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Abstract
The paper investigates the importance of modeling in *cay* estimations from a statistical and economic perspective by observing the stochastic trend, a thus far neglected component. In order to do this, we perform an empirical analysis on US secular annual data from 1900 to 2015 considering the *cay* with non-durables and services and the *cay* with total consumption expenditure. Findings show the usefulness of including the stochastic trend in *cay* estimation. Furthermore, out-of-sample statistical and economic significance tests show the ability of the *cay* model with trend to outperform the traditional *cay* measure.

Keywords
Cay, Trend, State-Space Model

JEL Codes
E21, C32

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Michele Costola [*] Lorenzo Frattarolo [†] Marcella Lucchetta [‡] Antonio Paradiso [§]

September 20, 2016

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1 Introduction

The study of US consumption dynamics and its determinants have played a central role in the macroeconomic debate for a long time. An important branch of research on this topic is represented by the so-called *cay* approach introduced by Lettau and Ludvigson (2001). These authors, with the use of the unobservable human wealth component, re-express the consumption-wealth relation introduced by Campbell and Mankiw (1989) and demonstrate, both mathematically and empirically, the existence of a cointegrating relation between (log) of non-durables and services (NDS) consumption (c_t^{NDS}), (log) aggregate assets (a_t), and (log) labor income (y_t).[1] In a recent follow-up, Lettau and Ludvigson (2015) show that cointegration still holds with total private consumption expenditure (c_t^{PCE}) instead of non-durable and services consumption (c_t^{NDS}), if the (log) of total (unobserved) flow consumption is cointegrated with (log) of total private consumption expenditure (PCE).
Bianchi et al. (2015) shows infrequent mean-shifts in the cointegration relation between c_t, a_t, and y_t through a two-state Markov-switching version of the *cay* (cay^{MS}) that accounts for these shifts. The authors find that, over time, the value of traditional *cay*, estimated either with NDS and PCE, became more persistent, whereas the new version with mean-shifts show a very low persistence representing a clear signal of a stationary pattern.

We aim to investigate the nature of these mean-shifts trend in *cay* built with c_t^{NDS} and c_t^{PCE} over a secular period for the US in further detail. In particular, we test if a stochastic trend in *cay* is confirmed by statistical and economic tests for both versions with NDS and PCE. To the best of our knowledge, we are the first to analyse the presence of a stochastic trend, from a statistical and economic perspective in the *cay* framework using US annual data spanning the last century. Our strategy, is therefore, as follows. First, we present a general formalization of *cay* with the trend modelled as an AR(1) process with time-varying drift for both versions of NDS ($cay_t^{NDS,T}$) and PCE ($cay_t^{PCE,T}$) consumption. This trend formalization encompasses the original formulation of Lettau and Ludvigson (2001) by enforcing the constraint of zero linear correlation in the trend. The linear restriction on model parameters is easily tested using a Wald-type statistic. Secondly, we test the out-of-sample forecasting ability of the two *cay* versions with trend in predicting the equity premium compared against the traditional *cay* without trend and historical average of equity premium. Thirdly, following Della Corte et al. (2010), we move beyond a statistical perspective and in a mean-variance framework we study the problem of allocating capital between risk-free assets and equity.

[1] More specifically, Lettau and Ludvigson (2001) express *cay* as a function on stationary terms. This implies that $cay \equiv c_t^{NDS} - \omega a_t - (1-\omega) y_t$ is also stationary, so that c_t, a_t and y_t must be cointegrated.

In this way, we are able to assess the economic relevance of the trend. In short, we investigate the capacity of *cay* with and without trend in generating higher economic gains compared against historical average.
Using US data, we find that the Wald-type test strongly supports the presence of a stochastic trend in the *cay* with both NDS and PCE. In addition, we show that the persistence of the trend is very high. Comparing the out-of-sample forecasting performance, we show that the *cay* version with stochastic trend outperform the traditional *cay*. Finally, while the economic analysis of a portfolio built with traditional *cay* shows that it cannot beat a portfolio built with the historical average of the equity premium, portfolio returns built using *cay* with trend strategies are, instead, larger than the gain obtainable from trading based on of the historical average. This result supports the economic relevance of the stochastic trend.
The paper is organized as follows: Section 2 describes the *cay* model with a stochastic trend and the out-of-sample tests; Section 3 presents the empirical results and Section 4 provide the conclusion to our findings.

2 Empirical strategy

In this Section, we present the *cay* model with a stochastic-trend formalization (Subsection 2.1). We also delineate the out-of-sample forecasting strategy (Subsection 2.2.1) to predict equity premium (i.e. the excess of stock market returns respect to the risk-free rate), and the framework to measure the economic value (Subsection 2.2.2) of information provided by *cay* models (i.e. the out-of-sample performance of a portfolio built upon the *cay* predictions).

2.1 *cay* model with stochastic trend

To investigate the presence of a stochastic-trend in *cay*, the following cointegrating relation is estimated:

$$c_t^i = \psi_t + \beta_a a_{t-1} + \beta_y yd_t + e_t \tag{1}$$

where i corresponds to different specifications of consumption (i.e. NDS or PCE) according to the consumption version investigated, ψ_t is the stochastic trend, and β_a and β_y are the long-run parameters of aggregate asset wealth and disposable income (yd_t). Details on data construction and sources are reported in Appendix A.

In line with empirical literature, we estimate an unobserved stochastic trend-component using the state-space representation and the Kalman filter method.[2] The state space representation of the Equation (1) is the following:

[2] The Kalman filter is a recursive algorithm that provides an optimal estimate of ψ_t conditional

$$
\begin{cases}
c_t^i = \psi_t + \beta_a a_{t-1} + \beta_y y d_t + e_t \\
\psi_t = \mu_{t-1} + \rho\psi_{t-1} + \epsilon_t \\
\mu_t = \phi\mu_{t-1} + \xi_t
\end{cases}
\tag{2}
$$

where e_t, ϵ_t and ξ_t are Gaussian independent errors.

A simple calculation yields the autocorrelation function of the ψ_t as a nonlinear function of ϕ and ρ

$$
\mathbb{C}\mathrm{orr}\left[\psi_t, \psi_{t+k}\right] = \left\{ \frac{\left(\phi^2 - 1\right)\rho^k(\rho(\phi(\rho - \phi) - 2) + \phi) + \left(\rho^2 - 1\right)\phi^{k+1}}{(\rho - \phi)\left(\phi^2(\rho\phi - 1) + 2\right)} \right\}
\tag{3}
$$

Under the null hypothesis $H_0 : \rho = \phi = 0$, we have $\mathbb{C}\mathrm{orr}\left[\psi_t, \psi_{t+k}\right] = 0$ for each k, $\psi_t \sim NID\left(0, 2\right)$ and the model is equivalent to the model without trend. This linear restriction can be tested using the usual Wald test. The analytical expression of the autocorrelation function can also be used to investigate the persistence of the trend once estimated parameters are used.

2.2 The out-of-sample predictive power of *cay*

2.2.1 Forecasting performance

This subsection tests the forecasting ability of the *cay* model, evaluating different versions of the following equation

$$
\underbrace{[Rm_t - Rf_t]}_{\equiv X_t} = \gamma_0 + \gamma_1 cay_{t-1}^i + \zeta_t
\tag{4}
$$

where $[Rm_t - Rf_t]$ is the equity premium (i.e. the return on the stock market (Rm_t) minus the return on a short-term risk-free T-bill (Rf_t), $\zeta_t \sim N(0, \sigma_\zeta^2)$.

We compute out-of-sample forecasting following the approach used by Della Corte et al. (2010). We use the first 50 observations for estimation purposes and we leave the remaining observations for the out-of-sample exercise. The target is to compare the forecasts of various *cay* respect to a benchmark represented by the simple historical average of equity premium. The out-of-sample statistics are:

1. The difference of the root-mean-squared error: $\Delta RMSE = \sqrt{MSE_{ave}} - \sqrt{MSE_{cay^i}}$;

on an information set and knowledge of the parameters of the state space.

2. The out-of-sample R^2: $R^2_{OS} = 1 - \frac{MSE_{cay^i}}{MSE_{ave}}$;

3. The equal mean square error F test proposed by McCracken (2007): $MSE-F = P \cdot \frac{MSE_{ave} - MSE_{cay^i}}{MSE_{cay^i}}$; and,

4. The forecast encompassing test proposed by Clark and McCracken (2001): $ENC = P \cdot \frac{\bar{c}}{MSE_{cay^i}}$;

where MSE_{cay^i} is the mean square error of the conditional forecasts based on various cay^i measures, MSE_{ave} is the mean square error based on the conditional forecasts based on the historical average of equity premium, P represents the number of forecasts, $\bar{c} = P \cdot \frac{P^{-1}\sum_t(\zeta^2_{ave,t} - \zeta_{ave,t} \cdot \zeta_{cay^i,t})}{MSE_{cay^i}}$, and $\zeta_{cay^i,t}$ and $\zeta_{ave,t}$ denote the out-of-sample forecast error for cay^i and historical average models, respectively.

2.2.2 Economic significance

The economic significance follows from portfolio decisions of a quadratic utility maximizing agent, as in the usual mean variance framework. The solution to the optimization problem of a two-asset investment problem with one risky and one risk-free asset delivers the following weight

$$w_t = \frac{1}{\lambda} \frac{\mathbb{E}_t[X_{t+1}]}{Var_t[X_{t+1}]} \tag{5}$$

where $\mathbb{E}_t[X_{t+1}]$ is the conditional expectation of equity premium X in time $t+1$, $Var_t[X_{t+1}]$ is the conditional variance of X_{t+1}, and λ is the relative risk aversion coefficient set equal to three as in Campbell and Thompson (2008).

Following Della Corte et al. (2010), we measure the average realized utility $\bar{U}\{\bullet\}$ as

$$\bar{U}\{\bullet\} = \frac{W_0}{T} \sum_{t=0}^{T-1} \left\{ R_{p,t+1} - \frac{\lambda}{2(1+\lambda)} R^2_{p,t+1} \right\} \tag{6}$$

where W_0 represents the initial wealth fixed to one for simplicity, and $R_{p,t+1} = Rf_t + w_t X_{t+1}$ is the realized return on portfolio in time $t+1$.

To measure the performance of the cay^i strategy compared to the naive-average (AVE), we follow Della Corte et al. (2010) and find the value of the parameter Φ that satisfies the following relation

$$\sum_{t=0}^{T-1} \left\{ (R^{cay^i}_{p,t+1} - \Phi) - \frac{\lambda}{2(1+\lambda)} (R^{cay^i}_{p,t+1} - \Phi)^2 \right\} = \sum_{t=0}^{T-1} \left\{ R^{AVE}_{p,t+1} - \frac{\lambda}{2(1+\lambda)} (R^{AVE}_{p,t+1})^2 \right\} \tag{7}$$

where $R^{cay^i}_{p,t+1}$ indicates the portfolio return constructed using the prediction of cay^i and $R^{AVE}_{p,t+1}$ is the portfolio return obtained following the historical average of equity premium. According to Equation (7), if cay_i contains no predictive information, then we should observe $\Phi \leq 0$; otherwise we should observe $\Phi > 0$.

Finally, a Sharpe ratio measure is employed. Following Goetzmann et al. (2007) and Della Corte et al. (2010), as a complement to the performance of Φ we calculate the abnormal return of cay^i strategy relative to the AVE

$$\Theta = \frac{1}{(1-\lambda)} \left\{ \ln \left[\frac{1}{T} \sum_{t=0}^{T-1} \left(\frac{R^{cay^i}_{p,t+1}}{Rf_t} \right)^{1-\lambda} \right] - \ln \left[\frac{1}{T} \sum_{t=0}^{T-1} \left(\frac{R^{AVE}_{p,t+1}}{Rf_t} \right)^{1-\lambda} \right] \right\} \tag{8}$$

3 Empirical Results

State-space model estimates of the system of equations (2) for the two cay versions with stochastic trend (i.e. NDS and PCE) are reported in Table 1. Results show that all parameters are statistically significant at the 0.01 level for both cases. Usual residual tests confirm that residuals of cay do not suffer from autocorrelation (Q-test) and that cointegration relation exists (ADF-test).
The most interesting finding for our purposes is the magnitude of the coefficient ϕ that represents the persistence of the stochastic trend. For both NDS and PCE cases, the value is very high and Wald test confirms that ϕ is highly statistically significant. In particular, both the parameter value estimate of ϕ and the Wald test statistic are higher for NDS, meaning that the inclusion of the stochastic trend is more relevant in this case. As an additional check, we calculate the autocorrelation function for NDS and PCE according to Equation 3. Figure 1 shows that the decay rates of ϕ are higher for NDS respect to PCE, but in both cases are very persistent. These results strongly confirm the presence of a stochastic trend in cay formulation from a statistical point of view, suggesting that Lettau and Ludvigson (2001) original formulation constraining the ϕ at zero is not supported by US data over a long time span. Our result is more line with Bianchi et al. (2015) who find the presence of mean-shifts in the cay relation. These shifts can be produced by the presence of a more general stochastic trend over a secular period.
The out-of-sample forecasting performance and economic significance are reported in Tables 2 and 3, respectively. We find that, for the forecast sample 1950-2015, the cay with trend is able to improve on the forecasting record of the AVE. In fact, $cay_t^{NDS,T}$ and $cay_t^{PCE,T}$ exhibit positive $\Delta RMSE$ and R^2_{OS}. The p-values for both the $MSE-F$ and the ENC are very low (i.e. lower than 5%). Differently, we find that traditional cay measures are unable to improve the forecasting performance provided by the AVE. In this case, the p-values for both forecast

Table 1: KALMAN FILTER ESTIMATION OF STATE SPACE (2).

	$c_t^{NDS,T}$	$c_t^{PCE,T}$
β_a	0.373*	0.407*
	(0.044)	(0.064)
β_y	0.547*	0.515*
	(0.048)	(0.075)
ρ	0.961*	0.850*
	(0.056)	(0.092)
ϕ	0.338*	0.410*
	(0.124)	(0.159)
Adj. R^2	0.9983	0.9982
Loglikelihood	275.01	267.61
AIC	−4.6958	−4.6071
$Wald$ test ($\rho = \phi = 0$)	391.89	198.39
	[0.000]	[0.000]
$Q(2)$	[0.171]	[0.245]
$Q(4)$	[0.227]	[0.057]
ADF	[0.000]	[0.000]

Notes: The sample is on annual data from 1900 to 2014. The estimation is the maximum likelihood obtained by the Newton-Raphson optimization procedure with Marquardt step. The reported SEs (in parentheses) are computed using the Huber-White method. * indicates statistical significance at the 0.01 level. $Q(p)$ is the Ljung-Box statistic based on the first p residual autocorrelations of standardized residuals. ADF represents the Augmented Dickey-Fuller test (with automatic lag-length selection based on SIC) conducted on the signal residuals (i.e. e_t of Equation (2)) for detecting cointegration. p-values for $Wald$ test, $Q(p)$ and ADF statistics are reported in square brackets.

Table 2: Out-of-sample statistical significance of forecast accuracy.

	Re-estimated cay			
	$\Delta RMSE$	R^2_{OS}	$MSE-F$	ENC
$1950-2015$				
cay_t^{NDS}	-0.00071	-0.00839	-0.54909	-0.17440
$cay_t^{NDS,T}$	0.00146	0.02999	2.04051**	1.14444*
cay_t^{PCE}	-0.00066	-0.00914	-0.59803	-0.22370
$cay_t^{PCE,T}$	0.00145	0.02931	1.99300**	1.13523*

Notes: The table reports the out-of-sample forecasting performance of predictive regression (4) relative to the historical average of equity premium $[Rm_t - Rf_t]$. The out-of-sample forecasts are generated using 50 years. $\Delta RMSE$, R^2_{OS}, $MSE-F$, and ENC are test statistics defined in Section 2.2.1. Asymptotic critical values (CVs) for the $MSE-F$ test are from Table 4 of McCracken (2007). Asymptotic CVs for the ENC test are from Table 1 of Clark and McCracken (2001). *, and ** indicate statistical significance at 0.1 and 0.05 level, respectively.

Table 3: Out-of-sample portfolio allocation.

	μ_p	σ_p	$\bar{U}$	Φ	Θ
$1950-2015$					
AVE	0.056	0.051	0.054		
cay_t^{NDS}	0.054	0.045	0.052	-0.002	-0.002
$cay_t^{NDS,T}$	0.058	0.049	0.056	0.003	0.003
cay_t^{PCE}	0.054	0.047	0.052	-0.001	-0.002
$cay_t^{PCE,T}$	0.060	0.054	0.058	0.004	0.004

Notes: The table reports the out-of-sample portfolio allocation performance built upon cay predictions relative to the simply historical average (AVE) strategy. cay performances are based on predictions using Equation (4). μ_p indicates the realized average portfolio return, σ_p the realized volatility, $\bar{U}$ the utility, Φ the fee that investors will pay to switch from AVE to cay strategy, and Θ the abnormal return of cay strategy relative to the AVE.

accuracy tests (i.e. $MSE - F$ and ENC) are higher than the commonly used 10% significance level. The evidence reported in Table 2 strongly supports the view that considering the stochastic trend in *cay* is very important to improve the forecasting ability of the historical average equity premium.[3]

Table 3 presents the realized average return (μ_p) and standard deviation (σ_p) of the portfolio obtained from various *cay* strategies and the simple AVE. In addition, the average utility ($\bar{U}$), the performance fee (ϕ), and the abnormal return (Θ) described in Section 2.2.2 are reported. Our results show that only *cay* with trend exhibit a larger returns than the AVE. Standard deviation of the portfolio built under the $cay_t^{PCE,T}$ strategy is higher than the portfolio standard deviation calculated under the AVE. The opposite happens for the $cay_t^{NDS,T}$ strategy. However, the realized utility is higher, relative to the AVE's portfolio, for both portfolios obtained from *cay* with the stochastic trend. The performance fees (ϕ) are positive for both *cay* with trend, suggesting a convenience in switching from a model using AVE to a model based on *cay* with trend. The abnormal return Θ is fully consistent with the results obtained from the performance fees. Portfolio allocation strategies based upon traditional *cay* performs poorly: all the statistics clearly show that the returns obtained following this strategy are lower compared to those obtained from a simple AVE.

4 Conclusions

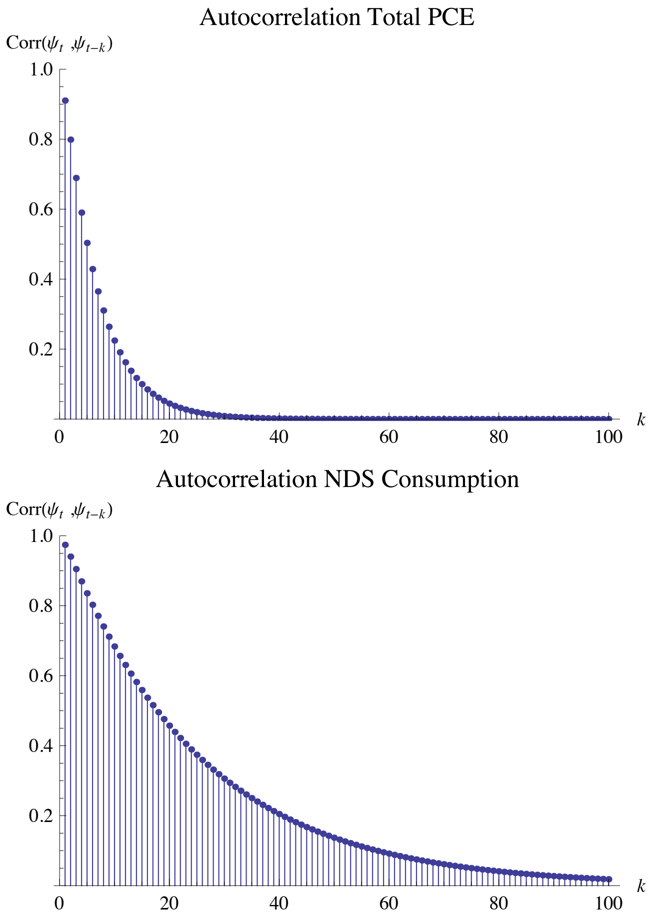
This paper clearly documents the need for a stochastic trend in a *cay* model using US annual data with a secular time range by both statistical and economic means. The ordinary *cay* without stochastic trend is nested in the proposed model, allowing to test the relevance of our proposal by a simple Wald test.

Different *cay* versions are tested with the same rigorous and consistent methodological intuition. Empirical and statistical results distinctly confirm that the stochastic trend needs to be considered in *cay* estimation. Moreover, our out-of-sample statistical and economic significance tests suggest that a *cay* model with trend outperform the traditional *cay* measure. The relevance of our findings beyond US will be left for future research.

Our work points towards an important research direction worthy of further development, suggesting that economic policy design requires the use a stochastic trend to correctly evaluate welfare and output when interventions are applied.

[3]The results are robust to the introduction of economic restrictions as described in (Campbell and Thompson, 2008). Results are reported in Table B.1.

Figure 1: Autocorrelation function for non-durables and services consumption and total private consumption expenditure



Note: $\mathbb{C}\text{orr}\left[\psi_t, \psi_{t+k}\right] = \left\{ \frac{\left(\phi^2-1\right)\rho^k\left(\rho(\phi(\rho-\phi)-2)+\phi\right)+\left(\rho^2-1\right)\phi^{k+1}}{(\rho-\phi)(\phi^2(\rho\phi-1)+2)} \right\}$.

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A Data Appendix

The following table describes the data used in this paper.

Variable	Symbol	Source
Log of real per capita total consumption expenditure	c_t^{PCE}	From 1900-1928 : Lebergott (1996). 1929-2014: Bureau of Economic Analysis (BEA).
Log of real per capita non-durable and services consumption	c_t^{NDS}	From 1900-1928 : Lebergott (1996). 1929-2014: Bureau of Economic Analysis (BEA).
Log of real per capita private (households and non-profit organizations) net wealth	a_t	From 1900 to 2010 : Piketty and Zucman (2014) . 2011-2014: Federal Reserve Economic Data (FRED).
Log of real private disposable income	yd_t	From 1900 to 2010 : Piketty and Zucman (2014) . 2011-2014: FRED.
Return on the stock market	Rm_t	From 1900 to 2015 : `http://www.econ.yale.edu/~shiller/data.htm`.
One-year interest rate	Rf_t	From 1900 to 2015 : `https://www.measuringworth.com/datasets/interestrates/`.

The net wealth a in Equation (1) is measured at the beginning of the period. This way of considering net wealth is consistent with the fact that we use a measure of income including property income (for example, the returns earned on financial wealth, dividends and interest, etc.) that may produce a "double-accounting" problem if wealth is measured at the end of the period t. More precisely, we consider private disposable income (yd_t) that corresponds to labor income (yl_t) *plus* business saving and property income. Considering yd_t in proxying the human capital h_t in Lettau and Ludvigson (2001) framework does not produce great problems as labor income and the other components (i.e. business saving and property income) vary closely with the GDP. A cointegration analysis between yl_t and yd_t confirms that the two variables move together over time. The results are reported in Table A.1.

Table A.1: Canonical Cointegrating Regression (CCR) Results.

Estimated equation: $yl_t = \beta_0 + \beta_1 yd_t + u_t$		
β_0	β_1	$H(0, 1)$
-0.3547	1.0015	1.5809
(0.2239)	(0.0226)	[0.2086]

Notes: The CCR estimator is implemented to compute Park's variable additional test of the null of cointegration. The sample is annual and spans the period from 1948 to 2014. HAC standard errors are in parentheses. $H(0, 1)$ has a $\chi^2(1)$ distribution; p value for this statistic is reported in square brackets. A rejection of the null at the 5% level is warranted if the p-value for the $H(0, 1)$ is less than 0.05.

B Additional Results

Table B.1: Out-of-sample statistical significance of forecast accuracy with economic restrictions.

	Re-estimated *cay* with Rest.			
	$\Delta RMSE$	R^2_{OS}	$MSE-F$	ENC
$1950-2015$				
cay_t^{NDS}	0.00000	0.00000	0.00000	0.00000
$cay_t^{NDS,T}$	0.00146	0.02999	2.04051**	1.14444*
cay_t^{PCE}	−0.00015	−0.00157	−0.10323	−0.05102
$cay_t^{PCE,T}$	0.00145	0.02931	1.99300**	1.13523*

Notes: The table reports the out-of-sample forecasting performance of the predictive regression (4) with the economic restrictions imposed by Campbell and Thompson (2008). The out-of-sample forecasts are generated using 50 years. $\Delta RMSE$, R^2_{OS}, $MSE-F$, and ENC are test statistics defined in Section 2.2.1. Asymptotic critical values (CVs) for the $MSE-F$ test are from Table 4 of McCracken (2007). Asymptotic CVs for the ENC test are from Table 1 of Clark and McCracken (2001). *, and ** indicate statistical significance at 0.1 and 0.05 level, respectively.