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Working Paper Series

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**Working Paper n. 4/2017
March 2017**

ISSN: 2239-2734

PSO-based tuning of MURAME parameters for creditworthiness evaluation of Italian SMEs

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(March 2017)

Abstract. In this work we use a MultiCriteria Decision Analysis (MCDA) model to evaluate the creditworthiness of a sample of Italian Small and Medium-sized Enterprises (SMEs), on the basis of their balance sheet data provided by the AIDA database. Our methodology is able to consider simultaneously different factors affecting the firms' solvency level, and can produce results in terms of scoring, classification into homogeneous rating classes and migration probabilities. In this contribution we compare the results obtained considering two scenarios. On one hand, we experience an exogenous specification of the parameters that describe the preference structure implicit in the used MCDA model. On the other hand, we consider the results obtained using a preference disaggregation method to endogenously determine some of the model parameters. Because of the complexity of the obtained mathematical programming problem, we use an heuristic methodology, namely Particle Swarm Optimization (PSO), which provides a reasonable compromise between the quality of the solution and the computational burden.

Keywords: MultiCriteria Decision Analysis, Small and Medium-sized Enterprises, Credit Risk, Particle Swarm Optimization.

JEL Classification Numbers: C38, C61, C63.

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In this paper we evaluate the creditworthiness of a large sample of Italian Small and Medium-sized Enterprises (SMEs) during a period embracing the beginning phase of the recent economic and financial crisis, from 2006 to 2008, by using a Multi-Criteria Decision Analysis (MCDA) approach. Italy is among the countries in the European Union with the highest number of micro enterprises and SMEs (see [20]). In the second quarter of 2008 Italy faced a recession period, and a remarkable number of SMEs went bankrupt or liquidate their operations. In this paper we will mainly focus our attention on improving some of the results obtained in [24] in terms of classification of bankrupt firms.

In the literature, the topic of creditworthiness evaluation of SMEs has been mainly addressed by using statistical and econometric techniques. Some of the recent contributions on this topic have been stimulated by the new Basel capital accord Basel II (see [9]), that permitted banks to distinguish separately the exposures to SMEs. Along this line of research, we can mention [30], that proposed an internal credit risk model for SME loans and compared the results obtained in terms of capital requirements with those derived from the advanced internal ratings-based (IRB) approach defined under Basel II. The effects of Basel II on the bank capital requirements have also been investigated by [3] in case of US, Italian and Australian SMEs. Moreover, the relationship between probabilities of default and asset correlations, and its comparison with what assumed by Basel II, have been analyzed by [31] for a set of German and French SMEs.

In particular, we underline the importance of modeling credit risk for SMEs, an interesting topic for our objective here. A pioneering analysis was made by [36]. However, this issue has received an increasing attention since the papers of [3] and [4] have explicitly pointed out the importance to develop credit risk models specifically for SMEs. Particularly, [4] identified a set of financial ratios that could influence SMEs creditworthiness. They used a logit regression technique and their empirical analysis focuses on data of U.S. SMEs showed the superiority of the model proposed in terms of default prediction accuracy, compared to a generic corporate model.

It is interesting to note that, even if statistical and econometric techniques have been mainly adopted for modeling credit risk, recently new methodologies have attracted the attention in the field of creditworthiness assessment, especially in the case of SMEs. The survey made by [58] reviewed credit and behavioral scoring[1], and underlined the importance not only of the traditional statistical tools but also of operational research methods based on mathematical programming techniques and artificial intelligence technologies. Following this stream of research, the recent contribution of [2] proposes a credit scoring model that simultaneously uses artificial neural network and fuzzy logic, whereas the most recent research trends on the use of evolutionary computing techniques for credit scoring are explored in [49] and in [23].

Indeed, both researchers and practitioners have recently been tackling the problem of evaluating credit risk through approaches which are based on robust quantitative methods and which are able to take simultaneously into account the largest possible information. Therefore, it is possible to observe a growing interest towards the use of Multi-Criteria De-

[1]The aim of the credit scoring models is basically to sort the applicants into two sets: those having high probability to maintain financial obligations and those having a low probability to maintain their obligations.

cision Analysis (MCDA) capable of providing general support to complex financial decisions (see [56], [62]). MCDA is a well-known family of approaches for supporting decision making in evaluating alternatives taking into account multiple (often conflicting) criteria. Many methodologies which fall within MCDA have been widely adopted to support several kinds of real-life decision problems, as showed in [38], including financial decisions. Among the financial applications of MCDA, those regarding creditworthiness evaluations play a relevant role (see [47], [33], [34], [8], [35], [39], [25], [23]). Moreover, a few recent studies built MCDA models to specifically evaluate SMEs creditworthiness. A first effort to use MCDA to classify SMEs into fixed homogeneous classes has been undertaken by [60], with an application to a set of 143 Greek industrial SMEs. More recently, [7] has employed a multicriteria approach to build a credit rating model that assigns innovative SMEs into risk categories, and has provided an application to 4 innovative SMEs with headquarters in Italy. Also the recent paper [23] used MCDA to deal with a creditworthiness evaluation problem of both large enterprises and SMEs, within the framework of preference disaggregation that aims at specifying the preference model of the decision maker.

This contribution extends the analysis presented in [23], where a ranking problem was considered, by employing a preference disaggregation approach in the case of a more complex creditworthiness classification problem. The methodology is then applied to the same data set used in [24], in order to improve the results obtained there with respect to the correct classification of bankrupt firms.

The remainder of this contribution is organized as follows. Section 1 describes the specific MCDA technique, namely the MURAME one, that we adopted to analyze the creditworthiness of Italian SMEs. In Section 2 we present the data used and we tackle the problem of the endogenous determination of some parameters of the adopted model, using an evolutionary computation approach. In Section 3 we present the results obtained in Section 2, and we compare them with the ones obtained using an exogenous determination of the same parameters as in [24]. Finally, in Section 4 some considerations conclude the study.

1 The MURAME methodology

MURAME is a multicriteria methodology that allows to obtain a scoring and consequently a complete ranking of a set of alternatives $A = \{a_1, \dots, a_m\}$, on the basis of a set of given criteria. It consists of a combination of two well-known multicriteria methods, ELECTRE III ([54]) and PROMETHEE ([16]). In order to determine a complete ranking of alternatives, in accordance with the outranking-based approaches, MURAME considers a series of indices (concordance, discordance and outranking indices) which lead to the computation of a final score for each alternative. It means that an alternative having a higher score should be considered to be better than another with lower score. The method makes it possible to evaluate the creditworthiness of the companies asking for bank loans on the basis of various indicators. In particular, the MURAME-based approach used in this study allows us to rank the firms according to their credit risk features, to sort them into a prefixed number of homogeneous creditworthiness classes and to calculate the probabilities of migration over time from one generic rating class to another.

It is worth emphasizing that the set of indicators is not determined exogenously, as happens in other methodologies for creditworthiness evaluation, but it can be specified by the Decision Maker (DM). Therefore, the methodology makes it possible to take into consideration the DM's preferences and even gives the possibility to the DM not only to express a strong preference towards a certain applicant or to consider two applicants indifferent from the point of view of their creditworthiness, but also to express simply a weak preference, that is a sort of indecision between two applicants (a quite realistic preference relation). Indeed, how to model preferences is a crucial question in decision-making problems. We refer the reader to [51] for an overview of different types of preference structures and for a discussion of the main issues related to preference modeling. We remind that in classical preference systems there are no thresholds and weights, and that the DM, when comparing two alternatives a_i, $a_k \in A$, with i, $k = 1, \dots, m$ and $i \neq k$, either states that one alternative is preferred to the other or shows the indifference between them[2]. I.e., there is not uncertainty in judgments.

Unlike the approaches based on classical preference structure, ELECTRE III and MURAME make both use of the concepts of indifference, preference and veto thresholds, allowing therefore to consider also the case of hesitation in which the DM is not completely sure to prefer a given alternative to another one. This leads to the concept of "weak preference" which deals with the uncertainty on the decision-making process between indifference and strict preference. In the following we describe such a non-classical preference structure in which the case of hesitation is taken into account.

Denoting by p_j the preference threshold and by q_j the indifference threshold associated to the criterion I_j, $j = 1, \dots, n$, with $0 \leq q_j \leq p_j$, the following preference relations with respect to I_j are considered:

$$
\begin{array}{lll}
a_i \ \mathbf{P} \ a_k \ (a_i \text{ is strictly preferred to } a_k) & \text{iff} & g_{ij} > g_{kj} + p_j \\
a_i \ \mathbf{Q} \ a_k \ (a_i \text{ is weakly preferred to } a_k) & \text{iff} & g_{kj} + q_j \leq g_{ij} \leq g_{kj} + p_j \\
a_i \ \mathbf{I} \ a_k \ (a_i \text{ is indifferent to } a_k) & \text{iff} & |g_{ij} - g_{kj}| \leq q_j
\end{array}
$$

where a_i, $a_k \in A$, g_{ij} represents the score of the alternative a_i in relation to criterion I_j (assumed to be maximized), and **P**, **Q** and **I** indicate the preference, the weak preference and the indifference relation with respect to I_j, respectively.

In the first phase, the MURAME aims at defining an outranking relation by building for each $a_i, a_k \in A$, with $i \neq k$, an outranking (or credibility) index.

Let us start by defining for each criteria the local concordance $C_j(a_i, a_k)$ and the local discordance $D_j(a_i, a_k)$ indexes as follows:

$$
C_j(a_i, a_k) = \begin{cases} 1 & \text{if } g_{kj} \leq g_{ij} + q_j \\ 0 & \text{if } g_{kj} \geq g_{ij} + p_j \\ \frac{g_{ij} - g_{kj} + p_j}{p_j - q_j} & \text{otherwise} \end{cases} \tag{1}
$$

and

[2]For simplicity's sake, in the following we omit to specify "with i, $k = 1, \dots, m$ and $i \neq k$", unless it creates interpretative problems.

$$D_j(a_i, a_k) = \begin{cases} 0 & \text{if } \; g_{kj} \leq g_{ij} + p_j \\ 1 & \text{if } \; g_{kj} \geq g_{ij} + v_j \\ \frac{g_{kj} - g_{ij} - p_j}{v_j - p_j} & \text{otherwise} \end{cases}, \tag{2}$$

where p_j and q_j are respectively the preference and the indifference thresholds defined above, and v_j in (2), while $v_j \geq p_j \geq q_j \geq 0$, is the so-called veto threshold associated to the criterion I_j. In particular, v_j represents the power given to I_j to put its veto when the difference between g_{kj} and g_{ij} is greater than itself, forcing the local discordance index to reach its maximal value of 1.

Let us continue by building the *global concordance* index $C(a_i, a_k)$ by aggregating as follows the local concordance indexes:

$$C(a_i, a_k) = \sum_{j=1}^{n} w_j C_j(a_i, a_k), \tag{3}$$

where $w_j \geq 0$, with $\sum_{j=1}^{n} w_j = 1$, represents the weight associated to criterion I_j.

Let us conclude by building for each $a_i, a_k \in A$ an outranking (or credibility) index $O(a_i, a_k)$ computed as follows:

$$O(a_i, a_k) = \begin{cases} C(a_i, a_k) & \text{if } \; D_j(a_i, a_k) \leq C(a_i, a_k) \;\; \forall \, j \\ C(a_i, a_k) \prod_{j \in T} \frac{1 - D_j(a_i, a_k)}{1 - C(a_i, a_k)} & \text{otherwise} \end{cases} \tag{4}$$

where $T \subseteq \{1, \ldots, n\}$ denotes the subset of criteria for which $D_j(a_i, a_k) > C(a_i, a_k)$. We can see that the outranking index is equal to the global concordance $C(a_i, a_k)$, unless the performance of an alternative with respect to at least a criterion is so bad that it poses a veto to the global outranking relation, so that the outranking index decreases. If there is maximum discordance for a single criterion, that is $D_j(a_i, a_k) = 1$ for a given j, the outranking index (4) is equal to zero.

In the second phase, the MURAME computes for each alternative a_i the following final score, the so-called *net flow*:

$$\varphi(a_i) = \sum_{k \neq i} O(a_i, a_k) - \sum_{k \neq i} O(a_k, a_i), \tag{5}$$

where $O(a_i, a_k)$ is the outranking index computed in (4).

Finally, we notice that a complete ranking of the alternatives $\{a_1, \ldots, a_m\}$ is obtained by ordering them according to the decreasing values of the final score (5).

The objectives of the last phase are first to classify the applicants into homogeneous classes, from the point of view of their creditworthiness, and then to provide information on the dynamics of the creditworthiness associated with a possible downgrading/upgrading of the rating of the applicants (a phenomenon known as *risk of migration*). In this case, the procedure computes the probability to migrate from a given class of rating to another one in the considered time interval.

Technically, both of these outputs can be achieved by coupling the so-called *reference profiles* to the alternatives analyzed in the previous phases, which represent a sort of benchmark profiles related to fictitious applicants. The reference profiles, which can also be specified by the DM, delimit contiguous rating classes and serve as comparison with the profiles of real applicants. Notice that, if we wish to classify applicants in l rating classes $R_1, \ldots, R_l$, a number of $l-1$ reference profiles $r_1, \ldots, r_{l-1}$ have to be specified and included among the alternatives. Then the classification of the firms is done according with this procedure:

$$a_i \in R_j \qquad \Longleftrightarrow \qquad \varphi(r_{j-1}) > \varphi(a_i) \geq \varphi(r_j) \tag{6}$$

for $j = 1, \ldots, l$, with $\varphi(r_0) = +\infty, \varphi(r_l) = -\infty$.

It is clear that, in order to effectively apply MURAME, one has to specify the values of the parameters q_j, p_j, v_j, w_j, and also to determine the reference profiles r_j. Different specifications of these values correspond to different (implicit) preference structure of the DM, and lead to different results in terms of classification of the alternatives.

2 Preference disaggregation for MURAME

This section reports both the description of our data set of firms, along with some basics of the methodology MURAME, used to suitably classify the firms within homogeneous subsets.

2.1 Data set and evaluation criteria

The three phases of the methodology outlined in the previous section have been applied to a large set of Italian SMEs with less than 250 employees and annual turnover not exceeding EUR 50 million[3].

Data have been collected for the triennium 2006-2008 from the professional database AIDA (by Bureau Van Dijk Electronic Publishing), which reports the balance sheet drafted according to the IV CEE directive and other information (such as location, sector, year of incorporation, ownership) of Italian firms. In particular, AIDA focuses on companies and corporations which are forced by the Italian law to compile financial statement. Therefore, other legal forms, such as cooperatives, consortium, several forms of partnerships, sole proprietors, family firms, have limited coverage in the database. For these reasons, we perform our investigation only on the SMEs which have the legal status of companies or corporations.

Within the SMEs category, for the purpose of present work, we considered the largest ones, in terms of the number of employees, that if the so-called *Mig* firms, with 50 to 249 employees. In order to be able to compare the results with the ones obtained in [24], we considered the same evaluation criteria. Starting from an initial set of 32 indicators that are frequently adopted in the literature to measure firm's profitability, liquidity, solvency, and other aspects of firm's profile, the selected indicators after a preliminary analysis of correlations are reported in Table 1.

[3]This is in line with the general definition of micro enterprises and SMEs established by the Commission of the European Communities, that defines the latter in terms of the number of employees and either in terms of turnover or total balance sheet (Commission recommendation 2003/361/EC, Article 2 of the Annex).

Table 1: Indicators considered in the creditworthiness analysis.

I_1	Cost of debt: Financial costs/Bank debts
I_2	Return on equity (ROE): Net profit before tax/Total equity
I_3	Total assets turnover: Sales/Total assets
I_4	R&D costs/Total asset
I_5	Income tax/Profit before taxes
I_6	Equity − Equipment
I_7	Rate of increase of revenues from sales and services
I_8	Liabilities/Total assets
I_9	Cash/Total assets
I_{10}	Working capital/Total assets
I_{11}	Intangible/Total assets
I_{12}	EBITDA/Total assets
I_{13}	Retained earnings/Total assets
I_{14}	Net income/Sales
I_{15}	Short term debt/Equity
I_{16}	EBITDA/Interest expenses
I_{17}	Account payable/Sales
I_{18}	Account receivable/Liabilities
I_{19}	Sales/Personnel costs

In Table 2 we show also the numerousness of the final sample of firms for which all the selected accounting indicators were available in each considered year.

Table 2: Numerousness of the sample of firms for each considered year.

Years	Active firms	Bankrupt firms	Total
2006	6625	1089	7714
2007	6766	925	7691
2008	6933	696	7629

2.2 MURAME parameters: exogenous determination

As described in the methodological section, in order to operatively apply MURAME and classify SMEs into homogeneous classes from the point of view of their creditworthiness, the parameter models (weights, thresholds and reference profiles) have to be specified. In this analysis we followed the exogenous direct specification adopted in [24] for reference profiles and thresholds.

The reference profiles are then obtained by directly considering the range of each indicator I_j, and assuming 5 rating classes. More specifically, for each indicator I_j, we first computed the empirical quintiles of its distribution, that we named $I_j^1,\ I_j^2,\ \ldots,\ I_j^4$. Then we aggregated the quintiles of the same order, obtaining 4 fictitious alternatives to be used as reference profiles:

$$\begin{cases} r_1 = (I_1^1, I_2^1, \dots, I_n^1) \\ \quad\vdots \\ r_4 = (I_1^4, I_2^4, \dots, I_n^4). \end{cases} \tag{7}$$

For the determination of the thresholds p_j, q_j and v_j for each criterion we adopted the standard setting of parameters suggested in [25], which consists in defining first the range $s_j = \max I_j - \min I_j$ for each indicator I_j and then computing the preference, indifference and veto thresholds as follows:

$$p_j = \frac{2}{3}s_j, q_j = \frac{1}{6}s_j, v_j = \frac{5}{6}s_j. \tag{8}$$

3 Using PSO evolutionary methodology to improve MURAME

As reported in [24], the MURAME methodology has shown good general performances in creditworthiness classification of SMEs. However, there is a drawback related to possible mis-classification of bankrupt firms. This can be an effect of the presence of incorrect data or outliers in the AIDA database (see for example [52]), but it is also probably a consequence of the standard generic specification of MURAME parameters, in particular of the weights. This could be observed when considering as evaluation criteria only the accounting indicators selected by [4] in the prediction of U.S. SMEs default (namely "Altman's variables"). Altman's variables are listed in Table 3 and correspond to the indexes I_{15}, I_9, I_{12}, I_{13} and I_{16} of Table 1. From a technical point of view, selecting Altman's variables is equivalent to a change of the preference structure implicit in the model in terms of importance assigned to the indicators. Indeed, in such a case the value of the weight of each Altman's variable becomes 1/5 and the value of the weight of any other variable becomes 0. Furthermore, the Altman's variables are more or less the 25% of all the variables in Table 1, as to say that the informative contents of the two sets of variables are so different that they have a significant impact on the rating.

Table 3: Variables entered in the U.S. SME model of [4].

Accounting ratio category	*Variable*
Leverage	Short term debt/Equity book value
Liquidity	Cash/Total assets
Profitability	EBITDA/Total assets
Coverage	Retained earnings/Total assets
Activity	EBITDA/Interest expenses

The results in terms of distribution of bankrupt firms in rating classes presented in [24, Tables 15, 32, 33] show that, in all the considered years, there is a slight decrease of the number of bankrupt firms in the first (best) class, and a slight increase of the number of the bankrupt enterprises in the last (worst) class. Therefore, it appears that MURAME methodology, when using only the Altman's variables, rates slightly better the bankrupt firms in the extreme classes than when using all the variables of Table 1.

This remark suggests that a different specification of the weights of the criteria, can improve the model performance. In this contribution we adopt an indirect procedure for the endogenous specification of the weights of the criteria, using a preference disaggregation methodology (see [45]). The key assumption of this methodology is that, when the direct specification of the parameters is not feasible, they can be inferred from a set of decision-examples on a reference set of alternatives.

In order to make this point clear, let us suppose that we have a reference set A' consisting of m' alternatives on which the classification is known *a priori*. The reference alternatives in this case, as it is often the case when the universe of alternatives A is large, will be a subset $A' \subseteq A$ of the whole set of the alternatives. In our particular implementation, it will consist of a subset of the bankrupt firms, for which we will assume that we know their *a priori* correct classification. Given this input, the objective is to determine the MURAME criteria weights that will minimize the inconsistencies between the model classification of bankrupt firms and their correct one. As a measure of the latter inconsistencies minimization, we consider the maximization of the number of bankrupt firms assigned to the worst rating class. In order to formalize the problem, suppose we are given a measure of inconsistency of our model $\mathcal{I}(w_1, \ldots, w_n)$, which ranges in the unit interval $[0, 1]$, with $\mathcal{I}(\bar{w}_1, \ldots, \bar{w}_n) = 0$ meaning that the weights $\bar{w}_1, \ldots, \bar{w}_n$ ensure the correct classification of the bankrupt firms. Then our aim is to solve the following mathematical programming problem:

$$\begin{aligned} \min_{w_1,\ldots,w_n} \quad & \mathcal{I}(w_1, \ldots, w_n) \\ \text{s.t.} \quad & w_j \geq 0 \qquad j = 1, \ldots, n \\ & \sum_{j=1}^{n} w_j = 1. \end{aligned} \tag{9}$$

This apparently simple mathematical programming problem hides its complexity in the objective function $\mathcal{I}(w_1, \ldots, w_n)$. Indeed, in our case every computation of $\mathcal{I}(w_1, \ldots, w_n)$ first requires the computation of the scores $\varphi(a_i)$ (see (5)) of each of the alternatives considered in the reference set, not only the bankrupt ones. Then, it requires that the firms are classified according to (6), and finally the measure of the inconsistency of the model is computed. Observe that it is quite hard to write an exact analytical expression for $\mathcal{I}(w_1, \ldots, w_n)$ in terms of its variables, so that the use of gradient methods for the optimization task is discouraged, and an evolutionary approach seems more appropriate in order to provide a fast solution.

In order to solve the optimization problem (9), we use a PSO-based solution algorithm. PSO is a bio-inspired iterative metaheuristics for the solution of nonlinear global optimization problems ([14] and [46]). The basic idea of PSO is to model the so called "swarm intelligence" ([15]) that drives groups of individuals belonging to the same species when they move all together looking for food. On this purpose, every member of the swarm explores the search area keeping memory of its best position reached so far, and it exchanges this information with the neighbors in the swarm. Thus, the whole swarm is supposed to converge eventually to the best global position reached by the swarm members.

From a mathematical point of view, every member of the swarm (formally a particle) represents a possible solution of the investigated optimization problem, and it is initially positioned randomly in the feasible set of the problem. To every particle is also initially assigned a random velocity, which is used to determine its initial direction of movement.

In the next section we give a description of the standard PSO metaheuristics, along with the implementation of our PSO-based solution algorithm in the investigated preference disaggregation context. Then in Section 3.2 we present the results obtained and we compare them with the ones in [24].

3.1 PSO algorithm, parameters, and initialization procedures

Let us denote with P the size of the swarm, and let in general be $f : \mathbb{R}^n \mapsto \mathbb{R}$ the function to minimize. For each particle $l = 1, \ldots, P$, given that its position at step $k \geq 0$ of the algorithm is $\mathbf{x}_l^k \in \mathbb{R}^n$, the new position at step $k+1$ is

$$\mathbf{x}_l^{k+1} = \mathbf{x}_l^k + \mathbf{v}_l^{k+1} \qquad l = 1, \ldots, P. \tag{10}$$

Then, the new search direction $\mathbf{v}_l^{k+1}$ is determined by

$$\mathbf{v}_l^{k+1} = \chi^k \left[w^k \mathbf{v}_l^k + \boldsymbol{\alpha}_l^k \otimes (\mathbf{p}_l^k - \mathbf{x}_l^k) + \boldsymbol{\beta}_l^k \otimes (\mathbf{p}_g^k - \mathbf{x}_l^k) \right] \tag{11}$$

where $\mathbf{v}_l^k$ is the previous search direction, $\mathbf{p}_l^k$ and $\mathbf{p}_g^k$ are respectively the best solution so far found by particle l and the whole swarm, respectively, that is

$$\mathbf{p}_l^k = \arg\min_{0 \leq h \leq k} \left\{ f(\mathbf{x}_l^k) \right\} \qquad l = 1, \ldots, P \tag{12}$$

$$\mathbf{p}_g^k = \arg\min_{1 \leq l \leq P} \left\{ f(\mathbf{p}_l^k) \right\} \tag{13}$$

and $\boldsymbol{\alpha}_l^k, \boldsymbol{\beta}_l^k \in \mathbb{R}^n$ are positive random vectors, with the symbol $\otimes$ denoting the component-wise product. The most common used specifications in the literature for $\boldsymbol{\alpha}_l^k, \boldsymbol{\beta}_l^k$, which we will adhere to, are

$$\boldsymbol{\alpha}_l^k = c_1 \mathbf{r}_1^k \tag{14}$$

$$\boldsymbol{\beta}_l^k = c_2 \mathbf{r}_2^k \tag{15}$$

where $\mathbf{r}_1^k, \mathbf{r}_2^k$ are vectors whose entriesr are uniformly randomly distributed in $[0, 1]$, and $c_1, c_2 \in (0, 2.5]$.

Since the PSO was conceived for unconstrained problems, its direct application to (9) can not prevent from generating infeasible particles' positions when constraints are considered. To avoid this problem, different strategies have been proposed in the literature, and most of them involve the repositioning of the particles ([61]) or the introduction of some external criteria to rearrange the components of the particles ([28]). On the contrary, in this paper we consider an approach which encompasses a nonlinear reformulation of problem (9), so that we can maintain PSO as in its original formulation, and in place of (9) we can solve the unconstrained problem

$$\min_{t_1, \ldots, t_n} \mathcal{I} \left[w_1(\mathbf{t}), \ldots, w_n(\mathbf{t}) \right], \tag{16}$$

where the mapping between the new and old variables is given by the following nonlinear transformation

$$w_j(\mathbf{t}) \leftarrow \frac{t_j^2}{\sum_{i=1}^{n} t_i^2}, \qquad j = 1, \ldots, n. \tag{17}$$

Observe that any global solution $\bar{\mathbf{t}} \in \mathbb{R}^n$ of (16) corresponds to a global solution $w \leftarrow w(\bar{\mathbf{t}})$ of (9), even though we introduce nonlinearities which in principle increase the computational complexity.

As for every evolutionary algorithm, PSO performance depends on the choice of its parameters χ, w, c_1, c_2 and on the initial positions and velocities of the swarm, that is $\mathbf{x}_l^0, \mathbf{v}_l^0 \in \mathbb{R}^n$ for $l = 1, \ldots, P$. While for the choice of the parameters we will comply with standard settings in the literature, for the initialization of the algorithm we will apply and compare three different approaches: the standard random one mainly adopted in the literature, and two novel deterministic ones, namely `Orthoninit` and `Orthoinit+` recently proposed in [21] and [32]. The idea behind these two novel initializations is to scatter particle trajectories in the search space in the early iterations, in order to better initially explore the search space, and to obtain approximate solutions that are not concentrated in a reduced subspace. In the following we will present a brief summary of the theoretical results supporting these initializations; we refer the reader to [32] for a more complete report.

Let us assume that $\mathbf{r}_1^k = r\mathbf{1}, \mathbf{r}_2^k = r_g\mathbf{1}$, with given $r, r_g > 0$ and $\mathbf{1} = (1, \ldots, 1)^T \in \mathbb{R}^n$, that is PSO evolution is deterministic. Assume then that $w^k = w > 0, \chi^k = \chi > 0$, and that the following relations hold:

$$\begin{aligned} &a = \chi w < 1 \\ &\omega = \chi(c_1 r + c_2 r_g) < 2(\chi w + 1) \\ &\omega \neq (1 \pm \sqrt{\chi w})^2. \end{aligned} \tag{18}$$

By denoting with

$$X_l(k) = \begin{pmatrix} \mathbf{v}_l^k \\ \\ \mathbf{x}_l^k \end{pmatrix} \in \mathbb{R}^{2n}$$

the state of particle l at iteration k, it is easy to prove that

$$X_l(k) = X_l^L(k) + X_l^F(k) \tag{19}$$

where $X_l^L(k)$ is the so-called *free response* and has the representation

$$X_l^L(k) = A^k X_l(0) \tag{20}$$

with

$$A = \begin{pmatrix} aI & -\omega I \\ \\ aI & (1-\omega)I \end{pmatrix} \in \mathbb{R}^{2n \times 2n}, \tag{21}$$

being $X_l^L(k)$ independent of $\mathbf{p}_l^k$ and $\mathbf{p}_g^k$.

Since the free response depends only on the initial state of the particle, an appropriate choice of the initial state can force $X_l^L(k)$ to retain specific properties. In particular, it is possible to prove that A has only two distinct eigenvalues λ_1 and λ_2; let us consider then, for a fixed $k \geq 0$,

$$\gamma_1(k) = \frac{\lambda_1^k(a-\lambda_2) - \lambda_2^k(a-\lambda_1)}{\lambda_1 - \lambda_2} \qquad \gamma_2(k) = \frac{\omega(\lambda_1^k - \lambda_2^k)}{\lambda_1 - \lambda_2} \tag{22}$$

and the $2n$ vectors in $\mathbb{R}^{2n}$ ($e_i \in \mathbb{R}^n$ represents the i-th unit vector)

$$\begin{aligned} z_i(k) &= \begin{pmatrix} \frac{\gamma_2(k)}{\gamma_1(k)} e_i \\ \\ e_i \end{pmatrix} \qquad i = 1, \ldots, n \\ z_{n+i}(k) &= \begin{pmatrix} -\frac{\gamma_1(k)}{\gamma_2(k)} e_i \\ \\ e_i \end{pmatrix} \qquad i = 1, \ldots, n. \end{aligned} \tag{23}$$

Assuming for simplicity $P = 2n$, if we adopt then the following initialization procedure (`Orthoinit`)

$$\begin{pmatrix} \mathbf{v}_i^0 \\ \\ \mathbf{x}_i^0 \end{pmatrix} = \rho_i z_i(k), \quad \rho_i \in \mathbb{R} \setminus \{0\}, \quad i = 1, \ldots, n \tag{24}$$

and

$$\begin{pmatrix} \mathbf{v}_{n+i}^0 \\ \\ \mathbf{x}_{n+i}^0 \end{pmatrix} = \rho_{n+i} z_{n+i}(k), \quad \rho_{n+i} \in \mathbb{R} \setminus \{0\}, \quad i = 1, \ldots, n \tag{25}$$

then, the first n entries of the free responses of the particles (i.e. the velocities $\mathbf{v}_i^0$, $i = 1, \ldots, 2n$) are orthogonal at step k of the deterministic PSO. To some extent this also tends to impose a near orthogonality of the particles trajectories at step k, as well as in the subsequent forthcoming iterations. While this initialization has the advantage of making the particle trajectories better scattered in the search space, it has the drawback, as it has been observed in [32], that the approximate solutions found by the algorithm are too sparse, that is only few components of the vector of solutions are nonzero. In order to possibly pursue a *dense* final solution, the following modification (`Orthoinit+`) has been proposed in [32]: replace in (24) and (25) the vectors $z_i(k)$, $\quad i = 1, \ldots, 2n$, with the following ones

$$\begin{aligned} \nu_i(k) &= \quad z_i(k) - \alpha \sum_{\substack{j=1 \\ j \neq i}}^{n} z_j(k) - \gamma \sum_{j=n+1}^{2n} z_j(k), \qquad i = 1, \ldots, n \\ \nu_{n+i}(k) &= \ z_{n+i}(k) - \beta \sum_{\substack{j=n+1 \\ j \neq n+i}}^{2n} z_j(k) - \delta \sum_{j=1}^{n} z_j(k), \qquad i = 1, \ldots, n, \end{aligned} \tag{26}$$

by choosing $\alpha \in \mathbb{R} \setminus \{-1, \frac{1}{n}\}$, $\quad \beta = \frac{2}{n-2}$, $\quad \gamma = 0$, $\quad \delta \in \mathbb{R} \setminus \{0, 1\}$. It is possible to prove that the vectors $\nu_1(k), \dots, \nu_{2n}(k)$ are still well scattered in $\mathbb{R}^{2n}$, as well as uniformly linearly independent (see [32]).

3.2 Results

The ideal application of the preference disaggregation methodology described in the previous section aims to consider the whole group of firms, that is to take the reference set A' as $A' = A$, and to determine then the weights that minimize $\mathcal{I}(\mathbf{w})$ for the entire population of firms, for each of the considered years. However, this implies a quite heavy computational task, because of the large number of criteria considered, since a single computation of $\mathcal{I}(\mathbf{w})$ requires on average 1000 seconds on a PC with a standard configuration. This means that for a single step of PSO with $P = 40$ particles, around 11 hours are required[4]. Then, on the basis of the results obtained in [23], we considered a smaller reference set of firms in order to achieve a good compromise between the quality of the obtained solution and the computational time required. Indeed, the larger the cardinality of the reference set, the better the classification performance for the whole group of considered firms, but also the longer the time to obtain it. Moreover, we also would like to check if, analogously to what presented in [21], also in this application the employment of `Orthoinit` and `Orthoinit+` allows a better minimization, with respect to the usual random initialization, of the objective function in the early iterations of PSO. The latter result could be very useful in finding better approximate solutions when the size of the used reference set is large, and consequently the time needed for each PSO iteration is quite long.

In table 4 we show the values of the parameters used in PSO experiments; they have been chosen according to the prevailing literature on PSO and analogously to [21], in order to be able to compare the obtained results with the same initial conditions. For the reference set cardinality, after some preliminary tests, we have selected the value $|A'| = 2500$. The reference set has been randomly selected from the whole population of the considered firms, preserving the same ratio between active and bankrupt firms as in the entire group. The total number of PSO iterations performed has been 500.

Table 4: Values used in PSO experiments.

c_1	1.49618
c_2	1.49618
$w^k = w$	0.7298
$\chi^k = \chi$	1
α	0.25
δ	0.75

For the measure of inconsistency of the model, we have followed this general principle: a good creditworthiness classification model should place as many bankrupt firms as possible in the worst rating class, and the number of bankrupt firms in each class should increase

[4]The time can be reduced by parallelization of the code used for the computations.

with the class, being the first one the best one, and the fifth one the worst one. By denoting with n_j^B the number of bankrupt firms positioned by the model in j-th class and with N^B the total number of bankrupt firms in our reference set, we considered then, according to the aforementioned principle, the following three measures of inconsistency:

$$\begin{aligned}\mathcal{I}_1 &= \frac{n_1^B}{N^B};\\ \mathcal{I}_2 &= 1 - \frac{n_5^B}{N^B};\\ \mathcal{I}_3 &= \frac{\sum_{j=1}^{4} n_j^B}{N^B}.\end{aligned} \tag{27}$$

In Tables 5-8 we show the results, in terms of classification of bankrupt firms and of the entire population of *Mig* firms, using data of year 2008. The percentages reported represent the distribution of the firms in the five rating classes obtainede using the weights $w_j(\mathbf{t})$ in (17) found by the algorithm using the two objective functions $\mathcal{I}_1$ and $\mathcal{I}_2$, using the three initialization procedures (random, `Orthoinit` and `Orthoinit+`) described in Section 3.1, and considering first all indicators of Table 1 and then only the Altman's variables of Table 3. We remark that the weights have been found applying the algorithm to a reference set of size $|A'| = 2500$, but the classification results reported here refer to the application of MURAME to all the population of firms. For comparison purposes, in the first row of each table we report the classification results obtained using instead the standard exogenous specification of weights, that is $w_i = 1/n$ for $i = 1, \ldots, n$, adopted in [24].

Table 5: Distribution of bankrupt and all firms, using $\mathcal{I}_1$ and all indicators.

		1	2	3	4	5
Standard	Bankrupt	20,40%	21,70%	20,98%	18,39%	18,53%
	All	23,59%	20,85%	20,71%	20,23%	14,62%
PSO-Random	Bankrupt	5,60%	12,36%	19,40%	31,03%	31,61%
	All	20,40%	18,17%	18,18%	23,38%	19,87%
PSO-`Orthoinit`	Bankrupt	1,29%	15,23%	24,57%	32,04%	26,87%
	All	13,07%	24,76%	25,32%	21,37%	15,48%
PSO-`Orthoinit+`	Bankrupt	9,63%	15,09%	18,53%	27,59%	29,17%
	All	21,09%	19,71%	19,29%	19,94%	19,96%

It can be seen that, when considering the whole sets of indicators of Table 1, using both the objective functions considered, the (sub)-optimal weights found by the PSO algorithm show improvements with respect to neutral weights specification in terms of the distribution of bankrupt firms, according to the specific goal of each objective function. Moreover, the global quality of bankrupt firms creditworthiness distribution depends on the PSO initialization procedure considered: indeed, in the case of `Orthoinit` initialization the classification obtained does not satisfy the general principle aforementioned, since it shows an higher concentration of bankrupt firms in the 4^{th} rating class. On the contrary, when considering only the five Altman's indicators of Table 3, we obtained very little improvements, regardless of

Table 6: Distribution of bankrupt and all firms, using $\mathcal{I}_2$ and all indicators.

		1	2	3	4	5
Standard	Bankrupt	20,40%	21,70%	20,98%	18,39%	18,53%
	All	23,59%	20,85%	20,71%	20,23%	14,62%
PSO-Random	Bankrupt	9,48%	11,78%	15,80%	23,28%	39,66%
	All	20,87%	20,79%	17,63%	18,67%	22,05%
PSO-`Orthoinit`	Bankrupt	2,16%	2,16%	6,18%	65,09%	24,43%
	All	14,17%	10,46%	9,04%	48,46%	17,87%
PSO-`Orthoinit+`	Bankrupt	9,34%	11,78%	16,24%	23,13%	39,51%
	All	20,89%	20,02%	18,29%	18,52%	22,28%

Table 7: Distribution of bankrupt and all firms, using $\mathcal{I}_1$ and Altman's indicators.

		1	2	3	4	5
Standard	Bankrupt	6,03%	13,07%	20,26%	27,30%	33,33%
	All	20,20%	20,04%	19,98%	19,90%	19,88%
PSO-Random	Bankrupt	5,60%	13,51%	19,97%	27,59%	33,33%
	All	20,09%	20,02%	20,02%	19,94%	19,94%
PSO-`Orthoinit`	Bankrupt	5,60%	13,36%	19,68%	27,87%	33,48%
	All	20,02%	19,99%	19,99%	20,00%	20,00%
PSO-`Orthoinit+`	Bankrupt	5,60%	13,51%	19,68%	27,73%	33,48%
	All	20,07%	20,02%	19,99%	19,98%	19,95%

Table 8: Distribution of bankrupt and all firms, using $\mathcal{I}_2$ and Altman's indicators.

		1	2	3	4	5
Standard	Bankrupt	6,03%	13,07%	20,26%	27,30%	33,33%
	All	20,20%	20,04%	19,98%	19,90%	19,88%
PSO-Random	Bankrupt	5,75%	13,36%	19,97%	27,59%	33,33%
	All	20,13%	20,02%	19,99%	19,94%	19,92%
PSO-`Orthoinit`	Bankrupt	5,60%	13,36%	19,68%	27,87%	33,48%
	All	20,02%	19,99%	19,99%	20,00%	20,00%
PSO-`Orthoinit+`	Bankrupt	5,75%	13,36%	19,68%	27,87%	33,33%
	All	20,12%	20,00%	19,96%	19,99%	19,92%

the objective function or initialization considered. These results suggested that when using just Altman's indicators the sole determination of MURAME weights (i.e. the unknowns in (16)) was not enough in order to improve the quality of the creditworthiness classification obtained, so that we enlarged the search space by considering also the indifference thresholds $q_j,\ j = 1, \ldots, n$ as variables of our optimization problem, while keeping for p_j and v_j the same setting of (8). Again, to prevent the possible generation of infeasible (i.e. negative) values for the unknowns $\{q_j\}$, we adopted the quadratic nonlinear transformation $q_j \leftarrow s_j^2$, $j = 1, \ldots, n$, and we solved by PSO an unconstrained optimization problem analogous to (16), for all the three objective functions (27). The results are shown in Tables 9-11.

Table 9: Distribution of bankrupt and all firms, using $\mathcal{I}_1$ and Altman's indicators. Optimization run including variables $\mathbf{w}, \mathbf{q}$.

		1	2	3	4	5
Standard	Bankrupt	6,03%	13,07%	20,26%	27,30%	33,33%
	All	20,20%	20,04%	19,98%	19,90%	19,88%
PSO-Random	Bankrupt	0,43%	18,25%	58,48%	19,83%	3,02%
	All	1,31%	19,67%	60,13%	16,53%	2,36%
PSO-`Orthoinit`	Bankrupt	0,57%	18,10%	10,34%	22,84%	48,13%
	All	7,20%	39,70%	14,84%	14,10%	24,16%
PSO-`Orthoinit+`	Bankrupt	0,43%	16,67%	63,22%	16,95%	2,73%
	All	1,27%	19,31%	62,76%	14,44%	2,22%

Table 10: Distribution of bankrupt and all firms, using $\mathcal{I}_2$ and Altman's indicators. Optimization run with variables $\mathbf{w}, \mathbf{q}$.

		1	2	3	4	5
Standard	Bankrupt	6,03%	13,07%	20,26%	27,30%	33,33%
	All	20,20%	20,04%	19,98%	19,90%	19,88%
PSO-Random	Bankrupt	2,01%	5,89%	15,23%	32,90%	43,97%
	All	20,02%	20,09%	20,06%	19,90%	19,94%
PSO-`Orthoinit`	Bankrupt	0,72%	20,11%	10,06%	21,98%	47,13%
	All	8,01%	39,93%	14,65%	13,41%	24,00%
PSO-`Orthoinit+`	Bankrupt	2,01%	5,89%	15,52%	32,76%	43,82%
	All	20,08%	20,08%	20,11%	19,83%	19,90%

In this case, the problem solved using $\mathcal{I}_1$ objective function, while producing a very good result in terms of the percentage of bankrupt firms in the best creditworthiness class, fails to satisfy the general principle of classification independently of the initialization procedure employed. The best results, and very similar, are obtained using $\mathcal{I}_2$ and Random or `Orthoinit+` initializations, and they show a remarkable improvement with respect to those of Tables 7-8.

As stated before, we wanted also to check the capability of the two deterministic initialization procedures in order to find better approximate solutions in the early iterations

Table 11: Distribution of bankrupt and all firms, using $\mathcal{I}_3$ and Altman's indicators. Optimization run with variables $\mathbf{w}, \mathbf{q}$.

		1	2	3	4	5
Standard	Bankrupt	6,03%	13,07%	20,26%	27,30%	33,33%
	All	20,20%	20,04%	19,98%	19,90%	19,88%
PSO-Random	Bankrupt	3,02%	5,75%	15,52%	32,61%	43,10%
	All	20,54%	20,06%	19,96%	19,73%	19,71%
PSO-`Orthoinit`	Bankrupt	0,57%	18,10%	10,34%	22,84%	48,13%
	All	7,20%	39,70%	14,84%	14,10%	24,16%
PSO-`Orthoinit+`	Bankrupt	5,03%	14,08%	11,49%	21,70%	47,70%
	All	27,19%	20,38%	15,40%	14,10%	22,93%

of PSO, which could be very useful in a possible application of the methodology to a more general problem of higher dimensionality. As an example, in Figures 1 and 2 we show the results obtained for $\mathcal{I}_2$, in terms of the early convergence of the algorithm applied to the reference set A', first in the case of the problem with all the indicators of Table 1 and with only $\mathbf{w}$ as search variables, then in the case of the problem with only the Altman's indicators and both $\mathbf{w}$ and $\mathbf{q}$ as search variables.

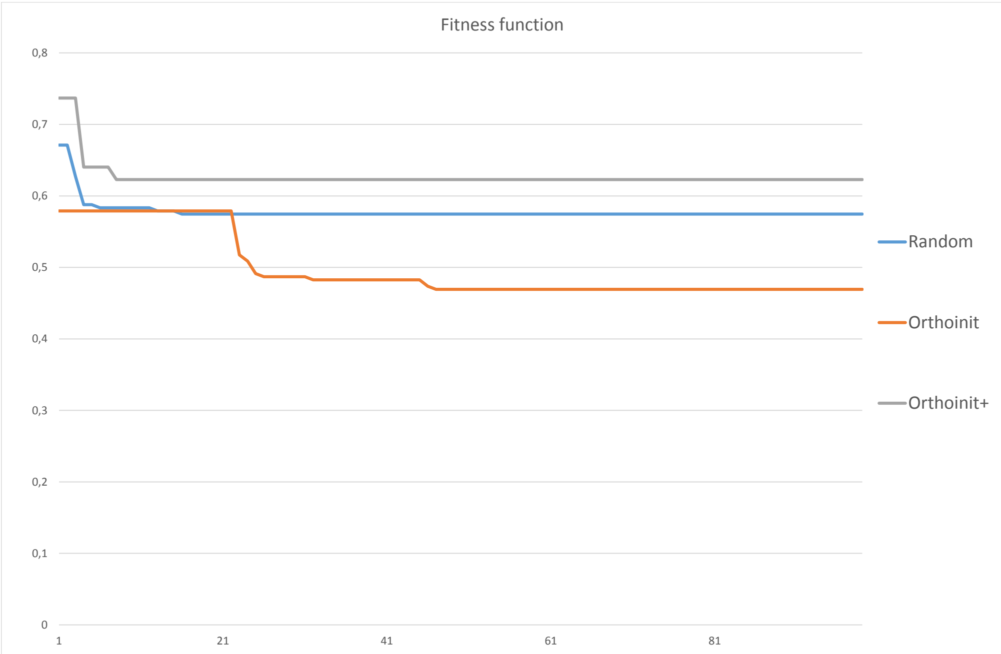

Figure 1: Values of $\mathcal{I}_2(\mathbf{w})$, for the first 100 PSO iterations.

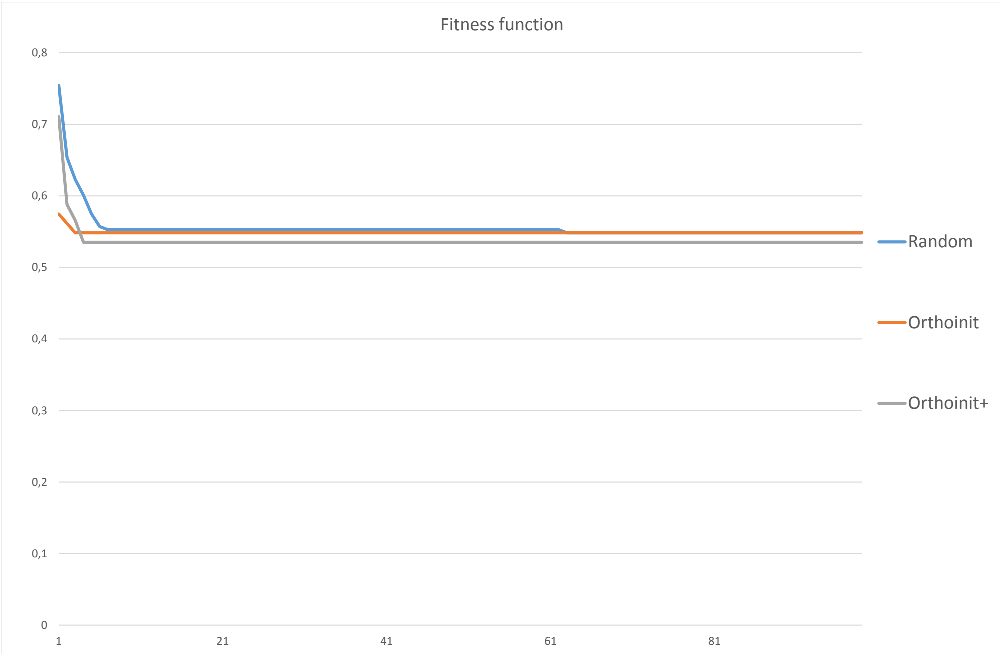

Figure 2: Values of $\mathcal{I}_2(\mathbf{w}, \mathbf{q})$, for the first 100 PSO iterations.

It can be seen that, especially when using `Orthoinit+` procedure, a fast decrease of the objective function is obtained after the very first steps of PSO, that is when the effects of the dense and uniformly linearly independent choice of the initial state of the particles are still relevant in terms of the free response velocities. After that there are no improvements in the solution found, which could mean that the evolutionary search process is entangled in a local minimum, and this happens regardless of the chosen initialization. However, this is not necessarily a drawback of our methodology, since we are not looking for an exact global minimum, and, as reported in the previous Tables, the quality of the creditworthiness classification obtained is remarkably improved.

We conclude this section by showing in Tables 12-13 the (sub)-optimal values of the model parameters obtained using our methodology and the objective function $\mathcal{I}_2$, first in the case of the general problem with all indicators and only the weights of the model as search variables, then in the case of the restricted Altman's problem with both weights and indifference thresholds as search variables. It emerges, from the displayed values, a problem related to the use of `Orthoinit` analogous to what reported in [21]: the sparsity of the achieved solutions. Indeed, while both with Random and `Orthoinit+` initialization we have a dense solution, meaning that the great majority of the weights and of the indifference thresholds have significantly non-zero values, the solutions provided by `Orthoinit` imply the adoption of a single-criteria or at most two-criteria classification model, whose performance we have seen from Tables 5-11 to be not satisfactory. This is an *a fortiori* reason in favor of the adoption of a MultiCriteria model. Finally, we want to remark that, even if the parameter values obtained only correspond to a sub-optimal solution of the optimization problem considered, the aim of a MCDA model is not to provide an exact solution to the decision problem at hand. Instead, especially when an iterative procedure for the determination of the parameters is employed as in this case, the principal aim is to help the Decision Maker in the whole decision process, providing tools for a possible interactive procedure which should enable him/her to enhance the final goal. From this perspective, the values of the weights obtained can represent a starting point for a better understanding of the relevance of each financial indicator and, analogously, the values of the indifference thresholds could represent a basis for a better understanding of the preference structure implicit in the model, in order to refine or to simplify it for subsequent applications.

Table 12: Weights of all indicators using $\mathcal{I}_2$.

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}
PSO-Random	0,01%	34,22%	14,76%	0,01%	11,56%	0,00%	4,84%	2,50%	10,92%	4,37%
PSO-`Orthoinit`	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%
PSO-`Orthoinit+`	0,06%	57,17%	38,41%	0,06%	3,45%	0,06%	0,06%	0,06%	0,06%	0,06%
	I_{11}	I_{12}	I_{13}	I_{14}	I_{15}	I_{16}	I_{17}	I_{18}	I_{19}	
PSO-Random	0,02%	0,03%	3,27%	0,59%	3,34%	2,75%	4,99%	0,41%	1,40%	
PSO-`Orthoinit`	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	0,00%	0,00%	0,00%	
PSO-`Orthoinit+`	0,06%	0,06%	0,06%	0,06%	0,06%	0,06%	0,06%	0,06%	0,06%	

Table 13: Weights and indifference thresholds of Altman's indicators using $\mathcal{I}_2$. Optimization run with variables $\mathbf{w}, \mathbf{q}$.

		I_9	I_{12}	I_{13}	I_{15}	I_{16}
PSO-Random	$\mathbf{w}$	0,92%	56,54%	15,33%	1,28%	25,93%
	$\mathbf{q}$	6,994	5,468	2,886	20,714	0,021
PSO-`Orthoinit`	$\mathbf{w}$	0%	0%	50,53%	0%	49,47%
	$\mathbf{q}$	0	0	0	0	1,549
PSO-`Orthoinit+`	$\mathbf{w}$	13,97%	21,51%	21,51%	21,51%	21,51%
	$\mathbf{q}$	0,161	0,161	0,161	22,829	0,161

4 Conclusions and future works

In this paper we have presented a MultiCriteria Decision Analysis methodology for the evaluation of the creditworthiness of Italian SMEs, that allows to consider simultaneously several different financial indicators derived from balance sheet data of the firms, and provides results in terms of classification of the firms in homogeneous rating classes. Then, we have applied an evolutionary computation methodology to determine some parameters of the model that can improve the classification performance of the bankrupt firms, and we have in particular studied the impact of some initialization methodologies for the PSO algorithm used with respect to the ability of generating early efficient approximate solutions. The results in both cases are satisfactory, however we plan in the future to extend in particular the study of the efficient determination of MURAME parameters, both in terms of the quality of the values found and in terms of the computational effort to achieve them. In particular we believe that some improvements could come by the determination not only of the weights of the criteria, but also of all the thresholds, by a suitable reduction of the complexity of the mathematical programming problem involved. For example one approach could be that of reducing the search space by excluding the criteria which result to have small weights in a first disaggregation procedure, and to subsequently determine one or more sets of criteria thresholds. We also plan to study the possibility to combine the deterministic PSO with the usual random one, both with respect to the initialization of the algorithm and with the respect to its evolution.

Acknowledgments G.Fasano thanks the National Research Council - Marine Technology Research Institute (CNR-INSEAN), Italy, for the support received. The work of G. Fasano is also partially supported by the Italian Flagship Project RITMARE, coordinated by the Italian National Research Council (CNR) and funded by the Italian Ministry of Education, within the National Research Program 2012-2016.

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