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Spatial resource wars: A two region example

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Abstract
We develop a spatial resource model in continuous time in which two agents strategically exploit a mobile resource in a two-location setup. In order to contrast the overexploitation of the resource (the tragedy of commons) that occurs when the player are free to choose where to fish/hunt/extract/harvest, the regulator can establish a series of spatially structured policies. We compare the three situations in which the regulator: (a) leaves the player free to choose where to harvest; (b) establishes a natural reserve where nobody is allowed to harvest; (c) assigns to each player a specific exclusive location to hunt. We show that when preference parameters dictate a low harvesting intensity, the policies cannot mitigate the overexploitation and in addition they worsen the utilities of the players. Conversely, in a context of harsher harvesting intensity, the intervention can help to safeguard the resource, preventing the extinction and also improving the welfare of both players.

Keywords : Spatial harvesting problems, Markov perfect equilibrium, Environmental protection policies, Differential Games

JEL Codes: Q28, C72, Q23, C61, R12

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SPATIAL RESOURCE WARS: A TWO REGION EXAMPLE

GIORGIO FABBRI, SILVIA FAGGIAN, AND GIUSEPPE FRENI

ABSTRACT. We develop a spatial resource model in continuous time in which two agents strategically exploit a mobile resource in a two-locations setup.

In order to contrast the overexploitation of the resource (the *tragedy of commons*) that occurs when the player are free to choose where to fish/hunt/extract/harvest, the regulator can establish a series of spatially structured policies. We compare the three situations in which the regulator: (a) leaves the player free to choose where to harvest; (b) establishes a natural reserve where nobody is allowed to harvest; (c) assigns to each player a specific exclusive location to hunt.

We show that when preference parameters dictate a low harvesting intensity, the policies cannot mitigate the overexploitation and in addition they worsen the utilities of the players. Conversely, in a context of harsher harvesting intensity, the intervention can help to safeguard the resource, preventing the extinction and also improving the welfare of both players.

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1. INTRODUCTION

Stationary Markov perfect Nash equilibria in models with a common property resource stock have been studied under different hypotheses in the literature (see e.g., Levhari and Mirman, 1980, Clemhout and Wan, 1985, Negri, 1989, Tornell and Velasco, 1992, Dockner and Sorger, 1996, Sorger, 1998, Tornell and Lane, 1999, Rowat and Dutta, 2007, Strulik, 2012a, 2012b, Mitra and Sorger, 2014, 2015, Dasgupta, Mitra and Sorger, 2017. Long, 2011, 2016, survey the literature). The typical setting is the one in which a homogeneous stock, whose growth function is known, is harvested by a finite number or a mass of identical agents who reap utility from consuming the resource. Since the analysis of the Markov perfect Nash equilibria has turned to be difficult, straightforward results have been obtained only for special growth and utility functions (see e.g., Dockner, Jorgensen, Van Long and Sorger, 2000, section 12.1 for the case of an exhaustible resource with a CRRA instantaneous utility function). Although there are a few exceptions, the usual conclusion in this literature is that non-cooperation leads to overexploitation of the resource (the so called "tragedy of the commons").

In view of the above technical difficulties, it is not surprising that the literature that considers strategic interaction when resources are heterogeneous is sparse. In recent years, however, both in growth theory and in environmental and resource economics there has been a growing interest in the economic effects that specifically arise when spatial distributed stocks are subject to diffusion or dispersal processes (see e.g., Smith, Sanchirico and Wilen, 2009; Xepapadeas, 2010; and Brock, Xepapadeas and Yannacopoulos, 2014, for surveys). Fish stocks, to take an obvious example, are spatially distributed and in many cases they move across different locations. Similarly, stocks of air or water pollutants are rarely stationary at the emission point, but diffuse in space. Even water reservoirs, and some exhaustible resources as oil deposits, have a spatial dynamics. Since the economic effects of the mobility in space of these stocks are sometimes relevant, the process of spatial diffusion they follow cannot be always abstracted from in the study of their exploitation. For example, territorial use rights fisheries (TURFs) can be more or less effective depending on the spatial externalities associated with the movements of fish stocks. In these cases, the analysis faces the challenging task of modeling the economic forces shaping the dynamics of extraction of moving spatially distributed stocks.

Although the literature is rapidly growing (Costello, Querou and Tomini, 2015, Herrera, Moeller and Neubert, 2016, Costello, Nkuiya and Querou, 2017, de Frutos and Martin Herran, 2018), dynamic strategic interaction is largely absent from the studies that have introduced spatial-dynamic processes in growth or resource models. In these works, the analysis proceeds either on the assumption that rent dissipates instantaneously (e.g., Sanchirico and Wilen, 1999), or on the assumption that the planner either controls the entire environment (e.g, Boucekkine, Camacho and Fabbri, 2013), or takes the spatial distributed stock path as given (e.g., Janmaat, 2005, Santanbrogio, Xepapedeas, Yannacoupolos, 2017). Clearly, these assumptions are not well suited for the analysis of the spatial externalities that arise when the resource is a moving spatial distributed stock, but access is restricted to a small number of extractors.

We develop in this work a simplified framework to study spatial resource wars. Our aim is to provide an analytically tractable model that highlights how difficult is to design an efficient systems for the management of the resource based on spatial property rights, if the spatial externalities stemming from the movements of the stocks are not completely internalized. We compare the behaviors of the agents in a initial *common property case*, where they can decide both where and how much to harvest, with their choices in policies-constrained cases, where the regulator can establish a natural reserve or assign an harvesting location to each agent. We show that implementing these policies can only be effective when the agents choose an high harvesting intensity/effort.

To have an analytically solvable model, some simplifications are made. First, we chose to study a two-regions, two-players case. Second, as is often assumed in the literature, we suppose the stock diffuses from the higher density to the lower density location at a constant rate. Third, as it can be expected, special growth and utility function are used, since, as it is well known, not even a mere existence results for Nash equilibria can be obtained in a general framework. Since we look for linear Markov equilibria, tight restrictions must be imposed on the primitives of the model and we use throughout the paper the family of CRRA utility functions and linear (re)production functions (see Gaudet and Lohoues, 2008, for similar conditions for the scalar common pool case).

For the case in which the preferences of the agents dictate low harvesting intensity, the existence of a symmetric local Markov perfect Nash equilibrium is proved and explicitly characterized in the three scenarios we study: (a) the initial *common property case*, where each agent can decide, at any times in which region(s) to harvest and how much; (b) the *reservoir case*, where the regulator forbid the agents to harvest in one of the two regions; (c) the *TURF Case*, where each player can only harvest in a specific (exclusive) region. For each situation we characterize the optimal response function of the players, the resource stock evolution (and in particular its (re)production rate at the equilibrium) and the utility of the players.

It turns out that, in case of low harvesting intensity, the mentioned spatial property rights policies cannot improve the growth rate of the resource and in particular they cannot prevent its depletion/extinction in case the implicit rate(s) of growth is positive, but very small. Furthermore their effect on the utilities of the agents is never positive and, in almost all the circumstances, they strictly worsen the utility of at least one of the players.

The analysis of the results allow us to show (see Subsection 3.4) that a *voracity effect* similar to that described by Tornell and Lane (1999) can arise in the spatial context we have, and to identify what kinds of "technological" shocks generate it. As expected, if voracity prevails, then an increase in any of the local intrinsic growth rates of the resource reduces growth. Notably, however, it turns out that also a *reduction* in the spatial mobility of the resource has the same effect.

Things change sharply when a policy induces the agents to choose their maximal effort (Section 4). In the high harvesting intensity case indeed the territorial policies we mentioned, and in particular the creation of a reservoir, can determine an effective reduction of the overexploitation and have a consequent positive impact on the rate of (re)production of the resource. The policies can, in suitable circumstances, prevent the (asymptotic) depletion/extinction of the resource that would occur under a regime of common property. Moreover, for suitable choices of the parameters they can also increase the utility of all the agents.

In comparison with existing literature, the model here is a spatial generalization of the classical Levhari and Mirman (1980) example of a "fish war"[1] and is closest to the models in Herrera, Moeller and Neubert (2016), De Frutos and Martin Herran (2017), and Costello, Nkuiya and Querou (2017). All three study Markov perfect Nash equilibria in models with a mobile spatially distributed stock. Differently from Herrera, Moeller and Neubert (2016), that is essentially a numerical paper, we do not only focus on steady states but we characterize equilibrium feedbacks, we describe the whole optimal trajectory and the corresponding transition dynamics. In this way we can also analyze how the welfare changes in the various specifications of the problem. Among these, we also include property rights on the various parts of the sea. In this last sense, our specification is more similar to the one used by De Frutos and Martin Herran (2017) which study a transboundary pollution linear-quadratic differential game. The approach is a little different in the contribution by Costello, Nkuiya and Querou (2017) where the authors use a two-patch discrete time model to study the strategic effects induced by the threat of a regime shift affecting the movement of the resource. They have asymmetric players and study a two-stage problem. However, their post-shift problem is structurally similar to our problem, although they assume the agents do not consume the harvest but sell it on local competitive markets. Their Markov perfect Nash equilibrium is linear but, differently from the equilibrium we find in our model, there is an initial jump to the stationary state and the extraction strategy of each agent is independent of the other region's stock.

The paper proceeds as follows: in Section 2 we describe the two-players two-locations model. In Section 3 we study it in the three mentioned specifications: the common property case (Section 3.1), the reservoir case (Section 3.2), the TURF case (Section 3.3). In addition to this we discuss the impact of policies comparing payoffs and evolution of stocks in these various cases (Section 3.4). In Section 4 we study the case of high/constrained efforts, showing

[1] Although to simplify the analysis we use linear growth functions instead of strictly concave functions with a finite carrying capacity.

the mechanism at work in a first example (Section 4.1) and opening a more general discussion in Section 4.2. Section 5 contains the conclusions.

2. The model

Assume a stock of movable resource is distributed on a given territory, partitioned in two contiguous subareas, zone 1 and zone 2. The stock distribution is given by a nonnegative column vector $x(t) = (x_1(t), x_2(t))'$, where $x_i(t) \geq 0$ is the biomass standing on zone i ($i = 1, 2$) at time t, $t \geq 0$. Natural conditions are such that the natural resource in the two subareas have different intrinsic reproduction rates Γ_1 and Γ_2, with $\Gamma_2 \geq \Gamma_1$ (with Γ_1 not necessarily positive).

Two agents compete for the exploitation of the resource. We denote by $c_i^j(t)$ the rate of extraction of the agent j in the location i at time t, with $j \in \{1, 2\}$.

Although no amount of the resource can be shifted by agents from one zone to the other, some living stock of the resource moves spontaneously between the regions, from the one with higher to the one with lower biomass concentration. More precisely we assume the diffusion process follows Fick's first law: the flow of the resource from region i to region $3 - i$ at time t is given by

$$\alpha(x_i(t) - x_{3-i}(t)), \tag{1}$$

where $\alpha > 0$ is the diffusion coefficient. The dynamics of the resource stock is then given by

$$\begin{cases} \dot{x}_1(t) = \Gamma_1 x_1(t) + \alpha(x_2(t) - x_1(t)) - c_1^1(t) - c_1^2(t), & x_1(0) = x_1^\circ \geq 0 \\ \dot{x}_2(t) = \Gamma_2 x_2(t) + \alpha(x_1(t) - x_2(t)) - c_2^1(t) - c_2^2(t), & x_2(0) = x_2^\circ \geq 0 \end{cases} \tag{2}$$

and positivity contraints on the control

$$c_i^j(t) \geq 0, \text{ for all } \ t \geq 0, \quad i, j \in \{1, 2\}. \tag{3}$$

and on the stock

$$x_i(t) \geq 0, \text{ for all } \ t \geq 0, \quad i \in \{1, 2\}. \tag{4}$$

The j-th player chooses the strategy $c^j(t) = (c_1^j(t), c_2^j(t))'$ to maximize either the functional

$$J^j(c^j) = \int_0^{+\infty} e^{-\rho t} \frac{\left(b_1 c_1^j(t) + b_2 c_2^j(t)\right)^{1-\sigma}}{1 - \sigma} dt, \tag{5}$$

where $\sigma > 0$, $\sigma \neq 1$, and $b_i \in [0, 1]$, or its logarithmic version

$$J^j(c^j) = \int_0^{+\infty} e^{-\rho t} \ln\left(b_1 c_1^j(t) + b_2 c_2^j(t)\right) dt. \tag{6}$$

The nonnegative constants b_1 and b_2 represent iceberg costs, with $b_1 \leq b_2$ if zone 1 harder to reach than zone 2.[2]

We also take into account the fact that extraction of a resource is more difficult in territories where the resource is scarser. More precisely we consider a Schaefer production function, assuming that the harvest of agent j in region i depends on the agent's effort $E_i^j(t)$, on the level of stock, and on a *catchability parameter* β_i combined linearly to obtain

$$c_i^j(t) = \beta_j E_i^j(t) x_i(t). \tag{7}$$

[2]The quantities $(1 - b_1)$ and $(1 - b_2)$ can be also interpreted as taxes, although revenue from taxes is not part of our model.

Assuming that the maximal total effort exerted by agent j, that is $E_i^j(t) + E_{3-i}^j(t)$, is finite and normalized to 1, from (7) we derive the constraint

$$\frac{c_1^j(t)}{\beta_1 x_1(t)} + \frac{c_2^j(t)}{\beta_2 x_2(t)} \leq 1. \tag{8}$$

We call an *equilibrium* any Markovian (possibly symmetric) equilibrium of the game described by the state equation (2), the payoff (5) or (6), and the constraints (3) (4) (8).[3]

In the next sections we provide existence of equilibria in two different sets of data. In Section 3 we study the *low harvesting intensity* case, *i.e.* the case when the set of parameters (ρ, σ, β_i, b_i, Γ_i, α) is such that constraints (8) are never binding. In Section 4 we study the case in which constraints (8) are binding.

3. Solution of the model in the low harvesting intensity case

In this section we study the game described by (2) (3) (4) (5) and (8) in three different scenarios: (a) the *common property case*, where each agent can decide, at any times in which region(s) to harvest and how much; (b) the *reservoir case*, in which the regulator forbid the agents to harvest in one of the two regions; (c) the *TURF Case*, in which each player can only harvest in a specific (exclusive) region. In all three cases we will choose a set of parameters such that (8) is not binding. This assumption translates, in terms of the problem, into the requirement that the value z, representing the intensity of harvest and defined differently in the three cases, is a sufficiently small positive number - summarizing, the intensity of harvesting is "low". A representation of this set of parameters is given in Figure 1 and commented in Remark 3.5.

It is useful to consider the vector representation of (2), that is

$$x'(t) = Mx(t) - C(t)e, \quad x(0) = (x_1^\circ, x_2^\circ)'$$

where, if $\{e_1, e_2\}$ is the canonical base of column vectors for $\mathbb{R}^2$, and

$$M := \begin{pmatrix} \Gamma_1 - \alpha & \alpha \\ \alpha & \Gamma_2 - \alpha \end{pmatrix}, \; C(t) = \begin{pmatrix} c_1^1(t) & c_1^2(t) \\ c_2^1(t) & c_2^2(t) \end{pmatrix}, \; e = e_1 + e_2.$$

The matrix M has two distinct eigenvalues, and we denote the largest with λ. From $\alpha > 0$ one derives that eigenvectors associated to λ are all proportional to $\eta = (1, \mu)'$, where $\mu \geq 1$ (and $\mu > 1$ when $\Gamma_2 > \Gamma_1$).[4]

3.1. The Common Property Case.

The scenario here considered is that in which players are free to choose to fish in zone 1 or zone 2 (or both). We consider the instance of the game described above with payoff (5), and we look for *symmetric* equilibria. What we will prove is that there exists an equilibrium where both the player fish in just one zone (the same for both) depending on the ratio $\frac{b_2}{b_1}$, at least for some values of the parameters and for initial data in a subset of $\mathbb{R}^2_+$. In order to qualify this set of parameters, we define for $z \geq 0$ the set S_z of $y \in \mathbb{R}$ satisfying

$$\begin{cases} \Gamma_1 - \alpha + \alpha y - 2z(1 + \mu y) = \Gamma_2 - \alpha + \alpha \frac{1}{y} \\ z(1 + \mu y) \leq \beta_1 \\ y \geq 0. \end{cases} \tag{9}$$

[3]We could equivalently describe strategies in terms of E_i^j rather than c_i^j.

[4]More explicitly, $\lambda = \frac{1}{2}\left[\Gamma_1 + \Gamma_2 - 2\alpha + \sqrt{4\alpha^2 + (\Gamma_2 - \Gamma_1)^2}\right]$, while $\mu = \frac{2\alpha}{\sqrt{4\alpha^2 + (\Gamma_2 - \Gamma_1)^2} - (\Gamma_2 - \Gamma_1)}$.

and z_1^* as the maximum z for which S_z is nonempty. Note that: (a) the second and third inequality imply $z \leq \beta_1$; (b) $z \geq 0$ implies, after some computation, $z < \alpha/(2\mu)$; (c) the first equation has two solutions and (b) implies that only one is (strictly) positive; (d) z_1^* is the maximum $z \geq 0$ such that

$$z\left[1+\frac{\mu}{2(\alpha-2\mu z)}\left(\Gamma_2-\Gamma_1+2z+\sqrt{(\Gamma_2-\Gamma_1+2z)^2+4\alpha(\alpha-2\mu z)}\right)\right] \leq \beta_1$$

from which one derives in particular that $z_1^* > 0$.

Analogously, we define z_2^* as the maximum $z \geq 0$ for which the set of $r \in \mathbb{R}$ such that

$$\left\{\begin{array}{l} \Gamma_1-\alpha+\alpha\frac{1}{r}=\Gamma_2-\alpha+\alpha r-2z(\frac{1}{\mu}r+1) \\ z(\frac{1}{\mu}r+1) \leq \beta_2 \\ r \geq 0 \end{array}\right. \tag{10}$$

is nonempty, with z_2^* resulting positive as well.

Theorem 3.1 *Set $z = \frac{\rho-(1-\sigma)\lambda}{2\sigma-1}$, and assume $\mu > \frac{b_2}{b_1}$, $0 < z \leq z_1^*$, and (x_1°, x_2°) in $C_1 = \{(x_1,x_2) \in \mathbb{R}^2_+ \,:\, \frac{x_2}{x_1} \leq \frac{1}{\mu}(\frac{\beta_1}{z}-1)\}$. Then a Markovian equilibrium is given by*

$$\begin{array}{ll} c_1^1(t) = z\left(x_1(t)+\mu x_2(t)\right), & c_2^1(t) \equiv 0 \\ c_1^2(t) = z\left(x_1(t)+\mu x_2(t)\right), & c_2^2(t) \equiv 0, \end{array} \tag{11}$$

and the utilities of players at equilibrium are, respectively, $v^1(x_1^\circ,x_2^\circ), v^2(x_1^\circ,x_2^\circ)$, where

$$v^2(x_1,x_2) = v^1(x_1,x_2) = b_1^{1-\sigma}\ \frac{(x_1+\mu x_2)^{1-\sigma}}{z^\sigma(1-\sigma)} \tag{12}$$

Similarly, if $\mu < \frac{b_2}{b_1}$ and $0 < z \leq z_2^*$, the equilibrium is given by

$$\begin{array}{ll} c_1^1(t) \equiv 0, & c_2^1(t) = \frac{z}{\mu}\left(x_1(t)+\mu x_2(t)\right) \\ c_1^2(t) \equiv 0, & c_2^2(t) = \frac{z}{\mu}\left(x_1(t)+\mu x_2(t)\right). \end{array}$$

and

$$v^2(x_1,x_2) = v^1(x_1,x_2) = \left(\frac{b_2}{\mu}\right)^{1-\sigma} \frac{(x_1+\mu x_2)^{1-\sigma}}{z^\sigma(1-\rho)}\ .$$

Remark 3.2 (a) Note that the theorem establishes the existence of a *local* Markovian equilibrium as defined for instance in Dockner and Wagener (2014), p. 588. In particular such equilibrium exists if z is small enough.

(b) In the case $b_1 = b_2$ the theorem says that, for initial data in the cone C and for a positive z an equilibrium policy is that in which both players fish only in the least productive zone 1, so that fish that reproduce at a higher rate $\Gamma_2 > \Gamma_1$ in zone 2 are not harvested.

Remark 3.3 Along the proof of the theorem we will show that

$$x_1(t)+\mu x_2(t) = \langle x(t),\eta\rangle = (x_1^\circ+\mu x_2^\circ)e^{gt} \tag{13}$$

with $g = \lambda - 2z = \frac{\lambda-2\rho}{2\sigma-1}$. In particular, the equilibrium strategies c_i^j and utilities v^j depend merely on the projection of $x(t)$ along the direction of the eigenvector η.

Remark 3.4 We note that the extraction/consumption policy of the symmetric interior equilibrium has the same form as the policy function of the homogeneous stock case, where each agent's consumption is a fixed proportion of the stock. Since the state here is a two dimensional vector, extraction is linear in the *value* of the stock, so μ is actually the relative price of the stock in zone 2. Intuitively, $\mu \geq 1$ because $\Gamma_2 \geq \Gamma_1$. To explain the coefficient of the policy function, note that in a multidimensional linear setting the one-sector "productivity of the stock" is substituted by the von Neumann maximum rate of growth, that in our single production framework coincides with the dominant eigenvalue of the production matrix M (see e.g., Freni, Gozzi and Salvadori, 2006).

Proof. We analyse the case $\mu > \frac{b_1}{b_2}$ since the one with opposite inequality can be treated similarly. We also procede neglecting constraints (4) and (8), and check they are satisfied *a posteriori*.

Step 1: Solution of the Hamilton-Jacobi-Bellman equation for player 1. We assume player 2 fishes only in zone 1, using a strategy $c_1^2(t) = w\,\langle \eta, x(t)\rangle$ with $w \in \mathbb{R}^+$, and $c_2^2(t) = 0$. Then player 1 has to choose c^1 so to maximize $J^1(c^1)$ given by (5) and subject to

$$\dot{x}(t) = Mx(t) - w\,\langle \eta, x(t)\rangle\, e_1 - c^1(t), \quad x(0) = x.$$

The Hamiltonian function for a couple $(x,p) \in \mathbb{R}^2 \times \mathbb{R}^2$ is given by

$$\begin{aligned} H(x,p) &= \langle p, Mx\rangle - w\,\langle \eta, x\rangle\,\langle p, e_1\rangle + \sup_{c_1^1, c_2^1 \geq 0} \left\{ \frac{\left(b_1 c_1^1 + b_2 c_2^1\right)^{1-\sigma}}{1-\sigma} - \left\langle p, c^1 \right\rangle \right\} \\ &= \langle p, Mx\rangle - w\,\langle \eta, x\rangle\,\langle p, e_1\rangle + \frac{\sigma}{1-\sigma} \min\left\{ \frac{p_1}{b_1}, \frac{p_2}{b_2} \right\}^{1-1/\sigma} \end{aligned}$$

where the supremum is attained on the boundary, either at

$$c_1^{1*} = \frac{1}{b_1}\left(\frac{p_1}{b_1}\right)^{-\frac{1}{\sigma}}, \quad c_2^{1*} = 0 \tag{14}$$

or at

$$c_1^{1*} = 0, \quad c_2^{1*} = \frac{1}{b_2}\left(\frac{p_2}{b_2}\right)^{-\frac{1}{\sigma}}$$

It is then easy to check that the HJB equation $\rho v(x) = H(x, \nabla v(x))$ associated to the problem has a solution of type $v(x) = \frac{1}{1-\sigma}\left(\beta\,\langle \eta, x\rangle\right)^{1-\sigma}$. Indeed, $\nabla v(x) = \beta^{1-\sigma}\,\langle \eta, x\rangle^{-\sigma}\,\eta$ and $M\eta = \lambda\eta$ imply

$$\langle \nabla v(x), Mx\rangle = \langle M\nabla v(x), x\rangle = \beta^{1-\sigma}\,\langle \eta, x\rangle^{-\sigma}\,\langle M\eta, x\rangle = \lambda\beta^{1-\sigma}\,\langle \eta, x\rangle^{1-\sigma},$$

moreover, since $\mu > \frac{b_1}{b_2}$, one has

$$\min\left\{ \frac{\partial_1 v(x)}{b_1}, \frac{\partial_2 v(x)}{b_2} \right\} = \beta^{1-\sigma}\,\langle \eta, x\rangle^{-\sigma} \min\left\{ \frac{1}{b_1}, \frac{\mu}{b_2} \right\} = \frac{1}{b_1}\beta^{1-\sigma}\,\langle \eta, x\rangle^{-\sigma}.$$

Hence HJB is verified when β satisfies

$$\beta = \left(\frac{\sigma}{\rho - (\lambda - w)(1-\sigma)}\right)^{\frac{\sigma}{1-\sigma}} b_1.$$

Step 2: Nash equilibrium. From (14), the optimal fishing strategy for player 1 is

$$c_1^{1*}(t) = \frac{\rho - (\lambda - w)(1-\sigma)}{\sigma}\,\langle \eta, x(t)\rangle, \quad c_2^{1*}(t) = 0.$$

We may repeat the argument from the standpoint of player 2, assuming player 1 has a strategy of type $c_1^1(t) = u\,\langle \eta, x(t)\rangle$ for some $u \in \mathbb{R}^+$ and $c_2^1(t) = 0$, deriving

$$c_1^{2*}(t) = \frac{\rho - (\lambda - u)(1-\sigma)}{\sigma}\,\langle \eta, x(t)\rangle, \quad c_2^{2*}(t) = 0.$$

Then $\left(c^{1*}, c^{2*}\right)$ is a Nash equilibrium if and only if $c_1 = c_1^*$ and $c_2 = c_2^*$, that is

$$\begin{cases} w = \frac{\rho-(\lambda-u)(1-\sigma)}{\sigma} \\ u = \frac{\rho-(\lambda-w)(1-\sigma)}{\sigma} \end{cases}$$

which implies

$$w = u = \frac{\rho - \lambda(1-\sigma)}{2\sigma - 1} = z, \quad \beta = z^{\frac{\sigma}{\sigma-1}}$$

and, as a consequence, (11) and (12).

Step 3: Constraints are satisfied. We now check (4) and (8) along the trajectories at equilibrium. Set $y(t) = x_2(t)/x_1(t)$, and note that (8) is, at equilibrium, equivalent to

$$z(1 + \mu y(t)) \le \beta_1, \quad \forall t \ge 0. \tag{15}$$

Since $\dot y(t)/y(t) = \dot x_2(t)/x_2(t) - \dot x_1(t)/x_1(t)$, then $y(t)$ satisfies

$$\dot y(t) = (2\mu z - \alpha)y^2(t) + (\Gamma_2 - \Gamma_1 + 2z)y(t) + \alpha.$$

which has, in view of (9), two stationary solutions, $y_1 < 0$ and $y_2 > 0$, with y_2 an attractor from all initial values $y(0) = x_2^\circ/x_1^\circ$. Moreover, if $y(0) > y_2$ then $y(t) \searrow y_2$, so that $(x_1^\circ, x_2^\circ) \in C_1$ implies that (15) is verified. When instead $y(0) < y_2$, then $y(t) \nearrow y_2$ where y_2 is the unique solution of (9), so that (15) is verified again. Positivity constraints (4) also follow from monotonicity of $y(t)$ and positivity of y_2.

Step 4: Verification Theorem. In order to prove that the solution of the HJB equation is in fact the value function of the problem, one has to prove a verification theorem. The proof of this fact is standard (and we omit it for brevity) once we have shown the transversality condition $\lim_{t\to+\infty} e^{-\rho t} v(x(t)) = 0$. Indeed along the equilibrium trajectories one has $d/(dt)\langle x(t), \eta\rangle = (\lambda - 2z)\langle x(t), \eta\rangle$, so that (13) holds true, and $e^{-\rho t} v(x(t))$ decreases exponentially to zero with rate $-\rho + (1-\sigma)(\lambda - 2z) = -z < 0$. □

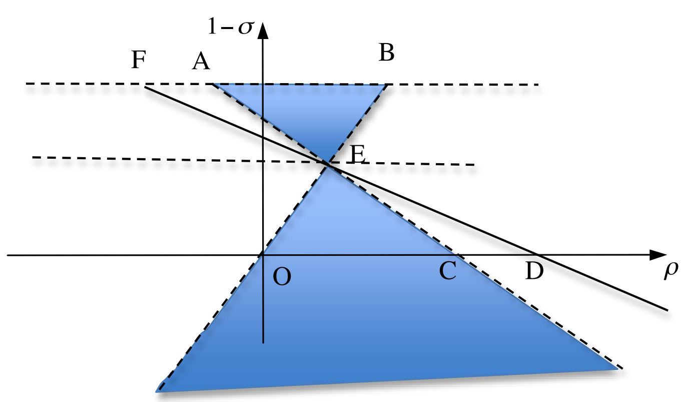

FIGURE 1. Varying parameters ρ and σ satisfying the assumptions of Theorem 3.1. Case $\mu > \frac{b_2}{b_1}$

Remark 3.5 In Figure 1 we represent parameters σ and ρ for which $z \in (0, z_1^*]$ is satisfied (blue region), in the case $\mu > \frac{b_2}{b_1}$. The line $z = 0$ is OB while $z = z_1^*$ is AC, and they cross at $(\lambda/2, 1/2)$ for all β_1. If β_1 increases, then AC rotates counterclockwise, and when $\beta_1 \to +\infty$

then AC tends to coincide with FD. At this limit value, x_1 becomes null. Note that "low intensity harvestig" takes place for couples $(\rho, 1-\sigma)$ close to the OB. On the other hand, the area AEB is where we have the *voracity effect* we better describe in Section 3.4.

Remark 3.6 For a logarithmic utility the situation is similar to that described in Theorem 3.1. For instance, when $\mu > \frac{b_2}{b_1}$ and $0 < z \equiv \rho \leq z_1^*$ the equilibrium is given by

$$\begin{array}{ll} c_1^1(t) = \rho\left(x_1(t) + \mu x_2(t)\right), & c_2^1(t) \equiv 0 \\ c_1^2(t) = \rho\left(x_1(t) + \mu x_2(t)\right), & c_2^2(t) \equiv 0, \end{array} \tag{16}$$

and the utility is $v^2(x_1, x_2) = v^1(x_1, x_2) = \frac{1}{\rho}\left(\frac{\lambda - 2\rho}{\rho} + \ln(b_1\rho) + \ln(x_1 + \mu x_2)\right)$.

Remark 3.7 When $\mu = \frac{b_2}{b_1}$ each player j is indifferent among all the strategies (c_1^j, c_2^j) such that

$$c_1^j(t) + \mu c_2^j(t) = \frac{\rho - (1-\sigma)\lambda}{2\sigma - 1}\left(x_1^\circ + \mu x_2^\circ\right).$$

Since several possible indifferent strategies are possible, the constraint on the initial datum is less stringent than in Proposition 3.1 and in particular a feasible equilibrium in the described set exists as long as at least as one of the second conditions of (9) or (10) is verified. In general, a local linear symmetric Markov equilibrium in which

$$c_1^j(t) = \theta\ \frac{\rho - (1-\sigma)\lambda}{2\sigma - 1}\left(x_1^\circ + \mu x_2^\circ\right)$$

$$c_2^j(t) = (1-\theta)\frac{\rho - (1-\sigma)\lambda}{2\sigma - 1}\ \frac{1}{\mu}\left(x_1^\circ + \mu x_2^\circ\right)$$

and $\theta \in [0, 1]$ exists under conditions less stringent than those in Theorem 3.1. The same argument applies to the case of logarithmic utility discussed in Remark 3.6.

3.2. **The Reservoir Case.** In a second scenario one region is kept as a reservoir and fishing is there forbidden. If that region is the one where agents would not fish even if free to choose, a Markovian equilibrium is obtained as in Section 3.1. If harvesting is forbidden in the zone where players would rather fish, then further investigation is necessary. We carry out the case when $\mu > b_2/b_1$, meaning that players would rather fish in zone 1, but fishing is there forbidden.

Theorem 3.8 *Let z_2^* as defined in* (10)*, and $z = \frac{\rho-(1-\sigma)\lambda}{2\sigma-1}$. Assume $0 < z \leq z_2^*$, $\mu > \frac{b_2}{b_1}$, and (x_1°, x_2°) in $C_2 = \{(x_1, x_2) \in \mathbb{R}_+^2 \ :\ x_1/x_2 \leq \mu(\beta_2/z - 1)\}$. Assume that harvesting is forbidden in zone 1. Then a Markovian equilibrium is given by*

$$\begin{array}{ll} c_1^1(t) = 0, & c_2^1(t) = \frac{z}{\mu}\left(x_1(t) + \mu x_2(t)\right) \\ c_1^2(t) = 0, & c_2^2(t) = \frac{z}{\mu}\left(x_1(t) + \mu x_2(t)\right), \end{array} \tag{17}$$

and the utilities of players at equilibrium are, respectively, $v^1(x_1^\circ, x_2^\circ), v^2(x_1^\circ, x_2^\circ)$, where

$$v^2(x_1, x_2) = v^1(x_1, x_2) = \left(\frac{b_2}{\mu}\right)^{1-\sigma}\frac{(x_1 + \mu x_2)^{1-\sigma}}{z^\sigma(1-\rho)}.$$

The proof is similar to that of Theorem 3.1 and we omit it. A similar result holds in the case when $\mu < b_2/b_1$, and $z \in (0, z_1^*]$, and harvesting is forbidden in zone 2. Extensions to the case of logarithmic utility, and to the case $\mu = \frac{b_2}{b_1}$ are possible, yielding results similar to those mentioned, respectively, in Remarks 3.6 and 3.7.

3.3. **The TURF Case.** In the third considered scenario each player has the right of exclusive exploitation of one of the two regions (say, player 1 of zone 1 and player 2 of zone 2). The evolution of stocks is now described by

$$\begin{cases} \dot{x}_1(t) = (\Gamma_1 - \alpha)x_1(t) + \alpha x_2(t) - c_1(t), & x_1(0) = x_1^\circ \\ \dot{x}_2(t) = (\Gamma_2 - \alpha)x_2(t) + \alpha x_1(t) - c_2(t), & x_2(0) = x_2^\circ \end{cases} \tag{18}$$

where $c_i(t)$ is the harvesting intensity chosen by player i, in zone i, at time t, so that player i is maximizing

$$J^i(c_i) = \int_0^{+\infty} e^{-\rho t}\, \frac{(b_i c_i(t))^{1-\sigma}}{1-\sigma} dt.$$

Consider $z \in [0, \frac{\alpha}{\mu}]$, and the set of solutions $y \in \mathbb{R}$ of the system

$$\begin{cases} \Gamma_1 - \alpha - z + (\alpha - z\mu)y = \Gamma_2 - \alpha - z + (\alpha - \frac{1}{\mu}z)\frac{1}{y} \\ z(1+\mu y) \leq \beta_1 \\ z(\frac{1}{y\mu} + 1) \leq \beta_2 \\ y \geq 0. \end{cases} \tag{19}$$

Define z_3^* as the maximum z for which the previous set is nonempty. Note that the second and third inequality imply $y \in [z(\beta_2 - z)/\mu, (\beta_1/z - 1)/\mu]$, and that this interval tends to the positive halfline for $z \to 0^+$.

Theorem 3.9 *Let $z = \frac{\rho - (1-\sigma)\lambda}{2\sigma - 1}$, and assume $0 < z \leq z_3^*$, and (x_1°, x_2°) in*

$$C_3 = \left\{ (x_1, x_2) \in \mathbb{R}_+^2 \; : \; \frac{1}{\mu}\frac{z}{\beta_2 - z} \leq \frac{x_2}{x_1} \leq \frac{1}{\mu}\frac{\beta_1 - z}{z} \right\}.$$

Then a Markovian equilibrium is given by

$$c_1(t) = z\left(x_1(t) + \mu x_2(t)\right), \qquad c_2(t) = \frac{z}{\mu}\left(x_1(t) + \mu x_2(t)\right), \tag{20}$$

and the utilities of players at equilibrium are, respectively, $v^1(x_1^\circ, x_2^\circ), v^2(x_1^\circ, x_2^\circ)$, where

$$v^1(x_1, x_2) = b_1^{1-\sigma}\, \frac{(x_1 + \mu x_2)^{1-\sigma}}{z^\sigma(1-\sigma)}, \qquad v^2(x_1, x_2) = \left(\frac{b_2}{\mu}\right)^{1-\sigma} \frac{(x_1 + \mu x_2)^{1-\sigma}}{z^\sigma(1-\sigma)} \tag{21}$$

Proof. We procede as in the proof of Theorem 3.1. Assume that the strategy of player 2 is of type $c_2(t) = w\langle \eta, x(t)\rangle$ with $w > 0$ given. Then player 1 has to choose c_1 so to maximize $J^1(c^1)$ and subject to

$$\dot{x}(t) = Mx(t) - w\left\langle \eta, x(t)\right\rangle e_2 - c_1(t)e_1, \quad x(0) = x^\circ.$$

The Hamiltonian function associated to the problem, for $(x, p) \in \mathbb{R}^2 \times \mathbb{R}^2$, is

$$\begin{aligned} H(x,p) &= \langle p, Mx\rangle - w\langle \eta, x\rangle \langle p, e_2\rangle + \sup_{c_1 \geq 0}\left\{ \frac{(b_1 c_1)^{1-\sigma}}{1-\sigma} - c_1 p_1 \right\} \\ &= \langle p, Mx\rangle - w\langle \eta, x\rangle p_2 + \frac{\sigma}{1-\sigma}\left(\frac{p_1}{b_1}\right)^{1-\frac{1}{\sigma}} \end{aligned}$$

with supremum is attained at

$$c_1 = \frac{1}{b_1}\left(\frac{p_1}{b_1}\right)^{-\frac{1}{\sigma}} \tag{22}$$

The HJB equation $\rho v(x) = H(x, \nabla v(x))$ has a solution of type $v(x) = \frac{1}{1-\sigma}\left(\beta \langle \eta, x\rangle\right)^{1-\sigma}$, with

$$\beta = b_1 \left(\frac{\rho - (\lambda - \mu w)(1-\sigma)}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}.$$

as it is easy to check by direct proof. From (22), the optimal fishing strategy for player 1 is

$$c_1^*(t) = \frac{\rho - (\lambda - w\mu)(1-\sigma)}{\sigma} \langle \eta, x(t)\rangle .$$

We may repeat the argument from the standpoint of player 2, assuming player 1 has a strategy of type $c_1(t) = u\langle \eta, x(t)\rangle$ for some $u \in \mathbb{R}^+$, deriving a solution of the HJB equation of type $v(x) = (1-\sigma)^{-1}(\gamma\langle \eta, x\rangle)^{1-\sigma}$, with

$$\gamma = \frac{b_2}{\mu}\left(\frac{\rho - (1-\sigma)(\lambda - u)}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}$$

and the optimal strategy

$$c_2^*(t) = \frac{\rho - (\lambda - u)(1-\sigma)}{\sigma\mu} \langle \eta, x(t)\rangle .$$

Then (c_1^*, c_2^*) is a Nash equilibrium if and only if $c_1 = c_1^*$ and $c_2 = c_2^*$, that is

$$\begin{cases} w = \frac{\rho-(\lambda-u)(1-\sigma)}{\sigma\mu} \\ u = \frac{\rho-(\lambda-\mu w)(1-\sigma)}{\sigma} \end{cases}$$

which implies

$$\mu w = u = \frac{\rho - \lambda(1-\sigma)}{2\sigma - 1} = z, \quad \beta = b_1\, z^{\frac{\sigma}{\sigma-1}}, \quad \gamma = \frac{b_2}{\mu}\, z^{\frac{\sigma}{\sigma-1}}$$

and, as a consequence, (20) and (21).

We now check (4) and (8) along the trajectories at equilibrium. Set $y(t) = x_2(t)/x_1(t)$, and note that (8) is, at equilibrium, equivalent to

$$\begin{cases} z(1+\mu y(t)) \le \beta_1, & \forall t \ge 0 \\ \frac{z}{\mu}\left(\frac{1}{y(t)} + \mu\right) \le \beta_2, & \forall t \ge 0 \end{cases} \tag{23}$$

and that $y(t)$ satisfies

$$\dot{y}(t) = -(\alpha - \mu z)y^2(t) + (\Gamma_2 - \Gamma_1)y(t) + \alpha - \frac{z}{\mu}.$$

The above equation has, in view of (19), two stationary solutions $y_1 < 0$ and $y_2 > 0$, with y_2 an attractor. Hence positivity constraints (4) are verified. Now note that if $y(0) > y_2$ then $y(t) \searrow y_2$. Hence, since $y(0) = x_2^\circ/x_1^\circ$, and $(x_1^\circ, x_2^\circ) \in C$, we have

$$z(1+\mu y(t)) \le z\left(1 + \mu\frac{x_2^\circ}{x_1^\circ}\right) \le \beta_1, \ \forall t \ge 0.$$

On the other hand, $y(t) \ge y_2$ for all $t \ge 0$ and y_2 solves (19), so that

$$\frac{z}{\mu}\left(\frac{1}{y(t)} + \mu\right) \le \frac{z}{\mu}\left(\frac{1}{y_2} + \mu\right) \le \beta_2 \ \forall t \ge 0.$$

and (23) holds true along the whole trajectory. Similarly one proves that (23) are satisfied in the case $y(0) < y_2$. Finally the proof of the transversality condition is similar to that of Theorem 3.1 and we omit it for brevity. □

3.4. **Impact of the policies in the low harvesting intensity case.** In the previous subsections we analysed the cases where: (a) players are free to choose to fish in both zone 1 and 2 (no harvesting policies), (b) a policy is introcuced by creation of a reservoir; (c) a policy is introduced by creation of TURFS. We now intend to compare how welfares of players change after the introduction of policies (b) and (c) in comparison to the case (a) of absence of policies.

To fix ideas, assume that $\mu > \frac{b_2}{b_1}$, set $z = \frac{\rho-\lambda(1-\sigma)}{2\sigma-1}$, and consider a set of parameters such that $0 < z \le \min\{z_1^*, z_2^*, z_3^*\}$, where z_i^* were defined in (9), (10) and (19). Assume also that the initial condition (x_1°, x_2°) belongs to $C_1 \cap C_2 \cap C_3$ where C_i were defined in theorems 3.1, 3.8 and 3.9.

In terms of impact on the welfare of the two players, the introduction of policies is not very encouraging. If in the case of no policy (common property case) we have

$$v_{cp}^2(x_1, x_2) = v_{cp}^1(x_1, x_2) = b_1^{1-\sigma} \; \frac{(x_1 + \mu x_2)^{1-\sigma}}{z^\sigma(1-\sigma)}$$

then:

- introducing the reservoir strictly decreases the welfare of all players, changing it from v_{cp}^j to
$$v_{res}^2(x_1, x_2) = v_{res}^1(x_1, x_2) = \left(\frac{b_2}{\mu}\right)^{1-\sigma} \frac{(x_1 + \mu x_2)^{1-\sigma}}{z^\sigma(1-\sigma)}$$
with $\mu > \frac{b_2}{b_1}$ implying $v_{res}^j < v_{cp}^j$.
- introducing TURFS leaves unchanged the utility of the player assigned to zone 1 (player 1, according to our choice) while it reduces the welfare of the player assigned to zone 2 (player 2) from v_{cp}^2 to
$$v_{turf}^2(x_1, x_2) = \left(\frac{b_2}{\mu}\right)^{1-\sigma} \frac{(x_1 + \mu x_2)^{1-\sigma}}{z^\sigma(1-\sigma)}$$
with $v_{turf}^2 = v_{res}^2 < v_{cp}^2$.

The introduction of policies is not satisfactory even in terms of safeguard of the natural resource. Indeed, even if the dynamics of the two stocks $x_1(t)$ and $x_2(t)$ can be different in the three cases (a) (b) (c), at the equilibrium the total stock of the resource $x_1(t) + x_2(t)$ satisfies (13) in all cases so that

$$\frac{1}{\mu}(x_1^\circ + \mu x_2^\circ)e^{tg} \le x_1(t) + x_2(t) \le (x_1^\circ + \mu x_2^\circ)e^{tg}.$$

where $g = \lambda - 2z = \frac{\lambda-2\rho}{2\sigma-1}$ may be positive or negative. This means that, although these policies could affect the level of the resource, they cannot affect its growth rate. In particular, when the resource is eventually exhausted in the common property case, exhaustion takes place even when the described policies are enforced.

We finally analyse the effect of policies that increase the rate of growth or reduce resource mobility between regions. We start by noting that, as in Tornell and Lane (1999), a voracity effect characterizes our interior equilibrium when $1 - \sigma > \frac{1}{2}$ (this is the analog of Tornell and Lane's (1999) condition (21) on p. 30). In this case, which occurs when the preference parameters belong to triangle AEB in Figure 1, a positive technological shock, *i.e.* an increase in the value of the eigenvalue λ, reduces the equilibrium rate of growth. Each agent reacts to the the shock by increasing extraction more than proportionally and this in the end results in a fall of the post-extraction growth of the resource stocks.

Indeed, note that the dominant root λ, has a simple interpretation in terms of the technological primitives of the model as a weighted average of the two intrinsic rates of growth Γ_1 and Γ_2, with the weights depending, if $\Gamma_1 \neq \Gamma_2$, on the diffusivity coefficient α (of course if $\Gamma_1 = \Gamma_2 \equiv \Gamma$ then $\lambda = \Gamma$). To see this, we eliminate μ between the two equations

$$\Gamma_1 + \alpha(\mu - 1) = \lambda \tag{24}$$

$$\Gamma_2\mu + \alpha(1 - \mu) = \lambda\mu \tag{25}$$

reaching in the case $\Gamma_1 < \Gamma_2$ the relationship

$$\alpha = \frac{(\lambda - \Gamma_1)(\Gamma_2 - \lambda)}{2\lambda - (\Gamma_1 + \Gamma_2)}. \tag{26}$$

Taking the derivative of this function we establish $\frac{d\alpha}{d\lambda} < 0$. Moreover, since from equations (24) and (25) we have $\lambda \in [\Gamma_1, \Gamma_2]$, we note that $\alpha > 0$ implies the tighter restriction $\lambda \in (\frac{\Gamma_1+\Gamma_2}{2}, \Gamma_2)$. Then, it can be easily seen that $\lambda \to \Gamma_2$ for $\alpha \to 0$ and that $\lambda \to \frac{\Gamma_1+\Gamma_2}{2}$ for $\alpha \to \infty$. The meaning of these results is that increasing resource mobility between the two regions diminishes the maximum rate of growth that can be obtained in the system.

Using again equations (24) and (25), we can also establish that λ is increasing with the implicit rates of growth of the two regions. This follows from the fact that μ increases with Γ_2 and decreases with Γ_1. That λ increases with Γ_1 then follows from equation (24), and that it increases also with Γ_2 follows from (25).

To summarize, if voracity prevails, then positive technological shocks that increase the implicit rates of growth of two regions or reduce resource mobility between the regions lead to strategic responses that actually reduces growth.

4. Effectiveness of the policy in the high harvesting intensity case

4.1. **A first example.** We begin by looking at a very specific case, showing which are the mechanisms at work when the constraint (8) are binding. Then we will discuss how the mechanisms at work are effective in a more general situation.

We assume $\Gamma_1 = \Gamma_2 = 2/3$, $\beta_1 = \beta_2 = 1$, $\alpha = 2/3$, $b_1 = b_2 = 1$ and $\rho \in (1/3, 1/2)$, to fix ideas $\rho = 5/12$. As a consequence, $\lambda = \Gamma$, $\mu = 1$. We also assume a logarithmic utility (6).

Firstly we consider the case in which the two players are free to fish in the two regions and free to choose how to distribute their effort among the two. Note that Remark 3.6 and Remark 3.7 apply, as long as the equilibria there described are feasible. That means that, if we denote with $s(t)$ the overall stock of fish, namely $s(t) = x_1(x) + x_2(t)$ then, for any $\theta \in [0, 1]$, a Markov equilibrium would be given by c_i^j satisfying

$$c_1^j(t) + c_2^j(t) \equiv \theta\rho s(t) + (1 - \theta)\rho s(t) = \rho s(t), \quad \forall t \geq 0, \ j = 1, 2.$$

We then check that with this choice of parameters and for any $(x_1^\circ, x_2^\circ) \in \mathbb{R}_+^2$ the equilibirum satisfies all constraints. In particular (8) reads as

$$\rho s(t) \left(\frac{\theta}{x_1(t)} + \frac{1 - \theta}{x_2(t)} \right) \leq 1 \tag{27}$$

and since $\frac{\theta}{x_1} + \frac{1-\theta}{s-x_1}$ is maximal when $x_1 = \theta s$, the constraint is more stringent when $x_1(t) = \theta s(t)$ and $x_2(t) = (1 - \theta)s(t)$, which plugged into (27) give the condition $\rho \leq 1/2$, satisfied by assumption. The positivity of stocks follows trivially.

Note now that $\dot{s}(t) = gs(t)$, with $g = \alpha - 2\rho < 0$ given that $\rho > 1/3$, so that the overall stock $s(t) = (x_1^\circ + x_2^\circ)e^{gt}$ is asimptotically decreasing to 0 (extinction). The corresponding utilities of players are both equal to

$$J^j(c^j) = \int_0^{+\infty} e^{-\rho t} \ln(\rho s(t)) dt = \frac{\ln(\rho(x_1^\circ + x_2^\circ)) + \lambda - 2\rho}{\rho}. \tag{28}$$

Secondly, we consider the case when a marine reserve is set up in zone 2, so that forcedly $c_2^1 = c_2^2 \equiv 0$. To fix ideas, we suppose $x_2^\circ \geq 2x_1^\circ$. We start by showing that strategies

$$c_1^1(t) = c_1^2(t) = x_1(t) \tag{29}$$

constitute a linear Markov equilibrium. To this extent, we assume player 2 adopts the strategy $c_1^2(t) = x_1(t)$ so that player 1 maximizes $J^1(c^1)$ subject to

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + \frac{2}{3}x_2(t) - c_1^1(t), & t \geq 0 \\ \dot{x}_2(t) = \frac{2}{3}x_1(t), & t \geq 0 \\ (x_1(0), x_2(0)) = (x_1^\circ, x_2^\circ) \end{cases} \tag{30}$$

Note that (8) is satisfied for player 2, while it imposes $c_1^1(t) \leq x_1(t)$ on player 1. The HJB equation associated to (30) and $J^1(c^1)$ is

$$\rho v(x_1, x_2) = \left\langle \begin{pmatrix} -x_1 + \frac{2}{3}x_2 \\ \frac{2}{3}x_1 \end{pmatrix}, \nabla v(x_1, x_2) \right\rangle + \sup_{c_1^1 \in [0, x_1]} \left\{ \ln(c_1^1) - \partial_{x_1} v(x_1, x_2) c_1^1 \right\} \tag{31}$$

We show next that $c_1(t) = x_1(t)$ is the best response of player 1. If that is true, the value function of the problem is

$$V(x^\circ) = \int_0^{+\infty} e^{-\rho t} \ln(x_1(t)) dt = \int_0^{+\infty} e^{-\rho t} \ln\langle e^{tA} e_1, x^\circ \rangle dt \tag{32}$$

where $x_1(t)$ is the first component of the solution $x(t) = e^{tA}x^\circ$ to

$$\begin{cases} \dot{x}_1(t) = -2x_1(t) + \frac{2}{3}x_2(t), & t > 0 \\ \dot{x}_2(t) = \frac{2}{3}x_1(t), & t > 0 \\ (x_1(0), x_2(0)) = x^\circ \end{cases} \tag{33}$$

where $A = \begin{pmatrix} -2 & \frac{2}{3} \\ \frac{2}{3} & 0 \end{pmatrix}$, that is

$$x_1(t) = \langle e^{tA} x^\circ, e_1 \rangle = \langle e^{tA} e_1, x^\circ \rangle.$$

Note also that $x_2(t)/x_1(t)$ satisfies an ordinary differential equation with stationary solutions $3/2 \pm \sqrt{13}/2$ with $3/2 + \sqrt{13}/2$ an attractor. This implies, in particular, that $e^{tA}e_1$ is a vector with positive coordinates, since it is the solution to (33) with initial condition $(1, 0)$.

Next we prove V solves the HJB equation. The partial derivatives of V are given by

$$\partial_{x_1} V(x) = \int_0^{+\infty} e^{-\rho t} \frac{\langle e^{tA} e_1, e_1 \rangle}{\langle e^{tA} e_1, x \rangle} dt, \quad \partial_{x_2} V(x) = \int_0^{+\infty} e^{-\rho t} \frac{\langle e^{tA} e_1, e_2 \rangle}{\langle e^{tA} e_1, x \rangle} dt$$

Note that, since we are assuming $x_2^\circ \geq 2x_1^\circ \geq 0$, that remains true along the entire trajectory, and we have

$$0 \leq \partial_{x_1} V(x) \leq \frac{1}{x_1} \int_0^{+\infty} e^{-\rho t} \frac{\langle e^{tA} e_1, e_1 \rangle}{\langle e^{tA} e_1, e_1 + 2e_2 \rangle} dt =: \frac{1}{x_1} I < \frac{1}{x_1}$$

where the value of I is strictly less than 1 ($I \simeq 0.69$, with $\rho = 5/12$). Thus

$$\arg \max_{c_1^1 \in [0,x_1]} \left(\ln(c_1^1) - \partial_{x_1} V(x_1, x_2) c_1^1 \right) = x_1$$

and

$$\max_{c_1^1 \in [0,x_1]} \left(\ln(c_1^1) - \partial_{x_1} V(x_1, x_2) c_1^1 \right) = \ln(x_1) - \partial_{x_1} V(x_1, x_2) x_1.$$

Using this fact we can see that the right hand side in (31) is equal to

$$(34) \quad \int_0^{+\infty} e^{-\rho t} \frac{\left\langle \left(-x_1 + \frac{2}{3} x_2\right) e_1 + \frac{2}{3} x_1 e_2, e^{tA} e_1 \right\rangle}{\langle e^{tA} e_1, x \rangle} dt + \ln(x_1) + \int_0^{+\infty} e^{-\rho t} \frac{\left\langle -x_1 e_1, e^{tA} e_1 \right\rangle}{\langle e^{tA} e_1, x \rangle} dt$$
$$= \int_0^{+\infty} e^{-\rho t} \frac{\left\langle Ax, e^{tA} e_1 \right\rangle}{\langle e^{tA} e_1, x \rangle} dt + \ln(x_1)$$

so that (31) is verified if and only if

$$\rho \int_0^{+\infty} e^{-\rho t} \ln \left\langle e^{tA} e_1, x \right\rangle dt = \int_0^{+\infty} e^{-\rho t} \frac{\left\langle Ax, e^{tA} e_1 \right\rangle}{\langle e^{tA} e_1, x \rangle} dt + \ln(x_1)$$

that is if and only if

$$\int_0^{+\infty} -\frac{d}{dt} \left[e^{-\rho t} \ln \left\langle e^{tA} e_1, x \right\rangle \right] dt = \ln(x_1)$$

which is trivially satisfied. By means of a standard verification theorem one can prove that $V(x)$ defined in (32) is the optimal output of player 1 so that the proof that $c_1^1(t) = x_1(t)$ is the optimal response of player 1 is complete. Simmetrically operating from the standpoint of player 2, one finally derives that (29) represents a linear symmetric Markov equilibrium. The dynamics of the system along the equilibrium is given by (33) with $x_2^\circ \geq 2x_1^\circ \geq 0$.

Finally we show that the reservoir policy preserves fish from extinction for initial data x° with $x_2^\circ \geq \frac{\sqrt{3}+3}{2} x_1^\circ$. Note that the eigenvalues of A are $-(\sqrt{13}+3)/3 < 0$ and $(\sqrt{13}-3)/3 > 0$ associated respectively to eigenvectors $v_1 = (-(\sqrt{13}+3)/2,\ 1)'$ and $v_2 = ((\sqrt{13}-3)/2,\ 1)'$. Then, for t growing to $+\infty$, one has

$$x(t) = e^{tA} x^\circ = \frac{\langle x^\circ, v_1 \rangle}{|v_1|^2} e^{-\frac{\sqrt{13}+3}{3} t} v_1 + \frac{\langle x^\circ, v_2 \rangle}{|v_2|^2} e^{\frac{\sqrt{13}-3}{3} t} v_2 \sim \frac{\langle x^\circ, v_2 \rangle}{|v_2|^2} e^{\frac{\sqrt{13}-3}{3} t} v_2$$

where $\langle x^\circ, v_2 \rangle v_2$ has positive components along both coordinate axes if and only if $x_2^\circ \geq (\sqrt{3}+3)/2\, x_1^\circ$. Hence, in the long run the total fish stock – and also the two stocks $x_1(t)$ and $x_2(t)$ separately – grow at a positive rate.

We can also compare the welfare of the two players. Note that with no reservoir (28) utilities depended from the overall stock $s(0) = x_1^\circ + x_2^\circ$, while in presence of a marine reserve the result may change, for better or worse, depending on the initial distribution among the two regions. In particular, for some sets of initial condition welfare increases in presence of the marine reserve (to have a numeric example, take $(x_1^\circ, x_2^\circ) = (1, 2)$).

4.2. **The mechanisms at work in the example and more general situations: a discussion.** Let us summarize why both spatial property rights and reserves are at best ineffective when the coefficient of the extraction/consumption policy of the interior symmetric equilibrium is low. Note that granting secure extraction rights to player 1 in zone 1 and to player 2 in zone 2 is equivalent to setting $b_1 = 0$ in the control problem of 2 and $b_2 = 0$ in the control problem of player 1. On the other hand, the creation of a reserve in zone 1 is equivalent to setting $b_1 = 0$ for both players. If $\frac{\rho-(1-\sigma)\lambda}{2\sigma-1} \leq \min\{z_1^*, z_2^*\}$, then each player reacts to these new regulations by adapting the linear policy that characterizes our interior

Markov equilibrium to the new set of parameters. The rate of growth of the resource is unaffected by this change od behavior and the welfare of each player is at best unchanged. In the above specific example we have $z_1^* = z_2^* = \frac{1}{3}$ and a logarithmic utility (i.e., $1-\sigma=0$), so we find policy-ineffectiveness if $\rho \leq \frac{1}{3}$.

However, we have also shown that if $\frac{\rho-(1-\sigma)\lambda}{2\sigma-1} > \min\{z_1^*, z_2^*\}$, but the interior equilibrium still exists under the common property or TURF regimes, creating a nature reserve in the zone with the maximum z^* can increase the rate of growth of the resource and the welfare of the players. Indeed an extreme equilibrium in which the effort constraint is binding can exist in this case. Although a complete analysis of the existence conditions of this extreme equilibrium is not given in this paper, our example is sufficient to show that the set parameters for which it exists is nonempty. More in general, even a TURF management system can be somewhat effective if, with its institution, the effort constraint of an player is binding.

5. Conclusions

In this paper we develop a continuous time model in which two players compete to exploit a resource which can move and diffuse among two zones. It provides an analytically tractable model which can capture the difficulties to design, in the context of spatially distributed resource, an efficient systems of spatial property rights.

We compare the behaviors of the players in a initial common property case, where they can decide both where and how much to harvest, with their choices in two main policies-constrained cases: the first where the regulator can establish a natural reserve (*i.e.*, where harvesting in one of the two zones is forbidden) and the second where each player has an exclusive exploitation rights on one of the two locations. We show that the policies are completely ineffective (and also harmful in terms of utilities of the players) when the conditions lead the players to choose a low exploitation intensity of the resource, while they can be useful for safeguarding the resource and also in terms of players' utility in case of high exploitation intensity.

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