Optimal Bailouts, Bank's Incentive and Risk*

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Abstract

We show how the impact of government's bailout in the form of liquidity assistance on a representative bank's ex ante effort depends on the volatility of its investment. Bank's investment delivers a cash flow that follows a geometric Brownian motion and the government guarantees bank's liabilities. To counter the bank's expectations of bailout, the government may choose a tighter liquidity policy when bank's effort is not observable than under full information. This tighter liquidity induces a more prudent ex ante behavior of the bank, but it may have the opposite effect when the investment volatility is high. This novel effect arises because the bank could be discouraged to be prudent precisely because the chances of receiving liquidity assistance are low.

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Kewords: liquidity assistance, bank closure, real option.

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1 Introduction

This paper revisits the relationship between government's bailouts and bank's risk taking, and shows how this relationship depends on the volatility of the bank's investment. The massive bailouts during the 2008-2009 financial crisis have reignited the debate about the danger of excessive bank's risk taking when governments rescue distressed banks. The bulk of the literature argues that the expectations of government's bailouts weaken market discipline and leads to excessive bank's risk taking culminating in banking crises. Demirgüç-Kunt and Detragiache (2002), Demirgüç-Kunt and Kane (2002) and Barth et al. (2004) provide evidence in favor of this view. Two papers find evidence that government's guarantees increase bank's risk taking in Germany: Dam and Koetter (2012) identify the risk taking effect of bailout expectations by exploiting regional political factors; Gropp et al. (2014) show that the banks that lost government's guarantees lowered credit risk by cutting off riskiest borrowers. Similarly, Brandao Marques et al. (2013) in an international sample of rated banks find that government's support is associated with more bank's risk taking, especially prior and during the recent financial crisis. On the theoretical side Farhi and Tirole (2012) argue that banks may find it optimal to take correlated risks if they believe that bailouts are more likely when many of them could fail simultaneously. Cordella, Dell'Ariccia and Marquez (2017) show that when bank capital is endogenous, public guarantees lead unequivocally to an increase in bank leverage and an associated increase in risk taking.

To assess the impact of the expectations of bailouts in the form of liquidity injections on bank's risk taking we embed the action of a bank in a dynamic model. We focus on just one bank to abstract from any consideration of contagion and systemic risk and to consider the strategic interaction between the bank and the policy maker along a single dimension. The bank has the possibility to make an investment (the "project" hereafter) of given size, financed with its own capital and government-insured deposits. Before deciding whether to invest the bank puts unobservable effort to screen projects to increase the probability that the investment is productive (See e.g. Holmström and Tirole 1997). The government needs the bank to screen projects and to provide payment services in the form of deposits. For the latter objective the government guarantees deposits. If the investment is productive its output follows a geometric Brownian motion. If the output exceeds the coupon to pay to the depositors, the shareholders of the bank keep the difference and consume it. If the output is insufficient to pay the coupon, that is if the bank is insolvent, either the government closes it or it injects liquidity into the bank to pay the difference and keep it alive. When it closes it, the government fully reimburses the depositors and sells the bank's assets.

The impact of the government's liquidity policy on bank's effort depends on the volatility of the project: a more generous bailout policy lowers bank's effort (induces the bank to be less prudent) when the project's output volatility is low, and increases effort when volatility is high (Proposition 1). The reason is that the bank anticipates that high volatility increases the probability of a bailout, and to take advantage of this, it puts more effort to increase the chances that the project is productive. The novel effect of a non-monotonic relationship between liquidity policy and bank's effort is at the heart of our model. It arises because we have cast the bank's effort choice in a dynamic model where the government has the option to decide when to stop injecting liquidity and close an insolvent bank.

The government's commitment to guarantee deposits can be interpreted as a call-like option of the bank to obtain liquidity if needed. We measure the value of this option using a real option framework. The value of the option feedbacks into the bank's risk taking. As effort is costly the bank puts more effort when the value of the option to obtain liquidity is higher. In fact, the longer the government allows the bank to stay in business (i.e. the looser is the liquidity policy), the higher the bank profits. When output volatility is low, the probability of a bailout is low and thus the advantage that the bank obtains from a bailout is low; hence the incentive to put effort to enjoy future profits is smaller.

Liquidity policy itself is determined by the government, trading off two frictions that work in opposite directions. First, bank closure entails a deadweight loss (e.g. fire sale of assets, negative externality). Second, the cost of liquidity injection increases with the amount of liquidity already injected. We compute how the government sets the optimal liquidity policy under full information (FI) (i.e. when the government chooses bank's effort) and when bank's effort is not observable. We show that even under FI and commitment to liquidity policy, it is optimal for the government to delay bank closure when the output falls short of the coupon; that is, it is optimal to bailout an insolvent bank for a while (Proposition 2).

Importantly, since both the bank's effort and the optimal liquidity policy depend on the level of the project volatility we can conduct some comparative static analysis. In particular, under FI as project volatility increases bank's effort increases and the optimal liquidity policy becomes tighter (Proposition 3). When bank's effort is not observable to counter the bank's expectations of bailout the government chooses a tighter liquidity policy than under FI (Proposition 4), a result that holds irrespective of the project volatility if the government savings from early closure are particularly large.

Our main result (Proposition 5) shows how project volatility affects the optimal liquidity policy and how this feedbacks into the bank's effort choice, and thus on the probability that the project is successful. When project volatility is low and the government savings from early closure are large, the optimal liquidity policy is less generous when the bank behavior is not observable than under FI. This induces the bank to exert a level of effort higher than socially desirable. However, when project volatility is high the opposite may happen if the optimal liquidity policy is very tight. This novel effect arises because the bank could be discouraged to exert a high level of effort as the chances of receiving liquidity assistance are very low. Therefore, the relationship between expectations of bailouts and bank's effort depends crucially on the volatility of the project, which, in turn, affects the value of the call-like option of the bank to obtain liquidity.

We stress that Proposition 5 makes the broader point that the impact of a public safety net policy depends on the exogenous conditions of the economy. During normal periods (when the variance of the returns is likely to be low) a restrictive policy has the intended effect of inducing more bank effort. Vice-versa, during systemic downturns (when the variance of the returns is likely to be high) the impact can be reversed. This holds true for a liquidity support policy as in our model, but also for a deposit insurance policy. Anginer et al. (2014), for a sample of 4109 banks in 96 countries over the period 2004-2009, show that deposits insurance has two effects: a moral hazard effect which prevails in good times, and a stabilization effect which prevails in crisis times. The latter effect shows up as deposit insurance increases depositors' confidence and lowers the probability of bank runs, so that bank's risk is lower in countries where the public safety net is more generous.

1.1 Related literature

Our paper is linked to several strands of literature. First of all our paper is linked to the studies that challenge the conclusion that expectations of more generous bailouts induce banks to be less prudent and suggest a complex relationship between prudential policy, the institutional framework governing bank resolution and bailouts, and bank's risk taking.

In particular our paper is closely related to Cordella and Levy-Yeyati (2003) and Dell'Ariccia and Ratnowski (2014). Cordella and Levy-Yeyati study a recursive model where bank managers can affect the risk-return profile of their portfolio to exploit the limited liability protection. They show that besides creating moral hazard, a bailout policy that reimburses the depositors contingent on the exogenous (unfavorable) state of nature can have a positive effect on the bank's charter value hence providing the bank with incentive to be more prudent. Dell'Ariccia and Ratnowski in a static model show that the expectation of government's support, while creating moral hazard, also entails a virtuous systemic insurance effect. This effect arises because bailouts isolate a bank from the endogenous risk of contagion from the failure of another bank. Absent bailouts banks take on too much risk because the contagion externality that their failure generates is not priced correctly at the margin. A bailout that prevents contagion can correct this externality and increase the bank's return from monitoring loans.

Although our paper shares with these two studies the idea that a more generous liquidity policy may encourage the bank to be more prudent our results and their sources differ in a number of dimensions. First, Cordella and Levy-Yeyati consider the simplified problem of the choice of the optimal risk exploiting the risk-return trade-off of a portfolio financed with debt. Therefore their model cannot capture the idea that, unlike in a portfolio of securities, to lower the risk of a loans portfolio the bank must exert effort to screen and or monitor loans. Second, in our model unlike Cordella and Levy-Yeyati the potential beneficial effect of bailouts does not depend on contingent market conditions, rather it is a function of the standard deviation of the output, an inherent feature of the project, known at the beginning. Third, we identify the source of the "charter value effect" in the call-like option of the bank to obtain liquidity assistance and we measure this value using a real option framework. Fourth, by casting the problem of liquidity injection as a real option we can determine the optimal stopping time, that is the optimal moment for the government to stop injecting liquidity as a function of the liquidity already injected. Fifth, our framework allows us to determine how output volatility, which is a crucial determinant of the bank's option to obtain liquidity, feedbacks into the government's liquidity decision and into the bank's risk taking. As bank's effort to prevent risk is costly the bank puts more effort when the option value is higher. Finally, unlike Dell'Ariccia and Ratnowski, in our model we do not need contagion risk to motivate liquidity support form the government.

In a model where intertemporal consumption risk and asset risk generate both panic runs and fundamental runs, Allen et al. (2014) study how government's guarantees affect a bank's risk choice, measured by the amount of liquidity held by the bank. They show that broader government's guarantees can be preferred if they lower the probability of both panic and fundamental crises, and that the guarantees do not always induce banks to take excessive risk.

The view of the policy makers is well represented by Geithner (2014) who warns against the "fundamentalism of moral hazard", namely the exaggerated concern that crisis response policies should always aim at avoiding moral hazard.

Another strand of literature argues that the government may not be able to commit to a bailout strategy. Mailath and Mester (1994) investigate whether the threat to close a bank that has chosen risky assets is ex post credible. Absence commitment, for certain parameter values, this threat is not credible, for once the bank has selected risky assets it will not be in the best interest of the government to close it. Indeed under some parameter restrictions a bank that has chosen a risky asset in the first period is less likely to choose a risky asset in the second period. That is, a government may forebear by not closing an insolvent bank. Acharya and Yorulmazer (2007, 2008) argue that ex ante regulators would like to be tough to prevent excessive risk taking. However, during systemic crises the costs associated with not providing assistance can be so high that regulators may feel compelled to provide assistance. Bailing out banks at taxpayer's expenses as opposed to early liquidation of collateral that may lose value over time, may also be socially optimal when the probability that the collateral loses value is low (Kocherlakota and Shim 2007). Shim (2011) shows that a combination of a risk-based deposit insurance premium and a book-value capital regulation with stochastic liquidation can implement a regulation akin to the Prompt Corrective Actions in the USA. Morrison and White (2013) show that a regulator may prefer not to close a unsound bank because of the fear of inducing contagion. The action of promptly closing a weak bank reveals that the regulator has less skill in screening banks than previously expected. This revelation reduces confidence in other banks screened by the same regulator, and, in some circumstances, triggers financial contagion and the closure of these banks, even though their intermediation remains socially valuable.

The rest of the paper is organized as follows: In Section 2 we introduce the model. In Section 3 we study how the bank's effort depends on output volatility and liquidity policy. In Section 4 we study how the government chooses effort and liquidity policy when it has FI about bank's effort. In Section 5 we study the problem that the government faces when it chooses the liquidity policy without observing bank's effort and we compare the resulting liquidity policy and bank's effort with the FI case. The proofs are in Appendix.

2 Model set up

This paper studies how the government's liquidity policy to guarantees a bank's liabilities affects the effort that the bank devotes to screen its investment. At $t = 0$ the bank, run by a shareholder-manager, has the opportunity to make an investment (the project) of given size 1. The project is financed with the exogenous wealth of the bank's shareholder k and deposits $1 - k$. Before deciding to invest the bank exerts unobservable effort q to screen projects more accurately ex ante. As in Holmström and Tirole (1997) effort increases the probability q that the project is productive. With complementary probability the project is not productive and the bank does not invest. Effort entails a disutility $\frac{a}{2}q^2$, $a > 0$, for the bank. This is a standard way to capture how bank's effort affects the quality of its investment in a static setting. See for example De Nicolò and Lucchetta (2009) and Carletti et al. (2016).

Once the investment is made, following Merton (1974) and Leland (1994) we assume that it generates a cash flow stream v_t at each t, $0 \leq t < \infty$. Under the risk-neutral probability measure, v_t evolves according to a geometric Brownian motion, i.e.:

$$dv_t/v_t = \mu dt + \sigma d\omega_t \text{ with } v_{t=0} = v_0 > 0, \tag{1}$$

where μ is the risk-neutral rate of drift, $\sigma > 0$ is the exogenous instantaneous volatility of cash flow and ω_t is a Wiener process.[1] The risk-neutral rate of

[1] The process (1) is quite standard in the banking literature, (see also Black and Cox 1976; Bhattacharya et al. 2002; Decamps et al. 2004; Sundaresan and Wang 2017; Hugonnier and Morellec 2017). More generally a bank owns a portfolio of risky assets

drift μ is less than the risk-free rate r such that there is a rate-of-return shortfall δ, i.e. $\delta = r - \mu \geq 0$.[2] Furthermore, to simplify the notation, in what follows we set the risk-neutral drift of (1) equal to zero. That is, on average, the project's rate of return equals r plus a risk premium.

The cash flow is the sole state variable and we assume that it is observable by both the government and the bank. However, there is an important contracting friction in this economy: namely we assume that cash flow cannot be stored. This implies that over time the bank cannot build reserves to offset output short fall. This assumption is not new in the banking literature. Parlour et al. (2012) assume that dividends must be consumed immediately and cannot be invested to become new capital. In a model with dynamic interactions between a banker and a regulator Shin (2011) assumes that the output is either consumed by the banker or paid to the deposit insurance fund. This assumption is also linked to the observation of Rajan and Myers

that generate cash flows. The portfolio volatility is measured by σ which is also the volatility of assets cash flow. Since in our model the investment has fixed characteristics, we do not explore how the governement intervention may affect the portfolio composition and its volatility. We share this assumption with most of the banking literature. For an analysis of the asset subsitution effect see, for example, Schneidar and Tornell (2004) and Pennachi (2006).

[2] We remind that a world where the expected growth rate is set equal to $(r - \delta)$ is referred to as a "*risk-neutral*" world (see e.g. Cox and Ross, 1976; Harrison and Kreps, 1979; McDonald and Siegel, 1986). The method of risk-neutral valuation suggests that any contingent claim on an asset, whether traded or not, can be evaluated in a world with systematic risk by replacing the actual growth rate of the cash flows with a certainty-equivalent growth rate (by subtracting a risk premium that would be appropriate in market equilibrium), and then behaving as if the world were risk-neutral (i.e. disconting the expected cash flows at the riskless rate.)

(1998) that liquid reserves could be easily wasted or subject to absconding. For the same reason we assume that the bank keeps no liquidity at $t = 0$, that is all the funds raised are invested.[3]

Depositors are promised a (per unit of deposit) return, a coupon, $r(1-k)$ per unit of time. The deposit market is perfectly competitive so that the bank will set $r(1-k)$ at the level the depositors require to recover their opportunity cost of funds and to be willing to participate. Since cash flow cannot be stored, when $v_t - r(1-k) > 0$, for $t > 0$, the bank consumes it immediately. Furthermore, for the rest of the paper, we assume that at $t = 0$ the project is viable, i.e. $\frac{v_0}{r} \geq 1$. This means that the bank funds an infinitely living project that is capable of paying back deposits and capital; i.e. $\frac{v_0 - r(1-k)}{r} - k > 0$. Depositors, the bank, and the government, are risk neutral.

2.1 Liquidity and probability of bank closure

We assume that the government acts on behalf of society, fully guarantees both the stock of deposits and their coupons, and provides liquidity to the bank until it closes it. Thus the government performs the functions of different agencies like the deposit insurance fund and the central bank. We assume away agency problems between any of these institutions and society, to focus only on the agency problem between the government and the bank.

When the output falls below the coupon $r(1-k)$, the bank is insolvent and the only way to pay the coupon is for the government to inject funds,

[3] In the Appendix we briefly discuss how relaxing this assumption would affect our main results.

that we denote "liquidity" for simplicity, to cover the shortfall. Examples of these government's interventions include collateralized lending by the central bank, revolving credit lines, publicly-funded recapitalizations, government's guarantees for new debt. As long as it injects funds the government bears the per period bank losses, $v_t - r(1-k) < 0$. As we will see later this entails a distortion.

Since deposits are fully insured, the stochastic process that we consider enables us to treat the liquidity injections as a buffered stochastic flow. More specifically, the process v_t has a lower barrier and the government wants to prevent the stochastic variable from falling below that barrier. In our model the lower barrier is the coupon $r(1-k)$. Accordingly, the government intervenes by means of instantaneous, infinitesimal funds injections never allowing v_t to go below the threshold $r(1-k)$. The process v_t is free to move as dictated by (1) as long as $v_t > r(1-k)$, but the instant v_t crosses $r(1-k)$ from above, it is reflected at $r(1-k)$.[4] Letting $\tilde{v}_t$ be a version of the process v_t reflected at $r(1-k)$ it can be defined as $\tilde{v}_t = v_t + L$. The process L_t represents the cumulative amount of funds injected up to t, has initial value $L_0 = 0$, and increases only when $v_t = r(1-k)$.

If the project is productive the bank receives a cash flow net of a coupon $\tilde{v}_t - r(1-k)$ until the government closes it. At closing time τ, the government reimburses to the bank the current value of deposits $1-k$, which in present value terms is $e^{-r\tau}(1-k)$. The present expected value of the cash flow net

[4] A reflected process has the same dynamics as the original process but is required to stay above a given barrier whenever the original process tends to fall below it. See Harrison (2013) for a formal definition of these processes.

of coupon accruing to the bank is:

$$\mathbb{E}[\int_0^{\tau} e^{-rt}(\tilde{v}_t - r(1-k))dt + e^{-r\tau}(1-k)], \tag{2}$$

where $\mathbb{E}[\cdot]$ denotes the expectation with respect to (1) conditional on the information at $t = 0$.

We denote with ψ the government's liquidity policy. Since the government has discretion to stop injecting liquidity, the closing time associated with the strategy $\tilde{v}_t = v_t + L_t$ is defined by:

$$\tau = \inf\left(t \geq 0 : \psi L_t = l\right). \tag{3}$$

By (3) the government closes the bank the first time that the process ψL_t hits the threshold l, where $\psi \in [0, \infty)$, the liquidity policy, is the weight that the government assigns to the cumulated liquidity injected up to date t. Now, assuming that the trigger l is a random variable described by an exponential distribution independent of (1), we are able to derive the probability of bank closure as:[5]

$$\Pr(\tau \leq t) = \Pr\left(\psi L_t > l\right) = 1 - e^{-\psi L_t}. \tag{4}$$

The process ψL_t depends on the amount of liquidity already injected L_t, and the importance (the weight), ψ, that the government assigns to L_t. In other words, the probability that the government continues to inject liquidity after time t, $e^{-\psi L_t}$, declines the more liquidity L_t it has already injected. If the government chooses $\psi = 0$, it guarantees liquidity forever and the bank will never be closed, i.e. $\Pr(\tau \leq t) = 0$. On the contrary, setting $\psi = \infty$ means that the government never injects liquidity and closes the bank the

[5] For the stopping time (3) see Harrison (2013, p. 159-160).

first time that v_t hits the boundary $r(1-k)$. This captures the notion that the government updates the probability of closing the bank as a function of both the liquidity injected and the level of output v_t. Notice that the government does not commit to a deterministic liquidity policy, rather it commits to how to respond to the evolution of the output v_t, which makes the closing time τ stochastic.[6]

When the government closes the bank, it reimburses depositors in full and cancels the shareholders's claims, two features that we encounter in most instances of bank closures. When the bank is closed the government does not terminate the project, rather it sells the bank's assets, and receives the proceeds from the assets sale.

We also assume that the government incurs a deadweight loss $Z > 0$, from closing a bank and that this cost, which is not internalized by the bank, is independent from the time of closure. This cost arises from several sources. First, a fire sale discount makes the resale value of the assets smaller than the expected present value of their output stream (Leland 1994). This is so because, for example, the incumbent bank management is more capable than anybody else to extract value from these assets (See for example Diamond and Rajan 2006), or because outsiders can observe the output only at a cost (Townsend 1979). Second, bank's closure generates a negative externality as it may induce financial instability by casting a doubt on the ability of the bank regulator to screen other banks as in Morrison and White (2013), and

[6] Dell'Ariccia and Ratnovski (2014) assume that the government can commit to a bailout strategy. In Shim (2011) stochastic liquidation after output shortfall provides the banker with incentives to continue to act in the interest of the regulator.

there may be costs to layoff the bank employees.[7]

3 Liquidity, effort, and output volatility

Recall that the bank exerts effort q with disutility $\frac{a}{2}q^2$ to increase the probability that the project is productive. Then, using (2), the objective function of the bank can be written as

$$V = \max_q q\left(\mathbb{E}[\int_0^\tau e^{-rt}(\tilde{v}_t - r(1-k))dt - \left(1 - e^{-r\tau}(1-k)\right)]\right) - \frac{a}{2}q^2, \quad (5)$$

where $1 - e^{-r\tau}(1-k)$ is the present value of the bank's net cash outflow for the investment and consists of the project cost 1, minus the present value of the deposits that the bank receives if the bank is closed $e^{-r\tau}(1-k)$.[8] By

[7] More formally, when the bank is closed, the realized assets will revert to the government. The value of these assets is $(1-\xi)\frac{v_\tau}{r}$ where $\xi\frac{v_\tau}{r}$ ($\xi \in [0,1)$) measures a fire sale cost (Leland, 1994). As at the closure time τ the cash flow is $v_\tau = r(1-k)$, and the salvage value is $(1-\xi)(1-k)$. Thus the deadweight Z loss is equal to $S - (1-\xi)(1-k) > 0$ where S is the closure cost, for example from staff layoff.

[8] Since (5) may appear counterintuitive, observe that if the government never closes the bank, that is if $\tau \to \infty$, the bank's objective function becomes

$$\lim_{\tau\to\infty} V = \lim_{t\to\infty} \max_q q\left(\mathbb{E}[\int_0^\tau e^{-rt}(\tilde{v}_t - r(1-k))dt - \left(1 - e^{-r\tau}(1-k)\right)]\right) - \frac{a}{2}q^2 =$$
$$\max_q q\left(\mathbb{E}\int_0^\infty e^{-rt}\tilde{v}_t dt + \frac{1}{r}r(1-k) - 1\right) - \frac{a}{2}q^2 =$$
$$\max_q q\left(\mathbb{E}\int_0^\infty e^{-rt}\tilde{v}_t dt - k\right) - \frac{a}{2}q^2.$$

That is, the bank spends k to obtain $\mathbb{E}\int_0^\infty e^{-rt}\tilde{v}_t dt$ which is the present expected value of the project cash flow.

using (3), we are able to write (5) as:

$$V = \max_q q \left(\mathbb{E} \int_0^\infty e^{-rt} e^{-\psi L_t} \left(\tilde{v}_t - r\left(1-k\right)\right) dt + (1-k)\mathbb{E} \int_0^\infty e^{-rt} \psi e^{-\psi L_t} dL_t - 1 \right) - \frac{a}{2} q^2. \tag{6}$$

Formally, in (6) we can regard the project a infinitely-lived one, where the payoffs at each time are multiplied by the probability that the bank is not closed up to time t, $e^{-\psi L_t}$, and

$$(1-k)\mathbb{E} \int_0^\infty e^{-rt} \psi e^{-\psi L_t} dL_t = (1-k)\mathbb{E}(e^{-r\tau}) \equiv D \tag{7}$$

is the discounted expected value of the deposits.[9] Thus, the bank chooses an effort level equal to:[10]

$$\begin{aligned} aq^B &= \mathbb{E} \int_0^\infty e^{-rt} e^{-\psi L_t} (\tilde{v}_t - r\left(1-k\right)) dt + (1-k)\mathbb{E} \int_0^\infty e^{-rt} \psi e^{-\psi L_t} dL_t - 1 \\ &= \frac{v_0 - r\left(1-k\right)}{r} + \underbrace{\mathbb{E} \int_0^\infty e^{-rt} e^{-\psi L_t} L_t dt}_{M} + \underbrace{(1-k)\mathbb{E} \int_0^\infty e^{-rt} \psi e^{-\psi L_t} dL_t)}_{D} - 1, \end{aligned} \tag{8}$$

where the term M is the expected present value of total liquidity injections. Since both M and D are positive, they increase bank's effort. Substituting (8) into (6) it is easy to see that

$$V = \frac{a}{2}(q^B)^2, \tag{9}$$

[9]Notice that the effect of (3) is similar to calculating the value of an investment opportunity with an uncertain expiration date. If the expiration date is described by a Poisson process with parameter ψ, Merton (1973) shows that the investment opportunity is equal to a perpetual one with the discount rate subsituted by $r + \psi$.

[10]When it is not necessary, for the rest of the paper we drop the dependence of q^B from the initial condition v_0.

so that maximizing V is equivalent to maximize (8).

To compute the discounted expectation in the second and third term on the R.H.S. of (8) we use the dynamic programming decomposition. We may split the conditional expectation in (8) into the contribution over the infinitesimal time interval 0 to dt and the integral from dt to ∞ with a particular condition at the (reflecting) barrier $r(1-k)$. The solution of (8) is given by the following Lemma.

Lemma 1: *The level of effort* (8) *is equal to:*

$$aq^B = \frac{v_0 - r(1-k)}{r} - \left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{(1+\psi r(1-k))}{\beta - \psi r(1-k)} - 1, \qquad (10)$$

where $\beta = \frac{1}{2} - \sqrt{(\frac{1}{2})^2 + \frac{2r}{\sigma^2}} < 0$, *and* $\frac{\partial \beta}{\partial \sigma} > 0$.

Proof: See Appendix.

Lemma 1 addresses the incentive compatibility problem of the bank, that is how the bank chooses effort as a function of government intervention (liquidity policy and deposit guarantee). By direct inspection of (8) and (10), $M + D$ is equal to:

$$M + D = -\left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{(1+\psi r(1-k))}{\underbrace{\beta - \psi r(1-k)}_{<0}} > 0, \qquad (11)$$

which depends on the output volatility σ, and the government's liquidity policy through the stopping rate ψ. In particular, since β, which recall is negative, is monotonically increasing in σ, the level of output volatility, may be equivalently characterized in terms of $\beta = \beta(\sigma)$. For the rest of the paper we say that a project is "high volatility" when $\mid \beta \mid$ is very low, i.e. close to

0, and "low volatility" when $\mid \beta \mid$ is very high. In both cases, however, unless otherwise specified, we exclude the extreme values $\beta = -\infty$, where the cash flow becomes constant over time, and $\beta = 0$, where the cash flow volatility is infinite.[11]

Making comparative statics analysis on σ we try to capture different macroeconomic conditions that could arise from banks operating in different countries, regions, industries, or phases of the business cycle.

By the real option theory (McDonald and Siegel 1984 and Dixit and Pindyck 1994), equation (11) can be seen as a sum of infinite set of call-like options. At each time t the bank has the option to use a unit of liquidity for free to prevent v_t to fall below $r(1-k)$. Then, the present value of the total liquidity (11) can be calculated by valuing each of these options and summing these values up by integrating over time t. In particular, as $\left(\frac{v_0}{r(1-k)}\right)^{\beta} = \mathbb{E}(e^{-rT}) < 1$ where T is the first time starting from v_0 that the process (1) hits the liquidity threshold $r(1-k)$, the term

$$-(1-k)\frac{(1+\psi r(1-k))}{\beta - \psi r(1-k)},$$

in (11) indicates the payoff the bank expects to receive from exercising these options. The expected present value $\mathbb{E}(e^{-rT})$ can be determined by using dynamic programming (see for example Dixit and Pindyck 1994, pp. 315-316).

[11] If $\beta = -\infty$ the model collapses to a static one without uncertainty as in Dell'Ariccia and Ratnovski (2014) and Carletti et al. (2016).

We are now able to write (10) as:

$$aq^B = \frac{v_0}{r} - 1 + (1-k)\left[-\mathbb{E}(e^{-rT})\underbrace{\frac{(1+\psi r(1-k))}{\beta - \psi r(1-k)}}_{<0} - 1\right]. \qquad (12)$$

Observe that as the value of these options increases with σ, the last term of (12) is higher when the project is high volatility. That is, for a high volatility project, i.e. $\mid \beta \mid$ close to 0, both the payoff of the options and the probability that they will be exercised, increase. On the contrary the value of these options tend to $-(1-k)$ for low volatility projects, i.e. $\mid \beta \mid$ is very high. Thus the last term on the R.H.S. of (12) shows the contribution of these options to increase the bank's effort above the value of insured deposits $(1-k)$. Finally, as $\frac{v_0}{r} - 1 \geq 0$, the constant a in (10) serves as a normalization.[12]

Taking the derivative of (10) with respect to ψ, we are able to investigate the effect of the government's liquidity policy on the bank's effort. In particular we prove the following proposition:

Proposition 1. *The sign of the effect on q^B of the government's liquidity*

[12] Note, however, that if the initial valuation of the project is high, it could be always worth for the bank to exert the maximum level of effort, i.e. $q^B \to 1$. On the contrary if the initial valuation of the project v_0 is close to the boundary $r(1-k)$, we obtain:

$$\lim_{v_0 \to r(1-k)} aq^B = -(1-k)\left[\frac{\beta+1}{\beta - \psi r(1-k)}\right],$$

and q^B is greater than zero only if $\beta + 1 > 0$. That is, if the initial condition on the cash flow is such that the project starts with low cash flow if the investment is made, the bank has an incentive to put effort only if the project volatility is "sufficiently" high to guarantee that the expected payoff from exercising the call-like options is positive.

policy, is given by:

$$sign\frac{\partial q^B}{\partial \psi} = sign[-r(1-k)(\beta+1)] \quad \text{for } \beta \in (-\infty, 0) \ . \tag{13}$$

Proof: See Appendix.

Proposition 1 is crucial to understanding why liquidity injections may or may not induce a bank to increase its screening effort depending on output volatility. To understand the $sign[-r(1-k)(\beta+1)]$, recall that $\beta < 0$ and $\frac{\partial \beta}{\partial \sigma} > 0$. Thus, if the project is high volatility, i.e. $\mid \beta \mid$ is close to zero such that $\beta + 1 > 0$, by (13) we have $\frac{\partial q^B}{\partial \psi} < 0$, that is the bank will increase effort if the government injects more liquidity. Conversely, if the project is low volatility, i.e. $\mid \beta \mid$ is very high, $\frac{\partial q^B}{\partial \psi} > 0$, that is the bank will reduce effort if the government injects more liquidity.

Intuitively, the reason a looser liquidity policy increases the incentive of the bank to put effort when the project is high volatility, is that the bank anticipates that liquidity support is more likely in that case, and that to take advantage of that it has to put more effort to increase the chances that the investment is made. This effect arises because we consider a dynamic model where the longer the government allows the bank to stay in business, the higher is the expected value of the call-like options to obtain liquidity.

When instead the project is low volatility, the probability of liquidity support is low and thus the advantage that the bank obtains from a bailout is low. Indeed, in this case, the looser is the liquidity policy and the lower the incentive to put effort to enjoy future profits. Both the received view that argues that expectations of generous bailouts induce banks to be less prudent, and the studies that challenge this conclusion, capture only a piece of the story, because they do not take into account how liquidity policy itself is set.[13] We now turn to determine the optimal liquidity policy as a function of output volatility. We do so first when the government has full information on bank's effort, and then when the government cannot observe bank's effort.

4 Full information regulation

4.1 The costs of intervention

When the government has full information on the effort of the bank, perhaps because of on-site inspections, it chooses both the level of effort and the liquidity policy to maximize its objective function which includes both the objective function of the bank, V, and the expected costs of the intervention.

Besides the reimbursement of deposits, which has no impact on welfare, the costs of intervention arise from two frictions with welfare implications: the deadweight cost of bank closure Z discussed above, and the increased risk of the government's portfolio from liquidity injections. As for the latter, we assume that the cost that the government faces to inject liquidity grows with the amount of liquidity already injected. This reflects the deterioration of the quality of the portfolio of the authority providing liquidity to the bank in distress. A bank in distress may lose market access and increase the use of central bank credit. Since the eligible collateral that central banks accept

[13] The effectiveness of a more generous liquidity policy in inducing effort if the project is financed with more equity depends on the output volatility. In particular if output volatility is high, as k increases, the marginal value of liquidity to induce the bank to provide effort, declines. The opposite is true when output volatility is low. See Appendix.

from banks in distress tends to be both of lower quality and more illiquid (Acharya et al. 2009, Nyborg 2015), as central bank lending becomes more concentrated on weaker counterparts, the average quality of the risks in the central bank's portfolio worsens (Bindseil and Jabłecki 2013). More generally, Hall and Reis (2015) argue that the unconventional monetary policies followed by the FED and the ECB after the crisis have exposed them to increased interest rate and default risks.

Recall that the government guarantees both the deposits and the coupons, and that the process v_t is reflected at the lower barrier $r(1-k)$ by liquidity injection. This reflection is costly: in particular, to model the cost of liquidity injection we assume that the government bears a cost of $c(\psi)$ units for each unit of liquidity injected. That is to say, liquidity supply has a time-invariant marginal cost $c(\psi)$ per unit of account. Formally this is equivalent to set:

$$dC_t = c(\psi) \times dL_t, \tag{14}$$

where dL_t is the increment of liquidity, if any, in the interval $(t, t+dt)$. (We express payments in present value terms, i.e. the liquidity cost has the dimension of the present value of one unit of account injected in the bank forever). We assume that the cost of liquidity is a decreasing function of ψ with the properties $c'(\psi) < 0$, and $c''(\psi) < 0$. That is, the cost for the government declines, although at a declining rate, the sooner it closes a bank in distress. Furthermore we assume that $c(\psi) \geq 1/r$ and that there is no cost over the market rate $1/r$ only if the government closes the bank the first time that the output falls short of the coupon; i.e. $c(\infty) = 1/r$.

4.2 How the government chooses effort

As in the previous section, indicating with $\tilde{v}_t$ a version of the process v_t reflected at $r(1-k)$, the government's objective function is:

$$W = \underbrace{\max_q q\left[\mathbb{E}[\int_0^\tau e^{-rt}(\tilde{v}_t - r(1-k))dt - \left(1 - e^{-r\tau}(1-k)\right)]\right] - \frac{a}{2}q^2}_{V}$$
$$-\left\{q\left[\mathbb{E}[\int_0^\tau e^{-rt}dC_t + e^{-r\tau}((1-k)+Z)]\right]\right\}$$
$$= \max_q V - q\left[\mathbb{E}[\int_0^\tau e^{-rt}dC_t + e^{-r\tau}((1-k)+Z)]\right]. \quad (15)$$

Using the stopping time (3) and going through the same steps as before, equation (15) can be reduced to:

$$W = \max_q q\left[\mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[(\tilde{v}_t - r(1-k))dt - dC_t] - Z\mathbb{E}\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t - 1\right] - \frac{a}{2}q^2, \quad (16)$$

where $Z\mathbb{E}\left[\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t\right] = Z\mathbb{E}(e^{-r\tau})$ is the discounted expected value of the cost to close the bank. Thus, the government chooses effort equal to:

$$aq^W = \mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[(\tilde{v}_t - r(1-k))dt - dC_t] - Z\mathbb{E}\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t - 1$$
$$= \frac{v_0 - r(1-k)}{r} + \underbrace{\mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[L_t dt - (c(\psi) + \psi Z)dL_t]}_{H} - 1. \quad (17)$$

The second term on the R.H.S. of (17), denoted by H is the difference between the present value of the liquidity injections, $M = E\int_0^\infty e^{-rt}e^{-\psi L_t}L_t dt$, and the additional costs to guarantee liquidity plus the expected discounted value of the deadweight cost of bank closure, $P \equiv E\int_0^\infty e^{-rt}e^{-\psi L_t}(c(\psi) + \psi Z)dL_t$. Therefore, $H = M - P$, and can be interpreted as the loss of project value due to the intervention.

Substituting (17) into (16) it is easy to see that:

$$W = \frac{a}{2}(q^W)^2, \tag{18}$$

so that also for the government, maximizing W is equivalent to maximize (17). To compute the discounted expectation in (17) we repeat the arbitrage calculation of Section 3. The solution of (17) is in the following Lemma.

Lemma 2: *Effort under FI is given by:*

$$aq^W = \frac{v_0 - r(1-k)}{r} + \overbrace{\left(\frac{v_0}{r(1-k)}\right)^\beta (1-k) \frac{\overbrace{c(\psi)r - 1 + \psi r Z}^{\geq 0}}{\underbrace{\beta - \psi r(1-k)}_{<0}}}^{H<0} - 1. \tag{19}$$

Proof: See Appendix.

By direct inspection of (17) and (19), H is negative, which induces the government to lower effort in (19).[14]

[14] Also for the government, similarly to what happens for the bank, when the initial valuation of the project is high it is worth demanding maximum effort, i.e. $q^W = 1$. On the contrary, if v_0 is close to $r(1-k)$, we obtain:

$$\lim_{v_0 \to r(1-k)} aq^W = \frac{(1-k)[c(\psi)r - 1 + \psi r Z]}{\beta - \psi r(1-k)} - 1 < 0.$$

That is, if the initial condition on the cash flow is such that if the project is productive, it will start with low cash flow, it will be never optimal for the government that the bank puts effort i.e. $q^W = 0$. In this case, even if the expected cash flows are sufficient to cover the coupons, the government prefers that the bank puts no effort, that is that it does not invest.

4.3 Optimal liquidity policy

We are now able to derive the optimal liquidity policy under FI. Taking the derivative of (19) with respect to ψ, and collecting the results we obtain:

Proposition 2: *Under FI, there exists an optimal liquidity policy* ψ*, such that* $0 < \psi < \infty$*, given by:*

$$c'(\psi^W)(\beta - \psi^W r(1-k)) + (c(\psi^W)r - 1)(1-k) = -Z\beta. \qquad (20)$$

Proof: See Appendix.

Several comments are in order. First, the result that the optimal ψ^W is finite, i.e. it is always optimal to inject some liquidity, means that the government does not close a bank immediately after an output shortfall, that is forbearance is optimal under FI. This shows that lack of commitment is not necessary to establish that the government does not close a bank as soon as it becomes insolvent.

Second, the L.H.S. of (20) is the net marginal benefit of tightening the liquidity policy, which is strictly positive, and the R.H.S. is the closure cost.[15] In Appendix we see that q^W is maximized if the loss of value due to government's intervention H is minimized. Then an increase of ψ entails a marginal reduction of H, due to saving the costs of liquidity, $c'(\psi^W)(\beta - \psi^W r(1-k)) + (c(\psi^W)r - 1)(1-k)$, against a marginal increase of H to the closure cost $-Z\beta$.

Third, to better grasp the role of the assumption on $c(\psi)$, it is useful to see what the optimal liquidity policy would be if the liquidity supply had instead a constant marginal cost equal to the market cost, $1/r$. By direct

[15] Taking the derivative of the L.H.S. of (20) , we obtain: $c''(\beta - \psi^W r(1-k)) > 0$.

inspection of (19) and recalling that $\beta < 0$, one can see that the sign of $\frac{\partial q^W}{\partial \psi}$ is always negative. Thus, the government would maximize the welfare function (16) by providing liquidity forever, $\psi^W = 0$. The intuition is evident from (19). Under FI, the government faces a trade-off between the cost of intervention and the level of bank's effort to increase the probability that the project is productive. However, if the government's cost of liquidity is equal to the market cost, for any given level of volatility, the government is able to reduce the cost of the intervention by supporting the project forever.

As for the effect of output volatility on bank's effort and the optimal liquidity policy we are able to prove that:

Proposition 3: *Under FI, both the effort q^W and the the stopping rate ψ^W decrease as output volatility increases:*

$$\frac{\partial q^W}{\partial \sigma} < 0 \quad \text{and} \quad \frac{\partial \psi^W}{\partial \sigma} < 0. \tag{21}$$

Proof: See Appendix.

Proposition 3 says that as output volatility increases, the government reduces the probability that the project is productive, $\frac{\partial q^W}{\partial \sigma} < 0$, but it increases the liquidity injected to keep the bank open once the investment is made, $\frac{\partial \psi^W}{\partial \sigma} < 0$. The intuition for $\frac{\partial q^W}{\partial \sigma} < 0$ is straightforward: a higher level of output volatility increases the probability that the government has to inject liquidity into the bank which, ceteris paribus, increases the cost of providing liquidity and makes q^W (and thus W) lower.

The explanation of $\frac{\partial \psi^W}{\partial \sigma} < 0$ is less intuitive. If we interpret $\frac{\partial q^W}{\partial \psi}$ as the marginal productivity of a restrictive liquidity policy, that is how a unit reduction of liquidity is able to affect the probability that the project is productive, then an increase in output volatility lowers the marginal productivity of a restrictive liquidity policy (i.e. $\frac{\partial^2 q^W}{\partial\psi\partial\sigma}(\psi(\sigma),\sigma) < 0$; see Appendix). Therefore, as output volatility increases, if the government wants that the productivity of liquidity increases, it has to increase the liquidity, i.e. $\frac{\partial\psi^W}{\partial\sigma} < 0$.

5 Unobservable effort

5.1 The government problem

In the previous section we have assumed that the government could perfectly observe bank's effort and thus it could dictate its desired level, q^W. Differently, in this section we take into account that the government cannot observe the effort that the bank chooses, q^B, and we explore how this affects the trade-off between bank's effort and the optimal liquidity policy. The government problem becomes:

$$W^h = \max_{\tau} q^B \left[\mathbb{E}[\int_0^{\tau} e^{-rt}\,[(\tilde{v}_t - r\,(1-k))dt - dC_t] - e^{-r\tau}Z] - 1\right] - \frac{a}{2}(q^B)^2, \tag{22}$$

subject to the incentive compatibility constraint of bank's effort

$$aq^B = \mathbb{E}[\int_0^{\tau} e^{-rt}(\tilde{v}_t - r\,(1-k))dt + e^{-r\tau}(1-k)] - 1, \tag{23}$$

and the bank's participation constraint, $V \geq 0$.

Constraint (23) indicates that the bank chooses effort to maximize its payoff taking as given the liquidity policy. Substituting (23) in (22) and rearranging, we obtain:

$$\begin{aligned} W^h &= \max_{\tau} q^B \left[\mathbb{E}[\int_0^{\tau} e^{-rt} \, [(\tilde{v}_t - r\,(1-k))dt - dC_t - e^{-r\tau} Z] - 1 \right] - \frac{a}{2}(q^B)^2 \\ &= \max_{\tau} \frac{a}{2}(q^B)^2 - q^B \mathbb{E}[\int_0^{\tau} e^{-rt} \, dC_t + e^{-r\tau}(1-k) + e^{-r\tau} Z]. \qquad (24) \end{aligned}$$

By using the stopping time (3) and going through the same steps as in the previous sections, we can write the last term in (24) as:

$$\mathbb{E}[\int_0^{\tau} e^{-rt} \, dC_t + e^{-r\tau}(1-k) + e^{-r\tau} Z] = \mathbb{E}[\int_0^{\infty} e^{-rt} \, e^{-\psi L_t} [c(\psi) + \psi Z + \psi(1-k)] dL_t, \qquad (25)$$

where the R.H.S. of (25) , is the expected costs of government's intervention $P + D$, which is given by the expected value of additional costs to guarantee liquidity, plus the expected discounted value of the deadweight cost of bank closure, and the discounted expected value of the deposits. Observe that by using (11) and (19) we obtain:

$$\begin{aligned} P + D &= (M + D) - (M - P) \\ &= -\left(\frac{v_0}{r(1-k)}\right)^{\beta} (1-k)(\frac{rc(\psi) + \psi r(1-k) + \psi r Z}{\underbrace{\beta - \psi r(1-k)}_{<0}}) > 0. \qquad (26) \end{aligned}$$

Substituting (25) in (24), we are able to write the government's objective function as:

$$W^h = \max_{\psi} \underbrace{\frac{a}{2}(q^B)^2}_{V} \quad \underbrace{-q^B(P + D)}_{\substack{\text{expected costs of} \\ \text{government intervention}}} \quad . \qquad (27)$$

The first term on the R.H.S. of (27) is the bank's ex-ante value of the project V, and the second is the expected costs of regulatory intervention, evaluated at the level of effort q^B chosen by the bank.

The government maximizes (27) by trading off the value of the project for the bank V and the expected costs of the intervention $q^B(P+D)$. Thus high bank's effort, on the one hand, increases the value of the project V, but on the other one, it increases the expected cost of liquidity, the cost of closure, and the cost of reimbursing depositors, $q^B(P+D)$. The government determines ψ in such a way that bank's effort maximizes V without leading to excessive expected costs of intervention.

Before maximizing (27), let's investigate the difference between q^B and q^W for a given liquidity policy. From Lemma 1 and Lemma 2, it is easy to show that:

$$a[q^B - q^W] = -\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\frac{\psi r(1-k)) + c(\psi)r + \psi rZ}{\beta - \psi r(1-k)} = P + D > 0. \tag{28}$$

The misalignment with respect to the FI case generates a deadweight loss for society. In particular, using (28) we are able to rewrite W^h in (27) as

$$W^h = \max_{\psi} \underbrace{\frac{a}{2}(q^W)^2}_{W} - \underbrace{\frac{a}{2}[q^B - q^W]^2}_{I}. \tag{29}$$

The first term W in (29) is the government's objective function under FI, and the second term I represents the distortion induced by the unobservable effort, that lowers the government's objective function.

Next step is to determine the government's optimal liquidity policy when effort is unobservable, that we denote ψ^h. If an optimal ψ^h exists, it should equate the marginal value of liquidity $\frac{\partial W}{\partial \psi}$ to its marginal cost $\frac{\partial I}{\partial \psi}$. Although a close form solution for ψ^h is difficult to obtain, we are able to compare ψ^h with ψ^W. In particular, since by Proposition 2 W has an interior maximum, $\frac{\partial W}{\partial \psi}_{|\psi=\psi^W} = 0$, it follows that $\psi^h > \psi^W$ if $\frac{\partial I}{\partial \psi} < 0$. From (28) and (29) we

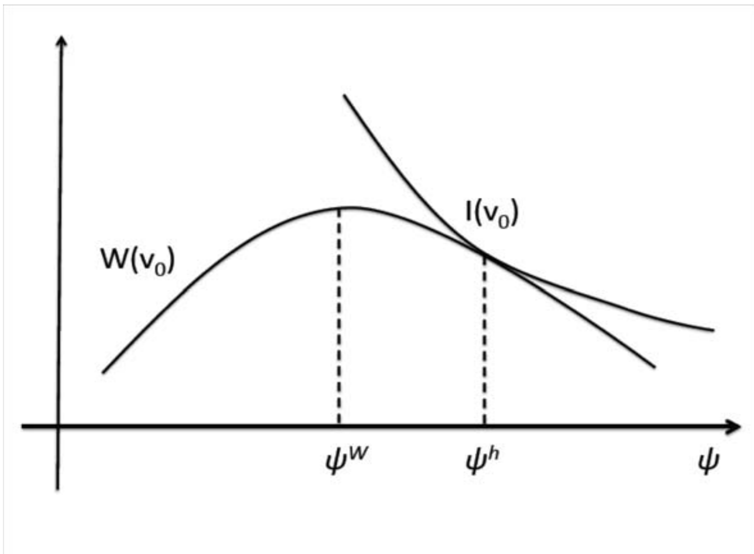

Figure 1: The government provides (weakly) less liquidity under MH than under FI; $\psi^W \leq \psi^h$.

obtain $\frac{\partial I}{\partial \psi} = a[q^B - q^W]\frac{\partial(P+D)}{\partial \psi}$ so that the sign of $\frac{\partial I}{\partial \psi}$ is dictated by the role played by the government's liquidity policy in reducing the expected costs of the intervention $P + D$. In this respect we are able to prove the following Proposition:

Proposition 4: *When bank's effort is not observable, a restrictive liquidity policy reduces (weakly) the expected costs of government's intervention, i.e.* $\frac{\partial(P+D)}{\partial \psi} \leq 0$, *when the government's savings from early closure are particularly large, i.e.*

$$-rc'(\psi) > r(1-k) + rZ, \tag{30}$$

or, when (30) *does not hold, if the volatility of the project's output is high (i.e.* $\mid \beta \mid$ *close to* 0*). In either case we obtain* $\psi^h \geq \psi^W$.

Proof: See Appendix.

Proposition 4 shows that when bank's risk taking is not observable the government wants to provide (weakly) less liquidity than under FI $\left(\psi^h \geq \psi^W\right)$ to reduce the social distortion, as illustrated in Figure 1. If condition (30) holds the result is straightforward and irrespective of project's volatility; even if condition (30) does not hold, we are able to show that we obtain $\psi^h \geq \psi^W$ for high volatility projects (See Appendix). Furthermore, the degree of liquidity restriction that the government implements to address the unobservablity of bank's effort plays a crucial role in determining the bank's ex ante behavior, as we show in the next subsection.

5.2 Liquidity support lowers bank's effort?

Finally, we are able to address the question we posed at the beginning of this paper, namely whether the expectations of government's liquidity support induce a bank to put less effort ex ante. To do so we compare the bank's effort $q^B(\psi^h)$ under the optimal liquidity policy ψ^h when bank's effort is not observable, and the bank's effort $q^W(\psi^W)$ under the optimal liquidity policy ψ^W when the bank's effort is observable. Denote by $\Delta q = q^B(\psi^h) - q^W(\psi^W)$, and $\Delta\psi = \psi^h - \psi^W$ the variable that measures the severity of the distortion in the optimal liquidity policy induced by the unobservability of effort. By taking the Taylor expansion of $q^B(\psi^h)$ around ψ^W we obtain:

$$\Delta q \simeq q^B(\psi^W) - q^W(\psi^W) + \frac{\partial q^B(\psi^W)}{\partial \psi}\Delta\psi. \qquad (31)$$

Thus, by (31) the difference Δq can be approximated by the sum of two terms. The first one, $q^B(\psi^W) - q^W(\psi^W) > 0$, is the difference $q^B - q^W > 0$ in

(28) evaluated at the optimal FI policy ψ^W from (29) , and measures the cost of intervention. The last term is related to the bank's incentive compatibility constraint as it measures how the government's liquidity policy affects the bank's effort.

If the volatility of the output is low we know from Proposition 1 that $\frac{\partial q^B(\psi^W)}{\partial \psi} > 0$ and that $\Delta\psi > 0$ if the condition (30) in Proposition 4 holds. In this case we conclude that $\Delta q > 0$.

On the contrary, if the volatility is high, from Proposition 1 we have $\frac{\partial q^B(\psi^W)}{\partial \psi} < 0$, and from Proposition 4 $\Delta\psi \geq 0$. Combining these results, if the problem posed by the unobservablity of effort is not too severe, i.e. $\Delta\psi \simeq 0$, we obtain $\Delta q > 0$. On the contrary, if that problem is severe, i.e. $\Delta\psi >> 0$, the reverse can happen.

We resort to numerical examples to better illustrate the role played by effort unobservability in determining the sign of (31) when volatility is high. We replace the expressions for $q^B(\psi^W)$, $q^W(\psi^W)$ and $\frac{\partial q^B(\psi^W)}{\partial \psi}$ into (31) . For the difference Δq to be positive it must be

$$\psi^h < f(\psi^W) \equiv \psi^W - \frac{(\beta - \psi^W r(1-k))[\psi^W r(1-k)) + c(\psi^W)r + \psi^W rZ]}{r(1-k)\,(\beta+1)}, \tag{32}$$

where, from Proposition 1, we assume that $\beta + 1 > 0$. This assumption reduces the complexity of numerical analysis without affecting the quality of the results (See Appendix).

To gain some insight about the impact of output volatility on the optimal liquidity policy, and thus on bank's effort, we calibrate the model. Let us assume that $r = 2\%$ while σ can take on three values: $25\%, 30\%, 35\%$. In this case we obtain $\beta(25\%) = -0.443\,40$, $\beta(30\%) = -0.333\,33$, and $\beta(35\%) =$

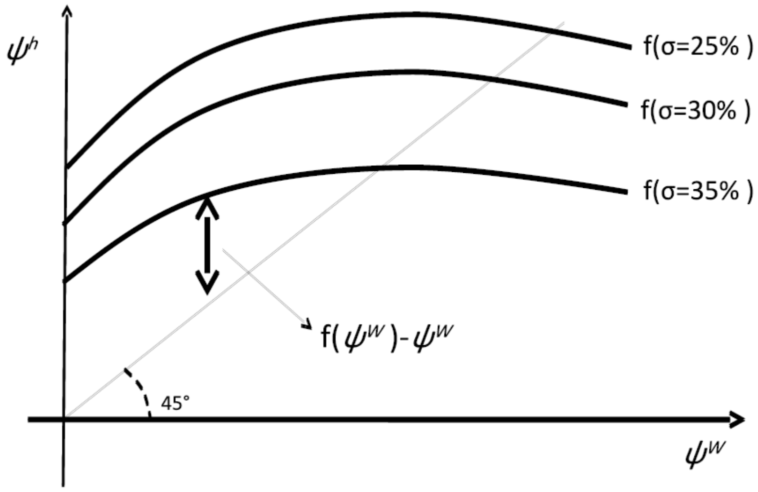

Figure 2: As $f(\psi^W) - \psi^W$ declines, the probability that a tighter liquidity policy reduces bank effort increases.

$-0.259\,30$. Furthermore, let us assume that the deadweight cost of bank closure is $Z = 0.5$, equity is $k = 0.1$ and, finally, for the cost of liquidity we adopt the following function $c(\psi) = \frac{G-\psi^2}{rG}$, with $\psi \in [0, \sqrt{G}]$ and $G = 100$.

In Figure 2 we plot the function $f(\psi^W)$, for different values of σ. From (32) we know that $\Delta q > 0$ if $\Delta\psi < f(\psi^W) - \psi^W$ where, recall, $\Delta\psi$ measures the severity of the problem caused by the unobservability of effort. We note that as σ increases the difference $f(\psi^W) - \psi^W$ declines, which, for a given value of $\Delta\psi$, increases the probability that $\Delta\psi < f(\psi^W) - \psi^W$ is violated. That is, as the difference $f(\psi^W) - \psi^W$ declines, the probability of having a liquidity policy ψ^h that affects negatively the bank's effort, $\Delta q < 0$, increases.

We summarize the above observations in the following proposition.

Proposition 5: *When the bank's effort is not observable, the government's optimal liquidity policy has a different impact on bank's effort depending on the level of output volatility. By collecting the results of the previous sections, we can state that:*

a) when the output volatility is low, and condition (30) *holds,* $\Delta q > 0$;

b) when the output volatility is high, but the optimal liquidity policy is not too restrictive, $\Delta q > 0$;

c) on the contrary, when the output volatility is high and the optimal liquidity policy is very restrictive, it could be that $\Delta q < 0$.

Several observations are in order. First, Proposition 5 establishes that the relationship between government's liquidity support and bank's effort depends crucially on the volatility of the output. When the latter is low and condition (30) holds (point a)), it is optimal for the government to provide a less generous liquidity policy when the bank's behavior is not observable than under FI, $\psi^h > \psi^W$. Anticipating this, the bank behaves even more prudently than socially desirable, $1 - q^B(\psi^h) < 1 - q^W(\psi^W)$.

A similar distortion arises when volatility is high (point b)) and it is optimal for the government to establish a liquidity policy which is not too restrictive, albeit more stringent than under FI, $\psi^h > \psi^W$. The combination of high output volatility and a restrictive liquidity policy, still induces the bank to behave even more prudently than socially desirable. Points a) and b) both illustrate the effect stressed in the literature that argues that less generous bailouts induce banks to behave more prudently.

A qualitatively new effect arises instead when output volatility is high and the severity of the distortion caused by the unobservability of effort calls for a very restrictive liquidity policy (point c)). When the optimal liquidity policy under unobservable effort is much tighter than under FI $\left(\psi^h >> \psi^W\right)$, the bank could find it optimal to be less prudent than socially desirable, $1 - q^B(\psi^h) > 1 - q^W(\psi^W)$. This novel effect stems from the fact that the bank anticipates that an output shortfall is likely given the high level of the output volatility. However, the bank knows that it cannot count on generous liquidity support, which could discourage it from being prudent ex ante. This illustrates the notion that when output volatility is high, addressing a severe distortion caused by unobservable effort with a very tight liquidity policy, although optimal from a welfare standpoint, may back fire in terms of stability. This effect is similar to the one stressed by the studies that challenge the received view that generous bailouts induce bank's risk taking, in particular Cordella and Levy-Yeayati (2003) and Dell'Ariccia and Ratnovski (2014).

Second, output volatility can differ for many reasons, among which, as argued, because banks may operate in different countries or be in different phases of the business cycle. As for the latter the novel effect arises when volatility is high, which is more likely the case in crises times; when markets are calm the effect stressed by the literature arises. In the case of countries, output volatility is more likely to be high in emerging economies subject to shocks potentially outside their controls. Proposition 5 shows that for those economies a tight liquidity policy could be further destabilizing.

Third, although endogenizing output volatility is not the aim of this paper, we can conjecture that when output volatility is high and the optimal liquidity policy is very tight, if the bank could it would lower output volatility because it knows that the chances of being illiquid are high. Therefore the bank would like to lower the risk of being illiquid precisely when liquidity support is unlikely. This, in turn, should lower the reduction of the equilibrium effort. More generally in a model where the output volatility is endogenous, effort and volatility could work as substitutes.

To sum up, Proposition 5 shows that while a less generous liquidity support induces for sure the bank to be more prudent when output volatility is low as the received view argues, this relationship is ambiguous when output volatility is high. In fact a tighter liquidity policy could be destabilizing and could induce the bank to be less prudent, precisely when the underlying volatility of its investment is already high, and the distortion induced by unobservable effort severe.

6 Conclusions

We are able to capture the complex interactions between cash flow volatility, expectations of government's support, in the form of liquidity injections, and bank's behavior, because we have cast the bank's and the government's choices in a dynamic model. In our model bank's output follows a continuous time process and the government's decision to end liquidity injections entails the exercise of a real option whose value crucially depends on the volatility of the process governing the output from the investment.

At the center of our analysis is the notion that the exogenous volatility of the bank' investment affects the severity of the problem induced by unobservable effort and the optimal liquidity policy to address it, which, in turn, affects the bank's ex ante behavior.

By making liquidity policy endogenous as a function on external conditions, our paper reconciles the studies that show that expectations of government's supports increase bank's risk taking, and the recent studies that challenge this view. When cash flow volatility is taken into consideration and liquidity supports are optimally set, the relationship between liquidity support and bank's incentives to behave prudently is not monotonic. In particular when volatility is low, and the optimal liquidity policy is tight, less generous liquidity supports induce banks to put more effort ex ante to screen the projects more carefully. Similarly, when volatility is high and the optimal liquidity policy is tight, less generous liquidity supports induce banks to be more prudent. However, when high volatility induces a severe distortion due to unobservable effort, the very tight liquidity requested to address the unobservable effort could make the bank less willing to exert effort.

7 Appendix

7.1 Liquidity buffers

We can model liquidity buffers introducing a second process Y_t that represents the bank's payouts strategy. Defining $\tilde{v}_t$ a version of the process v_t with the liquid reserves, this is given by:

$$\tilde{v}_t = v_t + L_t - Y_t \tag{33}$$

where Y_t is an adapted, left-continuous and non-decreasing process with initial value $Y_0 = 0$ which represents the bank's cumulative payouts to its shareholders. Now the two cumulative processes become:

$$Y_t = \max_{0 \le s \le t} \left[b - v_s - L_s \right]^- , \tag{34}$$

and

$$L_t = \max_{0 \le s \le t} \left[v_s - Y_s - r(1-k) \right]^- \tag{35}$$

where b is the upper level of v above which the bank distributes dividends. That is, the payout policy consists in distributing dividends to maintain liquid reserves at or below the target level b. With the process (33), the government intervenes only when $L_t - Y_t \ge 0$, i.e. when the liquidity buffer generated by Y_t is not sufficient to keep $\tilde{v}_t$ above $r(1-k)$. Under these assumptions the objective function of the bank is:

$$V = \max_q q \left[\max_b \left(\mathbb{E}[\int_0^\tau e^{-rt} (\tilde{v}_t - b) dt - \left(1 - e^{-r\tau} (1-k) \right)] \right) \right] - \frac{a}{2} q^2, \tag{36}$$

subject to (34) and (35).

In (36), the bank chooses both q and b, while the government chooses τ. If reserves do not pay interests (e.g. as in Hugonnier and Morellec 2017) it is easy to conjecture that the bank chooses $b = r(1-k)$. Indeed the government's liquidity support is akin to raise equity in the market; there is no reason to hold liquidity buffers if the bank can count on liquidity support for a while. In general, however, with a $b > r(1-k)$ the value of the project (36) is lowered, although qualitatively, nothing changes regarding the choice of q.

7.2 Proof of Lemma 1.

Defining:

$$Q^B(v_0) \equiv \mathbb{E}\int_0^{\infty} e^{-rt}e^{-\psi L_t}(\tilde{v}_t - r\,(1-k))dt + (1-k)\mathbb{E}\int_0^{\infty} e^{-rt}\psi e^{-\psi L_t}dL_t$$

the Bellman equation is:

$$rQ^B = v_0 - r\,(1-k) + lim_{dt\to 0}\frac{1}{dt}\mathbb{E}[dQ^B]. \tag{37}$$

Using the stochastic process (1) and Ito's Lemma on $\mathbb{E}[dQ^B(v_0)]$, we obtain the following partial differential equation:

$$\frac{1}{2}\sigma^2 v^2 Q^{B\prime\prime} - rQ^B = -(v_0 - r(1-k)) \quad \text{ for } v_0 \in [r(1-k), \infty), \tag{38}$$

with boundary conditions:

$$\lim_{v_0\to\infty}[Q^B - \frac{v_0 - r\,(1-k)}{r}] = 0, \tag{39}$$

$$Q^{B\prime}(r(1-k)) - \psi[Q^B(r(1-k)) - (1-k)] = 0, \tag{40}$$

where $Q^{B\prime}$ and $Q^{B\prime\prime}$ represent the first and the second derivative of $Q^B\,(v_0)$ w.r.t. v. Equation (39) states that, when cash flows go to infinity the effort must be bounded. In fact, the second term in (39) represents the discounted present value of excess returns over an infinite horizon starting from v_0. The boundary condition (40) means that when the cash flows reach the lower boundary $r(1-k)$, to continue to keep the bank open the marginal value of one extra unit of liquidity must not fall below the bank's cost to increase effort by one unit represented by $\psi[Q^B(r(1-k)) - (1-k)]$.[16] By the linearity

[16] The boundary condition (40) requires a linear combination of the unknown function $q^B(v)$ and its first derivative $q^{B\prime}(v)$ at $v = r(1-k)$. In differential equation therory this

of the differential equation (38) and making use of (39), the general solution of (38) takes the form:

$$Q^B = \frac{v_0 - r(1-k)}{r} + A(v_0)^\beta, \tag{41}$$

where A is a constant to be determined and β, with $-\infty < \beta < 0$, is the negative root of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1)-r = 0$. The boundary condition (40), yields the value of the constant A:

$$[\frac{1}{r} + \beta A(r(1-k)^{\beta-1}] - \psi A[r(1-k)]^\beta + \psi(1-k) = 0$$

$$A = -\underbrace{\frac{(1-k)(1+\psi r(1-k))}{\beta - \psi r(1-k)}}_{<0}[r\,(1-k)]^{-\beta} > 0. \tag{42}$$

Then, the expected present value of the total liquidity supplied is equal to:

$$Av_0^\beta = -\left(\frac{v_0}{r(1-k)}\right)^\beta \frac{(1-k)(1+\psi r(1-k))}{\beta - \psi r(1-k)} > 0. \tag{43}$$

Finally, substituting (43) in (41), we are able to write the effort as in the text.

7.3 Proof or Proposition 1.

Taking the derivative of the effort level (10) with respect to ψ, we obtain:

$$\frac{\partial q^B}{\partial \psi} = -\left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{r(1-k)(\beta-\psi r(1-k)) + (1+\psi r(1-k))r(1-k)}{(\beta - \psi r(1-k))^2} \tag{44}$$

$$= -\Gamma(\beta) r(1-k)\,(\beta+1)\,,$$

condition is called *Robin* (or third type), boundary condition. See Harrison (2013) p. 159-160 for an application of this condition in a context similar to ours.

where

$$\Gamma(\beta) \equiv \left(\frac{v_0}{r(1-k)}\right)^{\beta} \frac{1-k}{(\beta - \psi r(1-k))^2} > 0. \tag{45}$$

By (44) it is easy to show that:

$$\frac{\partial q^B}{\partial \psi} > 0 \quad \text{if} \quad \beta < -1; \qquad \frac{\partial q^B}{\partial \psi} < 0 \quad \text{if} \quad \beta > -1.$$

Moreover, taking the limits:

$$\lim_{\beta \to 0} \frac{\partial q^B}{\partial \psi} = -\frac{1}{(\psi r)^2} r < 0; \qquad \lim_{\beta \to -\infty} \frac{\partial q^B}{\partial \psi} = 0.$$

7.4 Equity and liquidity policy

In the model we have assumed that the project is financed with the exogenous equity of the bank's shareholder k and deposits $1-k$. In what follows we explore the effectiveness of the liquidity policy in inducing effort if the project is financed with more equity. Recall that the impact of liquidity on effort is given by

$$\frac{\partial q^B}{\partial \psi} = -\Gamma(\beta) r(1-k)(\beta+1),$$

where recall $\Gamma(\beta)$ is defined in (45). Taking the derivative of $\Gamma(\beta)$ with respect to k we obtain:

$$\frac{\partial \Gamma(\beta)}{\partial k} = \Gamma(\beta) \left[\frac{\beta - 1}{(1-k)} + \frac{2\psi r}{(\beta - \psi r(1-k))}\right] < 0.$$

Therefore

$$\frac{\partial}{\partial k}\left(\frac{\partial q^B}{\partial \psi}\right) = r(\beta+1) \underbrace{\Gamma(\beta)}_{>0} \underbrace{\left[-\beta + 2 - \frac{2\psi r(1-k)}{(\beta - \psi r(1-k))}\right]}_{>0} \tag{46}$$

whose sign depends on the sign of $\beta + 1$. If the project's volatility is high, i.e. $\mid \beta \mid$ is close to zero such that $\beta + 1 > 0$, then the sign of (46) is positive. Intuitively, in Proposition 1 we have established that if volatility is high the decision of the government to provide a more generous liquidity support (that is to lower ψ) induces the bank to increase effort ($\frac{\partial q^B}{\partial \psi} < 0$). Then if k increases, the marginal value of liquidity to induce the bank to provide effort declines, i.e. $\frac{\partial}{\partial k}\left(\frac{\partial q^B}{\partial \psi}\right) > 0$.

On the contrary if the project volatility is low, when the government increases liquidity it induces the bank to lower effort ($\frac{\partial q^B}{\partial \psi} > 0$). In this case, since if $\beta \to -\infty$ we have $\beta + 1 < 0$, then the larger is the equity, the lower will be the reduction of effort, $\frac{\partial}{\partial k}\left(\frac{\partial q^B}{\partial \psi}\right) < 0$. That is, again the marginal value of liquidity on the bank's effort declines. To sum up the effectiveness of a more generous liquidity policy in inducing effort declines with effort.

7.5 Proof of Lemma 2.

Defining

$$Q^W(v_0) \equiv \mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[(\tilde{v}_t - r(1-k))dt - dC_t] - Z\mathbb{E}\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t,$$

the solution for Q^W is obtained by solving the following Bellman equation:

$$\frac{1}{2}\sigma^2 v^2 Q^{W\prime\prime} - rQ^W = -(v_0 - r(1-k)) \quad \text{for } v_0 \in [r(1-k), \infty), \tag{47}$$

with boundary conditions:

$$\lim_{v_0 \to \infty}[Q^W - \frac{v_0 - r(1-k)}{r}] = 0, \tag{48}$$

$$Q^{W\prime}(r(1-k)) - \psi[Q^W(r(1-k)) + Z] = c(\psi). \tag{49}$$

While (48) is equal to (39), and has the same meaning, condition (49) replaces the boundary condition (40). In fact, since liquidity is costly for the government, at each liquidity injection the marginal value of continuing to keep the bank open must not fall below the marginal cost, that now includes both the cost of liquidity $c(\psi)$ as well as the deadweight cost of bank closure Z, i.e.:

$$c(\psi) + \psi[Q^W(r(1-k)) + Z].$$

Again, by the linearity of the differential equation (47) and making use of (48), the general solution takes the form:

$$Q^W = \frac{v_0 - r(1-k)}{r} + Bv_0^\beta, \tag{50}$$

where B is a constant to be determined and $\beta < 0$ is still the negative root of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1) - r = 0$. Using (49) we obtain:

$$B = \frac{(1-k)[(c(\psi)r-1) + \psi r Z]}{\underbrace{\beta - \psi r(1-k)}_{<0}}(r(1-k))^{-\beta} < 0,$$

from which:

$$Bv_0{}^\beta = \left(\frac{v_0}{r(1-k)}\right)^\beta \frac{(1-k)[(c(\psi)r-1) + \psi r Z]}{\beta - \psi r(1-k)} < 0, \tag{51}$$

which is H, the loss of project value due to government's intervention. Since (51) is negative it concurs to lower the level of effort. Finally, substituting (51) in (50) we are able to obtain the expression in the text.

7.6 Proof of Propositions 2 and 3.

To prove Proposition 2, recall that from (18) maximizing W is equivalent to maximize q^W, and that from (19) the probability of success is

$$aq^W = \frac{v_0 - r(1-k)}{r} + H(\psi, \beta) - 1,$$

where, recall,

$$H(\psi,\beta)=\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\frac{(c(\psi)r-1)+\psi rZ}{\beta-\psi r(1-k)}<0. \tag{52}$$

To maximize q^W we look for a ψ^W that minimizes H. Let us consider the F.O.C.:

$$\frac{\partial H}{\partial \psi}=\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\frac{(c'(\psi)r+rZ)(\beta-\psi r(1-k))+[c(\psi)r-1+\psi rZ]r(1-k)}{(\beta-\psi r(1-k))^2} \tag{53}$$

$$=\Gamma(\beta)\left[c'(\psi)r\beta+rZ\beta-\psi r(1-k)c'(\psi)r+(c(\psi)r-1)r(1-k)\right]=0,$$

where recall $\Gamma(\beta)$ is defined in (45). Hence we obtain:

$$\frac{\partial H}{\partial \psi}=\Gamma(\beta)r[c'(\psi)(\beta-\psi r(1-k))+(c(\psi)r-1)(1-k)+Z\beta]=0. \tag{54}$$

In addition the S.O.S.C. is always satisfied, i.e.:

$$\frac{\partial^2 H}{\partial \psi^2}=\frac{\partial\Gamma(\beta)}{\partial\psi}\underbrace{[...]}_{=0\text{ by F.O.C.}}+\Gamma(\beta)\left[c''(\psi)r\beta-r(1-k)c'(\psi)r-\psi r(1-k)c''(\psi)r\right] \tag{55}$$

$$=\underbrace{\Gamma(\beta)\left[rc''(\psi)(\beta-\psi r(1-k))-r(1-k)c'(\psi)r\right]}_{SOSC}>0.$$

This proves Proposition 2.

To prove Proposition 3 we first analyze the effect of σ on the level of effort q^W. In particular, taking the derivative of (10), i.e. (52), with respect to σ we obtain:

$$\frac{\partial H}{\partial\sigma}=\frac{\partial H}{\partial\psi^W}\frac{\partial\psi^W}{\partial\sigma}+\frac{\partial H}{\partial\beta}\frac{\partial\beta}{\partial\sigma}.$$

Taking the derivative of H with respect to β, and observing that $\frac{\partial H}{\partial \psi^W} = 0$ by (54) , we obtain:

$$\frac{\partial H}{\partial \beta}\frac{\partial \beta}{\partial \sigma} =$$
$$\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\frac{c(\psi)r-1+\psi rZ}{\beta-\psi r(1-k)}\left[\log\frac{v_0}{r(1-k)}-\frac{1}{\beta-\psi r(1-k)}\right]\frac{\partial \beta}{\partial \sigma}<0,$$

from which it follows that $\frac{\partial q^W}{\partial \sigma} < 0$. Now, totally differentiating (54) with respect to σ and using (55) we obtain:

$$\frac{\partial \psi^W}{\partial \sigma} = -\frac{\frac{\partial (.)}{\partial \sigma}}{\frac{\partial (.)}{\partial \psi}} = -\frac{r[c'(\psi)+Z]\frac{\partial \beta}{\partial \sigma}}{SOSC} = -\frac{\overbrace{[\frac{-(c(\psi)r-1)r(1-k)-rZ\psi r(1-k)}{\beta-\psi r(1-k)}]}^{>0}\overbrace{\frac{\partial \beta}{\partial \sigma}}^{>0}}{SOSC>0} < 0. \tag{56}$$

To gain the intuition for why $\frac{\partial \psi^W}{\partial \sigma} < 0$ observe that the regulator maximizes W w.r.t. ψ by maximizing q^W. So, at the maximum we have:

$$\frac{\partial q^W}{\partial \psi}(\psi(\sigma),\sigma) = 0. \tag{57}$$

Taking the total differential of (57) w.r.t. σ, we obtain:

$$\frac{\partial^2 q^W}{\partial \psi^2}(\psi(\sigma),\sigma)\frac{\partial \psi^W}{\partial \sigma} + \frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma),\sigma) = 0,$$

from which

$$\frac{\partial \psi^W}{\partial \sigma} = -\frac{\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma),\sigma)}{\frac{\partial^2 q^W}{\partial \psi^2}(\psi(\sigma),\sigma)} < 0. \tag{58}$$

Now, since $\frac{\partial^2 q^W}{\partial \psi^2}(\psi(\sigma),\sigma) < 0$, from (58) we have $\frac{\partial \psi^W}{\partial \sigma} < 0$ if $\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma),\sigma) < 0$. If we interpret $\frac{\partial q^W}{\partial \psi}$ as the marginal productivity of a restrictive liquidity policy, then $\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma),\sigma) < 0$ denotes that an increase in output volatility lowers the marginal productivity of a restrictive liquidity policy. Therefore, as output volatility increases, if the government wants that the productivity of liquidity increases, it has to increase the liquidity, i.e. $\frac{\partial \psi^W}{\partial \sigma} < 0$.

7.7 Proof of Proposition 4.

We prove $\psi^h \geq \psi^W$ for high volatility projects first, and then we add a sufficient condition for $\psi^h \geq \psi^W$ for low volatility projects. We proceed in steps.

First step. Recall that $W^h = W - I$. The F.O.C. is:

$$\frac{\partial W^h}{\partial \psi} = \frac{\partial W}{\partial \psi} - \frac{\partial I}{\partial \psi} = 0. \tag{59}$$

As W is concave with $\frac{\partial W}{\partial \psi}_{|\psi=\psi^W} = 0$, if a value ψ^h that satisfies (59) exists, for $\psi^h > \psi^W$ it must be that $\frac{\partial I}{\partial \psi} < 0$. In particular, since $a[q^B - q^W] = P + D > 0$, we have $\frac{\partial I}{\partial \psi} = a[q^B - q^W]\frac{\partial (P+D)}{\partial \psi}$. Then, comparing ψ^h with ψ^W, we obtain $\psi^h \geq \psi^W$ if $\frac{\partial (P+D)}{\partial \psi} \leq 0$.

Second step. The sign of $\frac{\partial (P+D)}{\partial \psi} = a\frac{\partial [q^B - q^W]}{\partial \psi}$ is given by:

$$\frac{\partial (P+D)}{\partial \psi} = \tag{60}$$
$$-\Gamma(\beta) r \left\{ [1 - k + c'(\psi) + Z](\beta - \psi r(1-k)) + [\psi r(1-k)) + c(\psi) r + \psi r Z](1-k) \right\},$$

where $\Gamma(\beta) > 0$ is defined in (45). Expression (60) is continuous in σ (i.e. β). In addition, $\frac{\partial (P+D)}{\partial \psi}$ from (60) is ≤ 0 if:

$$-[c'(\psi) + ((1-k) + Z)]\beta \leq [c(\psi) - c'(\psi)\psi] r(1-k)). \tag{61}$$

Since the R.H.S. of (61) is always positive, there exists a value of σ in the region $[0, \infty)$ that satisfies (61). In the specific, if the project is high volatility (i.e. $\mid \beta \mid$ is close to 0), (61) is easily satisfied for any acceptable range of ψ.

Third step. Since $q^B - q^W > 0$, combining with $\frac{\partial (P+D)}{\partial \psi} \leq 0$, for high volatility projects we obtain $\psi^h \geq \psi^W$.

Fourth step. For a low volatility project (i.e. when $\mid \beta \mid$ is high), the sign of $\frac{\partial(P+D)}{\partial\psi}$ is harder to determine. However, from (61) a sufficient condition for $\frac{\partial(P+D)}{\partial\psi} \leq 0$ regardless of the project's volatility is (30), that is

$$c'(\psi) + (1-k) + Z < 0 \Leftrightarrow -rc'(\psi) > r(1-k) + rZ. \tag{62}$$

Thus, if (62) holds then $\psi^h > \psi^W$ for all value of σ. This proves Proposition 4.

7.8 Analysis of equation (32).

Replacing the expressions for $q^B(\psi^W)$, $q^W(\psi^W)$ and $\frac{\partial q(\psi^W)}{\partial\psi}$ into (31), we obtain:

$$\begin{aligned} \Delta q = & -\Gamma(\beta)\left[\psi^W r(1-k)) + c(\psi^W)r + \psi^W rZ\right] \\ & -\Gamma(\beta)\frac{r(1-k)(\beta-\psi^W r(1-k)) + (1+\psi^W r(1-k))r(1-k)}{(\beta-\psi^W r(1-k))}\Delta\psi. \end{aligned} \tag{63}$$

Rearranging (63) it is easy to see that for the difference Δq to be positive it must be:

$$\Delta\psi = \psi^h - \psi^W < -\frac{(\beta-\psi^W r(1-k))[\psi^W r(1-k)) + c(\psi^W)r + \psi^W rZ]}{r(1-k)\,(\beta+1)},$$

where we assume, like in Proposition 1, that if the project is high volatility, i.e. $\mid \beta \mid$ is close to zero, then $\beta + 1 > 0$.

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