



Ca' Foscari  
University  
of Venice

Department  
of Economics

Working Paper

**Martina Nardon and  
Paolo Pianca**

**European option pricing with  
constant relative sensitivity  
probability weighting  
functions**

ISSN: 1827-3580  
No. 25/WP/2014





## European option pricing with constant relative sensitivity probability weighting function

**Martina Nardon**

*University of Venice at Ca' Foscari*

and

**Paolo Pianca**

*University of Venice at Ca' Foscari*

### **Abstract**

We evaluate European financial options under continuous cumulative prospect theory. Within this framework, it is possible to model investors' attitude toward risk, which may be one of the possible causes of mispricing. We focus on probability risk attitudes and consider alternative probability weighting functions. In particular, curvature of the weighting function models optimism and pessimism when one moves from extreme probabilities, whereas elevation can be interpreted as a measure of relative optimism. The *constant relative sensitivity* weighting function is the only one, amongst those in the literature, which is able to model separately curvature and elevation. We are interested in studying the effects of both these features on options prices.

### **Keywords**

Behavioral finance, cumulative prospect theory, curvature, elevation, European option pricing.

### **JEL Codes**

C63, D81, G13

### *Address for correspondence:*

**Martina Nardon**  
Department of Economics  
Ca' Foscari University of Venice  
Cannaregio 873, Fondamenta S.Giobbe  
30121 Venezia - Italy  
Phone: (+39) 041 2347414  
Fax: (+39) 041 2349176  
e-mail: [mnardon@unive.it](mailto:mnardon@unive.it)

*This Working Paper is published under the auspices of the Department of Economics of the Ca' Foscari University of Venice. Opinions expressed herein are those of the authors and not those of the Department. The Working Paper series is designed to divulge preliminary or incomplete work, circulated to favour discussion and comments. Citation of this paper should consider its provisional character.*

# 1 Introduction

Prospect theory has recently begun to attract attention in the literature on financial options valuation; when applied to option pricing in its continuous cumulative version, it seems a promising alternative to other models, for its potential to explain option mispricing with respect to theoretical Black and Scholes prices. Empirical studies on quoted options highlight systematic differences between the market prices and the Black and Scholes model; this may be due to different causes, such as assumptions regarding the price dynamics (volatility, in particular), markets frictions, information imperfections, and investors' attitude toward risk. Normally one tries to improve the performance of models considering more complex dynamics for the prices of the underlying assets, but leaving unchanged decision maker's preferences. An alternative approach is to price options considering behavioral aspects of the operators.

According to prospect theory, individuals do not always take their decisions consistently with the maximization of expected utility. Decision makers are risk averse when considering gains and risk-seeking with respect to losses. They are loss averse: people are much more sensitive to losses than they are to gains of comparable magnitude. Gambles are evaluated based on potential gains and losses relative to a *reference point*, rather than in terms of final wealth. Decision makers tend to underweight high probabilities and overweight low probabilities<sup>1</sup>. Risk attitude, loss aversion and subjective probabilities are described by two functions: a value function and a weighting function, which models probability perception.

Shiller (1999) argues that the weighting function may be one of the possible causes of overpricing of out-of-the-money and in-the-money options, thus it may explain the options smile. This phenomenon could be explained in terms of the distortion in probabilities represented by the weighting function: due to the overestimation of small probabilities and underestimation of medium and large probabilities. The weighting function might even explain the down-turned corners that some smiles exhibit if at these extremes the discontinuities at the extremes of the weighting function become relevant (Shiller, 1999).

---

<sup>1</sup>Kahneman and Tversky (1979) provide empirical evidence of such behaviors.

The literature on behavioral finance<sup>2</sup> and prospect theory is huge, whereas a few studies in this field focus on financial options. A first contribution which applies prospect theory to options valuation is the work of Shefrin and Statman (1993), who consider covered call options in a one period binomial model. A list of paper on this topic should include: Poteshman and Serbin (2003), Abbink and Rockenbach (2006), Breuer and Perst (2007), and more recently Versluis *et al.* (2010). Following this direction, Nardon and Pianca (2013) apply the cumulative prospect theory in the continuous case in order to evaluate European plain vanilla options, extending the model of Versluis *et al.* (2010) to the European put option; the authors also consider both the positions of the writer and the holder.

In this contribution, we focus on the effects on European option prices of the probability weighting function. Such a function models probabilistic risk behavior; its *curvature* is related to the risk attitude towards probabilities. Empirical evidence suggests a particular shape of probability weighting functions which turns out in a typical *inverse-S shape*: the function is initially concave (probabilistic risk seeking or *optimism*) for small probabilities and convex (probabilistic risk aversion or *pessimism*) for medium and large probabilities. A linear weighting function describes probabilistic risk neutrality or objective sensitivity towards probabilities, which characterizes Expected Utility. Empirical findings indicate that the intersection between the weighting function and the linear function (*elevation*) is for probability around 0.33. Curvature of the weighting function models optimism and pessimism when one moves from extreme probabilities, whereas elevation can be interpreted as a measure of relative optimism. The *constant relative sensitivity* weighting function proposed by Abdellaoui *et al.* (2010) is the only one, amongst those in the literature, which is able to model separately curvature and elevation. We are interested in studying the effects of both these features on options prices.

The rest of the paper is organized as follows. Section 2 synthesizes the main features of prospect theory. Section 3 focuses on the probability weighting function. Section 4 present the option pricing models under continuous CPT. In Section 5 numerical results are provided and discussed. Section 6 concludes.

## 2 Prospect Theory

Prospect theory<sup>3</sup> (PT), in its formulation proposed by Kahnemann and Tversky (1979), is based on the subjective evaluation of *prospects*. Prospects assign to any possible outcome  $x_i$  a probability  $p_i$ ; originally PT deals only with a limited

---

<sup>2</sup>See e.g. Barberis and Thaler (2003) and Subrahmanyam (2007) for a survey

<sup>3</sup>See the book of Wakker (2010) for a thorough treatment on prospect theory.

set of prospects. Let  $\mathcal{P}$  denote the set of all prospects, a preference relation is introduced over  $\mathcal{P}$ .

With a finite set of potential future outcomes  $X = \{x_1, x_2, \dots, x_n\}$ , a prospect is a vector<sup>4</sup>

$$(\Delta x_1, p_1; \Delta x_2, p_2; \dots; \Delta x_n, p_n)$$

of pairs  $(\Delta x_i, p_i)$ ,  $i = 1, 2, \dots, n$ . Assume  $\Delta x_i \leq \Delta x_j$  for  $i < j$ ,  $i, j = 1, 2, \dots, n$ , and  $\Delta x_i \leq 0$  ( $i = 1, 2, \dots, k$ ) and  $\Delta x_i > 0$  ( $i = k + 1, \dots, n$ ).

Outcome  $\Delta x_i$  is defined relative to a certain *reference point*  $x^*$ ; being  $x_i$  the absolute outcome, we have  $\Delta x_i = x_i - x^*$ . An important difference between Expected Utility (EU) and PT is that in the former results are evaluated considering the final wealth, whereas in the latter results are evaluated through a value function  $v$  which considers only outcomes. In many applications, zero is taken as a reference point. Later, in order to simplify the notation, it will be convenient to write  $x_i$  instead of  $\Delta x_i$  for the outcomes, but still considering outcomes interpreted as deviations from a reference point.

A value function alone is not able to capture the full complexity of observed behaviors: the degree of risk aversion or risk seeking appears to depend not only on the value of the outcomes but also on the probability and ranking of outcome. Subjective values  $v(\Delta x_i)$  are not multiplied by objective probabilities  $p_i$ , but using *decision weights*  $\pi_i = w(p_i)$ .

The shape of the value function and the weighting function becomes significant in describing actual choice patterns. It is also relevant to separate gains from losses, as negative and positive outcomes may be evaluated differently: the function  $v$  is typically convex in the range of losses and concave and steeper in the range of gains; whereas subjective probabilities may be evaluated through a weighting function  $w^-$  for losses and  $w^+$  for gains, respectively.

Let us denote with  $\Delta x_i$ , for  $-m \leq i < 0$  negative outcomes and with  $\Delta x_i$ , for  $0 < i \leq n$  positive outcomes, with  $\Delta x_i \leq \Delta x_j$  for  $i < j$ . Subjective value of a prospect is displayed as follows:

$$V = \sum_{i=-m}^n \pi_i \cdot v(\Delta x_i), \quad (1)$$

with decision weights  $\pi_i$  and values  $v(\Delta x_i)$ . In the case of EU, the weights are  $\pi_i = p_i$  and the utility function is not based on relative outcomes.

*Cumulative prospect theory* (CPT) developed by Tversky and Kahnemann (1992) overcomes some drawbacks (such as violation of stochastic dominance) of the original PT. In CPT, decision weights  $\pi_i$  are differences in transformed

---

<sup>4</sup>Infinitely many outcomes may also be considered. See Schmeidler (1989).

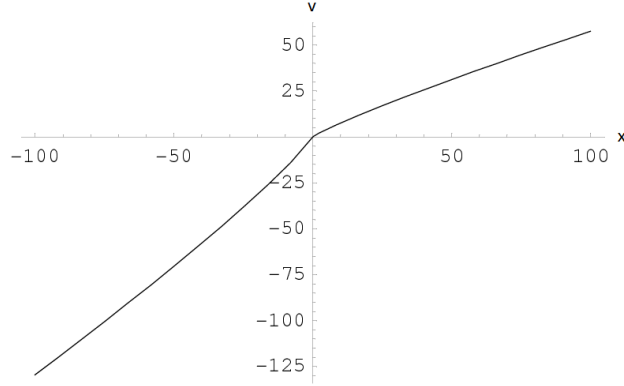


Figure 1: Value function (3) with parameters  $\lambda = 2.25$  and  $a = b = 0.88$

(through a weighting function) cumulative probabilities of gains or losses. Formally,

$$\pi_i = \begin{cases} w^-(p_{-m}) & i = -m \\ w^-\left(\sum_{j=-m}^i p_j\right) - w^-\left(\sum_{j=-m}^{i-1} p_j\right) & i = -m+1, \dots, -1 \\ w^+\left(\sum_{j=i}^n p_j\right) - w^+\left(\sum_{j=i+1}^n p_j\right) & i = 0, \dots, n-1 \\ w^+(p_n) & i = n. \end{cases} \quad (2)$$

Specific parametric forms have been suggested for the value function; some examples are reported in Table 1. Let  $x$  be an outcome, a function which is used in many empirical studies is

$$\begin{aligned} v^- &= -\lambda(-x)^b & x < 0, \\ v^+ &= x^a & x \geq 0, \end{aligned} \quad (3)$$

with positive parameters which control risk attitude ( $0 < a \leq 1$  and  $0 < b \leq 1$ ) and loss aversion ( $\lambda \geq 1$ );  $v^-$  and  $v^+$  denote the value function for losses and gains, respectively. Function (3) has zero as reference point; it is concave for positive outcomes and convex for negative outcomes, it is steeper for losses. Parameters values equal to one imply risk and loss neutrality. Figure 1 shows an example of the value function defined by (3).

In financial applications, and in particular when dealing with options, prospects may involve a continuum of values; hence, prospect theory cannot be applied directly in its original or cumulative versions. Davis and Satchell (2007) provide

Table 1: Alternative value functions

Linear	$v(x) = x$
Logarithmic	$v(x) = \ln(a + x)$
Power	$v(x) = x^a$
Quadratic	$v(x) = ax - x^2$
Exponential	$v(x) = 1 - e^{-ax}$
Bell	$v(x) = bx - e^{-ax}$
HARA	$v(x) = -(b + x)^a$

the continuous cumulative prospect value:

$$V = \int_{-\infty}^0 \Psi^- [F(x)] f(x) v^-(x) dx + \int_0^{+\infty} \Psi^+ [1 - F(x)] f(x) v^+(x) dx, \quad (4)$$

where  $\Psi = \frac{dw(p)}{dp}$  is the derivative of the weighting function  $w$  with respect to the probability variable,  $F$  is the cumulative distribution function (cdf) and  $f$  is the probability density function (pdf) of the outcomes.

### 3 The weighting function

Prospect theory involves a probability weighting function which models probabilistic risk behavior. A weighting function  $w$  is uniquely determined, it maps the probability interval  $[0, 1]$  into  $[0, 1]$ , and is strictly increasing, with  $w(0) = 0$  and  $w(1) = 1$ . In this work we will assume continuity of  $w$  on  $[0, 1]$ , even though in the literature discontinuous weighting functions are also considered.

The *curvature* of the weighting function is related to the risk attitude towards probabilities. Empirical evidence suggests a particular shape of probability weighting functions: small probabilities are overweighted  $w(p) > p$ , whereas individuals tend to underestimate large probabilities  $w(p) < p$ . This turns out in a typical *inverse-S shaped* weighting function: the function is initially concave (probabilistic risk seeking or *optimism*) for probabilities in the interval  $(0, p^*)$ , and convex (probabilistic risk aversion or *pessimism*) in the interval  $(p^*, 1)$ , for a certain value of  $p^*$ . A linear weighting function describes probabilistic risk neutrality or objective sensitivity towards probabilities, which characterizes Expected Utility. Empirical findings indicate that the intersection (*elevation*) between the weighting function and the 45 degrees line,  $w(p) = p$ , is for  $p^*$  in the interval  $(0.3, 0.4)$ .

The sensitivity towards probability is increased if<sup>5</sup>

$$\frac{w(p)}{p} > 1, \quad p \in (0, \delta) \quad \text{and} \quad \frac{1-w(p)}{1-p} > 1, \quad p \in (1-\varepsilon, 1),$$

for some arbitrary small  $\delta > 0$  and  $\varepsilon > 0$ .

A weighting functions exhibits decreased sensitivity if

$$\frac{w(p)}{p} < 1, \quad p \in (0, \delta) \quad \text{and} \quad \frac{1-w(p)}{1-p} < 1, \quad p \in (1-\varepsilon, 1),$$

for some arbitrary small  $\delta > 0$  and  $\varepsilon > 0$ .

Some weighting functions<sup>6</sup> display *extreme sensitivity*, in the sense  $w(p)/p$  and  $(1-w(p))/(1-p)$  are unbounded as  $p$  tends to 0 and 1, respectively.

As already noticed, empirical studies on probability perception suggest the typical inverse-S shaped form for  $w$ , which combines the increased sensitivity with concavity for small probabilities and convexity for medium and large probabilities. In particular, such a function captures the fact that individuals are extremely sensitive to changes in (cumulative) probabilities which approach to 0 and 1. Abdellaoui et al. (2010) discuss how optimism and pessimism are possible sources of increased sensitivity.

Different parametric forms for the weighting function with the above mentioned features have been proposed in the literature, and their parameters have been estimated in many empirical studies. Single parameter probability weighting functions are those proposed by Karmarkar (1978, 1979), Rell (1987), Currim and Sarin (1989), Tversky and Kahneman (1992), Luce et al. (1993), Hey and Orme (1994), Prelec (1998), Safra and Segal (1998), and Luce (2000). Two (or more) parameters probability weighting functions have been proposed by Bell (1985), Goldstein and Einhorn (1987), Currim and Sarin (1989), Lattimore et al. (1992), Wu and Gonzales (1996), Prelec (1998), Diecidue et al. (2009), and Abdellaoui et al. (2010). Some examples are reported in Table 2.

Tversky and Kahneman (1992) use the Quiggin's (1982) functional of the form

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}, \quad (5)$$

where  $\gamma$  is a positive constant (with some constraint in order to have an increasing function). Note that  $w(0) = 0$  and  $w(1) = 1$ . The parameter  $\gamma$  captures the degree of sensitivity toward changes in probabilities from impossibility (zero probability) to certainty (Tversky and Kahneman, 1992). When  $\gamma < 1$ , one obtains the typical

---

<sup>5</sup>See Abdellaoui *et al.* (2010).

<sup>6</sup>E.g. the functions suggested by Goldstein and Einhorn (1987), Tversky and Kahneman (1992) and Prelec (1998).



Table 2: Alternative probability weighting functions

Linear	$w(p) = \alpha p + \beta, w(0) = 0, w(1) = 1$
Power	$w(p) = p^\gamma$
Karmarkar (1978)	$w(p) = \frac{p^\gamma}{p^\gamma + (1-p)^\gamma}$
Goldstein and Einhorn (1987)	$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$
Tversky and Kahneman (1992)	$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$
Wu and Gonzales (1996)	$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^\delta}$
Prelec (1998)	$w(p) = e^{-\delta(-\ln p)^\gamma}$
Prelec-single parameter	$w(p) = e^{-(-\ln p)^\gamma}$

inverse-S shaped form; the lower the parameter, the higher is the curvature of the function.

Considering function (5), in equation (4) we have:

$$\Psi = \frac{dw(p)}{dp} = \gamma p^{\gamma-1} [p^\gamma + (1-p)^\gamma]^{-1/\gamma} - p^\gamma [p^{\gamma-1} - (1-p)^{\gamma-1}] [p^\gamma + (1-p)^\gamma]^{-(\gamma+1)/\gamma}. \quad (6)$$

Prelec (1998) suggests a two parameter function of the form

$$w(p) = e^{-\delta(-\ln p)^\gamma}, \quad p \in (0, 1), \quad (7)$$

with  $w(0) = 0$  and  $w(1) = 1$ . The parameter  $\delta$  (with  $0 < \delta < 1$ ) governs elevation of the weighting function relative to the 45° line, while  $\gamma$  (with  $\gamma > 0$ ) governs curvature and the degree of sensitivity to extreme results relative to medium probability outcomes. When  $\gamma < 1$ , one obtains the inverse-S shaped function. In this model, the parameter  $\delta$  influences the tendency of over- or under-weighting the probabilities, but it has no direct meaning.

As an alternative, we also consider the more parsimonious single parameter Prelec's weighting function

$$w(p) = \exp[-(-\ln p)^\gamma], \quad p \in (0, 1), \quad (8)$$

which only allows for curvature to be varied. Note that in this case, the unique solution of equation  $w(p) = p$  for  $p \in (0, 1)$  is  $p = 1/e \simeq 0.367879$  and does not depend on the parameter  $\gamma$ .

For function (7) one easily obtains

$$\Psi(p) = \frac{\delta\gamma}{p} (-\ln p)^{\gamma-1} e^{-\delta(-\ln p)^\gamma}. \quad (9)$$

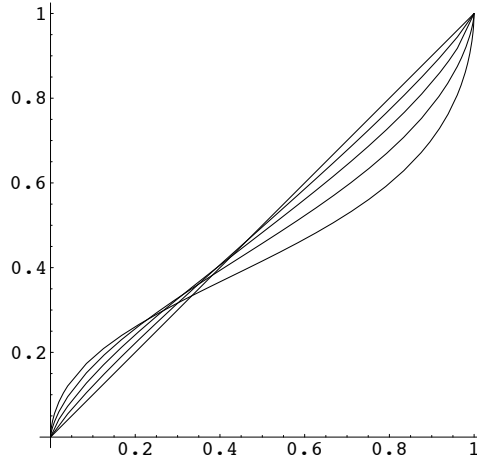


Figure 2: Weighting function (5) for different values of the parameter  $\gamma$ . As  $\gamma$  approaches the value 1, the  $w$  tends to the linear function

Figures 2 and 3 show some examples of weighting functions defined by (5) and (8) for different values of the parameters. As the parameters tend to the value 1, the weight tends to the objective probability and the function  $w$  approaches the 45° line. One can assume different parameters for probabilities when the outcome is in the domain of gains or losses.

In their empirical study, Wu and Gonzales (1999) consider both the Prelec (1998) weighting function and the *linear in log odds* function proposed by Goldstein and Einhorn (1987),

$$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}, \quad (10)$$

and used in a variant functional form by Lattimore *et al.* (1992). Function (10) has also been used by Tversky and Fox (1995), Birnbaum and McIntosh (1996), and Kilka and Weber (2001). The weighting function proposed by Karmarkar (1978, 1979) is the special case of (10) with  $\delta = 1$ .

An interesting parametric function is the *switch-power weighting function*<sup>7</sup> proposed by Diecidue *et al.* (2009), which consists in a power function for probabilities below a certain value  $\hat{p} \in (0, 1)$  and a dual power function for probabilities above  $\hat{p}$ ; formally  $w$  is defined as follows:

$$w(p) = \begin{cases} cp^a & \text{if } 0 \leq p \leq \hat{p}, \\ 1 - d(1-p)^b & \text{if } \hat{p} < p \leq 1, \end{cases} \quad (11)$$

<sup>7</sup>Diecidue *et al.* (2009) provide preference foundation for such a family of parametric weighting functions and inverse-S shape under rank dependent utility (RDU) based on testable preference conditions.

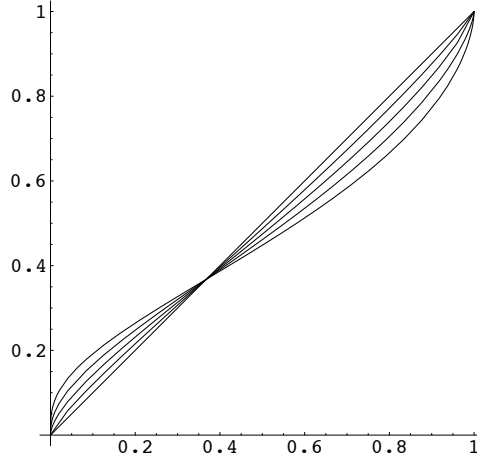


Figure 3: Prelec's weighting function (8) for different values of the parameter  $\gamma$

with five parameters  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $\hat{p}$ . All the parameters are strictly positive, assuming continuity and monotonicity of  $w$ . When  $\hat{p}$  approaches 1 or 0,  $w$  reduces to a power or a dual power probability weighting function, respectively.

Parameters reduce to three ( $a$ ,  $b$ , and  $\hat{p}$ ) by assuming continuity of  $w(p)$  at  $\hat{p}$  and differentiability. Hence one obtains

$$c = \hat{p}^{1-a} \left( \frac{b\hat{p}}{b\hat{p} + a(1-\hat{p})} \right), \quad (12)$$

and

$$d = (1-\hat{p})^{-b} \left( \frac{a(1-\hat{p})}{b\hat{p} + a(1-\hat{p})} \right). \quad (13)$$

For  $a, b \leq 1$ , the function  $w$  is concave on  $(0, \hat{p})$  and convex on  $(\hat{p}, 1)$  (hence it has an inverse-S shaped form), while for  $a, b \geq 1$  the weighting function is convex for  $p < \hat{p}$  and concave for  $p > \hat{p}$  (hence it has an S-shaped form). Both parameters  $a$  and  $b$  govern the curvature of  $w$  when  $a \neq b$ . In particular, parameter  $a$  describes probabilistic risk attitude for small probabilities; whereas parameter  $b$  describes probabilistic risk attitude for medium and large probabilities. In the case when  $a \neq b$ , parameter  $\hat{p}$ , which signals the point where probabilistic risk attitudes change from risk aversion to risk seeking (in the case of an inverse-S shaped weighting function), may not lie on the  $45^\circ$  line, hence it has not the meaning of dividing the region of over- and under-weighting of the probability.

When  $a = b$ , then  $w$  intersects the  $45^\circ$  line at  $\hat{p}$ . In such a case, one obtains the following two parameter probability weighting function

$$w(p) = \begin{cases} \hat{p}^{1-a} p^a & \text{if } 0 \leq p \leq \hat{p}, \\ 1 - (1-\hat{p})^{1-a} (1-p)^a & \text{if } \hat{p} < p \leq 1. \end{cases} \quad (14)$$

This is the same form as the *constant relative sensitivity* weighting function considered by Abdellaoui *et al.* (2010). Parameter  $\hat{p}$  separates the regions of over- and under-weighting of probabilities.

Abdellaoui *et al.* (2010) propose the family of weighting functions of the form

$$w(p) = \begin{cases} \delta^{1-\gamma} p^\gamma & \text{if } 0 \leq p \leq \delta, \\ 1 - (1 - \delta)^{1-\gamma} (1 - p)^\gamma & \text{if } \delta < p \leq 1, \end{cases} \quad (15)$$

with  $\gamma > 0$  and  $\delta \in [0, 1]$ . For  $\gamma < 1$  and  $0 < \delta < 1$  it has an inverse-S shape. The derivative of  $w$  at  $\delta$  equals  $\gamma$ ; this parameter controls for the curvature of the weighting function. The parameter  $\delta$  indicates whether the interval for over-weighting probabilities is larger than the interval for underweighting, and therefore controls for the elevation. Hence, such a family of weighting functions allows for a separate modeling of these two features.

Remember that a convex weighting function characterizes probabilistic risk aversion and a concave weighting function characterizes probabilistic risk proneness<sup>8</sup>. Then the role of  $\delta$  is to demarcate the interval of probability risk seeking from the interval of probability risk aversion<sup>9</sup>. In such a case, overweighting corresponds to risk seeking (or optimism) and underweighting corresponds to risk proneness (or pessimism). Elevation represents the relative strength of optimism vs. pessimism, hence it is a measure of relative optimism, and  $\delta$  may be interpreted as an index of relative optimism.

The intersection between the weighting function and the 45 degrees line,  $w(p) = p$ , is for  $p$  in the interval (0.3, 0.4). Gonzales and Wu (1999) and Abdellaoui *et al.* (2010) find that the weighting function is more elevated for losses than for gains. In Abdellaoui *et al.* (2010) the relative index of optimism for gains  $\delta^+$  is lower than the relative index of pessimism for losses  $\delta^-$ .

Curvature is a measure of the degree of sensitivity to changes from impossibility to possibility (Tversky and Kahneman, 1992), it represents the diminishing effect of optimism and pessimism when moving away from extreme probabilities 0 and 1. Hence parameter  $\gamma$ , controlling for curvature, measures relative sensitivity of the weighting function. This suggests an interpretation for the parameter  $\gamma$  as a measure of relative risk aversion. The index of relative sensitivity (see

---

<sup>8</sup>A linear weighting function characterizes probabilistic risk neutrality.

<sup>9</sup>This is not the case for weighting function (11); when  $a \neq b$ , both parameters controls for curvature and all parameters may influence elevation.

Abellaoui *et al.*, 2010) of  $w$  as defined in (15) is

$$\begin{aligned}
 RS(w, p) &= -\frac{p \frac{\partial^2 w(p)}{\partial p^2}}{\frac{\partial w(p)}{\partial p}} && \text{for } p \in (0, \delta], \\
 RS(w, p) &= -\frac{(1-p) \frac{\partial^2 (1-w(p))}{\partial (1-p)^2}}{\frac{\partial (1-w(p))}{\partial (1-p)}} && \text{for } p \in (\delta, 1),
 \end{aligned} \tag{16}$$

which is constant on the interval  $(0, 1)$  and equals  $1 - \gamma$ . For this reason, probability functions of the form (15) are called *constant relative sensitivity* (CRS) weighting functions.

Gonzales and Wu (1999) discuss the importance of modeling curvature and elevation independently, providing psychological interpretation. To our knowledge, the functional form in (15) is the only one, amongst those in the literature, which is able to capture separately the effects of curvature and elevation.

## 4 European options valuation

We evaluate European financial options within continuous CPT; in particular, in the applications we use the CRS weighting function defined in the previous section.

Versluis *et al.* (2010) provide the prospect value of writing call options, considering different time aggregation of the results. Their results are extended to the case of put options in Nardon and Pianca (2013); the authors also consider the problem both from the writer's and holder's perspective.

Let  $S_t$  be the price at time  $t$  (with  $t \in [0, T]$ ) of the underlying asset of a European option with maturity  $T$ ; in a Black-Scholes setting, the underlying price dynamics is driven by a geometric Brownian motion. Let  $c$  be the call option premium with strike price  $X$ . At time  $t = 0$ , the option's writer receives  $c$  and can invest the premium at the risk-free rate  $r$ , obtaining  $c e^{rT}$ . At maturity, he has to pay the amount  $S_T - X$  if the option expires in-the-money.

Considering zero as a reference point (*status quo*), the prospect value of the writer's position in the *time segregated* case is

$$V_s = v^+ (c e^{rT}) + \int_X^{+\infty} \Psi^- (1 - F(x)) f(x) v^- (X - x) dx, \tag{17}$$

with  $f$  and  $F$  being the pdf and the cdf<sup>10</sup> of the future underlying price  $S_T$ , and  $v$

<sup>10</sup>The probability density function (pdf) of the underlying price at maturity  $S_T$  is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp\left(-\frac{[\ln(x/S_0) - (\mu - \sigma^2/2)T]^2}{2\sigma^2 T}\right), \tag{18}$$

is defined as in (3).

In equilibrium, we equate  $V_s$  at zero and solve for the price  $c$ :

$$c = e^{-rT} \left( \lambda \int_X^{+\infty} \Psi^- (1 - F(x)) f(x) (x - X)^b dx \right)^{1/a}, \quad (20)$$

which requires numerical approximation of the integral.

When considering the *time aggregated* prospect value, one obtains

$$\begin{aligned} V_a &= w^+ (F(X)) v^+ (c e^{rT}) + \\ &+ \int_X^{X+c \exp(rT)} \Psi^+ (F(x)) f(x) v^+ (c \exp(rT) - (x - X)) dx + \\ &+ \int_{X+c \exp(rT)}^{+\infty} \Psi^- (1 - F(x)) f(x) v^- (c \exp(rT) - (x - X)) dx. \end{aligned} \quad (21)$$

In this latter case, the option price in equilibrium has to be determined numerically.

In order to obtain the value of a European put option, we can no longer use put-call parity arguments. Let  $p$  be the put option premium at time  $t = 0$ ; the prospect value of the writer's position in the time segregated case is

$$V_s = v^+ (p e^{rT}) + \int_0^X \Psi^- (F(x)) f(x) v^- (x - X) dx, \quad (22)$$

and one obtains

$$p = e^{-rT} \left( \lambda \int_0^X \Psi^- (F(x)) f(x) (X - x)^b dx \right)^{1/a}. \quad (23)$$

In the time aggregated case the put option value is implicitly defined equating at zero the following expression

$$\begin{aligned} V_a &= \int_0^{X-pe^{rT}} \Psi^- (F(x)) f(x) v^- (p e^{rT} - (X - x)) dx + \\ &+ \int_{X-pe^{rT}}^X \Psi^+ (1 - F(x)) f(x) v^+ (p e^{rT} - (X - x)) dx + \\ &+ w^+ (1 - F(X)) v^+ (p e^{rT}), \end{aligned} \quad (24)$$

which has to be solved numerically for  $p$ .

where  $\mu$  and  $\sigma > 0$  are constants, and the cumulative distribution function (cdf) is

$$F(x) = \Phi \left( \frac{\ln(x/S_0) - (\mu - \sigma^2/2)T}{\sigma\sqrt{T}} \right), \quad (19)$$

where  $\Phi(\cdot)$  is the cdf of a standard Gaussian random variable.

## 4.1 Option valuation from holder's perspective

When one considers the problem from the holder's viewpoint, the prospect values both in the time segregated and aggregated cases change. Holding zero as reference point, the prospect value of the holder's position for a call option in the time segregated case is

$$V_s^h = v^- (-c e^{rT}) + \int_X^{+\infty} \Psi^+ (1 - F(x)) f(x) v^+ ((x - X)) dx, \quad (25)$$

with  $f$  and  $F$  being the pdf and the cdf defined in (18) and (19) of the future underlying price  $S_T$ , and  $v$  is defined as in (3).

We equate  $V_s^h$  at zero and solve for the price  $c$ , obtaining:

$$c_s^h = e^{-rT} \left( \frac{1}{\lambda} \int_X^{+\infty} \Psi^+ (1 - F(x)) f(x) (x - X)^a dx \right)^{1/b}. \quad (26)$$

In the time aggregated case, the prospect value has the following integral representation:

$$\begin{aligned} V_a^h &= w^- (F(X)) v^- (-c e^{rT}) + \\ &+ \int_X^{X+c \exp(rT)} \Psi^- (F(x)) f(x) v^- ((x - X) - c \exp(rT)) dx + \\ &+ \int_{X+c \exp(rT)}^{+\infty} \Psi^+ (1 - F(x)) f(x) v^+ ((x - X) - c \exp(rT)) dx. \end{aligned} \quad (27)$$

In order to obtain the call option price in equilibrium, one has to solve numerically for  $c$ .

In an analogous way one can derive the put option prospect values for the holder's position. In the segregated case the prospect value is

$$V_s^h = v^- (-p e^{rT}) + \int_0^X \Psi^+ (F(x)) f(x) v^+ ((X - x)) dx. \quad (28)$$

Equating at zero and solving for the price  $p$ , one obtains

$$p_s^h = e^{-rT} \left( \frac{1}{\lambda} \int_0^X \Psi^+ (F(x)) f(x) (X - x)^a dx \right)^{1/b}. \quad (29)$$

Finally, in the time aggregated setting, the prospect value from holder's viewpoint is In the time aggregated case, the prospect value has the following integral representation:

$$\begin{aligned} V_a^h &= w^- (1 - F(X)) v^- (-p e^{rT}) + \\ &+ \int_{X-p \exp(rT)}^X \Psi^- (1 - F(x)) f(x) v^- ((X - x) - p \exp(rT)) dx + \\ &+ \int_0^{X-p \exp(rT)} \Psi^+ (F(x)) f(x) v^+ ((X - x) - p \exp(rT)) dx. \end{aligned} \quad (30)$$

The put option value is implicitly defined by the equation  $V_a^h = 0$ .

## 5 Results and sensitivity analysis

In this contribution, we perform a wide sensitivity analysis on call and put options values considered from writer's perspective, computed with the models presented in the previous section. We have calculated the options prices both in the time segregated and aggregated case. We applied alternative weighting functions and, in particular, we report the results for the CRS weighting function (15) proposed by Abdellaoui et al. (2010).

We let vary the parameters  $\gamma \in [0.7, 1.0]$  and  $\delta \in [0.3, 0.4]$ , considering also different sensitivity to probability risk for positive and negative outcomes ( $\gamma^+ \neq \gamma^-$  and  $\delta^+ \neq \delta^-$ ). For the value function, we compared different parameters sets, ranging from TK sentiment (see Tversky and Kahnemann, 1992) to more *moderate sentiment*; a linear function (with  $a = b = 1$  and  $\lambda = 1$ ) is considered as a limiting case (no sentiment). We computed the option prices for several values of the volatility and the strike price  $X$ . The choice of the values of the parameters  $\gamma^+$  and  $\gamma^-$  is motivated in order to obtain realistic option prices. TK sentiment parameters yield too high options prices, in particular in the segregated case<sup>11</sup>; 10% and 20% of the TK sentiment yield results more in line with market prices. The choice of  $\delta$  is suggested by empirical evidence, as noticed above.

It is worth noting that, when we set  $\mu = r$ ,  $a = b = 1$ ,  $\lambda = 1$ , and  $\gamma = 1$ , we obtain the same results as in the Black-Scholes (BS) model.

Numerical results suggest that option prices are increasing with  $\delta$  (elevation) within the interval  $[0.3, 0.4]$ ; prices increase at a decreasing rate<sup>12</sup>; the effect is more important the lower is  $\gamma$  (the higher the curvature).

The effect of  $\gamma$  (curvature) is non-trivial, depending on the moneyness and the model (time-aggregated or segregated) which is used. In particular, in the time-aggregated model (writer's perspective), option prices are decreasing with respect to  $\gamma$ ; in the time-segregated model (writer's perspective), option prices are decreasing with respect to  $\gamma$ , with the exception of deep-in-the-money calls and puts.

Tables 3–8 report the results for the European calls and puts in the time-aggregated models, from writer's perspective, for different strikes and elevation.

---

<sup>11</sup>See Versluis *et al.* (2010) and Nardon and Pianca (2013).

<sup>12</sup>Note that this is true with some rare exceptions, which may be due to possible round-off errors in the numerical procedure applied in order to approximate the integrals and to numerically solve the equations presented in the previous section.

Another exception is the case of deep-in-the-money puts, from holder's perspective, as highlighted in Table 14



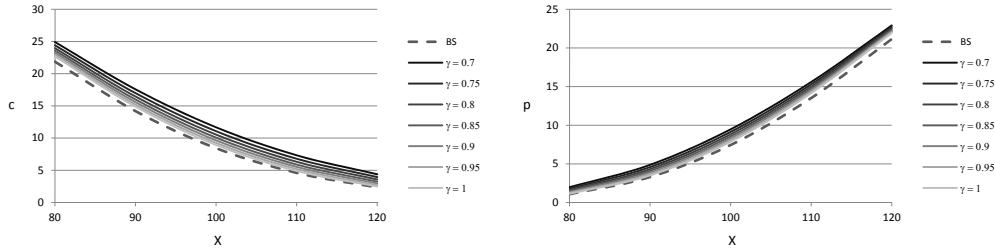


Figure 4: Sensitivity of the call (left) and put (right) option prices (writer’s position in the time-aggregated model) to the curvature of the probability weighting function,  $\gamma \in [0.7, 1.0]$ , with  $\delta = 0.325$ . BS is the Black-Scholes price (with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$ ). The option parameters are:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ ; the parameters of the value function are:  $a = b = 0.976$ , and  $\lambda = 1.125$

Similar results are obtained in the time-segregated model, but are not reported in the paper. Here we focus on the effect of elevation. In these examples, the parameters of the value function  $a$ ,  $b$  and  $\lambda$  are fixed; we assume moderate sensitivity of the value function. We consider  $\delta^+ = \delta^-$  and  $\gamma^+ = \gamma^-$ . The parameter  $\delta$  is letting vary in the interval  $[0.3, 0.4]$ . As regards the parameter  $\gamma$ , we calculate option prices for a wide interval ranging from  $\gamma = 0.7$  (which is closer to the value used by Tversky and Kahnemann, 1992) to  $\gamma = 1$  (in this latter case the only effect of the value function applies). We observe that for lower values of  $\gamma$ , options prices deviates sensitively from Black-Scholes prices.

Tables 9–14 report the results for the European calls and puts in the time-aggregated models, from holder’s perspective, for different strikes and elevation.

Figure 4 shows some results for the call and put options in the time-aggregated model; in these cases, option premia are decreasing with curvature. Note that writer’s prices are always above BS prices<sup>13</sup>.

<sup>13</sup>This is not the case when we consider holder’s perspective. If one considers the pricing problem both from the writer’s and holder’s perspective, it is possible to obtain an interval for the prices of call and put options for certain values of the sentiment parameters which are of practical interest. Balck-Scholes price lies in the interval bounded by the holder’s price from below and the writer’s price from above. The range of such an interval depends on the value of the parameters which govern investor’s sentiment (attitude toward risk and loss aversion and probability bias). More moderate sentiment implies smaller estimate intervals.

## 6 Concluding remarks

In this contribution we applied the constant relative sensitivity weighting function proposed by Abdellaoui *et al.* (2010), within the framework of CPT in its continuous version, to price European options. The CRS weighting function allow for separate modeling of curvature and elevation, which have an interesting interpretation in terms of probabilistic optimism and pessimism. We performed a number of numerical experiments in order to study the effect of curvature and elevation on option prices.

## References

- [1] Abbink K., Rockenbach B. (2006) Option pricing by students and professional traders: A behavioral investigation, *Managerial and Decision Economics* **27**, 497–510
- [2] Abdellaoui M., L’Haridon O., Zank H.: Separating curvature and elevation: A parametric probability weighting function, *J. of Risk and Uncertainty* **41**, 39–65 (2010)
- [3] Barberis N., Thaler R. (2003) A survey of behavioral finance. In: Constantinides G.M., Harris M., Stulz R. (eds) *Handbook of the Economics of Finance*, Elsevier Science, 1051–1121
- [4] Bell D.E. (1985) Disappointment in decision making under uncertainty, *Operations Research* **33** 1–27
- [5] Birnbaum M.H., McIntosh W.R. (1996) Violations of branch independence in choices between gambles, *Organizational Behavior and Human Decision Processes* **67**, 91–110
- [6] Black F., Scholes M.: The pricing of options and corporate liabilities, *Journal of Political Economy* **81(3)**, 637–654 (1973)
- [7] Breuer W., Perst A. (2007) Retail banking and behavioral financial engineering: The case of structured products, *Journal of Banking and Finance* **31**, 827–844
- [8] Currim I.S., Sarin R.K. (1989) Prospect versus utility, *Management Science* **35**, 22–41
- [9] Davies G.B., Satchell S.E. (2007) The behavioural components of risk aversion, *Journal of Mathematical Psychology* **51(1)**, 1–13

- [10] Diecidue E., Schmidt U., Zank H. (2009) Parametric weighting functions, *Journal of Economic Theory*, **144**, 1102–1118
- [11] Goldstein W.M., Einhorn H.J. (1987) Expression theory and the preference reversal phenomena, *Psychological Review* **94**, 236–254
- [12] Gonzalez R., Wu G. (1999) On the shape of the probability weighting function, *Cognitive Psychology*, **38**, 129–166
- [13] Hey J.D., Orme C. (1994) Investigating generalizations of expected utility theory using experimental data, *Econometrica* **62**, 1291–1326
- [14] Kahneman D., Tversky A. (1979) Prospect theory: An analysis of decision under risk, *Econometrica* **47**, 263–292
- [15] Karmarkar U.S. (1978), Subjectively weighted utility: A descriptive extension of the expected utility model, *Organizational Behavior and Human Performance* **21**, 61–72
- [16] Karmarkar U.S. (1979), Subjectively weighted utility and the Allais Paradox, *Organizational Behavior and Human Performance* **24**, 67–72
- [17] Kilka M., Weber M. (2001) What determines the shape of the probability weighting function under uncertainty? *Management Science* **47**, 1712–1726
- [18] Lattimore P.K., Baker J.R., Witte A.D. (1992) The influence of probability on risky choice: A parametric examination, *Journal of Economic Behavior and Organization* **17**, 377–400
- [19] Luce D.R. (2000) *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches*, Lawrence Erlbaum Publishers, London
- [20] Luce D.R., Mellers B.A., Chang S-J. (1993) Is choice the correct primitive? On using certainty equivalents and reference levels to predict choices among gambles, *Journal of Risk and Uncertainty* **6**, 115–143
- [21] Nardon M., Pianca P. (2013) A behavioural approach to the pricing of European options. In: Corazza M., Pizzi C. (Eds), *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, Springer-Verlag, Milano, 217–228
- [22] Poteshman A.M., Serbin V. (2003) Clearly irrational financial market behavior: Evidence from the early exercise of exchange traded stock options, *Journal of Finance* **58**, 37–70

- [23] Prelec D. (1998) The probability weighting function, *Econometrica* **66**, 497–527
- [24] Quiggin J. (1982) A theory of anticipated utility, *Journal of Economic Behavior and Organization* **3**, 323–343
- [25] Rell A. (1987), Risk aversion in Quiggin and Yaaris rank-order model of choice under uncertainty, *The Economic Journal* **97**, 143–159
- [26] Safra Z., Segal U. (1998) Constant risk aversion, *Journal of Economic Theory* **83**, 19–42
- [27] Schmeidler D. (1989) Subjective probability and expected utility without additivity, *Econometrica*, **57**, 571–587
- [28] Shefrin H., Statman M. (1993) Behavioral aspects of the design and marketing of financial products, *Financial Management* **22**, 123–134
- [29] Shiller R.J. (1999) Human behavior and the efficiency of the financial system. In: Taylor J.B., Woodford M. (eds) *Handbook of Macroeconomics* Vol. 1C, Elsevier Amsterdam, 1305–1340
- [30] Subrahmanyam A. (2007) Behavioural finance: A review and synthesis, *European Financial Management*, **14**(1), 12–29
- [31] Tversky A., Fox C.R. (1995) Weighing risk and uncertainty, *Psychological Review* **102**, 269–283
- [32] Tversky A., Kahneman D. (1992) Advances in prospect theory: Cumulative representation of the uncertainty, *Journal of Risk and Uncertainty* **5**, 297–323
- [33] Versluis C., Lehnert T., Wolff C.C.P.: A cumulative prospect theory approach to option pricing. *Working paper* LSF Research Working Paper Series 09–03, Luxembourg School of Finance (2010)
- [34] Wakker P.P.: *Prospect Theory: For Risk and Ambiguity*, Cambridge University Press, Cambridge (2010)
- [35] Wu G., Gonzalez R. (1996) Curvature of the probability weighting function, *Management Science*, **42**, 1676–1690
- [36] Wu G., Gonzalez R. (1999) Nonlinear decision weights in choice under uncertainty, *Management Science*, **45**, 74–85

Table 3: Sensitivity of the call option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	21.863306	24.861584	24.923291	24.974970	25.017342	25.051069
	90	14.192920	17.484446	17.573262	17.647340	17.708431	17.760472
	100	8.433319	11.576634	11.681723	11.774326	11.855848	11.927408
	110	4.610115	7.238857	7.334244	7.420773	7.499478	7.571065
	120	2.340649	4.311143	4.383860	4.450910	4.512931	4.570532
0.75	80	21.863306	24.363985	24.412815	24.453852	24.487621	24.514551
	90	14.192920	16.906403	16.978012	17.037933	17.087182	17.127181
	100	8.433319	10.991237	11.076160	11.150956	11.216763	11.274468
	110	4.610115	6.715597	6.791161	6.859693	6.921994	6.978708
	120	2.340649	3.889247	3.945155	3.996691	4.044361	4.088572
0.8	80	21.863306	23.933953	23.971090	24.002404	24.028263	24.048956
	90	14.192920	16.399516	16.455020	16.501542	16.539773	16.570767
	100	8.433319	10.476298	10.542324	10.600456	10.651570	10.696350
	110	4.610115	6.258459	6.316063	6.368287	6.415754	6.458992
	120	2.340649	3.526005	3.567396	3.605515	3.640748	3.673406
0.85	80	21.863306	23.559569	23.586259	23.608502	23.627080	23.642001
	90	14.192920	15.951804	15.992142	16.026031	16.053917	16.076425
	100	8.433319	10.020030	10.068253	10.110690	10.147981	10.180630
	110	4.610115	5.855977	5.897232	5.934653	5.968584	5.999493
	120	2.340649	3.210706	3.239504	3.265993	3.290452	3.313108
0.9	80	21.863306	23.231494	23.246752	23.262619	23.274666	23.284062
	90	14.192920	15.553823	15.579896	15.601702	15.619949	15.634517
	100	8.433319	9.613175	9.644532	9.672116	9.696345	9.717579
	110	4.610115	5.499174	5.525487	5.549318	5.570961	5.590649
	120	2.340649	2.935039	2.952883	2.969278	2.984433	2.998391
0.95	80	21.863306	22.942305	22.950338	22.957170	22.962866	22.967472
	90	14.192920	15.198026	15.210766	15.221380	15.230157	15.237244
	100	8.433319	9.248181	9.263653	9.277121	9.288944	9.299281
	110	4.610115	5.180922	5.193530	5.204942	5.215301	5.224721
	120	2.340649	2.692457	2.700765	2.708388	2.715410	2.721901
1	80	21.863306	22.686044	22.686044	22.686044	22.686044	22.686044
	90	14.192920	14.878309	14.878309	14.878309	14.878309	14.878309
	100	8.433319	8.919549	8.919549	8.919549	8.919549	8.919549
	110	4.610115	4.895488	4.895488	4.895488	4.895488	4.895488
	120	2.340649	2.477741	2.477741	2.477741	2.477741	2.477741

Table 4: Sensitivity of the call option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.988$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	21.863306	24.915260	24.976614	25.027902	25.069876	25.111850
	90	14.192920	17.557159	17.642500	17.716152	17.777261	17.826650
	100	8.433319	11.658732	11.763494	11.855805	11.937080	12.008392
	110	4.610115	7.323857	7.419200	7.505696	7.584332	7.655917
	120	2.340649	4.388088	4.461086	4.528415	4.590715	4.648509
0.75	80	21.863306	24.413041	24.461614	24.502359	24.535824	24.562465
	90	14.192920	16.971327	17.042771	17.102312	17.151369	17.191092
	100	8.433319	11.068954	11.153606	11.228165	11.293773	11.351277
	110	4.610115	6.795979	6.871529	6.940045	7.002328	7.059024
	120	2.340649	3.961005	4.017182	4.068960	4.116847	4.161254
0.8	80	21.863306	23.978925	24.015884	24.046793	24.072623	24.093095
	90	14.192920	16.460432	16.515688	16.561971	16.599950	16.637928
	100	8.433319	10.550108	10.615925	10.673867	10.724815	10.769450
	110	4.610115	6.334742	6.392341	6.444561	6.492021	6.535217
	120	2.340649	3.593138	3.634750	3.673067	3.708480	3.741303
0.85	80	21.863306	23.600912	23.627306	23.649718	23.668011	23.682801
	90	14.192920	16.009002	16.049189	16.082902	16.110600	16.132948
	100	8.433319	10.090335	10.138400	10.180697	10.217885	10.250410
	110	4.610115	5.928580	5.969855	6.007242	6.041210	6.072103
	120	2.340649	3.273687	3.302650	3.329292	3.353907	3.376674
0.9	80	21.863306	23.269595	23.286368	23.300569	23.312375	23.321828
	90	14.192920	15.607657	15.633641	15.655484	15.673467	15.687929
	100	8.433319	9.680301	9.711574	9.739050	9.763198	9.784325
	110	4.610115	5.568454	5.594790	5.618609	5.640256	5.659947
	120	2.340649	2.994281	3.012224	3.028721	3.043939	3.058016
0.95	80	21.863306	22.977500	22.985434	22.992499	22.997950	23.002511
	90	14.192920	15.248803	15.261410	15.272101	15.280788	15.287824
	100	8.433319	9.312573	9.327837	9.341261	9.353045	9.363348
	110	4.610115	5.247183	5.259796	5.271212	5.281574	5.290996
	120	2.340649	2.748291	2.756642	2.764317	2.771390	2.777924
1	80	21.863306	22.718622	22.718622	22.718622	22.718622	22.718622
	90	14.192920	14.926291	14.926291	14.926291	14.926291	14.926291
	100	8.433319	8.981136	8.981136	8.981136	8.981136	8.981136
	110	4.610115	4.958994	4.958994	4.958994	4.958994	4.958994
	120	2.340649	2.530452	2.530452	2.530452	2.530452	2.530452

Table 5: Sensitivity of the call option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.988$ , and  $\lambda = 1.25$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	21.863306	25.861804	25.930944	25.988828	26.036219	26.074269
	90	14.192920	18.375361	18.469813	18.548712	18.614281	18.668483
	100	8.433319	12.310931	12.421032	12.518220	12.603955	12.679400
	110	4.610115	7.791825	7.892499	7.983881	8.066999	8.142708
	120	2.340649	4.691004	4.768755	4.840462	4.906815	4.968370
0.75	80	21.863306	25.323377	28.769214	25.424061	25.462671	25.493051
	90	14.192920	17.760254	17.836616	17.900430	17.953246	17.996689
	100	8.433319	11.689715	11.778632	11.857081	11.926238	11.987037
	110	4.610115	7.232977	7.312788	7.385192	7.451040	7.511013
	120	2.340649	4.235866	4.295745	4.350933	4.401975	4.449310
0.8	80	21.863306	24.857056	24.899117	24.934613	24.963909	24.987339
	90	14.192920	17.220517	17.279811	17.329465	17.370435	17.403975
	100	8.433319	11.143085	11.212178	11.273111	11.326790	11.373978
	110	4.610115	6.744468	6.805333	6.860532	6.910721	6.956425
	120	2.340649	3.843586	3.887971	3.928843	3.966617	4.001664
0.85	80	21.863306	24.450214	24.480395	24.505956	24.527147	24.544117
	90	14.192920	16.743524	16.786701	16.822945	16.852836	16.877183
	100	8.433319	10.658564	10.708998	10.753456	10.792601	10.826957
	110	4.610115	6.314100	6.357708	6.397240	6.433172	6.465887
	120	2.340649	3.502736	3.533652	3.562092	3.588348	3.612669
0.9	80	21.863306	24.092922	24.112224	24.128597	24.142215	24.153465
	90	14.192920	16.319305	16.347255	16.370808	16.390215	16.405965
	100	8.433319	10.226395	10.259137	10.288021	10.313479	10.335729
	110	4.610115	5.932349	5.960175	5.985386	6.008335	6.029138
	120	2.340649	3.204441	3.223622	3.241243	3.257496	3.272572
0.95	80	21.863306	23.777274	23.786636	23.794516	23.801080	23.806389
	90	14.192920	15.939886	15.953469	15.964937	15.974406	15.982024
	100	8.433319	9.838641	9.854479	9.868740	9.880972	9.892003
	110	4.610115	5.591649	5.604989	5.617067	5.628034	5.638013
	120	2.340649	2.941706	2.950649	2.958852	2.966445	2.973393
1	80	21.863306	23.497214	23.497214	23.497214	23.497214	23.497214
	90	14.192920	15.598806	15.598806	15.598806	15.598806	15.598806
	100	8.433319	9.489099	9.489099	9.489099	9.489099	9.489099
	110	4.610115	5.285913	5.285913	5.285913	5.285913	5.285913
	120	2.340649	2.708953	2.708953	2.708953	2.708953	2.708953

Table 6: Sensitivity of the put option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	1.067293	1.950139	1.984390	2.016064	2.045450	2.072786
	90	3.297405	4.818069	4.877687	4.931473	4.980092	5.024074
	100	7.438302	9.401855	9.472155	9.533033	9.585377	9.630052
	110	13.515596	15.542022	15.591561	15.632406	15.665773	15.691992
	120	21.146629	22.883593	22.905272	22.923219	22.937835	22.949424
0.75	80	1.067293	1.765543	1.791986	1.816419	1.839077	1.860147
	90	3.297405	4.536561	4.584629	4.628010	4.667236	4.702735
	100	7.438302	9.075300	9.133168	9.183436	9.226659	9.263529
	110	13.515596	15.242759	15.282934	15.316223	15.343316	15.364748
	120	21.146629	22.668919	22.685685	22.699886	22.710957	22.719998
0.8	80	1.067293	1.604601	1.624224	1.642338	1.659123	1.674723
	90	3.297405	4.285593	4.322828	4.356434	4.386828	4.414343
	100	7.438302	8.782374	8.828068	8.867910	8.902176	8.931395
	110	13.515596	14.975948	15.007206	15.033155	15.054323	15.071109
	120	21.146629	22.482180	22.494615	22.504952	22.513418	22.520176
0.85	80	1.067293	1.463376	1.477047	1.489650	1.501316	1.512149
	90	3.297405	4.060563	4.087622	4.112043	4.134131	4.154130
	100	7.438302	8.518258	8.552065	8.581657	8.607131	8.628846
	110	13.515596	14.736777	14.759567	14.778516	14.794006	14.806321
	120	21.146629	22.318861	22.327500	22.334479	22.340598	22.345327
0.9	80	1.067293	1.338737	1.347212	1.355014	1.362228	1.368919
	90	3.297405	3.857746	3.875238	3.891021	3.905296	3.918220
	100	7.438302	8.279023	8.301245	8.320775	8.337612	8.351957
	110	13.515596	14.521333	14.536099	14.548338	14.558460	14.566483
	120	21.146629	22.176408	22.180662	22.185106	22.188762	22.191461
0.95	80	1.067293	1.228160	1.232105	1.235731	1.239078	1.242180
	90	3.297405	3.674096	3.682582	3.690237	3.697159	3.703425
	100	7.438302	8.061430	8.072381	8.082043	8.090392	8.097500
	110	13.515596	14.326404	14.333578	14.339557	14.344439	14.348379
	120	21.146629	22.048639	22.051105	22.052927	22.054859	22.056225
1	80	1.067293	1.129592	1.129592	1.129592	1.129592	1.129592
	90	3.297405	3.507094	3.507094	3.507094	3.507094	3.507094
	100	7.438302	7.862781	7.862781	7.862781	7.862781	7.862781
	110	13.515596	14.149325	14.149325	14.149325	14.149325	14.149325
	120	21.146629	21.936370	21.936370	21.936370	21.936370	21.936370



Table 7: Sensitivity of the put option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.988$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	1.067293	1.985795	2.020189	2.051987	2.081484	2.108919
	90	3.297405	4.864164	4.923609	4.977236	5.025707	5.069552
	100	7.438302	9.444568	9.514460	9.574955	9.626962	9.671343
	110	13.515596	15.570631	15.619871	15.660506	15.693426	15.719327
	120	21.146629	22.894149	22.915655	22.933419	22.948091	22.959261
0.75	80	1.067293	1.799193	1.825764	1.850312	1.873073	1.894237
	90	3.297405	4.581145	4.629085	4.672346	4.711462	4.746860
	100	7.438302	9.116706	9.174167	9.224124	9.267072	9.303703
	110	13.515596	15.269770	15.310232	15.342763	15.369598	15.390777
	120	21.146629	22.677954	22.694594	22.708369	22.719591	22.728496
0.8	80	1.067293	1.636394	1.656127	1.674340	1.691214	1.706895
	90	3.297405	4.328761	4.365903	4.399424	4.429739	4.457182
	100	7.438302	8.822371	8.867826	8.907425	8.941479	8.970510
	110	13.515596	15.001460	15.032559	15.058327	15.079303	15.095902
	120	21.146629	22.489834	22.502181	22.512421	22.520785	22.527350
0.85	80	1.067293	1.493448	1.507203	1.519885	1.531622	1.542520
	90	3.297405	4.102400	4.129397	4.153761	4.175797	4.195747
	100	7.438302	8.557003	8.590642	8.620060	8.645377	8.666953
	110	13.515596	14.760884	14.783570	14.802394	14.817750	14.829920
	120	21.146629	22.325260	22.333841	22.340968	22.346804	22.351466
0.9	80	1.067293	1.367208	1.375742	1.383598	1.390861	1.397598
	90	3.297405	3.898330	3.915785	3.931534	3.945778	3.958675
	100	7.438302	8.316581	8.338699	8.358118	8.374851	8.389107
	110	13.515596	14.544124	14.558873	14.571046	14.581031	14.588945
	120	21.146629	22.180590	22.185886	22.190290	22.193905	22.196802
0.95	80	1.067293	1.255140	1.259116	1.262770	1.266143	1.269268
	90	3.297405	3.713499	3.721968	3.729608	3.736517	3.742771
	100	7.438302	8.097863	8.108766	8.118374	8.126671	8.133737
	110	13.515596	14.347961	14.355138	14.361053	14.365919	14.369796
	120	21.146629	22.052862	22.055312	22.057351	22.059028	22.060376
1	80	1.067293	1.155185	1.155185	1.155185	1.155185	1.155185
	90	3.297405	3.545382	3.545382	3.545382	3.545382	3.545382
	100	7.438302	7.898146	7.898146	7.898146	7.898146	7.898146
	110	13.515596	14.169725	14.169725	14.169725	14.169725	14.169725
	120	21.146629	21.939648	21.939648	21.939648	21.939648	21.939648

Table 8: Sensitivity of the put option prices (writer's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.25$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	1.067293	2.126384	2.163094	2.197028	2.228503	2.257777
	90	3.297405	5.173926	5.236501	5.292969	5.344031	5.390247
	100	7.438302	9.951707	10.024905	10.076129	10.142944	10.189754
	110	13.515596	16.257373	16.311567	16.356527	16.393093	16.421937
	120	21.146629	23.723414	23.751460	23.774928	23.794214	23.809750
0.75	80	1.067293	1.927037	1.955422	1.981643	2.005954	2.028557
	90	3.297405	4.874618	4.925103	4.970675	5.011898	5.049224
	100	7.438302	9.606455	9.666776	9.643248	9.764192	9.802801
	110	13.515596	15.937467	15.981545	16.018218	16.048142	16.071818
	120	21.146629	23.484628	23.506048	23.524592	23.539896	23.552173
0.8	80	1.067293	1.753008	1.774104	1.793575	1.811615	1.828376
	90	3.297405	4.607578	4.646707	4.682033	4.713994	4.742940
	100	7.438302	9.296604	9.344289	9.385746	9.421484	9.452063
	110	13.515596	15.652019	15.686409	15.715095	15.738573	15.757214
	120	21.146629	23.274757	23.291436	23.305494	23.317145	23.326535
0.85	80	1.067293	1.600112	1.614830	1.628398	1.640955	1.652614
	90	3.297405	4.367960	4.396412	4.422097	4.445338	4.466390
	100	7.438302	9.017102	9.052417	9.083218	9.109773	9.132484
	110	13.515596	15.395949	15.421091	15.442109	15.459360	15.473104
	120	21.146629	23.090960	23.102766	23.113222	23.121066	23.127795
0.9	80	1.067293	1.465017	1.474154	1.482565	1.490341	1.497554
	90	3.297405	4.151841	4.170244	4.186854	4.201884	4.215497
	100	7.438302	8.763826	8.787061	8.807400	8.824943	8.839940
	110	13.515596	15.165126	15.181753	15.195139	15.206396	15.215393
	120	21.146629	22.928683	22.936119	22.942428	22.947696	22.951884
0.95	80	1.067293	1.345033	1.349292	1.353207	1.356820	1.360168
	90	3.297405	3.956014	3.964948	3.973008	3.980301	3.986905
	100	7.438302	8.533376	8.544837	8.554907	8.563602	8.571030
	110	13.515596	14.956152	14.964107	14.970782	14.976288	14.980701
	120	21.146629	22.784784	22.788296	22.791284	22.793788	22.795796
1	80	1.067293	1.237972	1.237972	1.237972	1.237972	1.237972
	90	3.297405	3.777827	3.777827	3.777827	3.777827	3.777827
	100	7.438302	8.322919	8.322919	8.322919	8.322919	8.322919
	110	13.515596	14.766208	14.766208	14.766208	14.766208	14.766208
	120	21.146629	22.656664	22.656664	22.656664	22.656664	22.656664

Table 9: Sensitivity of the call option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	21.863306	22.681439	22.725841	22.761584	22.792778	22.816570
	90	14.192920	15.567518	15.642843	15.705574	15.756829	15.797857
	100	8.433319	10.027595	10.121590	10.204082	10.276384	10.339428
	110	4.610115	6.101676	6.185452	6.261400	6.330390	6.393125
	120	2.340649	3.549761	3.611752	3.668963	3.721951	3.771070
0.75	80	21.863306	22.270943	22.305482	22.334201	22.357680	22.376281
	90	14.192920	15.068153	15.128783	15.179035	15.220300	15.261565
	100	8.433319	9.518055	9.594112	9.660844	9.719276	9.770223
	110	4.610115	5.652979	5.719279	5.779372	5.833955	5.883588
	120	2.340649	3.196096	3.243645	3.287493	3.328061	3.365718
0.8	80	21.863306	21.918247	21.944072	21.965593	21.983183	21.997157
	90	14.192920	14.630805	14.677433	14.716373	14.748296	14.773736
	100	8.433319	9.070215	9.129396	9.181359	9.226781	9.266407
	110	4.610115	5.261548	5.312037	5.357784	5.399328	5.437100
	120	2.340649	2.892302	2.927412	2.959757	2.989681	3.017371
0.85	80	21.863306	21.615966	21.631089	21.646213	21.658592	21.668437
	90	14.192920	14.244942	14.278685	14.307181	14.330041	14.348521
	100	8.433319	8.673711	8.716960	8.754919	8.788124	8.817033
	110	4.610115	4.917394	4.953518	4.986232	5.015962	5.042925
	120	2.340649	2.629184	2.653548	2.675963	2.696664	2.715837
0.9	80	21.863306	21.346984	21.358268	21.367572	21.375510	21.381800
	90	14.192920	13.902279	13.923998	13.942337	13.957109	13.969040
	100	8.433319	8.320447	8.348562	8.373265	8.394867	8.413656
	110	4.610115	4.612708	4.635737	4.656590	4.675459	4.692635
	120	2.340649	2.399613	2.414675	2.428512	2.441274	2.453084
0.95	80	21.863306	21.113882	21.119194	21.123630	21.127204	21.130172
	90	14.192920	13.595494	13.606695	13.615481	13.622708	13.628490
	100	8.433319	8.003913	8.017631	8.029705	8.040257	8.050808
	110	4.610115	4.341292	4.352310	4.362275	4.371311	4.379519
	120	2.340649	2.197991	2.204989	2.211407	2.217319	2.222782
1	80	21.863306	20.908512	20.908512	20.908512	20.908512	20.908512
	90	14.192920	13.321362	13.321362	13.321362	13.321362	13.321362
	100	8.433319	7.718852	7.718852	7.718852	7.718852	7.718852
	110	4.610115	4.098165	4.098165	4.098165	4.098165	4.098165
	120	2.340649	2.019864	2.019864	2.019864	2.019864	2.019864

Table 10: Sensitivity of the call option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.988$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	21.863306	22.743531	22.787917	22.824725	22.854658	22.878310
	90	14.192920	15.644711	15.720249	15.782384	15.828295	15.874205
	100	8.433319	10.114173	10.207972	10.290293	10.362419	10.425361
	110	4.610115	6.186511	6.270402	6.346439	6.415510	6.478316
	120	2.340649	3.622682	3.685075	3.742657	3.795955	3.845406
0.75	80	21.863306	22.328116	22.362669	22.391410	22.414753	22.433260
	90	14.192920	15.140506	15.201050	15.251131	15.292050	15.324958
	100	8.433319	9.600081	9.675980	9.742557	9.800866	9.852091
	110	4.610115	5.733044	5.799441	5.859633	5.914278	5.963977
	120	2.340649	3.263797	3.311692	3.355864	3.396702	3.434590
0.8	80	21.863306	21.971085	21.996939	22.018449	22.036000	22.049927
	90	14.192920	14.698845	14.745415	14.784384	14.815921	14.841352
	100	8.433319	9.148151	9.207223	9.259043	9.304397	9.343911
	110	4.610115	5.337369	5.387942	5.433609	5.475371	5.513202
	120	2.340649	2.955381	2.990765	3.023357	3.052382	3.081407
0.85	80	21.863306	21.661963	21.680122	21.695249	21.707608	21.717418
	90	14.192920	14.309116	14.342832	14.370976	14.394032	14.412599
	100	8.433319	8.748010	8.791189	8.829049	8.862183	8.891031
	110	4.610115	4.989416	5.025604	5.058376	5.088127	5.115172
	120	2.340649	2.688128	2.712697	2.735298	2.755398	2.775498
0.9	80	21.863306	21.392565	21.403915	21.413377	21.421122	21.427274
	90	14.192920	13.962965	13.984682	14.002821	14.017700	14.029584
	100	8.433319	8.391452	8.419513	8.444161	8.465712	8.484465
	110	4.610115	4.681303	4.704365	4.725238	4.744179	4.761387
	120	2.340649	2.454839	2.470036	2.483996	2.496872	2.508785
0.95	80	21.863306	21.157854	21.162647	21.166179	21.169822	21.172715
	90	14.192920	13.653729	13.664225	13.673001	13.680218	13.685959
	100	8.433319	8.071922	8.085633	8.097663	8.108192	8.117347
	110	4.610115	4.406781	4.417821	4.427807	4.436863	4.445089
	120	2.340649	2.249855	2.256920	2.263399	2.269366	2.274883
1	80	21.863306	20.948292	20.948292	20.948292	20.948292	20.948292
	90	14.192920	13.376023	13.376023	13.376023	13.376023	13.376023
	100	8.433319	7.784128	7.784128	7.784128	7.784128	7.784128
	110	4.610115	4.160821	4.160821	4.160821	4.160821	4.160821
	120	2.340649	2.068673	2.068673	2.068673	2.068673	2.068673

Table 11: Sensitivity of the call option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.25$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	21.863306	21.732672	21.769259	21.799369	21.823676	21.843045
	90	14.192920	14.745360	14.814477	14.871852	14.918620	14.955785
	100	8.433319	9.377297	9.466401	9.544467	9.612712	9.672103
	110	4.610115	5.636931	5.715678	5.787053	5.851858	5.910762
	120	2.340649	3.246649	3.304167	3.357280	3.406415	3.452059
0.75	80	21.863306	21.359127	21.387244	21.410386	21.428894	21.443729
	90	14.192920	14.279164	14.334517	14.380596	14.418031	14.447839
	100	8.433319	8.899768	8.971886	9.035078	9.090290	9.138303
	110	4.610115	5.219346	5.281610	5.338055	5.389335	5.435890
	120	2.340649	2.920782	2.964847	3.005489	3.043119	3.077982
0.8	80	21.863306	21.039217	21.059979	21.077064	21.090856	21.101672
	90	14.192920	13.871075	13.913677	13.949099	13.978015	14.001001
	100	8.433319	8.480210	8.536327	8.585535	8.628537	8.665851
	110	4.610115	4.855242	4.902657	4.945607	4.984596	5.019943
	120	2.340649	2.641147	2.673646	2.703589	2.731270	2.756932
0.85	80	21.863306	20.763222	20.780662	20.789438	20.798775	20.806464
	90	14.192920	13.511183	13.540749	13.567630	13.588455	13.605082
	100	8.433319	8.108895	8.149906	8.185863	8.217278	8.244558
	110	4.610115	4.535341	4.569246	4.599945	4.627803	4.653116
	120	2.340649	2.399202	2.421705	2.442433	2.461576	2.479307
0.9	80	21.863306	20.523560	20.532425	20.539711	20.545583	20.550180
	90	14.192920	13.191711	13.211485	13.227935	13.241376	13.252072
	100	8.433319	7.778168	7.804790	7.828220	7.848664	7.866408
	110	4.610115	4.252292	4.273886	4.293456	4.311147	4.327245
	120	2.340649	2.188247	2.202160	2.214939	2.226728	2.237636
0.95	80	21.863306	20.315503	20.318332	20.321698	20.324408	20.326528
	90	14.192920	12.906642	12.915987	12.923830	12.930406	12.935570
	100	8.433319	7.481910	7.494918	7.506343	7.516337	7.525005
	110	4.610115	4.000295	4.010628	4.019972	4.028438	4.036126
	120	2.340649	2.003147	2.009605	2.015527	2.020981	2.026023
1	80	21.863306	20.130439	20.130439	20.130439	20.130439	20.130439
	90	14.192920	12.650421	12.650421	12.650421	12.650421	12.650421
	100	8.433319	7.215178	7.215178	7.215178	7.215178	7.215178
	110	4.610115	3.774689	3.774689	3.774689	3.774689	3.774689
	120	2.340649	1.839746	1.839746	1.839746	1.839746	1.839746

Table 12: Sensitivity of the put option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	1.067293	1.598390	1.627460	1.654361	1.679334	1.702577
	90	3.297405	4.079989	4.132958	4.180724	4.223867	4.262854
	100	7.438302	8.223892	8.287030	8.342020	8.389100	8.428952
	110	13.515596	13.972983	14.013135	14.044625	14.069601	14.089418
	120	21.146629	21.015280	21.022472	21.027758	21.031353	21.034207
0.75	80	1.067293	1.444091	1.466457	1.487136	1.506322	1.524170
	90	3.297405	3.836487	3.879139	3.917609	3.952370	3.983795
	100	7.438302	7.937775	7.989638	8.035000	8.073930	8.106869
	110	13.515596	13.719600	13.750507	13.776118	13.796387	13.812205
	120	21.146629	20.854450	20.859164	20.862487	20.864758	20.866244
0.8	80	1.067293	1.309933	1.326477	1.341757	1.355922	1.369089
	90	3.297405	3.619770	3.652765	3.682527	3.709424	3.733748
	100	7.438302	7.681350	7.722224	7.758120	7.789020	7.815160
	110	13.515596	13.492848	13.516921	13.536485	13.552131	13.564424
	120	21.146629	20.716558	20.719384	20.721240	20.722389	20.723038
0.85	80	1.067293	1.192512	1.204003	1.214600	1.224412	1.233526
	90	3.297405	3.425763	3.449711	3.471311	3.490830	3.508484
	100	7.438302	7.450329	7.480516	7.507130	7.530118	7.549569
	110	13.515596	13.289924	13.307347	13.321523	13.332809	13.341629
	120	21.146629	20.597755	20.599223	20.599999	20.600455	20.600562
0.9	80	1.067293	1.089129	1.096233	1.102774	1.108822	1.114432
	90	3.297405	3.251175	3.266636	3.280579	3.293176	3.304570
	100	7.438302	7.241221	7.261071	7.278564	7.293760	7.306629
	110	13.515596	13.107984	13.118619	13.127711	13.134976	13.140639
	120	21.146629	20.494963	20.495542	20.495756	20.495732	20.495548
0.95	80	1.067293	0.997617	1.000915	1.003947	1.006745	1.009337
	90	3.297405	3.093318	3.100809	3.107563	3.113663	3.119181
	100	7.438302	7.051152	7.060902	7.069546	7.077091	7.083476
	110	13.515596	12.942512	12.947918	12.952293	12.955795	12.958519
	120	21.146629	20.405689	20.405795	20.406019	20.405600	20.405408
1	80	1.067293	0.916216	0.916216	0.916216	0.916216	0.916216
	90	3.297405	2.949971	2.949971	2.949971	2.949971	2.949971
	100	7.438302	6.877734	6.877734	6.877734	6.877734	6.877734
	110	13.515596	12.792906	12.792906	12.792906	12.792906	12.792906
	120	21.146629	20.327902	20.327902	20.327902	20.327902	20.327902

Table 13: Sensitivity of the put option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.988$ , and  $\lambda = 1.125$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	1.067293	1.632161	1.661429	1.688506	1.713639	1.737025
	90	3.297405	4.127799	4.180701	4.228401	4.271482	4.310411
	100	7.438302	8.272999	8.335887	8.390604	8.437431	8.477066
	110	13.515596	14.012384	14.051048	14.082425	14.107467	14.126998
	120	21.146629	21.036261	21.043574	21.048205	21.052836	21.055545
0.75	80	1.067293	1.475775	1.498310	1.519142	1.538466	1.556441
	90	3.297405	3.882545	3.925151	3.963579	3.998299	4.029686
	100	7.438302	7.985223	8.036896	8.082047	8.120771	8.153534
	110	13.515596	13.755683	13.786816	13.812084	13.832257	13.847946
	120	21.146629	20.873492	20.878315	20.881628	20.884079	20.885627
0.8	80	1.067293	1.339704	1.356388	1.371793	1.386071	1.399342
	90	3.297405	3.664200	3.697168	3.726904	3.753776	3.778077
	100	7.438302	7.727248	7.768061	7.803725	7.834465	7.860467
	110	13.515596	13.527167	13.551222	13.570740	13.586323	13.598447
	120	21.146629	20.733854	20.736775	20.738711	20.739929	20.740634
0.85	80	1.067293	1.220528	1.232124	1.242816	1.252715	1.261909
	90	3.297405	3.468678	3.492611	3.514197	3.533703	3.551346
	100	7.438302	7.494779	7.524875	7.551379	7.574252	7.593601
	110	13.515596	13.322611	13.340031	13.354155	13.365283	13.374260
	120	21.146629	20.613478	20.615020	20.615920	20.616372	20.616527
0.9	80	1.067293	1.115527	1.122701	1.129306	1.135413	1.141078
	90	3.297405	3.292675	3.308131	3.322066	3.334659	3.346048
	100	7.438302	7.284242	7.304073	7.321536	7.336659	7.349460
	110	13.515596	13.138587	13.149795	13.158883	13.166123	13.171759
	120	21.146629	20.509266	20.509896	20.510136	20.510171	20.510041
0.95	80	1.067293	1.022522	1.025855	1.028918	1.031746	1.034365
	90	3.297405	3.133494	3.140985	3.147738	3.153836	3.159352
	100	7.438302	7.092975	7.102711	7.111328	7.118824	7.125176
	110	13.515596	12.972282	12.977692	12.982065	12.985560	12.988271
	120	21.146629	20.418710	20.418842	20.418815	20.418691	20.418644
1	80	1.067293	0.939740	0.939740	0.939740	0.939740	0.939740
	90	3.297405	2.988908	2.988908	2.988908	2.988908	2.988908
	100	7.438302	6.918363	6.918363	6.918363	6.918363	6.918363
	110	13.515596	12.821369	12.821369	12.821369	12.821369	12.821369
	120	21.146629	20.339763	20.339763	20.339763	20.339763	20.339763

Table 14: Sensitivity of the put option prices (holder's position in the aggregated model) to the elevation of the weighting function, for different values of  $\gamma$  (curvature), with  $\gamma^+ = \gamma^-$ . Parameters of the value function:  $a = b = 0.976$ , and  $\lambda = 1.25$ . Option parameters:  $S_0 = 100$ ,  $X \in [80, 120]$ ,  $r = 0.01$ ,  $\sigma = 0.2$ ,  $T = 1$ . BS is the Black-Scholes price with  $\gamma = 1$ ,  $a = b = 1$ , and  $\lambda = 1$

$\gamma$	$X$	BS	$\delta = 0.3$	$\delta = 0.325$	$\delta = 0.35$	$\delta = 0.375$	$\delta = 0.4$
0.7	80	1.067293	1.459005	1.485914	1.510823	1.533953	1.555484
	90	3.297405	3.776014	3.826030	3.871124	3.911842	3.948621
	100	7.438302	7.722118	7.781866	7.834035	7.878737	7.916432
	110	13.515596	13.290047	13.322624	13.351765	13.372524	13.389001
	120	21.146629	20.186118	<b>20.186885</b>	<b>20.186545</b>	<b>20.185532</b>	<b>20.181487</b>
0.75	80	1.067293	1.317041	1.337716	1.356835	1.374578	1.391085
	90	3.297405	3.548618	3.588868	3.625166	3.657952	3.687579
	100	7.438302	7.453349	7.502393	7.545393	7.582366	7.613547
	110	13.515596	13.054244	13.081381	13.103115	13.120239	13.133387
	120	21.146629	<b>20.048508</b>	<b>20.047881</b>	<b>20.046507</b>	<b>20.044741</b>	<b>20.042867</b>
0.8	80	1.067293	1.193750	1.209022	1.223130	1.236210	1.248371
	90	3.297405	3.346384	3.377503	3.405567	3.430921	3.453838
	100	7.438302	7.212544	7.251172	7.285174	7.314518	7.339279
	110	13.515596	12.844569	12.865431	12.882107	12.895224	12.905237
	120	21.146629	<b>19.931690</b>	<b>19.930261</b>	<b>19.928359</b>	<b>19.926265</b>	<b>19.924202</b>
0.85	80	1.067293	1.085954	1.096548	1.106320	1.115369	1.123774
	90	3.297405	3.165468	3.188041	3.208396	3.226785	3.243409
	100	7.438302	6.995654	7.024167	7.049359	7.071181	7.089618
	110	13.515596	12.657075	12.672107	12.684094	12.693502	12.700700
	120	21.146629	<b>19.832105</b>	<b>19.830399</b>	<b>19.828420</b>	<b>19.826377</b>	<b>19.824437</b>
0.9	80	1.067293	0.991141	0.997683	1.003707	1.009277	1.014444
	90	3.297405	3.002768	3.017335	3.030466	3.042326	3.053049
	100	7.438302	6.799384	6.818088	6.834672	6.849089	6.861292
	110	13.515596	12.489434	12.498189	12.505844	12.511836	12.516409
	120	21.146629	<b>19.746913</b>	<b>19.745390</b>	<b>19.743733</b>	<b>19.742081</b>	<b>19.740546</b>
0.95	80	1.067293	0.907290	0.910324	0.913113	0.915687	0.918072
	90	3.297405	2.855753	2.862809	2.869165	2.874905	2.880095
	100	7.438302	6.621024	6.630177	6.638409	6.645549	6.651608
	110	13.515596	12.330813	12.341039	12.344601	12.347562	12.349738
	120	21.146629	19.672287	19.672924	<b>19.671901</b>	<b>19.670942</b>	<b>19.670062</b>
1	80	1.067293	0.832772	0.832772	0.832772	0.832772	0.832772
	90	3.297405	2.722333	2.722333	2.722333	2.722333	2.722333
	100	7.438302	6.458320	6.458320	6.458320	6.458320	6.458320
	110	13.515596	12.198465	12.198465	12.198465	12.198465	12.198465
	120	21.146629	19.610962	19.610962	19.610962	19.610962	19.610962