Derivative-Free Global Design Optimization in Ship Hydrodynamics by Local Hybridization

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Abstract

A derivative-free global design optimization of the DTMB 5415 model is presented, using local hybridizations of two global algorithms, DIRECT (DIviding RECTangles) and PSO (Particle Swarm Optimization). The optimization aims at the reduction of the calm-water resistance at Fr = 0.25, using six design variables modifying hull and sonar dome. Simulations are conducted using potential flow with a friction model. Hybrid algorithms show a faster convergence towards the global minimum than the original global methods and are a viable option for design optimization, especially when computationally expensive objective functions are involved. A resistance reduction of 16% was achieved.

1. Introduction

Simulation-based design (SBD) optimization in ship hydrodynamics usually involves computationally expensive objective functions, whose values are often provided by systems of partial differential equations solved by black-box tools. In this context, the objective function is likely noisy and its derivatives are not directly provided. Furthermore, the existence of local minima cannot be excluded. For these reasons, derivative-free global optimization algorithms have been developed and applied, providing a global approximate solution at a reasonable computational cost. Even if global optimization approaches are a good compromise between exploration and exploitation of the research space, they could get trapped in a local minimum and the convergence to the global minimum cannot be proved. On the other hand, if the research region to explore is known a priori, local optimization approaches can give an accurate approximation of the local minimum, although their convergence may be computationally expensive. For these reasons, the hybridization of global optimization algorithms with local search methods is an interesting research field, where the qualities of both methods can be efficiently and robustly coupled.

Here, four derivative-free optimization algorithms are presented and applied. Two algorithms are global optimization approaches, specifically (a) the DIRECT (DIviding RECTangles) algorithm, *Jones et al. (1993*), and (b) a deterministic version of the particle swarm optimization method (DPSO), *Serani et al. (2014)*. The other two algorithms are global/local hybrid techniques for (a) and (b) respectively, enhancing the global methods with proved stationarity of the final solution: a hybrid DIRECT method coupled with line search-based derivative-free optimization, namely DIRMIN-2, *Campana et al. (2014)*, and a hybrid particle swarm algorithm coupled with line search-based derivative-free optimization, namely LS-DF_PSO, *Serani et al. (2015)*.

The SBD application presented herein pertains to the hull-form optimization of a USS Arleigh Burke-class destroyer, namely the DTMB 5415 model, an early version of the DDG-51. The DTMB 5415 model, Table I, has been widely investigated through towing tank experiments, *Stern et al. (2000)*, *Longo and Stern (2005),* and SBD studies, including hull-form optimization, *Tahara et al. (2008*). Recently, the DTMB 5415 model has been selected as the test case for the SBD activities within the NATO AVT-204 "Assess the Ability to Optimize Hull Forms of Sea Vehicles for Best Performance in a Sea Environment", aimed at the multi-objective design optimization for multi-speed reduced resistance and improved seakeeping performance, e.g. *Serani et al. (2015)*. Herein, a single-speed single-objective SBD example is shown, aimed at the reduction of the total resistance in calm water at 18 kn, corresponding to Froude number Fr = 0.25. An orthogonal representation of the shape modification is used. Specifically, two sets of orthogonal functions are applied for the modification of the hull and the sonar dome shapes, and controlled by a total number of design variables N_{dv} = 6. The constraints include fixed displacement and length, along with a $\pm 5\%$ maximum variation of beam and draft. The solver used is a linear potential flow code, *Bassanini et al. (1994)*, allowing for the evaluation of the wave resistance by transversal wave cut, *Telste and Reed (1994)*. The resistance due to friction is estimated by a local approximation based on flat-plate theory, *Schlichting et al. (2000)*.

2. Optimization problem and algorithms

Consider the following objective function:

$$f(\mathbf{x}):\mathbb{R}^n \to \mathbb{R}$$

and the global optimization problem

$$\min_{\mathbf{x}\in\mathcal{L}} f(\mathbf{x}),\ \mathcal{L}\subset\mathbb{R}^n$$

where $\mathcal{L}$ is a closed and bounded subset of $\mathbb{R}^n$. The global minimization of the objective function f requires finding a vector $\mathbf{a}\in\mathcal{L}$ so that:

$$\forall\mathbf{b}\in\mathcal{L}: f(\mathbf{a})\le f(\mathbf{b})$$

Then, $\mathbf{a}$ is a global minimum for the function $f(\mathbf{x})$ over $\mathcal{L}$. Since the solution of the minimization problem is in general a NP-hard problem, the exact identification of a global minimum might be very difficult. Therefore, solutions with sufficient good fitness, provided by heuristic procedures, are often considered acceptable for several practical purposes. In the optimization algorithm considered in the following, the candidate solutions will be denoted by $\mathbf{x}\in\mathcal{L}$, with associated fitness $f(\mathbf{x})$. Moreover, in this paper the compact set $\mathcal{L}$ is identified by box constraints.

The two global algorithms (DIRECT and DPSO) and their global/local hybridizations (DIRMIN-2 and LS-DF_PSO) are presented in the following.

2.1 The DIRECT algorithm

DIRECT is a sampling global derivative-free optimization algorithm and a modification of the Lipschitizian optimization method, *Jones et al. (1993)*. It begins the optimization by transforming the domain of the problem into the unit hyper-cube. At the first step of DIRECT, $f(\mathbf{x})$ is evaluated at the center of the search domain $\mathcal{D}$; the hyper-cube is then partitioned into a set of smaller hyper-rectangles and $f(\mathbf{x})$ is evaluated at their centers. Let the partition of $\mathcal{D}$ at iteration k be defined as

$$\mathcal{H}_k=\{\mathcal{D}^i : i\in I_k\}, \text{ with } \mathcal{D}^i=\{x\in\mathbb{R}^n : l^I \le x\le u^i\}, \forall i\in I_k$$

where l^i, $u^i\in[0,1]$, $i\in I_k$, and I_k is the set of indices identifying the subsets defining the current partition. At a generic k-th iteration of the algorithm, starting from the current partition $\mathcal{H}_k$ of $\mathcal{D}$ into hyper-rectangles, a new partition, $\mathcal{H}_{k+1}$, is built by subdividing a set of promising hyper-rectangles of the previous partition $\mathcal{H}_k$. The identification of potentially optimal hyper-rectangles is based on some measure of the hyper-rectangle itself and on the value of f at its center. The refinement of the partition continues until a prescribed number of function evaluations have been performed, or another stopping criterion is satisfied. The minimum of f over all the centers of the final partition, and the corresponding centers, provide an approximate solution to the problem. It may be noted that the box constraints are automatically satisfied.

2.2 Local hybridization of the DIRECT algorithm: DIRMIN-2

DIRMIN-2 is a global/local hybridization of the DIRECT algorithm. It is a more efficient variant of DIRMIN, *Lucidi and Sciandrone (2002)*. A single local minimization is performed starting from the best point produced by dividing the potentially-optimal hyper-rectangles. This strategy should result in a more efficient algorithm, which is less demanding in terms of number of function evaluations and preserves a good capability to find global solutions. DIRMIN-2's local minimization is used when the number of function evaluations reaches the activation trigger $\gamma b(\mathrm{N}_{dv})$, with $\gamma \in (0,1)$. The local minimization proceeds until either the number of function evaluations exceeds $b(\mathrm{N}_{dv})$ or the step size Δ falls below a given tolerance β. The local search is not allowed to violate the box constraints. *Campana et al. (2014)* has studied the performance of the algorithm varying the tolerance β and the activation trigger γ, and applied DIRMIN-2 to a ship optimization problem. Herein the suggestion by *Campana et al. (2014)* is used, setting $\gamma = 10^{-1}$ and $\beta = 10^{-4}$.

2.3 The DPSO algorithm

Particle Swarm Optimization (PSO) was originally introduced by *Kennedy and Eberhart (1995)*, based on the social-behavior metaphor of a flock of birds or a swarm of bees searching for food. PSO belongs to the class of heuristic algorithms for single-objective evolutionary derivative-free global optimization. In order to make PSO more efficient for use within SBD, a deterministic version of the algorithm (DPSO) was formulated by *Campana et al. (2009)* as follows

$$\begin{cases} \mathbf{v}_i^{k+1} = \chi\left[\mathbf{v}_i^k + c_1(\mathbf{x}_{i,pb} - \mathbf{x}_i^k) + c_2(\mathbf{x}_{gb} - \mathbf{x}_i^k)\right] \\ \mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \end{cases}$$

The above equations update velocity and position of the i-th particle at the k-th iteration, where χ is the constriction factor; c_1 and c_2 are the social and cognitive learning rate; $\mathbf{x}_{i,pb}$ is the personal best position ever found by the i-th particle and $\mathbf{x}_{gb}$ is the global best position ever found by all particles. *Serani et al. (2014)* made a systematic study on the performance and the use of DPSO, defining a guideline, successfully applied on a ship design optimization problem. Herein the setup suggested by *Serani et al. (2014)* is used: (a) number of particles (N_p) equal to 4 times the number of design variables; (b) particles initialization by Hammersly sequence sampling (HSS) distribution on domain and bounds with non-null velocity; (c) set of coefficient by *Clerc (2006)*, i.e., $\chi =$ 0.721, $c_1 = c_2 = 1.655$; (d) semi-elastic wall-type approach for box constraints. Specifically, in the case a particle violates one of the box constraints, then the particle is moved on the active boundary while the associated velocity component is redefined as $v_i^j = -v_i^j\left[\chi(c_1 + c_2)\right]^{-1}$.

2.4 Local hybridization of the DPSO algorithm: LS-DF_PSO

Global convergence properties of a modified PSO scheme may be obtained by properly combining PSO with a line search-based derivative-free method, so that convergence to stationary points can be forced at a reasonable cost. *Serani et al. (2015)* provides a robust method to force the convergence of a subsequence of points toward a stationary point, which satisfies first order optimality conditions for the objective function.

The method, namely LS-DF_PSO, starts by coupling the DPSO scheme with a line search-based method. Specifically, a Positively Spanning Set (PSS) is used, where the set of search directions (D) is defined by the unit vectors $\pm e_i$, $i = 1, \ldots, n$, as shown in the following equation and in Fig.1.

$$D = \left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$$

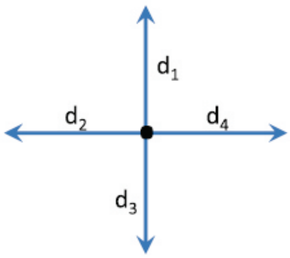


Fig. 1: Example of PSSs in $\mathbb{R}^n$

After each DPSO iteration, the local search is performed if the swarm has not find a new global minimum. The initial step size (α^k) for the local search is set equal to 0.25 times the variable domain range, and it is reduced by $\theta = 0.5$ at each local search iteration. Local searches continue in each direction until the step size is greater than $\gamma = 10^{-3}$. If the local search stops without providing a new global minimum, the actual global minimum is declared as a stationary point. The line search method is not allowed to violate the box constraints.

3. Ship design problem

Fig. 2 shows the geometry of a 5.720 m length DTMB 5415 model used for towing tank experiments, as seen at CNR-INSEAN, *Stern et al. (2000)*. The main particulars of the full scale model and tests conditions are summarized in Tables I and II, respectively.

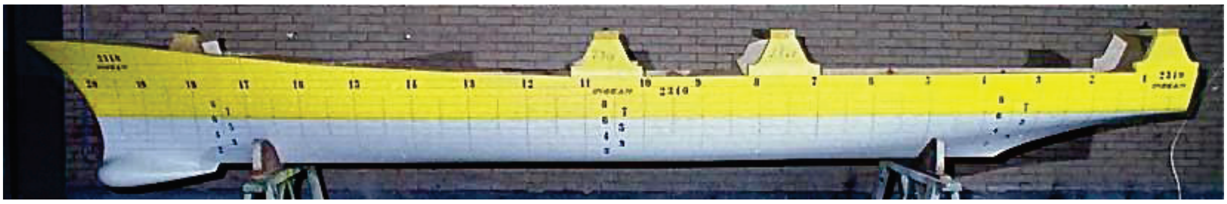


Fig. 2: A 5.720 m length model of the DTMB 5415 (CNR-INSEAN model 2340)

Table I: DTMB 5415 model main particulars (full scale)

Displacement	∇	8636 t
Length between perpendiculars	LBP	142.00 m
Beam	B	18.90 m
Draft	T	6.16 m
Longitudinal center of gravity	LCG	71.60 m
Vertical center of gravity	VCG	1.39 m

Table II: Test conditions

Speed	U	kn	18.00
Water density	ρ	kg/m^3	998.5
Kinematic viscosity	ν	m^2/s	$1.09 \cdot 10^{-6}$
Gravity acceleration	g	m/s^2	9.803

The objective function is the total resistance, R_T, in calm water at Fr = 0.25. A six design space is considered. Design modifications are defined in terms of orthogonal functions, $\boldsymbol{\psi}_j$ $(j=1,\ldots,6)$, defined over surface-body patches as

$$\begin{cases} \boldsymbol{\psi}_j(\xi,\eta) := \alpha_j \sin\left(\dfrac{p_j \pi \xi}{A_j} + \phi_j\right) \sin\left(\dfrac{q_j \pi \eta}{B_j} + \chi_j\right) \mathbf{e}_{k(\mathrm{j})} \\ (\xi,\eta) \in [0;A] \times [0;\mathrm{B}] \end{cases}$$

The coefficient α_j is the corresponding (dimensional) design variable; p_j and q_j define the order of the function in ξ and η direction, respectively; ϕ_j and χ_j are the corresponding spatial phases; A_j and B_j define the patch dimension; and $\mathbf{e}_{k(j)}$ is a unit vector. Modifications may be applied in x, y or z direction setting k(j) = 1, 2 or 3, respectively. Specifically, four orthogonal functions and design variables are used for the hull, whereas two functions/variables are used for the sonar dome, as summarized in Table III. The corresponding functions used for shape modification are shown in Figs.3 and 4. Upper and lower bounds used for dimensional (α_j) and non-dimensional design variables, $x_j = 2(\alpha_j - \alpha_{j,min})/(\alpha_{j,max} - \alpha_{j,min}) - 1$, are included in Table III.

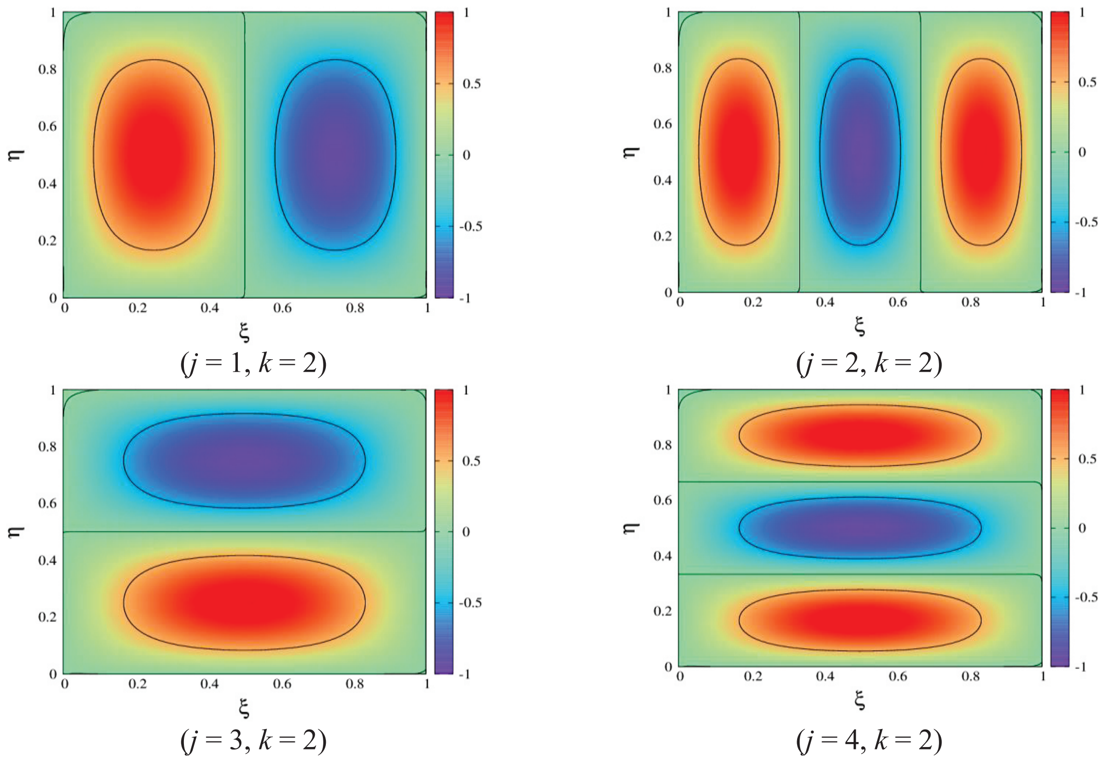
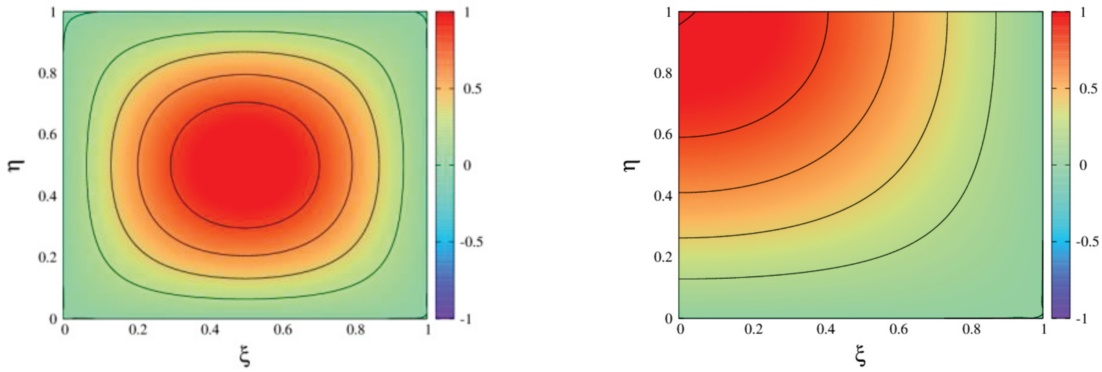


Fig. 3: Orthogonal functions $\boldsymbol{\psi}_j(\xi,\eta)$, $j = 1,\ldots,4$ for the hull modifications

Table III: Orthogonal functions parameters, for shape modification

	j	p_j	ϕ_j	q_j	χ_j	$k(j)$	$\alpha_{j,\min}$	$\alpha_{j,\max}$	$x_{j,\min}$	$x_{j,\max}$
Hull modification	1	2.0	0	1.0	0	2	-2.0	2.0	-1.0	1.0
	2	3.0	0	1.0	0	2	-2.0	2.0	-1.0	1.0
	3	1.0	0	2.0	0	2	-1.0	1.0	-1.0	1.0
	4	1.0	0	3.0	0	2	-1.0	1.0	-1.0	1.0
Sonar dome modification	5	1.0	0	1.0	0	2	-0.6	0.6	-1.0	1.0
	6	0.5	$\pi/2$	0.5	0	3	-1.0	1.0	-1.0	1.0



($j = 5$, $k = 2$) ($j = 6$, $k = 3$)

Fig. 4: Orthogonal functions $\boldsymbol{\psi}_j(\xi,\eta)$, $j = 5,6$ for the sonar dome modifications

Geometric constraints include fixed length between perpendicular (LBP) and fixed displacement (∇), with beam (B) and draft (T) varying between $\pm 5\%$ of the original hull. Fixed LBP and ∇ are satisfied by automatic geometry scaling, while constraints for B and T are handled using a penalty function method.

Simulations are conducted using the code WARP (Wave Resistance Program), developed at CNR-INSEAN. Wave resistance computations are based on linear potential flow theory; details of equations, numerical implementations and validation of the numerical solver are given in *Bassanini et al. (1994)*. The wave resistance is evaluated with the transverse wave cut method, *Telste and Reed (1994)*, whereas the frictional resistance is estimated using a flat-plate approximation, based on the local Reynolds number, *Schlichting and Gersten (2000)*. Simulations are performed for the right demi-hull, taking advantage of symmetry about the *xz* plane. The computational domain for the free surface is defined within 1 hull length upstream, 3 lengths downstream and 1.5 lengths aside, as shown in Fig. 5. The associated panel grid used, Fig. 5, is summarized in Table IV and guarantees solution convergence. The validation of CFD analyses performed by WARP for the original hull versus experimental data collected at CNR-INSEAN is shown in Fig. 6, showing a reasonable agreement especially for low speeds. For optimization, a limit to the maximum number of function evaluations is set equal to 1536, i.e. $256 \cdot N_{dv}$.

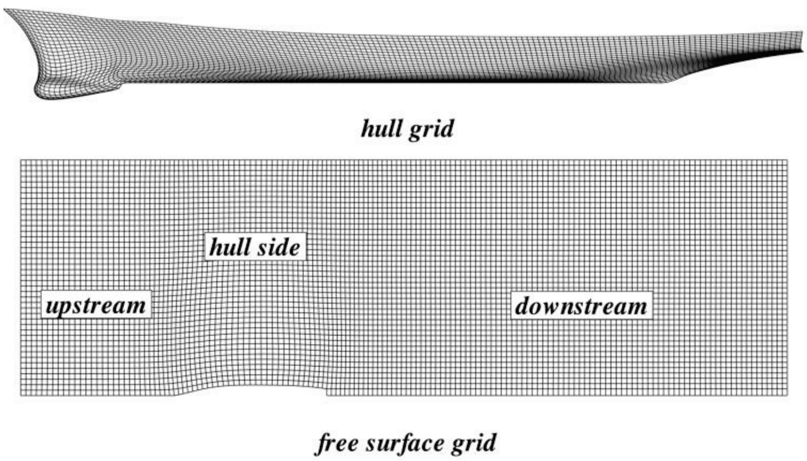


Fig. 5: Computational panel grid

Table IV: Panel grid used for WARP

Hull	Free surface			Total
	Upstream	Hull side	Downstream	
150 × 30	30 × 44	30 × 44	90 × 44	11k

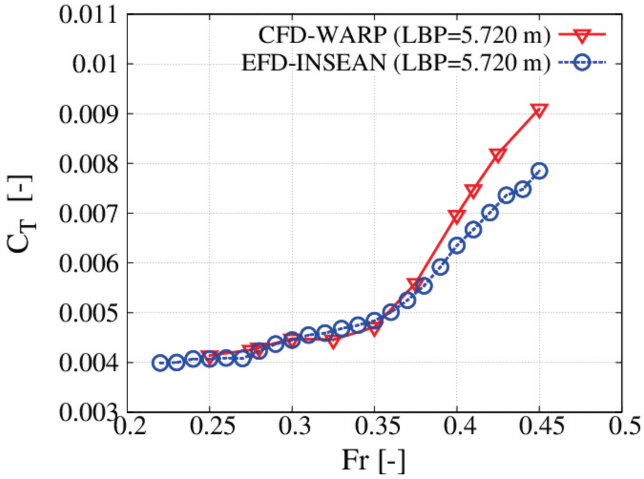


Fig. 6: Total resistance coefficient $C_T = 0.5 R_T / \rho U^2 S$ in calm water versus Fr

4. Numerical results

A preliminary sensitivity analysis for each design variable is presented in Fig. 7, showing the associated percent resistance reduction (Δ_{obj}) with respect to the parent hull. Unfeasible designs are not reported in the plot. Changes in Δ_{obj} are found significant, revealing a possible reduction of the total resistance at Fr = 0.25 close to 10%.

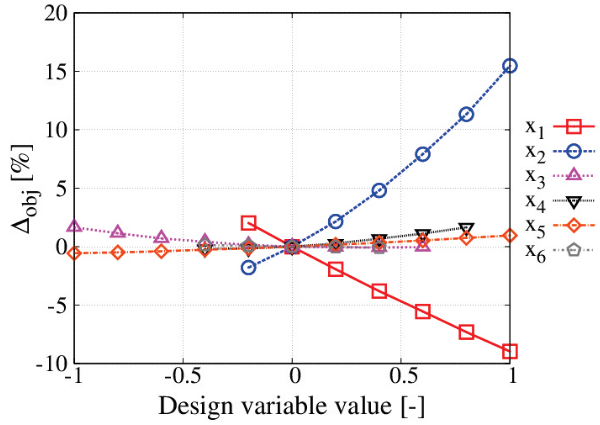


Fig. 7: Sensitivity analysis of design variables for DTMB 5415 model optimization

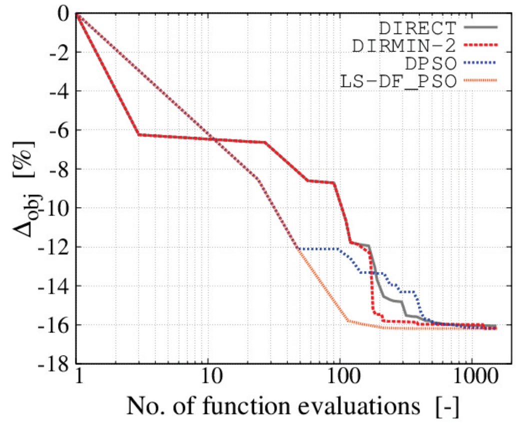
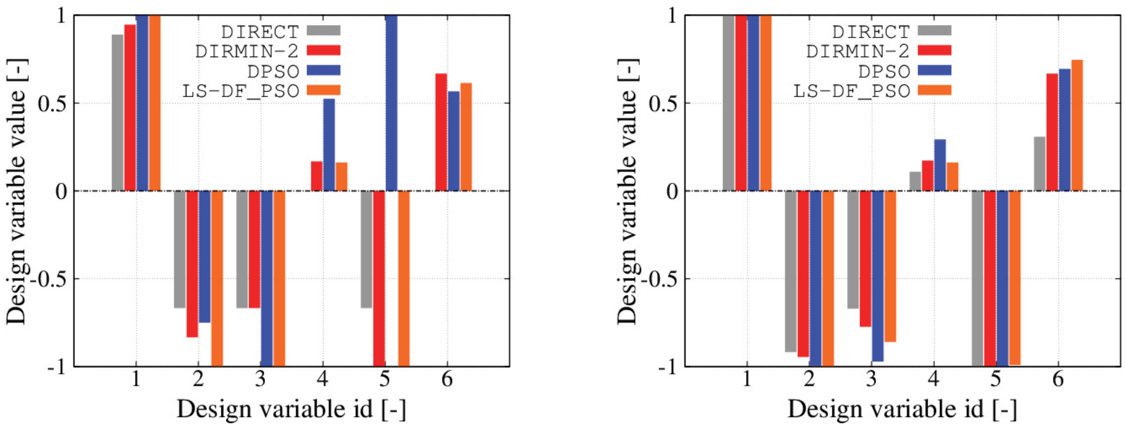


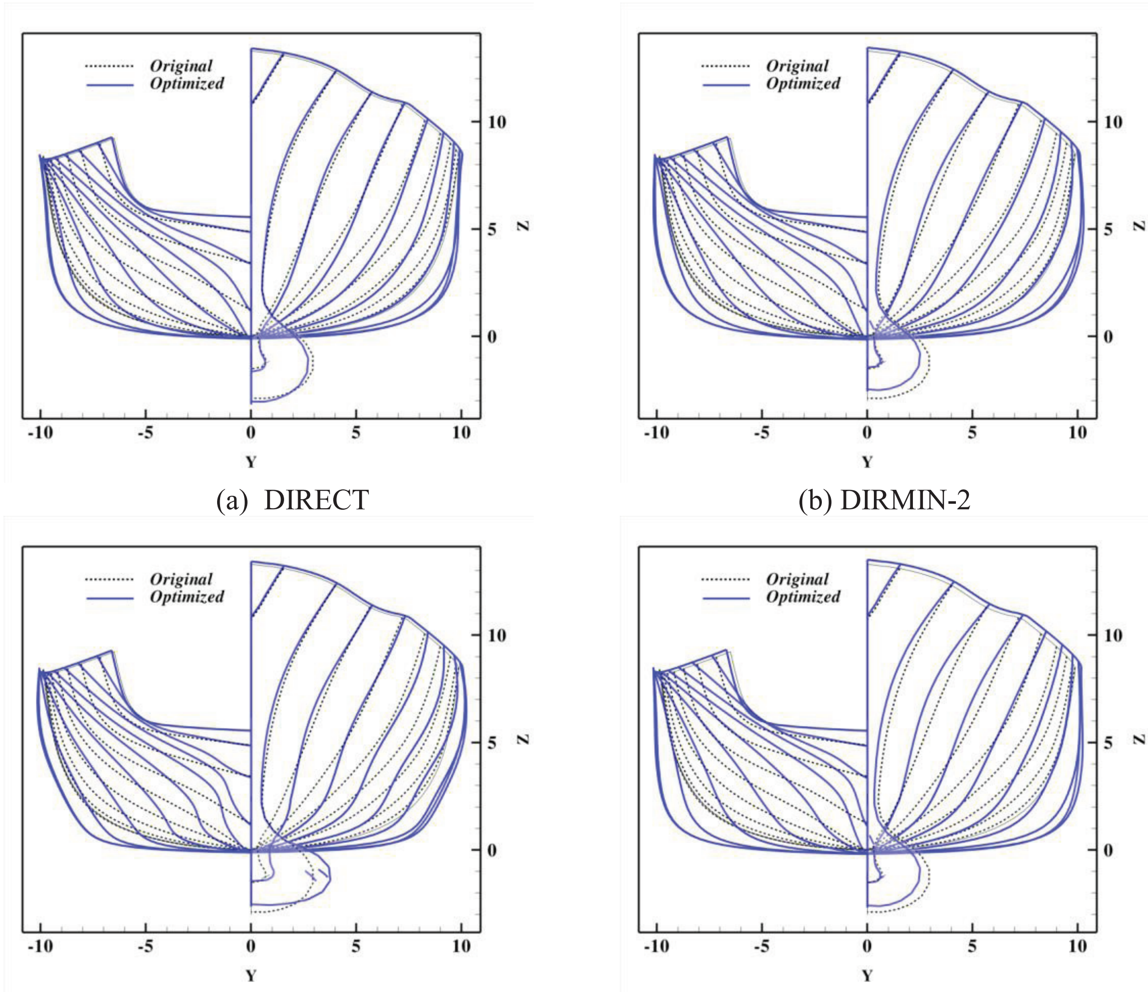
Fig. 8: Objective function convergence history



(a) 192 function evaluations

(b) 1536 function evaluations

Fig.9: Objective function convergence of optimal design variables



(a) DIRECT (b) DIRMIN-2

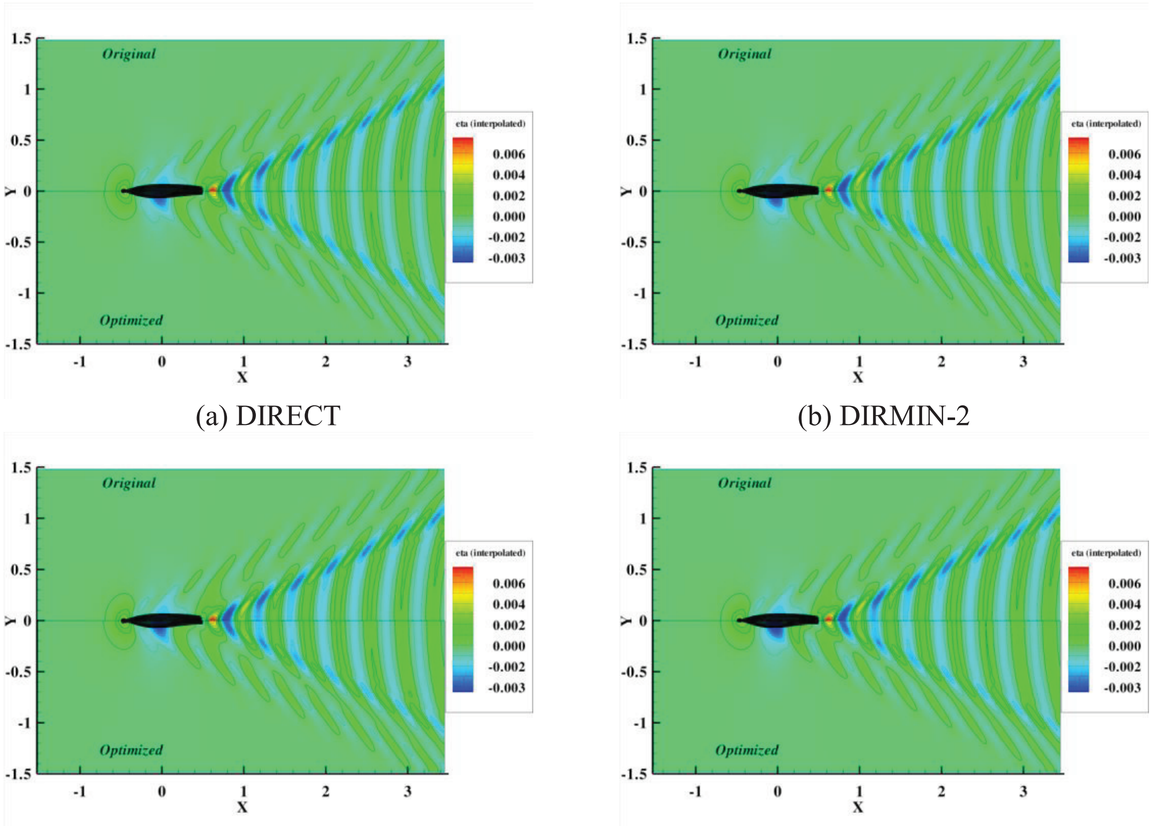
(c) DPSO (d) LS-DF_PSO

Fig.10: Optimal hull-form shape compared to the original using 192 function evaluations

The analysis of SBD results is conducted setting apart results (i) for a low budget of 192 function evaluations (which corresponds to $32 \cdot N_{dv}$, an eighth of the full budget), and (ii) for the full budget of 1536 function evaluations (which corresponds to $256 \cdot N_{dv}$).

For the case (i), the SBD optimization procedure achieves a resistance reduction of 13.68 and 15.45% using DIRECT and DIRMIN-2 respectively, and a reduction of 13.46 and 16.00% using DPSO and LS-DF_PSO respectively. The two global/local hybrid algorithms outperform their global version. In particular, LS-DF_PSO is found the most efficient algorithm for the present SBD problem, achieving the best design with the fastest convergence rate, as shown in Fig. 8. Fig. 9 presents the values of the optimal design variables, showing appreciable differences. Fig. 10 shows the corresponding optimized shapes, compared to the original. The reduction of the resistance is consistent with the reduction of the wave elevation pattern, both in terms of transverse and diverging Kelvin waves, Fig. 11. Fig. 12 shows the pressure field on the optimized hulls compared to the original, showing a better pressure recovery towards the stern.

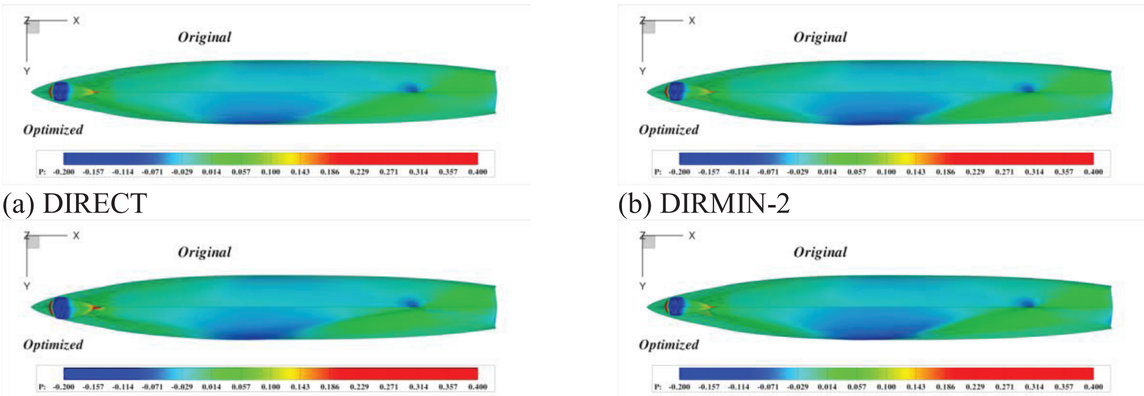
For the case (ii), the SBD optimization procedure achieves a resistance reduction of 16.04 and 16.17% using DIRECT and DIRMIN-2 respectively, and a reduction of 16.19 and 16.20% using DPSO and LS-DF_PSO respectively. The convergence history of the objective function towards the minimum is shown in Fig. 8, confirming the efficiency and robustness of the two hybrid global/local approaches DIRMIN-2 and LS-DF_PSO. More in detail, LS-DF_PSO achieves the most significant reduction of the objective function overall, although all the solutions are very close in this case.



(a) DIRECT (b) DIRMIN-2

(c) DPSO (d) LS-DF_PSO

Fig.11: Wave patterns produced by optimal hull forms at Fr = 0.25 compared with original, 192 function evaluations

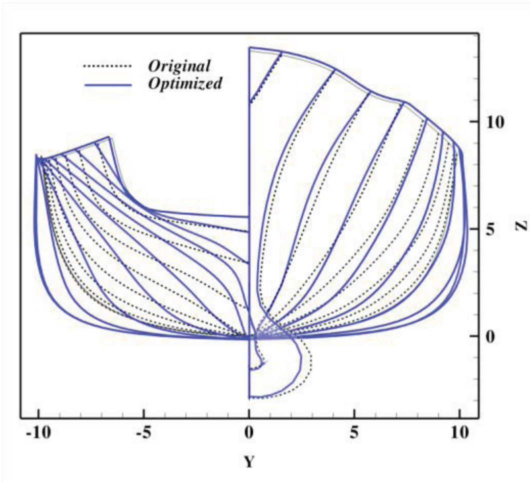


(a) DIRECT (b) DIRMIN-2

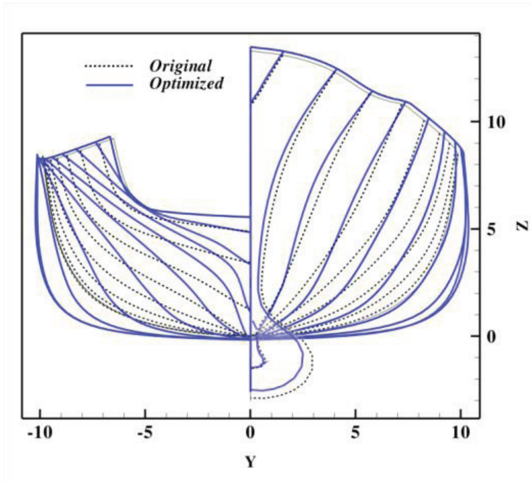
(c) DPSO (d) LS-DF_PSO

Fig.12: Pressure field on optimal hull forms at Fr = 0.25 compared with original, 192 function evaluations

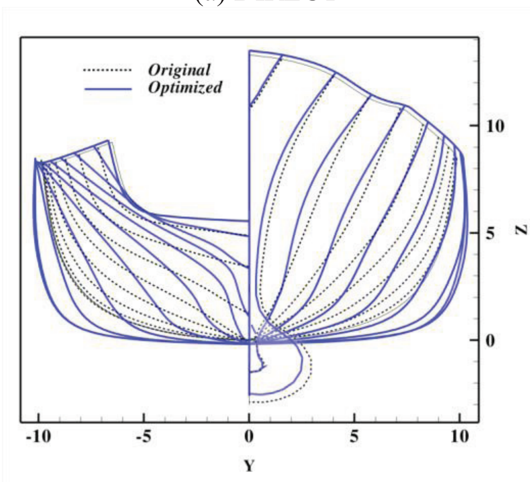
Fig. 9 presents the values of the corresponding optimal design variables and Fig. 13 shows the optimized shapes compared to the original. The close agreement of the solutions obtained by the different algorithms indicates that the global minimum region has been likely achieved. The reduction of the wave elevation pattern of the optimized shapes, both in terms of transverse and diverging Kelvin waves, is significant, as shown in Fig. 14. Fig. 15 presents the pressure field on the optimized hulls compared to the parent hull, showing a better pressure recovery towards the stern. Table V summarizes the optimization results.



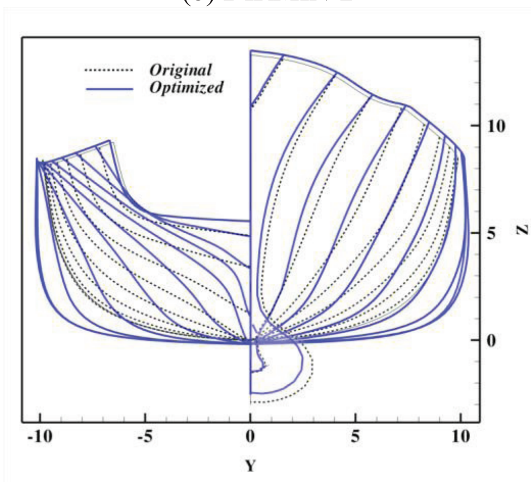
(a) DIRECT



(b) DIRMIN-2



(c) DPSO



(d) LS-DF_PSO

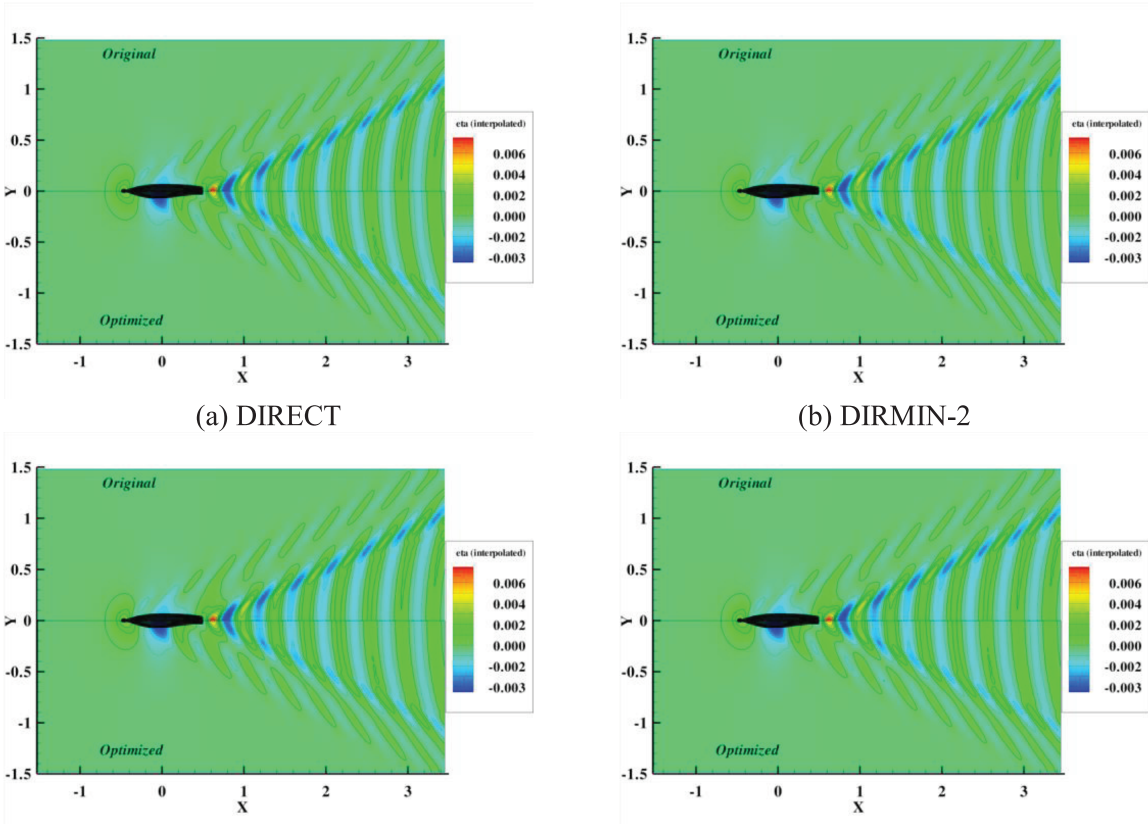
Fig.13: Optimal hull-form shape compared to the original, 1536 function evaluations

Table V: Summary of the optimization results for DTMB 5415 model

	Design variables (non-dimensional)							$R_T \times 10^5 (N)$	
N. of funct. eval.	Algorithm	x_1	x_2	x_3	x_4	x_5	x_6	value	Δ_{obj}(%)
192	DIRECT	0.889	-0.667	-0.667	0.000	-0.667	0.000	2.964	-13.68
	DIRMIN-2	0.944	-0.833	-0.667	0.167	-1.000	0.667	2.906	-15.45
	DPSO	1.000	-0.749	-0.998	0.523	1.000	0.564	2.975	-13.46
	LS-DF_PSO	1.000	-1.000	-1.000	0.161	-1.000	0.612	2.885	-16.00
1536	DIRECT	0.999	-0.917	-0.669	0.108	-0.999	0.307	2.883	-16.04
	DIRMIN-2	1.000	-0.944	-0.774	0.172	-0.998	0.667	2.878	-16.17
	DPSO	1.000	-0.993	-0.971	0.291	-1.000	0.693	2.877	-16.19
	LS-DF_PSO	1.000	-1.000	-0.859	0.161	-0.990	0.745	2.877	-16.20

5. Conclusions

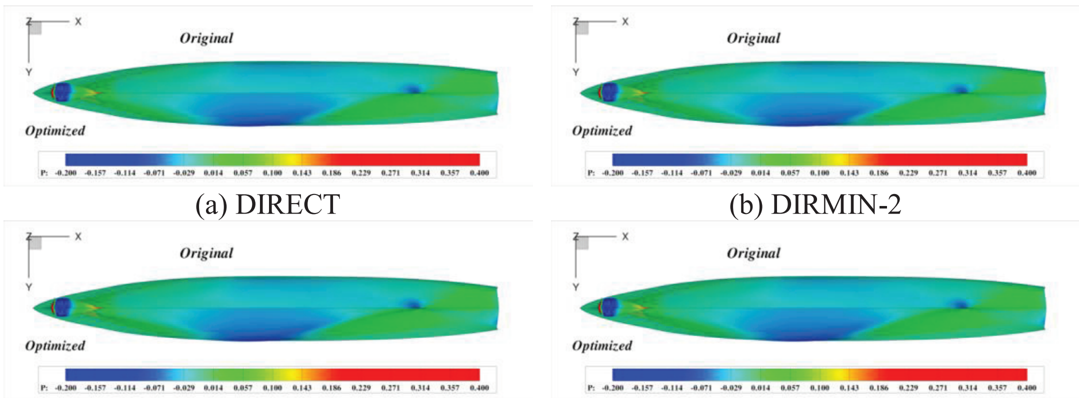
A derivative-free global design optimization of the DTMB 5415 model has been shown, using local hybridizations by line search methods of two well-known global algorithms, DIRECT and PSO respectively. The optimization aimed at the reduction of the total resistance in calm water at Fr = 0.25, using six design variables modifying the hull and the sonar dome shapes. Computer simulations were conducted using a linear potential flow code. The wave resistance has been assessed by the transversal wave cut method, whereas the frictional resistance has been estimated by a local approximation based on flat-plate theory. A resistance reduction of 16% has been achieved by the optimized designs.



(a) DIRECT (b) DIRMIN-2

(c) DPSO (d) LS-DF_PSO

Fig.14: Wave patterns produced by optimal hull forms at Fr = 0.25 compared with original, 1536 function evaluations



(a) DIRECT (b) DIRMIN-2

(c) DPSO (d) LS-DF_PSO

Fig.15: Pressure field on optimal hull forms at Fr = 0.25 compared with original, 1536 function evaluations

The local hybridization methods, DIRMIN-2 and LS-DF_PSO, outperformed their original global algorithms, DIRECT and PSO respectively. This result has been found significant (about 2% difference in resistance reduction achieved) especially for low budgets of function evaluations. Hybrid algorithms have shown their capability to combine effectively the characteristics of global and local approaches, resulting in a faster (and computationally less expensive) convergence towards the global minimum. This, along with their derivative-free formulation and implementation, makes the present local hybridization methods a viable and effective option for SBD optimization, especially when computationally expensive objective functions are involved.

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