. The reason is that, asymptotically, candidates utilities is null, so it is the effect of career concerns. Very extremist candidates would be incomparably better off by maximizing their first election probability.

8 Conclusions

The paper has presented a model of electoral competition under incomplete information in which candidates care about both office and the policy. It introduces incomplete information and the dynamic aspects of a double election and it proves that electoral campaign is able to convey relevant information to voters even when campaigning is not costly. The work opens a possibility for endogenous commitment. The result is driven by both candidates' career concern and the threat of failed reelection. The impossibility for candidates to sustain policies that are too faraway from their ideal ones shapes then policy

[12] Excluding the case of a totally out of equilibrium policy, which defines the MD1 criterion.

they carry out in the first term. Innovating on Harrington (1993), we find that not only reelection pressure but also policy motivation can give relevance to electoral promises, and they both concur in shaping implemented policies. Centrists' electoral opportunism cannot be eliminated. It can be only be reduced if candidates' degree of policy implication is high enough. This is consistent with the empirical literature which estimates that a part (even if relevant) of policies are responsive to electoral compromises.

The investigation can be extended in different directions. On the one hand toward the study of more complex models of competition. In our model the " world ends" after the second election. So just before there is no place for meaningful electoral competition before the last election. Allowing for repeated interactions should make it relevant. A suitable model would be the one of an overlapping generation of politicians that can stay in the office for a fixed number of terms. The threat to reelection imposed on the incumbent would probably be reinforced, and so the degree of commitment.

On the other hand, a partially unexplored field is the nature itself of electoral campaign. It is usually modeled as a one-shot policy announcements (either costly or cheap). Despite of it, in the real world, electoral campaigns are complex and longer interactions between electors and politicians. Voters are continuously exposed to announcements. Politicians invest many resources in polls to discover electors' intentions and tastes. Parties try both to send reliable messages and to get information about electors. The empirical literature considers these aspects as an important part of the process of information transmission ((see for instance Alvarez 1998), while theoretical investigation about such phenomena is little and not conclusive (see Meirowitz 2005, who studies polls manipulability by electors). Models of repeated and costless communication (see Krishna and Morgan 2004) could provide useful tools to deal with the topic.

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QTOappendix

Proofs

Preliminary Results on Monotonic Equilibria

Proof of Lemma 1: Let $0 \leq \alpha < \alpha'$. Set $t = s(\alpha, x)$, $t' = s(\alpha', x)$, $\pi = \pi_2(x, s(\alpha, x))$ and $\pi' = \pi_2'(x, s(\alpha', x))$.

(i) The proof of the claim is by contradiction. Assume that $t' < t$. From incentive compatibility it follows:

$$\begin{aligned} -k(t-\alpha)^2 + \pi\delta y &\geq -k(t'-\alpha)^2 + \pi'\delta y. \\ -k(t-\alpha')^2 + \pi'\delta y &\geq -k(t-\alpha')^2 + \pi\delta y. \end{aligned}$$

Which is:

$$\begin{aligned} (\pi - \pi')\,\delta y + k\left[(t'-\alpha)^2 - (t-\alpha)^2\right] &\geq 0. \\ (\pi' - \pi)\,\delta y + k\left[(t-\alpha')^2 - (t'-\alpha')'^2\right] &\geq 0. \end{aligned}$$

Summing up the two inequalities:

$$(t'-\alpha)^2 - (t-\alpha)^2 + (t-\alpha')^2 - (t'-\alpha') \geq 0.$$

Simplifying:

$$(\alpha - \alpha')\,(t - t') \geq 0$$

that yields a contradiction because $\alpha < \alpha'$.

(ii) The proof of the claim is by contradiction. Assume that $\pi > \pi'$. From (i) and from the definition of monotonic equilibrium it follows that it cannot be the case that α and α' belong to different policy pools or that α and α' belong the same policy pool or that α' pools with some other type while α separate.

It must be the case that α pools and α' separates. From Remark 1, the pool α belongs to is an interval (α_1, α_2) (or $[\alpha_1, \alpha_2]$, or $(\alpha_1, \alpha_2]$, or $[\alpha_1, \alpha_2)$). In such a case the decisive voter for α is:

$$e(\alpha) = \frac{1}{2} \frac{\int_{\alpha_1 0}^{\alpha_2} \beta^2 f(\beta)\, d\beta - (F(\alpha_2) - F(\alpha_1)) \int_0^D \beta^2 f(\beta)\, d\beta}{\int_{\alpha_1}^{\alpha_2} \beta f(\beta)\, d\beta + (F(\alpha_2) - F(\alpha_1)) \int_0^D \beta f(\beta)\, d\beta}$$

while the decisive voter for $\alpha' > \alpha$ is:

$$e(\alpha') = \frac{1}{2} \frac{(\alpha')^2 - \int_0^D \beta^2 f(\beta)\, d\beta}{\alpha' + \int_0^D \beta f(\beta)\, d\beta} > e(\alpha)$$

which yields a contradiction.

Proof of Lemma 2: From Remark 1, part (3) $m(\alpha)$ constant on (α_1, α_2). There is no loss of generality in assuming that (α_1, α_2) is the interior of the corresponding policy pool. From monotonicity $s(\alpha_2) \geq x$. First, consider the case $s(\alpha_2) > x$. Then policies in $(x, s(\alpha_2))$ are not used in equilibrium.

Now let $s(\alpha_2) = x$ and set $\widehat{x} = \lim_{\alpha \searrow \alpha_2} s(\alpha) = \inf_{\alpha > \alpha_2} s(\alpha)$. Observe that $s(\alpha) > x$ for $\alpha > \alpha_2$. By contradiction, suppose that $\widehat{x} = x$. Set $\pi_{1\varepsilon} = \pi_1(\alpha_2 + \varepsilon)$, $\pi_1 = \pi_{10}$, $\pi_{2\varepsilon} = \pi_2(\alpha_2 + \varepsilon)$, $\pi_2 = \pi_{20}$. It must be the case that $\pi_{2\varepsilon} < \pi_2$ and $0 < \pi_{1\varepsilon} \leq \pi_1$ for all $\varepsilon > 0$. The difference $\pi_2 - \pi_{2\varepsilon}$ is bounded below by some positive constant c. Furthermore, $(s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon))^2 > (x - \alpha_2 - \varepsilon)^2$ for all $\varepsilon > 0$ Otherwise, $\alpha_2 + \varepsilon$ could profitably deviate by mimicking α_2.

For all $0 < \varepsilon < \varepsilon^*$ set

$L(\varepsilon) = \pi_1 \left(y - k(x - \alpha_2 - \varepsilon)^2 + \pi_2 \delta y \right) - \pi_{1\varepsilon} \left\{ y - k\left[s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon)\right]^2 + \pi_{2\varepsilon} \delta y \right\}$

$L(\varepsilon)$ is the net loss or the net gain to type $\alpha_2 + \varepsilon$ from imitating type α_2. At equilibrium $L(\varepsilon) \leq 0$ for all $\varepsilon > 0$.

$L(\varepsilon) \geq \pi_1 k \left[(s(\alpha_2 + \varepsilon) - (\alpha_2 + \varepsilon))^2 - (x - \alpha_2 - \varepsilon)^2 \right] + \pi_{1\varepsilon^*}(\pi_2 - \pi_{2\varepsilon})\delta y \geq$

$\pi_1 k \left[(s(\alpha_2+\varepsilon) - (\alpha_2+\varepsilon))^2 - (x-\alpha_2-\varepsilon)^2 \right] + \pi_{1\varepsilon^*} c\delta y$.
$\inf_{\varepsilon>0} \pi_1 k \left[(s(\alpha_2+\varepsilon) - (\alpha_2+\varepsilon))^2 - (x-\alpha_2-\varepsilon)^2 \right] = 0$, then for ε small enough $L(\varepsilon) > 0$, a contradiction.

Proof of Lemma 3: We first prove that $s(\alpha) \leq \alpha$ for all $\alpha \in [\alpha_1, \alpha_2]$. By contradiction, assume that $s(\alpha) > \alpha$. Consider type $\alpha' = s(\alpha) > \alpha$. By monotonicity $s(\alpha') > s(\alpha)$ because agents in $[\alpha_1, \alpha_2)$ separate. But then α' could profitably imitate α: the probabilities of election at both stages weakly increase because of monotonicity and she would not pay policy costs.
We ow prove the second claim. By contradiction, suppose that $s(\widehat{\alpha}) = \widehat{\alpha}$ for some $\widehat{\alpha} \in (\alpha_1, \alpha_2)$. Let $\varepsilon \geq 0$ and set $\pi_{1\varepsilon} = \pi_1(\widehat{\alpha}+\varepsilon,)$, $\pi_1 = \pi_{10}$, $\pi_{2\varepsilon} = \pi_{2R}(\widehat{\alpha}+\varepsilon)$ $\pi_2 = \pi_{20}$. As s is strictly increasing $\pi_{2\varepsilon} < \pi_2$ and $\pi_{1\varepsilon} \leq \pi_1$for all $\varepsilon > 0$. Let $L(\varepsilon)$ be the net loss or the net gain to type $\widehat{\alpha}+\varepsilon$ from imitating type $\widehat{\alpha}$ as defined in the proof of Lemma 2. At equilibrium $L(\varepsilon) \leq 0$ for all $\varepsilon > 0$.
$L(\varepsilon) = \pi_1 \left(y - k\varepsilon^2 + \pi_2 \delta y\right) - \pi_{1\varepsilon} \left\{ y - k\left[s(\widehat{\alpha}+\varepsilon) - (\widehat{\alpha}+\varepsilon)\right]^2 + \pi_{2\varepsilon}\delta y \right\}$
$L(\varepsilon) \geq \pi_1 \left(y - k\varepsilon^2 + \pi_2 \delta y\right) - \pi_{1\varepsilon} \left\{ y + \pi_{2\varepsilon}\delta y \right\} \geq \pi_{1\varepsilon^*} \left[-k\varepsilon^2 + (\pi_2 - \pi_{2\varepsilon})\delta y \right]$,
for some fixed $\varepsilon^* > 0$. $\pi_{1\varepsilon^*} > 0$ and $\pi_{2\varepsilon} = \pi(\widehat{\alpha}+\varepsilon, f(\cdot))$.
Set $B(\varepsilon) = \pi_{1\varepsilon^*} \left[-k\varepsilon^2 + (\pi_2 - \pi_{2\varepsilon})\delta y \right]$, then:
$\frac{dB(\varepsilon)}{d\varepsilon} = -\pi_{1\varepsilon^*} \left(2k\varepsilon + \frac{d\pi_{2\varepsilon}}{d\varepsilon} \right)$ and
$\frac{dB(\varepsilon)}{d\varepsilon}_{|\varepsilon=0} = -\pi_{1\varepsilon^*} \left(\frac{d\pi(\widehat{\alpha}+\varepsilon, f(\cdot))}{d\varepsilon}_{|\varepsilon=0} \right) > 0$ from Example 1.
As $B(0) = 0$, then $B(\varepsilon) > 0$ for ε small enough. But then type $\widehat{\alpha}+\varepsilon$ could profitably mimic type $\widehat{\alpha}$, for ε small enough, yielding a contradiction.

Proof of Lemma 4: Let $0 \leq \alpha < \alpha' < \alpha''$. By contradiction, assume that some types in $(\alpha, \alpha') \subset (\alpha_i, \alpha_{i+1})$ send message m and that the types in $(\alpha', \alpha'') \subset (\alpha_i, \alpha_{i+1})$ send message m'. Let $\alpha'+\varepsilon$ imitate type $\alpha'-\varepsilon$. The gain in the probability of being elected at the first stage is bounded below by a strictly positive constant. The gain in second election probability is non negative. For $\varepsilon \to 0$ the loss in policy term goes to 0 by continuity, so the deviation would be profitable for ε small enough.

Equilibrium characterization and existence

When there is no risk of ambiguity we omit the arguments β, x, π_2 and we write simply $T(\alpha)$ for $T(\alpha, \beta, x, \pi_2)$ and $T'(\alpha)$ for $\frac{\partial T}{\partial \alpha}(\alpha, \beta, x, \pi_2)$.

So we have:

$$T'(\alpha) = 2k\left[\pi_1(\alpha)(\alpha - s(\alpha)) - \pi_1(\beta)(\alpha - x)\right]. \tag{3}$$

From Lemma 2 follows that if some types $(\alpha', \alpha'') \subset (\alpha_i, \alpha_{i+1})$ are in the same policy pool then (α_i, α_{i+1}) is included in the same policy pool. So if (α_i, α) with $\alpha \leq \alpha_{i+1}$ are separating then types in (α_i, α_{i+1}) are all separating.

Lemma 5 *In every SMD1 equilibrium*

(i) $s(0) = 0$

(ii) If types in (α_i, α_{i+1}) *are in the same policy pool then agents in* $(\alpha_{i+1}, \alpha_{i+2})$ *separate.*

(iii) If types in (α_i, α_{1+1}) *separate then* $\alpha_{i+1} = D$

(iv) If D *is separating in policy* $s(D) = D$.

Proof: For every $i \geq 1$ set $\underline{s_i} = \lim_{\alpha \nearrow \alpha_i} s(\alpha)$ and set $\overline{s_i} = \lim_{\alpha \searrow \alpha_i} s(\alpha)$. Set $\underline{s_0} = 0$, $\overline{s_0} = s(0)$ $\underline{s_D} = \lim_{\alpha \nearrow D} s(\alpha)$ and set $\overline{s_D} = s(D) \leq D$. By definition $\underline{s_i} < \overline{s_i}$ for $i = 1, 2....$ and $\underline{s_0} \leq \overline{s_0}$, $\underline{s_D} \leq \overline{s_D}$. From equilibrium monotonicity it follows that for all $i \geq 1$ and for all $x \in \left(\underline{s_i}, \overline{s_i}\right)$ $T_1(\alpha, \beta, x, \pi) < 0$ if $\alpha > \alpha_i$ and $T_1(\alpha, \beta, x, \pi) > 0$ for $\alpha < \alpha_i$. Then from $\mu(\alpha_i \mid m(\beta), x) = 1$. For $x \in \left(\underline{s_0}, \overline{s_0}\right)$ and $\alpha > 0$, $T_1(\alpha, \beta, x, \pi) < 0$ so $\mu(\alpha_i \mid m(\beta), x) = 1$. Finally, for all $x \in \left(\underline{s_D}, \overline{s_D}\right) \cup (\overline{s_D}, D)$, $T_1(\alpha, \beta, x, \pi) > 0$ then for all $x \in \left(\underline{s_D}, \overline{s_D}\right) \cup (\overline{s_D}, D)$, $\mu(D \mid m(\beta), x) = 1$.

(i) By contradiction, suppose that $s(0) > 0$. From the result above, for ε small

enough $\mu(0 \mid m(0), \varepsilon) = 1$. Then ε can profitably deviate by sending $(m(0), \varepsilon)$.
(ii) Let $s^* = s(\alpha)$ for all $\alpha \in (\alpha_i, \alpha_{i+1})$. By contradiction assume that the types in $(\alpha_{i+1}, \alpha_{i+2})$ are pooling. For ε small enough $\alpha_{i+1}+\varepsilon$ can profitably deviate by implementing policy $\overline{s_i}-\delta$, with δ small enough $\mu(\alpha_{i+1} \mid m(\alpha_{i+1}+\varepsilon), \overline{s_i}-\delta) = 1$, a contradiction. It is because, from the continuity of x on $(\alpha_{i+1}, \alpha_{i+2})$, the gain in second election probability is bounded below by a positive constant, while loss in policy term is of order δ^2.
(iii) If types in (α_i, α_{i+1}) are separating and types in $(\alpha_{i+1}, \alpha_{i+2})$ are pooling then type $\alpha_{i+1} + \varepsilon$ can profitably deviate by sending $(m(\alpha_{i+1} - \varepsilon), \overline{s_i} - \delta)$. $\mu(\alpha_{i+1} \mid (m(\alpha_{i+1} - \varepsilon), \overline{s_i} - \delta)) = 1$. For δ small enough, the loss in policy term is compensated by the gain in election probability. If types in (α, α_{i+1}) are pooling then, for ε small enough $\alpha_{i+1} + \varepsilon$ can profitably deviate by implementing policy $\overline{s_i} - \delta$, with δ small enough. $\mu(\alpha_{i+1} \mid m(\alpha_{i+1} + \varepsilon), \overline{s_i} - \delta) = 1$ The loss in policy term is of order δ^2, the gain in second election probability is bounded below by a positive constant.
(iv) By contradiction let $s(D) < D$. For all $x \in [s(D), D]$, $\pi_{lR}(x) = \pi_{hR}(x) = \pi_2(D)$. So,once observed x, the voters must assign positive probability to type $\alpha = D$. Then any type $\alpha = D - \varepsilon$, for ε small enough can profitably deviate by implementing policy $x = D - \varepsilon$. The loss in second term election probability would be at most infinitesimal (π_2 is continuous in a neighborhood of D), the gain in policy term is bounded below by $\min_{[\alpha_i, D]} k(\alpha - s(\alpha))^2 > 0$, from (iii).

Proof of Theorem 3: It suffices to show that $\alpha^* > 0$. By contradiction, assume that $\alpha^* = 0$. In this the equilibrium would have a monotonic electoral equilibrium with separating policies contradicting Proposition 2.

If π_2 is C^1 and strictly decreasing the problem defined by the differential equation 1 and the terminal condition $S(D) = D$ has a unique solution such that $s(\alpha) < \alpha$ on $[0, D)$. The result follows directly from Lemma 6 below.

Furthermore $s(0) < 0$. Otherwise the graph of s should crosses the diagonal at some $\alpha^* > 0$. In this case $\lim_{\alpha \to \alpha^{*+}} s'(\alpha) = \infty$. This is impossible: if the graph cross the diagonal it must be from below because $s(\alpha) < \alpha$ on $(0, D)$.

Lemma 6 *Let f be a strictly negative C^1 functions defined on $[0, D] \times B$ where B is a real interval such that $[0, D] \subsetneqq B \subset (-\infty, D]$. Then there exists a solution, defined on $[0, D]$, to the following ordinary differential equation problem:*

$$\begin{cases} y'(y - x) = f(x, y) \\ y(D) = D \\ y(x) \leq x \end{cases}$$

Furthermore, if there exists $\delta > 0$ such that $f_y(x, y) \geq 0$ for every $(x, y) \in \{(x, y) \in B : ||(x, y) - (D, D)|| < \delta,\ y < x,\}$, then the solution is unique.

Proof: The problem does not satisfy the local Lipschitz conditions in a neighborhood of D. The existence part of the Proof is by approximation. Let y_ε be the solution of the following Cauchy problem:

$$\begin{cases} y'(x)(y(x) - x) = f(x, y(x)) \\ y(D) = D - \varepsilon \end{cases}$$

Here the local existence and uniqueness theorem applies. In order to prove that $y_\varepsilon(x)$ can be extended to the interval $[0, D]$ it suffices to show that there exists no $x^* \in [0, D)$, such that $\lim_{x \to x^{*+}} y'_\varepsilon(x) = \infty$. In this case the extension theorem applies. First observe that if y_ε is defined and C^1in the interval $(x^*, D]$. From $y_\varepsilon(D) = D - \varepsilon$ and $y'_\varepsilon(x)(y_\varepsilon(x) - x) < 0$ it follows that $y'_\varepsilon(x) > 0$ and $y_\varepsilon(x) < x$ on $(x^*, D]$. If $\lim_{x \to x^{*+}} y'_\varepsilon(x) = \infty$, then $\lim_{x \to x^{*+}} y_\varepsilon(x) = x^*$. It follows that, for $\delta > 0$ small enough, $y'_\varepsilon(x) > 2$ on $(x^*, x^* + \delta]$. Let $0 < \delta' < \delta$. By the intermediate value theorem $y_\varepsilon(x^* + \delta) - (x^* + \delta) =$

$y_\varepsilon(x^* + \delta') - (x^* + \delta') + (y'_\varepsilon(x^* + \delta'') - 1)\left(\delta - \delta^{'}\right)$ for some $\delta' < \delta'' < \delta$ but then $y_\varepsilon(x^* + \delta) - (x^* + \delta) > y_\varepsilon(x^* + \delta') - (x^* + \delta') + \left(\delta - \delta^{'}\right)$. Let $\delta' \to 0$. From the previous observations it follows that the RHS converges to 2δ while the LHS is independent of $\delta' < \delta$. Then $y_\varepsilon(x^* + \delta) - (x^* + \delta) > \delta > 0$, which yields a contradiction. $y_\varepsilon(x)$ is C^1 with respect to ε on $[0, D)$ (Pontryagin 196), ch. 23). $y_\varepsilon(D) \to D$ for $\varepsilon \to 0$. By contradiction, assume that, for some $x \in [0, D)$, $y_\varepsilon(x)$ is not converging for $\varepsilon \to 0$. In particular, for some $0 < \delta < D$, the Ascoli-Arzelá Theorem does not apply in $[0, D - \delta]$. The family $\{y_\varepsilon\}_{\varepsilon>0}$ is uniformly bounded in $[0, D - \delta]$ (because $y_\varepsilon(x) \leq x$ on $[0, D - \delta]$). It must be the case that $\{y_\varepsilon\}_{\varepsilon>0}$ it is not uniformly continuous then $\sup_{\varepsilon>0} y'_\varepsilon = \infty$. As above, it follows that $y_\varepsilon(x) > x$ for some ε and some $x \in [0, D - \delta]$, a contradiction. So y_ε converges uniformly to some y in each interval $[0, D - \delta]$. Each y_ε satisfies $y'(x)(y(x) - x) = f(x, y(x))$, and y'_ε converges uniformly to some continuous z. Then $y' = z$. The local existence and uniqueness theorem implies that y is independent of the choice of δ. The function y is defined and differentiable on $[0, D)$ and satisfies $y'(x)(y(x) - x) = f(x, y)$ because each y_ε satisfies it. $y(x) < x$ on $[0, D)$ otherwise $y'(x) \to \infty$ for $x \to x^*$, some x^* against the uniform convergence of y'_ε. The existence part is proved by setting $y(D) = \lim_{x \to D} y(x) = 0$.

Now we prove uniqueness. Let f such that that, for some $\delta > 0$, $f_y(x, y) \geq 0$ for every $(x, y) \in \{(x, y) \in B : ||(x, y) - (D, D)|| < \delta,\ y < x, \}$. By contradiction, assume that y_1 and y_2 two different solutions of the problem. The local existence and uniqueness theorem implies that the graphs of the function cross only at (D, D). There is no loss of generality then in assuming that $y_1(x) < y_2(x)$ on $[0, D)$. Then, for some δ small enough $y'_1(x) > y'_2(x)$ for all $x \in [D - \delta, D)$. For x next to D, we have $y'_2(x)(y_2(x) - x) \geq y'_1(x)(y_1(x) - x) = f(x, y_1(x)) \geq f(x, y_2(x))$ with at least one strict inequality. This yields a contradiction because y_2 solves the ODE problem.

Proof of Theorem 4: We will always assume that, whenever beliefs are not imposed by Bayesian, consistency or by the MD1 refinement, if a candidate announces a policy and implements a policy that is expected from a different type, then the median voter will not confirm her. This is consistent as we allow beliefs to be correlated (actually it is sufficient that the incumbent is voted with probability $\pi_2(D, f(\cdot))$. Let us consider the different possibilities.

(a) The first case is $\alpha^* = D$ so that the equilibrium is equivalent to an equilibrium in which all types are pooling together at 0 a and at both stages they are elected with probability $\frac{1}{2}$, and after the first election all pool on policy 0. The payoff for type α is $\frac{1}{2}\left[\left(1+\frac{\delta}{2}\right)y - k\alpha^2\right]$. This is an MD1 equilibrium if and only if $\frac{1}{2}\left[\left(1+\frac{\delta}{2}\right)y - kD^2\right] \geq \frac{1}{2}y$ and $\frac{\delta}{2}y - kD^2 \geq \delta y \pi_2(D)$, otherwise type D could profitably separate by implementing policy D (at the campaign and at the policy stage, respectively) which is as far as:

$$k \leq \min\left\{\frac{y\delta}{2D^2}, y\delta\left(\frac{1}{2} - \pi_2(D)\right)\right\} = y\delta\left(\frac{1}{2} - \pi_2(D)\right) = k_0$$

where $\pi_2(D)$ is the probability a candidate is elected at the second stage if perceived as type D and the other candidate is selected from F_L, which is with probability $\left[1 - G\left(\frac{1}{2}\frac{D^2 - \int_0^D \beta^2 f(\beta)d\beta}{D + \int_0^D \beta f(\beta)d\beta}\right)\right] < \frac{1}{2}$, because of the symmetry of G. For $k < k_0$ a (continuous of) pooling equilibrium exists but it does not satisfies the MD1 criterion.

(b) The second case is that $\alpha^* < D$, and all types pool at the first stage.

In such a case all types are elected with probability $\frac{1}{2}$ at the first election. At the second stage type $\alpha \in [0, \alpha^*]$ is elected with probability:

$$\pi_2([0, \alpha^*]) = [1 - G(e([0, \alpha^*], f(\cdot)))]$$

where:

$$e\left([0,\alpha^*], f(\cdot)\right) = \frac{1}{2}\frac{\int_0^{\alpha^*} \beta^2 f(\beta)\, d\beta - F(\alpha^*) \int_0^D \beta^2 f(\beta)\, d\beta}{\int_0^{\alpha^*} \beta f(\beta)\, d\beta + F(\alpha^*) \int_0^D \beta f(\beta)\, d\beta}.$$

$e\left([0,\alpha^*], f(\cdot)\right)$ is the decisive voter when the types in $[0,\alpha^*]$ are pooling and matched to a challenger selected from the original distribution. Elementary analysis shows that $-D < e\left([0,\alpha^*], f(\cdot)\right) < D$ for $\alpha^* > 0$ and $e\left([0,\alpha], f(\cdot)\right)$ it is strictly increasing. Furthermore, $\lim_{\alpha^*\to 0^+} e\left([0,\alpha^*], f(\cdot)\right) = -\frac{1}{2}\frac{\int_0^D \beta^2 f(\beta) d\beta}{\int_0^D \beta f(\beta) d\beta} \in [-D, 0]$. $\lim_{\alpha^*\to D^-} e\left([0,\alpha^*], f(\cdot)\right) = 0$. So $\pi_2(\alpha)$ is strictly decreasing and differentiable in α.

A type $\alpha \in (\alpha^*, D]$ is elected at the second stage with probability:

$$\pi_2(\alpha) = 1 - G\left(e\left(\alpha, f(\cdot)\right)\right)$$

where:

$$e\left(\alpha, f(\cdot)\right) = \frac{1}{2}\frac{\alpha^2 - \int_0^D \beta^2 f(\beta)\, d\beta}{\alpha + \int_0^D \beta f(\beta)\, d\beta}.$$

We have $-D < e\left([0,\alpha^*], f(\cdot)\right) < e\left(\alpha, f(\cdot)\right) < D$. So $\pi_2(\alpha) < \pi_2([0,\alpha^*])$ for $\alpha > \alpha^*$. $e\left(\alpha, f(\cdot)\right)$ is strictly increasing on $(\alpha^*, D]$. $\lim_{\alpha^*\to D^-} e\left(\alpha, f(\cdot)\right) = \frac{1}{2}\frac{D^2 - \int_0^D \beta^2 f(\beta) d\beta}{D + \int_0^D \beta f(\beta) d\beta} \in (0, D)$. So $\pi_2(\alpha)$ is strictly decreasing and continuously differentiable in α^*.

If $s(\alpha)$ is separating on $(\alpha^*, D]$, it must satisfy:

$$2ks'(\alpha))\left(s(\alpha) - \alpha\right) = \pi_2'(\alpha)\,\delta y$$

with the final condition $s(D) = D$. Furthermore, $\alpha^* = \alpha_1(k) > 0$ must be indifferent between separating and pooling, then:

$$\left(\frac{1}{2} + \delta\pi_2([0,\alpha^*])\right) y - k\alpha^{*2} = \left(\frac{1}{2} + \delta\pi_2(\alpha^*)\right) y - k\left(s(\alpha^*) - \alpha^*\right)^2.$$

Set $H(\alpha) = \left(\frac{1}{2} + \delta\pi_2([0,\alpha])\right) y - k\alpha^2 - \left(\frac{1}{2} + \pi_2(\alpha)\delta\right) y + k(s(\alpha) - \alpha)^2$. $s(0) < 0$. So $H(0) > 0$. $H(D) = [\pi_2([0,D]) - \pi_2(D)]\delta y - kD^2 = \left[\frac{1}{2} - \pi_2(D)\right]\delta y - kD^2 \leq 0$ if $k \geq k_0$. $H'(\alpha) < 0 = \frac{d\pi_2([0,\alpha])}{d\alpha}\delta y - 2ks(\alpha) < 0$[13]. It is easily seen that $s(\alpha_1(k)) > 0$ because if $s(\alpha) = 0$ then $H(\alpha) > 0$.

Through implicit differentiation

$$\frac{dH(\alpha_1(k))}{dk} = H_\alpha(\alpha_1(k))\frac{d\alpha_1(k)}{dk} + H_k(\alpha_1(k)) = 0$$

so

$$\frac{d\alpha_1(k)}{dk} = \frac{-H_k(\alpha_1(k))}{H_\alpha(\alpha_1(k))} = -\frac{s^2(\alpha_1(k)) - 2\alpha_1(k)s(\alpha_1(k))}{\frac{d\pi_2([0,\alpha_1(k)])}{d\alpha}\delta y - 2ks(\alpha_1(k))} < 0$$

Then $\alpha_1(k)$ is strictly decreasing in k.

From $H(\alpha_1(k)) = 0$ follows $\alpha_1(k) \to 0$ as $k \to \infty$.

(c) In the third case there are two campaign pools $[0,\alpha^*)$ and$(\alpha^*, D]$, with the second separating in policies. $[0,\alpha^*)$ types' election probabilities are

$$\overline{\pi}_1([0,\alpha^*]) = \frac{1}{2}F(\alpha^*) + (1 - F(\alpha^*))\left[1 - G(e([0,\alpha^*), s))\right].$$

and $\pi_2([0,\alpha^*))$, respectively, where $(e([0,\alpha^*), s))$ is the decisive voter of pool $[0,\alpha^*)$ against pool $(\alpha^*, D]$: her location is:

$$\frac{1}{2}\frac{(1-F(\alpha^*))\int_0^{\alpha^*}\beta^2 f(\beta)\,d\beta - F(\alpha^*)\int_{\alpha^*}^D s^2(\beta)f(\beta)\,d\beta}{(1-F(\alpha^*))\int_0^{\alpha^*}\beta^2 f(\beta)\,d\beta + F(\alpha^*)\int_{\alpha^*}^D s(\beta)f(\beta)\,d\beta}$$

which is, simplifying:

Observe that $G(e([0,\alpha^*), s)) \leq \frac{1}{2}$, because $e([0,\alpha^*), (\alpha^*, D]) \leq 0$.

$(\alpha^*, D]$'s election probabilities are:

$$\overline{\pi}_1((\alpha^*, D]) = (F(\alpha^*))(1 - G(e(s,[0,\alpha^*)))) + \frac{1}{2}(1 - F(\alpha^*))$$

[13] Because s solves the differential equation.

and $\pi_2(\alpha)$, respectively, where:

$$e\left(\left(\alpha^*, D\right], \left[0, \alpha^*\right)\right) = \frac{1}{2}\frac{F(\alpha^*)\int_{\alpha^*}^{D} s^2(\beta) f(\beta)\, d\beta - (1 - F(\alpha^*))\int_0^{\alpha^*} \beta^2 f(\beta)\, d\beta}{F(\alpha^*)\int_{\alpha^*}^{D} s(\beta) f(\beta)\, d\beta + (1 - F(\alpha^*))\int_0^{\alpha^*} \beta^2 f(\beta)\, d\beta}$$

From the symmetry of the distribution G, $G\left(e\left(\left[0, \alpha^*\right), \left(\alpha^*, D\right]\right)\right) = 1 - G\left(e\left(\left(\alpha^*, D\right], \left[0, \alpha^*\right)\right)\right) = 1 - G \leq \frac{1}{2}$, so:

$$\overline{\pi}_1\left(\left(\alpha^*, D\right]\right) = \left(F(\alpha^*)\right)\left(G\left(e\left(\left[0, \alpha^*\right), s\right)\right)\right) + \frac{1}{2}\left(1 - F(\alpha^*)\right)$$

and $\overline{\pi}_1\left(\left[0, \alpha^*\right]\right) = \overline{\pi}_1\left(\left(\alpha^*, D\right]\right) + \frac{1}{2} - G\left(e\left(\left(\alpha^*, D\right], \left[0, \alpha^*\right)\right)\right) \geq \overline{\pi}_1\left(\left(\alpha^*, D\right]\right)$.

As above, on $\left(\alpha^*, D\right]$ s must satisfy:

$$2ks'(\alpha))\left(s(\alpha) - \alpha\right) = \pi_2'(\alpha)\,\delta y$$

and α^* must satisfy:

$\overline{\pi}_1\left[0, \alpha^*\right]\left[1 + \pi_2\left(\left[0, \alpha^*\right]\right)\delta y - k\alpha^{*2}\right] =$

$\overline{\pi}_1\left(\left(\alpha^*, D\right]\right)\left[1 + \pi_2(\alpha^*)\,\delta y - k\left(s(\alpha^*) - \alpha^*\right)^2\right]$

Set $H(\alpha, k) = \left\{\left(\overline{\pi}_1\left(\left[0, \alpha^*\right]\right) + \delta\pi_2\left(\left[0, \alpha^*\right]\right)\right) y - k\alpha^{*2}\right\} -$

$-\left\{\left(\overline{\pi}_1\left(\left(\alpha^*, D\right]\right) + \pi_2(\alpha^*)\,\delta\right) y - k\left(s(\alpha^*) - \alpha^*\right)^2\right\}$

As above, it can be shown, that a unique solution to $H(\alpha_2(k), k) = 0$ exists if and only if $H(D) > 0$ which is if and only if $k \geq k_1^* > k_0$ where $H(D, k_1^*) = 0$. $\alpha_2(k)$ is strictly decreasing and $\alpha_2(k) \to 0$ as $k \to \infty$.

It must be checked that type D does not want to imitate type $\alpha_2(k)$ in the campaign and then implement D, which is $\overline{\pi}_1\left[0, \alpha_2(k)\right] y \leq \overline{\pi}_1\left(\left(\alpha_2(k), D\right]\right)\left[\left(1 + \pi_2(D)\right)\delta y\right]$, or, equivalently $\frac{1}{2} - G - \overline{\pi}_1\left(\alpha_2(k), D\right)\pi_2(D)\,\delta \leq 0$ but $\overline{\pi}_1\left(\alpha_2(k), D\right) = -F\left(\alpha_2(k)\right)\left(\frac{1}{2} - G\right) + \frac{1}{2}$.

Then the condition is $\left(\frac{1}{2} - G\right)\left(1 + F\left(\alpha_2(k)\right)\pi_2(D)\,\delta\right) \leq \frac{\pi_2(D)\delta}{2}$.

Consider the function $R(\alpha) = \left(\frac{1}{2} - G\left(e\left(\left[0, \alpha\right), \left(\alpha, D\right]\right)\right)\right)\left(1 + F(\alpha)\,\pi_2(D)\,\delta\right) -$

$\frac{\pi_2(D)\delta}{2}$. $R(0) = -\frac{\pi_2(D)\delta}{2} < 0$, $R' > 0$. As $\alpha_2(k) \searrow 0$ as $k \to \infty$, there exists a unique $k^* > 0$ such that this kind of equilibrium exists only for $k \geq k^*$. Set $k_1 = \max\{k^*, k_1^*\}$.

(d) In this part we prove that there exists no equilibrium where politicians with types $[0, \alpha^*]$ pool in campaign and policy, and agents in $(\alpha, D]$ separate in campaign and in policy. By contradiction assume that such an equilibrium exists Then the probability of first stage election of types in $[\alpha^*, D]$ is:

$$\pi_1(s(\alpha), s(\cdot)) = \left[1 - G\left(\frac{s(\alpha)}{2}\right)\right] F(\alpha^*) + \int_{\alpha^*}^{D} \left[1 - G\left(\frac{s(\alpha) - s(\rho)}{2}\right)\right] f(\rho)\, d\rho$$

where the first term of the sum is the expected probability of being elected against a politician in $[-\alpha^*, D]$ while the second term represents the probability of being elected against a politician in $[-D, \alpha^*]$.

The probability of second stage election is, instead:

$$\pi_2(\alpha) = 1 - G\left(\frac{1}{2}\frac{\alpha^2 - \int_0^D \rho^2 f(\rho)\, d\rho}{\alpha + \int_0^D \rho f(\rho)\, d\rho}\right)$$

Observe that, for every s

$$\frac{d\pi_1(s, s(\cdot))}{ds} = -\frac{1}{2}\left[g\left(\frac{s}{2}\right)\int_0^{\alpha^*} f(\rho)\, d\rho + \int_{\alpha^*}^{D} g\left(\frac{s - z(\rho)}{2}\right) f(\rho)\, d\rho\right] < 0$$

and

$$\frac{d\pi_2(\gamma)}{d\gamma} = -\frac{1}{2} g\left(\frac{1}{2}\frac{\gamma^2 - \int_0^D \beta^2 f(\beta)\, d\beta}{\gamma + \int_0^D \beta f(\beta)\, d\beta}\right)\left[\frac{\gamma^2 + 2\gamma\int_0^D \beta f(\beta)\, d\beta + \int_0^D \beta^2 f(\beta)\, d\beta}{\left(\gamma + \int_0^D \beta f(\beta)\, d\beta\right)^2}\right] < 0$$

So the expected utility, for a type α mimicking type γ in $[\alpha^*, D]$.

$$
\begin{aligned}
U\left(s\left(\gamma\right),\alpha,s\left(\cdot\right)\right) \quad = \quad & \left\{\left[1-G\left(\frac{s\left(\alpha\right)}{2}\right)\right]F\left(\alpha^{*}\right)+\int_{\alpha^{*}}^{D}\left[1-G\left(\frac{s-s\left(\rho\right)}{2}\right)\right]f\left(\rho\right)d\rho\right\}* \\
& \left\{y-k\left[\alpha-s\left(\alpha\right)\right]^{2}+\delta y\left[1-G\left(\frac{1}{2}\frac{\gamma^{2}-\int_{0}^{D}\rho^{2}f\left(\rho\right)d\rho}{\gamma+\int_{0}^{D}\rho f\left(\rho\right)d\rho}\right)\right]\right\}
\end{aligned}
$$

Set:

$$
\begin{aligned}
\widehat{U}(s,\alpha,\gamma,s\left(\cdot\right)) \quad = \quad & \left\{\int_{0}^{\alpha^{*}}\left[1-G\left(\frac{s}{2}\right)\right]f\left(\rho\right)d\rho+\int_{\alpha^{*}}^{D}\left[1-G\left(\frac{s-s\left(\rho\right)}{2}\right)\right]f\left(\rho\right)d\rho\right\}* \\
& \left\{y-k\left[\alpha-s\right]^{2}+\delta y\left[1-G\left(\frac{1}{2}\frac{\gamma^{2}-\int_{0}^{D}\rho^{2}f\left(\rho\right)d\rho}{\gamma+\int_{0}^{D}\rho f\left(\rho\right)d\rho}\right)\right]\right\}
\end{aligned}
$$

Then $U\left(s\left(\gamma\right),\alpha,s\left(\cdot\right)\right)=\widehat{U}\left(s\left(\gamma\right),\alpha,\gamma,s\left(\cdot\right)\right)$.

At equilibrium $\alpha=\arg\max_{\gamma}\widehat{U}\left(s\left(\gamma\right),\alpha,\gamma,s\left(\cdot\right)\right)$. So a.e.

$$
\frac{d\widehat{U}(s\left(\gamma\right),\alpha,\gamma,s\left(\cdot\right))}{d\gamma}_{|\gamma=\alpha}=0
$$

Using the chain rule

$$
\frac{d\widehat{U}(s\left(\gamma\right),\alpha,\gamma,s\left(\cdot\right))}{d\gamma}_{|\gamma=\alpha}=\widehat{U}_{s}\left(s\left(\alpha\right),\alpha,\alpha,s\left(\cdot\right)\right)s'\left(\alpha\right)+\widehat{U}_{\gamma}\left(s\left(\alpha\right),\alpha,\alpha,s\left(\cdot\right)\right)
$$

We obtain that

$$
\frac{ds}{d\gamma}_{|\gamma=\alpha}=\frac{-\widehat{U}_{\gamma}\left(s\left(\alpha\right),\alpha,\alpha,s\left(\cdot\right)\right)}{\widehat{U}_{s}\left(s\left(\alpha\right),\alpha,\alpha,s\left(\cdot\right)\right)}
$$

The solution s has to be increasing and $s\left(D\right)=D$. We have

$$
\widehat{U}_{s}(s,\alpha,\gamma,s\left(\cdot\right))=\frac{d\pi_{1}\left(s\right)}{ds}\left[y-k\left(\alpha-s\right)^{2}+\delta y\pi_{2}\left(\gamma\right)\right]+2k\pi_{1}\left(s\right)\left(\alpha-s\right)
$$

and

$$\widehat{U}_\gamma(s,\alpha,\gamma,s(\cdot)) = \delta y\pi_1(s)\frac{d\pi_2(\gamma)}{d\alpha} < 0$$

Observe that

$$\widehat{U}_s(s(D),s(D),s(D),s(\cdot)) = \frac{d\pi_1(s(D))}{ds}[y + \delta y\pi_2(D)] < 0$$

Let s be any solution. The function s is increasing and $s(D) = D$, but

$$\frac{ds}{d\gamma}(D) = -\frac{\widehat{U}_\gamma(s(D),s(D),s(D),s(\cdot))}{\widehat{U}_s(s(D),s(D),s(D),s(\cdot))} < 0$$

which yields a contradiction.

The possibility of an expressive campaign

Proof of Proposition 5:

Consider the following set

$$\Im(\alpha^*,z^*) = \{z \mid z : [\alpha^*,D] \to \mathbf{R},\ z \text{ continuous and increasing},\ z(t) \leq t,\ z(D) = z^*\}$$

with the supnorm it is a complete metric space.

Consider the following functional:

For every $z \in \Im$ set

$$H(z)(t) = D + \int_t^D \frac{U_\gamma^z(z(\alpha),\alpha,\alpha)}{U_s^z(z(\alpha),\alpha,\alpha)}d\alpha D$$

Observe that

1. If $z^* < D$ and if k is big enough $\widehat{U}_s(z(D),z(D),z(D),z(\cdot)) \geq 0$

2. π_1^s, π_2 and their derivatives are bounded below and above by positive constant, because g and f are C^1

3. If α^* is big enough H is well defined and maps $\Im$ in itself (for α^* big enough $U_s^s(z, \alpha, \alpha)$ is bounded away from zero).

4. If α^* is big enough $\frac{U_\gamma^z(z,\alpha,\alpha)}{U_s^z(z,\alpha,\alpha)}$ is Lipschitz in z because g and f are C^1.

Then for α^* big enough H is a contraction so it has a unique fixed point, z. Working like in the proof of Lemma 6 it can be proved that the solution can be extended to $[0, D)$. The proof of the existence of $\alpha^*(k)$ and its asymptotic properties is the same as in Theorem 4.

Figures

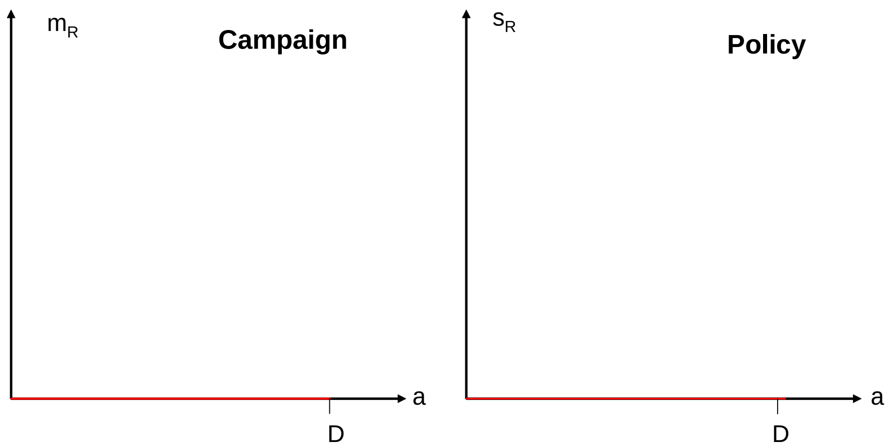

Figure 1:

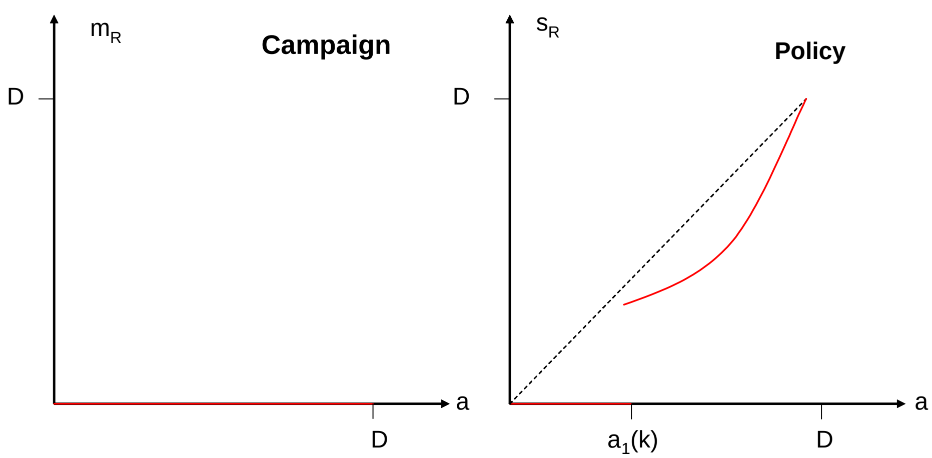

Figure 2:

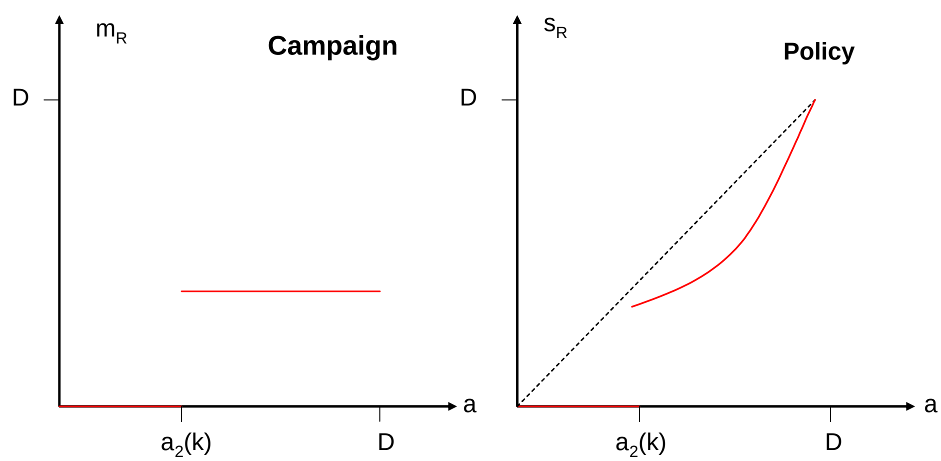

Figure 3:

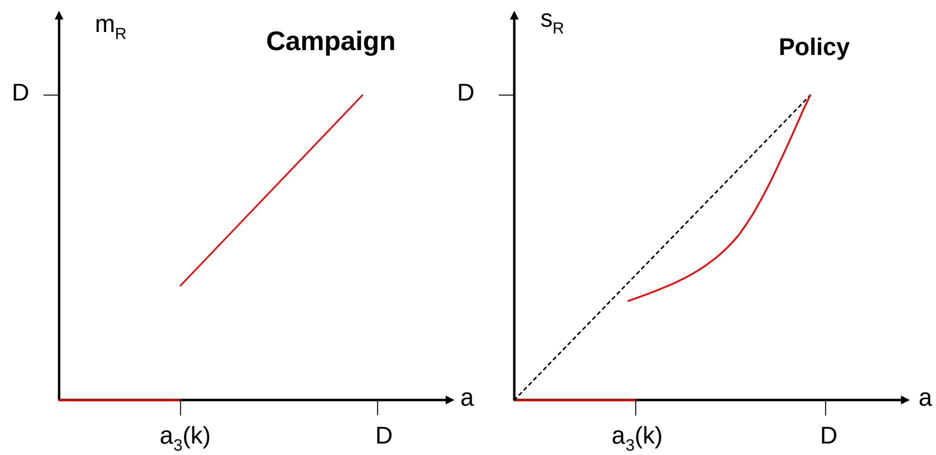

Figure 4:

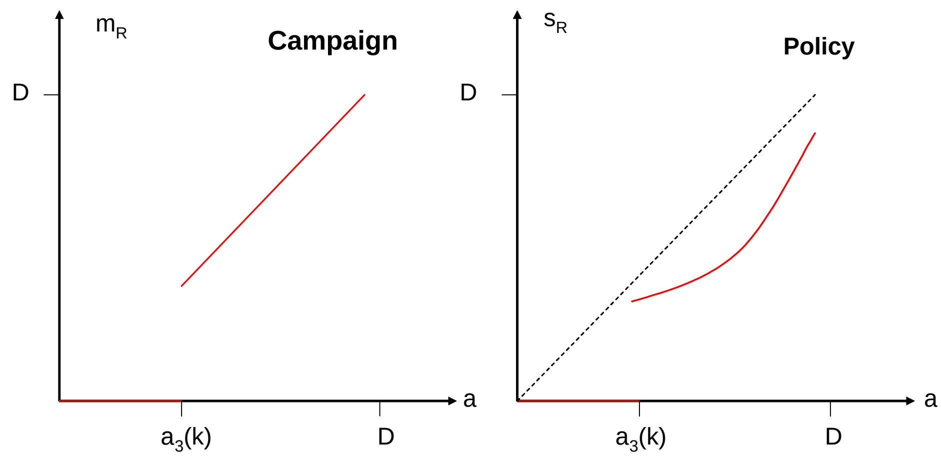

Figure 5: