Ca' Foscari University of Venice

Department of Economics

Working Paper

Paolo Pellizzari and Dino Rizzi

Citizenship and Power in an Agent-based Model of Tax Compliance with Public Expenditure

ISSN: 1827/3580
No. 24/WP/2012

Working Papers
Department of Economics
Ca' Foscari University of Venice
No. 24/WP/2012
ISSN 1827-3580

Citizenship and Power in an Agent-based Model of Tax Compliance with Public Expenditure

Paolo Pellizzari - Dino Rizzi
Department of Economics
Ca' Foscari University of Venice

Data: October 2012

Abstract
In this paper we present a model of tax compliance with heterogeneous agents who maximize their individual utility based on income and the conjectured level of per capita public expenditure. We formally include psychological drivers in this model. These drivers affect individual behavior, such as risk aversion, together with appreciation of public expenditure, expectations about peers' compliance and a natural inclination to comply, all of which we summarize in a quality termed "citizenship". The enforcement system, based on random inspections, is standard and only partially known to agents.
The agent-based model is simulated under a variety of settings, representing different "societies". We use the artificial data produced by the model to estimate the effects of taxpayers' traits on personal tax behavior and to build a compliance societal slippery slope. At the individual level, we find a positive dependence of compliance on all variables, with the significant exception of the tax rate, which has a negative impact. As far as societies are concerned, we show how aggregate tax compliance depends on composite indices of citizenship and power, and we find that the former is more important than the latter.

Keywords: Tax evasion, public expenditure, agent-based models, slippery slope model

JEL Codes: H26, H40, C63

Address for correspondence:
Paolo Pellizzari
Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta S.Giobbe
30121 Venezia - Italy
Phone: (++39) 041 2346924
Fax: (++39) 041 2347444
e-mail: paolop@unive.it

This Working Paper is published under the auspices of the Department of Economics of the Ca' Foscari University of Venice. Opinions expressed herein are those of the authors and not those of the Department. The Working Paper series is designed to divulge preliminary or incomplete work, circulated to favour discussion and comments. Citation of this paper should consider its provisional character.

The Working Paper Series
is available only on line
(www.dse.unive.it/pubblicazioni)
For editorial correspondence, please contact:
wp.dse@unive.it

Department of Economics
Ca' Foscari University of Venice
Cannaregio 873, Fondamenta San Giobbe
30121 Venice Italy
Fax: ++39 041 2349210

1. Introduction

The problem of tax compliance has always been central to the theory of public finance, and its relevance has increased over time. The seminal models by Allingham and Sandmo (1972) and Yitzhaki (1974) assume a neat rational framework where individual decisions are taken based on a cost–benefit analysis under uncertainty driven by the probability of auditing and the effects of fines. An impressive body of research has built on this approach to overcome limitations, and now incorporates other determinants of behavior and provides more realistic descriptions of observed data. A particularly important issue is excessive tax compliance observed in the real world with respect to the level that the standard model of tax evasion would predict[1]. Another significant issue is the impact of the level of tax rates on compliance which, in Allingham, Sandmo and Yitzhaki's works, is ambiguous or opposite to what is currently supported by common sense[2] or experimental findings. It was influentially pointed out that all tax-related decisions are not purely individual affairs, but depend on the quantity, efficiency and fairness of public expenditure, which should ultimately be financed by tax revenues: with no hope of being exhaustive, see Cowell and Gordon (1988), Cowell (1992), Bordignon (1993) and Rablen (2010) for up-to-date descriptions. Pyle (1991), Andreoni et al. (1998) and, more recently, Sandmo (2005) provide well-written summaries of further developments and research themes. After seminal contributions by Schmölders in the 1960's, see Frank and Kirchler (2006), recognition that psychological factors may be relevant for understanding and modeling tax behavior has gained tremendous momentum in the past decade, see Kirchler (2007). Compliance, the result of complex interactions between taxpayers and their respective Government, is situated on a slippery slope, where both trust in authorities and power shape the behavior of a collectivity of agents. Tax morale, see Alm and Torgler (2006), Torgler (2007), and knowledge/perception of the enforcement system are intuitively useful to explain how taxpayers behave; their importance is empirically and experimentally grounded. In our opinion, fewer works have explored how such elements can be introduced to formal models of individual responses. One exception is the rich strand of agent-based models that offer useful insights through numerical simulations of the complex behavior of heterogeneous agents. The seminal contribution in this field can be traced back to Mittone and Patelli (2000), who state that tax evasion can spread among agents like

[1] See Bernasconi (1998).
[2] See Bernasconi and Zanardi (2004).

outbreaks. Different types of agents can copy other agents' actions, if this is myopically deemed more convenient and, as a result, in the absence of a sufficient number of audits, evaders multiply at the expense of more honest taxpayers. Since then, other authors have taken similar approaches. For example, a set of distinct types is exogenously assumed to exist, see for instance Hokamp and Pickhardt (2010), who also have agents making random errors in their reported income, and Antunes et al. (2007) who consider different bounded rational agents. Davis et al. (2003) analyze a model with honest and susceptible agents together with evaders and show that if an initially compliant population shifts to evasion, it is difficult to reverse the situation and the number of audits must be increased beyond the level that would have achieved acceptable compliance in the first place.

Agent-based models are able to explore several issues that are difficult to tackle using analytical models. Repeated interaction among heterogeneous agents or between the fiscal administration and the taxpayers can typically be simulated. The model presented in Korobow et al. (2007) is an attempt to consider geographical spillovers and "contagion" effects, as agents are networked in localized structures and are aware of their neighbours' actions, facing peer pressure and conformity pressure. It is shown in the paper that substantial sharing of payoff information and tax practices can lead to less compliant behavior. While it is not entirely surprising that one bad apple can spoil the whole basket, the result also illustrates that a certain degree of taxpayers' "impressionability" may be conducive to a relatively high degree of compliance, even in the event of modest levels of auditing.

Despite the risk of increasing the number of parameters, the flexibility of agent-based methodology facilitates great sophistication in the depiction of many realistic features of taxpayers and of the data available to the fiscal authority. Bloomquist (2006) provides a good introduction to the field. He describes the Tax Compliance Simulator where audit efficacy and celerity, together with a host of other parameters, can be changed to test overall compliance and the success of specific auditing schemes with regard to both direct and indirect effects (due to additional revenues from fines and greater compliance of other "forewarned" taxpayers, respectively). Bloomquist (2011) builds on some of the previous ideas to present a situated agent-based model with 85,000 agents who are calibrated using realistic anonymized public data from the US Internal Revenue Service.

At the other extreme, a number of recent papers in the econophysics literature simplify relationships bewteen agents and present terse models of interaction based essentially on the Ising model of ferromagnetism. In Zaklan et al. (2009), 1,000,000 agents with "spin" are in contact on a grid; they are subject to local interactions and, possibly, social forces (due to mass media or cultural biases, say). As is typical of similar models, there are critical thresholds of the model's parameters that trigger phase changes. In Pickhardt and Seibold's (2011) recent paper, it is shown that Ising-like models can to some extent reproduce situations in which different types of taxpayer are present.

This paper presents a model with heterogeneous agents who maximize their individual utility based on (after-tax) income and the conjectured level of per capita public expenditure. Agents have different levels of risk aversion, a distinct relative preference for public expenditure, varying confidence in the likelihood that others will pay the amounts due and an innate attitude to comply, which can be associated with cultural traits, social constraints or shame. Hence, their final decisions rest on micro-founded rational behavior, personal characteristics and subjective judgements. The enforcement system, based on random inspections, is only partially known to agents. These agents establish noisy estimates of the auditing probability by meeting other taxpayers and exchanging information about income. In each period, an individual can optimally conceal a certain amount of income based on conjectures about the perceived probability of being audited, the perceived level of public expenditure and the perceived amount of tax paid by other individuals.

We contribute to the literature in two main ways. Firstly, we formally include psychological drivers in the model that affect individual behavior. Some parameters, broadly related to "trust" and tax morale, capture the appreciation of public expenditure and the natural inclination to comply. The combination of the previous ingredients with the expectation that peers will reciprocate appears to be related to "good citizenship", a quality that helps agents to fulfill their civic duties. Hence, our work suggests ways to link conceptual dimensions to specific personal parameters[3] and shows how to combine them to make a citizenship index. Secondly, we use simulated data drawn from a variety of "societies" to estimate a compliance slippery slope and then discuss how individual micro-motives aggregate in a variety of social macro-behaviors. Such a link is commonly drawn in agent-based approaches but, as far as we know, this is the first time a full-fledged slippery slope has been analyzed. On a more methodological note, the model generates artificial data that is then used to build a surface that correlates well to some stylized facts, even though this is not yet meant to describe any specific situation.

---

[3] In some sense, our agent-based model departs considerably from the econophysics paradigm, in which agents are electrons with +1 or -1 spins and have virtually no interiority, to allow for a good deal of nuanced peculiarities to emerge at the individual level.

The paper is structured as follows. Section 2 describes the agents' behavior leaving some technical details for Section 3, which is devoted to presenting our simulations and the parameters used. We stress that data can be examined at both the personal and the aggregate level; results are discussed in Sections 4 and 5 bearing in mind this duality. Section 6 offers a summary and a number of final remarks.

2. The model

We consider two types of agents, namely individuals and the Government. Individuals maximize their utility under a fixed monetary income. The utility function depends on after-tax income and the perceived level of public expenditure, together with individual attitudes to comply, which can be thought of as tax morale. Agents choose the fraction of income to be declared in order to pay income tax. Hence, based on heterogeneous parameters, they have the option to underreport their income to illegally reduce their tax burden.

The Government determines the income tax rate, collects all tax payments and uses the entire revenue to fund public expenditure, ensuring its budget is always balanced.

The Government can also counter tax evasion by controlling a fraction of individuals and by fining tax evaders.

Agents face the standard problem of deciding how much of their income they wish to declare for income taxation. Each individual $i=1, \ldots, N$ has a cardinal utility function:

[1] $$U_i = U(y_i, G_i, \theta_i),$$

where y_i is the net (after-tax) monetary income, G_i is the perceived value of public expenditure, and $\theta_i = \{\theta_{i1}, \ldots, \theta_{ik}\}$ comprises individual characteristics. We anticipate that θ_i allows agents to differ in their risk aversion, relative preference for public expenditure, innate attitude to comply and expectations about other agents' behaviors (conformity).

We suppose the utility function satisfies the following assumptions:

Assumption 1: positive marginal utility of income and public expenditure:

[2] $$\frac{\partial U}{\partial y_i} = U_y > 0 \text{ and } \frac{\partial U}{\partial G_i} = U_G > 0.$$

Assumption 2: concavity of utility function:

[3] $$\frac{\partial U_y}{\partial y_i} = U_{yy} < 0, \ \frac{\partial U_G}{\partial G_i} = U_{GG} < 0 \text{ and } U_{yy}U_{GG} - U_{yG}^2 > 0.$$

Assumption 3: minimum level of utility reached when either $y_i = 0$ or $G_i = 0$:

[4] $$U_{\min} = U(y,0) = U(0,G) = U(0,0).$$

The main idea behind our assumptions is that both private income and public expenditure are required in a society to provide material and immaterial satisfaction to agents.

One specification that satisfies all the previous assumptions is:

[5] $$U(y_i, G_i, \theta_i) = \phi(y_i, \theta_i)\varphi(G_i, \theta_i),$$

with $f_y > 0, f_{yy} < 0$, $j_G > 0, j_{GG} < 0$ and $U(0,0,\theta_i) = \phi(0,\theta_i)\varphi(0,\theta_i)$. In particular, we assume that:

[6] $$U_i = A_i \left[y_i G_i^{\alpha_i} \right]^{1-\rho_i},$$

where $\phi(y_i, \theta_i) = y_i^{1-\rho_i}$ and $\varphi(G_i, \theta_i) = G_i^{\alpha_i(1-\rho_i)}$, $r_i > 0$ is the relative risk aversion parameter, A_i is an individual scale factor, and $a_i > 0$ represents the relative intensity of individual preference for public expenditure with respect to net income.[4] This preference parameter encompasses a variety of individual considerations about the Government's behavior regarding the use of revenue. For instance, the preference can reflect individual judgements about efficiency, fairness, the mix in the provision of public goods and other public expenditure.

To evaluate their utility U_i, agents require a certain amount of knowledge of the term in square brackets, which in turn requires the likelihood and effects of audits to be taken into account and an estimate of public expenditure. Income tax is charged at a constant rate t on the exogenous amount of income I_i, which is known to the taxpayer only. Hence, the amount of tax paid by individual *i* is:

[7] $$T_i = \tau d_i I_i,$$

[4] With this Cobb-Douglas-type utility function, concavity requires $1 - \rho_i < 1$, $\alpha_i(1-\rho_i) < 1$, and $(1+\alpha_i)(1-\rho_i) < 1$.

where $0 \le d_i \le 1$ is the compliance rate, i.e. the share of gross income declared to the Government. Each agent can choose their decision variable d_i, opting for full ($d_i = 0$), partial ($d_i \neq 0$) or no ($d_i = 1$) tax evasion.

Given the control activity of the Government, each individual faces a probability q of being controlled. If individuals are audited, their tax evasion will definitely be discovered and a fine of f times the amount of evaded tax will be payable:

[8] $$F_i = f(1-d_i)\tau I_i .$$

In the simulations of the model presented in Section 3, individuals do not exactly know the probability of control *q*, but estimate their own subjective p_i at any time following random interaction with other agents.

If the taxpayer is not controlled, her net income is

[9] $$W(d_i) = I_i - T_i = I_i(1-d_i\tau).$$

If, instead, the taxpayer is audited, her net income is:

[10] $$Z(d_i) = I_i - T_i - F_i = I_i(1-\tau) - f(1-d_i)\tau I_i .$$

Each individual knows that, in aggregate, the Government budget constraint holds, hence public expenditure is equal to the total tax revenue[5]:

[11] $$G = \sum_i \tau d_i I_i .$$

However, no agents have full information on the current G and a conjecture must be used to obtain a reasonable tentative estimate. Not surprisingly, it turns out that the way in which G is estimated has a considerable impact on decisions. Per capita public expenditure[6] can be split into two parts: the part paid by individual *i* and the part paid by all others, so that:

[12] $$\frac{G}{N} = \frac{1}{N}\tau d_i I_i + \frac{1}{N}\sum_{k=1,k\neq i}^{N} \tau d_k I_k .$$

If we define the per capita public expenditure financed by all other individuals as:

[5] We assume that fees and revenue raised through enforcement are not used to finance public expenditure. At the same time, enforcement costs are not taken into account.

[6] Cowell and Gordon (1988) explicitly consider the degree of rivalness of public expenditure. Given the particular utility function chosen, we omit this feature in the present model because a constant degree of rivalness does not alter individuals' optimal choices.

[13] $$g_i = \frac{1}{N-1}\sum_{\substack{k=1 \\ k \neq i}}^{N} \tau d_k I_k$$

and, in a society with a large number of agents, we approximate 1/*N* to 0 and (*N*-1)/*N* to 1, we get:

[14] $$\frac{G}{N} = g_i \,.$$

It is worthwhile noting that the constant factor *N* does not affect the optimal choice and, without loss of generality, we consider in the following the simplified utility function:

[15] $$U_i = A_i \left(y_i g_i^{\alpha_i} \right)^{1-\rho_i} .$$

When choosing the rate of compliance, each taxpayer has to estimate how other individuals react to variations in her contribution[7]. As g_i is not known, the *i*-th agent must form her own expectation $\tilde{g}_i$ about other people's average contribution. This is modeled as an individual adjustment of the level of the previous period per capita public expenditure, g_{-1}, which we assume to be publicly observed with a conjecture about other people's reaction to their variation of the rate of compliance, so that:

[16] $$\tilde{g}_i = \gamma_i \left(\tau d_i \tilde{I}_i \right) + \left(1 - \gamma_i \right) g_{-1},$$

where $\tilde{I}_i$ is the agent's estimate of the average income, γ_i $(0 \leq \gamma_i \leq 1)$ is a "conformity"[8] parameter, as in Cowell-Gordon (1988), and g_{-1} is the observed level of per capita public expenditure in the previous period.

If γ_i approaches 1, the agent believes that others will react in the same direction (*positive conjectural variation*). In other words, if the agent increases her compliance rate, she believes that all other individuals will behave in the same way. If, on the contrary, γ_i is close to 0, the

---

[7] See Cornes and Sandler (1986), page 151: "The Nash assumption is that each individual expects no response by the rest of the community to variation in his or her own contribution. However, unless the reaction curve of the rest of the community is in fact horizontal, reflecting an actual response of zero, such a conjecture is not consonant with observable facts."

[8] This definition of conformity differs from "group conformity", defined in Gordon (1989) and Myles and Naylor (1996), in which it is assumed that individuals are more likely to evade the more others evade.

individual believes that other agents will not change their behavior (*zero conjectural variation*)[9] and therefore assumes that the per capita public expenditure will remain constant.

We expect that the higher the compliance rate, the higher the conformity parameter g_i will be. With respect to Kirchler et al. (2008), we note that conformity is a form of "trust" that has to do with other citizens and not with the central government, being related to the perceptions of how other taxpayers behave in a given institutional and social framework.

We finally introduce citizens' attitudes towards paying taxes to the utility function, assuming that scale factor A_i depends on an individual parameter $k_i \geq 0$:

[17] $$A_i = (1 + d_i)^{k_i} .$$

For a given level of compliance, factor $(1 + d_i)^{k_i}$ leads to higher increases in utility the higher k_i is. At the same time, a decrement in the compliance rate produces a larger decline in utility for individuals with higher k_i. Hence, this parameter can be thought of as a measure of the agent's tax morale or, alternatively, of the "shame" arising due to tax evasion.

The above definitions lead to the following full specification:

[18] $$U_i = A_i \left(y_i \tilde{g}_i^{\alpha_i} \right)^{1-\rho_i} = (1 + d_i)^{k_i} y_i^{1-\rho_i} \left[\gamma_i \left(\tau d_i \tilde{I}_i \right) + (1 - \gamma_i) g_{-1} \right]^{\alpha_i (1-\rho_i)} .$$

A higher level of compliance then has multiple effects on the level of utility: it decreases utility due to a lower net income but simultaneously increases it via the expected per capita public expenditure and through the tax morale multiplicative factor.

Each taxpayer chooses which share of income to declare, d_i, in order to maximize the expected utility:

[19] $$EU(d_i) = p_i U_i \left(Z_i, \tilde{g}_i \right) + (1 - p_i) U_i (W_i, \tilde{g}_i) .$$

We assume that updates of both p_i and $\tilde{g}_i$ depend on random meetings with other agents, from which the required sample information about the actual auditing frequency and income is obtained. A more formal description is given in the next section.

[9] See Cornes and Sandler (1986)

3. Compliance in a variety of societies with heterogeneous agents

This section is devoted to describing how the previous model can be simulated in an agent-based framework with many heterogeneous agents. To the best of our knowledge, in fact, it is impossible to derive clear implications from the standard comparative statics exercise based on first-order conditions. Also, in particular, it is extremely difficult to characterize the aggregate behavior of a society of diverse taxpayers with respect to several parameters. Hence, numerical simulation appears to be the methodological tool that can shed light on the main features of this model more effectively.

We assume that agents are able to decide which best action is best for them in terms of the compliance rate and, indeed, that they can optimize their utility given their personal traits. However, their rationality is bounded as they do not exactly know the probability of being audited and the amount of tax actually paid by other agents.

Each agent gathers the necessary information by meeting n other individuals in each period. Encounters are random and independent across agents and along time, i.e., each taxpayer will meet the same number of newly sampled agents at each time. Although this matching mechanism may be oversimplified, it retains the ability to describe settings in which agents have volatile estimates of some important variables. From each of her n meetings, the agent learns:

- whether the other taxpayer has been controlled; and
- the other agent's level of income.

Agents do not exchange any information about the amount of tax due or the degree of compliance, which seem to be more sensitive pieces of information. As a whole, these pairings allow the agent to compute a noisy estimate p_i of the true audit probability q and of the population's average income. In realistic situations, we expect the number of meetings n to be much smaller than the number of agents N. Letting n_i^c (out of n) be the number of encountered agents who have been controlled, the subjective audit probability for agent i at time t is n_i^c/n. As the Government can change its audit policy at any time, the agent re-estimates the subjective probability in each period, and we assume that the value p_i used in the individual utility maximization is a weighted average of her previous estimate (with weight π) and the new estimate n_i^c/n (with weight $1-\pi$). Therefore, the *i*-th agent's subjective probability of being controlled is:

[20] $$p_{it} = \pi p_{i,t-1} + (1-\pi)\frac{n_i^c}{n}.$$

The very same mechanism is used to estimate the average income, and we assume that the sample average income $\tilde{I}_i$ observed in the meetings is:

[21] $$\tilde{I}_i = \frac{1}{n}\sum_{j \in J(i)} I_j ,$$

where $J(i)$ is the set of agents that i meets at time t (which is omitted for notational simplicity).

The Government is in charge of:

- selecting the tax policy by choosing the level of income tax rate τ;
- providing public goods and services to be financed by tax revenues; and
- adopting an adequate auditing policy comprising the choice of the fraction q of individuals to control and the rate of penalties f.

With respect to the evasion control policy, we simply assume a purely random auditing scheme, so that a share q of all agents is randomly controlled in each period. Furthermore, we assume that the Government chooses its fine rate by comparing the punishment for tax evasion to those for other offenses, so that both f and q are exogenous parameters in the model.

One period of the simulation evolves according to the following steps:

- agents inherit previous period's parameters, maximize utility to decide their tax compliance, and pay taxes;
- the Government funds public expenditure, performs the random control of taxpayers, and punishes tax evaders; and
- agents gather information while meeting other taxpayers and revise their estimates of the probability of being controlled and of the average income of the population.

To this end, we simulate a framework in which agents interact and the levels of public expenditure and compliance are stochastic, eventually stabilizing around values driven by the distribution of the personal features of taxpayers and by the Government's enforcement policy (in addition to a handful of other "technical" parameters, such as n and π).

In each simulation we sample agents from distributions characterized by different individual parameters in order to represent different "societies" with specific institutional settings.

In other words, we can interpret a society as a specific set of values for some key parameters that shape the inhabitants' features and the institutional aspects of the tax collection system. Clearly, different societies are likely to exhibit sundry behaviors with respect to aggregate tax compliance and public expenditure, and one of the aims of this work is to clarify which societal dimensions are the most relevant for developing an illuminating description of potentially dissimilar tax-related conducts. Our model allows for diversity both *across* and *within* societies and it is likely, say, that some free riders coexist with a population of taxpayers that, as a whole, is very compliant. Obviously, our approach does not allow us to justify why parameters have specific values or how they evolved to reach those values and, in this sense, we still have a "black box" issue to address. To partially overcome this problem, we study a variety of outcomes and explore possible causal effects by sampling many static societies.

We run 250 simulations in which we pick random institutional parameters and minimum and maximum values that characterize the distributions of individual parameters. To avoid transient effects and taking into account the fact that the system stabilizes in a few tens of periods, we run 200 rounds in each simulation and only keep the data obtained in the last simulation.

Table 1 – Values of parameters used in the simulations

Number of societies	S	250		
Number of agents	N	1000		
Number of meetings per agent	n	10		
Institutional settings				
		Distribution	*Values*	
Tax rate	τ	Uniform	$\tau \in [0.2, 0.5]$	
Probability of control	q	Uniform	$q \in [0.01, 0.1]$	
Penalty per € of tax evaded	f	Uniform	$f \in [1.0, 5.0]$	
Individual characteristics				
		Distribution	*Values*	
Income	I_i	Lognormal	mean: 30,000 €	Standard deviation of log: log(2)
Relative risk aversion	ρ_i	Uniform $[\rho_{\min}, \rho_{\max}]$	$\rho_{\min} \in [0.0,0.5]$	$\rho_{\max} \in [0.5,1.0]$
Preference for public expenditure	α_i	Uniform $[\alpha_{\min}, \alpha_{\max}]$	$\alpha_{\min} \in [0.0,0.5]$	$\alpha_{\max} \in [0.5,1.0]$
Conformity	γ_i	Uniform $[\gamma_{\min}, \gamma_{\max}]$	$\gamma_{\min} \in [0.0,0.5]$	$\gamma_{\max} \in [0.5,1.0]$
Attitude to comply	k_i	Uniform $[k_{\min}, k_{\max}]$	$k_{\min} = 0.0$	$k_{\max} \in [0.0,0.5]$
Starting values				
Weight π in p_i	p	Constant	0.5	
Previous year public expenditure	g_{-1}	Constant	0 €	
Subjective probability of control	p_i	Constant	0	
Estimate of average income	$\tilde{I}_i$	I_i		

Table 1 shows the values of parameters used in the simulations. Although agent-based models are attractive due to their ability to allow for interaction and the massive heterogeneity of agents, this typically results in a large number of parameters. This is sometimes referred to as "the wilderness of bounded rationality"[10] and is one on the main reasons why it is difficult to analytically solve agent-based models. One way to overcome the problem and to reduce the number of effective degrees of freedom is to focus on *meta-parameters* that describe the distribution from which individual traits are sampled. Equivalently, meta-parameters can be thought of as descriptions of the range of individual parameters, and each society is indeed fully described by some values for its meta-parameters. In the following, we will consistently use the word "parameter" when individual quantities, such as α_i and γ_i, are referred to,

[10] See Hommes (2006) and Sims (1980) for the original idea.

whereas with "meta-parameters" we denote societal values, such as $\alpha_{\min}$, $\gamma_{\min}$ and $k_{\min}$, described in Table 1.

The results of the simulations are basically the agents' rates of compliance and the levels of per capita public expenditure. We expect that societies with low (high) values of $\alpha_{\min}$, $\alpha_{\max}$, $\gamma_{\min}$, $\gamma_{\max}$, $k_{\min}$, $k_{\max}$, namely those with low (high) values of preference for public expenditure, conformity and attitude to comply, will show low (high) aggregate compliance. In a related fashion, societies where low (high) auditing probabilities and penalty levels are in place are expected to display a tendency for lower (higher) compliance.

The results of our simulations are presented in the next sections. In particular, Section 4 focuses on individuals and provides a descriptive analysis of a cross-section of taxpayers' compliance, together with a regression analysis intended to show how parameters affect the actions of single agents. We also comment on three fictitious taxpayers with low, average and high "citizenship" to exemplify different (but somewhat typical) behaviors. Section 5 leaves aside individuals and focuses on the study of tax compliance in different societies. We identify and discuss the main drivers of aggregate behavior through regressions, and develop indices of power and citizenship in terms of the meta-parameters that are largely able to explain how societies position themselves on a slippery slope of tax compliance.

4 Individual compliance

This section discusses the individual data produced from our 250 simulations. Some descriptive statistics are reported in Table 2, which shows the mean, standard deviation, minimum and maximum values for each parameter. As expected, the values in the "Min" and "Max" columns are close to the extremes of the sample ranges described by the meta-parameters of Table 1. Note that our taxpayers exhibit compliance that spans the whole [0–100%] interval[11], with an average of 82.15%.

The distribution of individual compliance rates is depicted in Figure 1*a*, where it is apparent that a large fraction (54.6%) of agents are nearly fully compliant. However, many agents massively underreport their income, producing a remarkably left-tailed distribution.

[11] The numerical routine used to maximize [19] under the constraint $0 \le d_i \le 1$ yields results in the range $[\varepsilon, 1-\varepsilon]$. We assume that full compliance is achieved when $d_i > 99\%$ and that total evasion prevails when $d_i < 1\%$. Other reasonable definitions of full compliance and total evasion would yield the same results.

Per capita public expenditure values vary widely, too, with a certain amount of concentration between 6,000 and 12,000 (see part *b* of Figure 1). To give an idea of the variability in our results, parts *c* and *d* of Figure 1 show individual compliance against two individual characteristics (attitude to comply and conformity) for a random sample of 5,000 individuals.

Table 2 – Values of variables used in the regression model

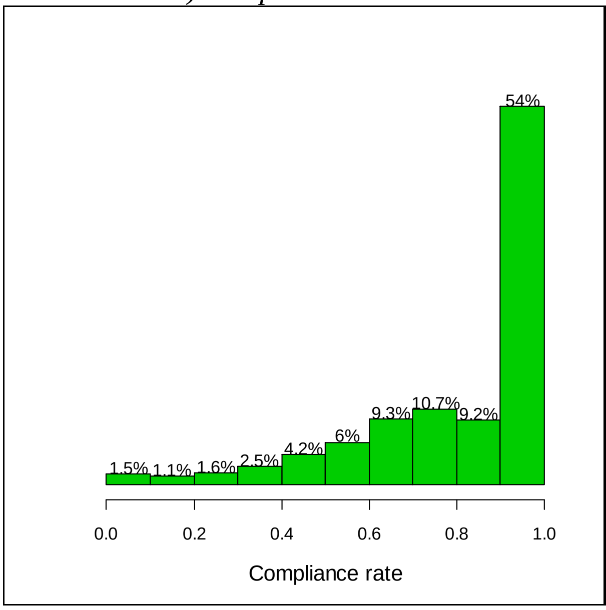
Variable	*Mean*	*Std. Dev.*	*Min*	*Max*
Compliance rate: d_i	0.821	0.234	0	1
Income: I_i	29919.1	23486.7	936.7	558711.1
Risk aversion: ρ_i	0.502	0.192	0.001	0.997
Preference for public expenditure: α_i	0.494	0.185	0.004	0.999
Conformity: γ_i	0.504	0.189	0.001	0.998
Attitute to comply: k_i	0.128	0.111	0.000	0.496
Subjective probability: p_i	0.055	0.048	0.000	0.396
Estimate of average income: $\tilde{I}_i$	29916.4	7077.8	11705.6	98064.4
Previous year's public expenditure: g_{-1}	8592.7	2059.2	4104.8	13904.4
Tax rate: τ	0.354	0.085	0.200	0.499
Penalty: f	3.007	1.156	1.034	4.980

To understand the relative importance of individual parameters, we treat the results of the simulations as if they were individual observations of a cross-section of 250,000 agents. Being the dependent variable d defined in the range [0,1], we use a tobit regression to tackle the large number of censored observations and to estimate linear and quadratic models where the compliance rate depends on the parameters listed in Table 3. Other specifications were tested and led to inferior results. The censored models presented here appear to be adequate in terms of goodness of fit and have an approximately normal residuals distribution.

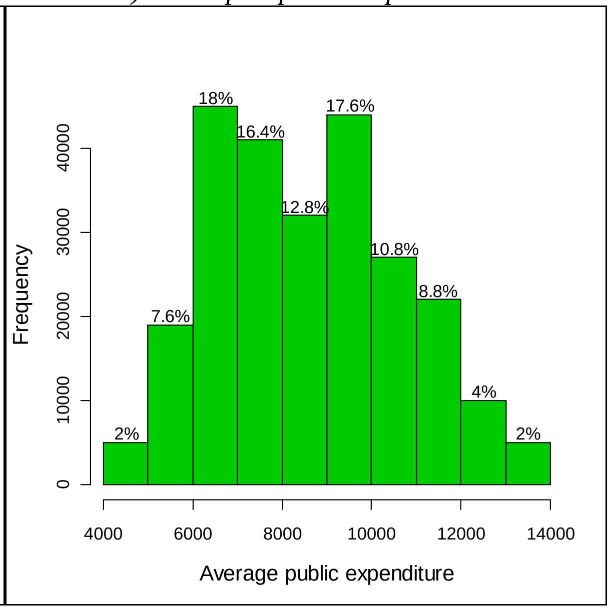
All other coefficients apart from income are extremely significant and there is a positive dependence of individual compliance on all individual characteristics and enforcement policy parameters. In agreement with intuition, instead, tax rate τ has a negative impact on declared income. The quadratic model in Table 3 has a slightly better fit but, for the sake of exposition, we focus on the less complex linear model in the remaining part of the section.

Figure 1 – Distribution of individual results (all societies)

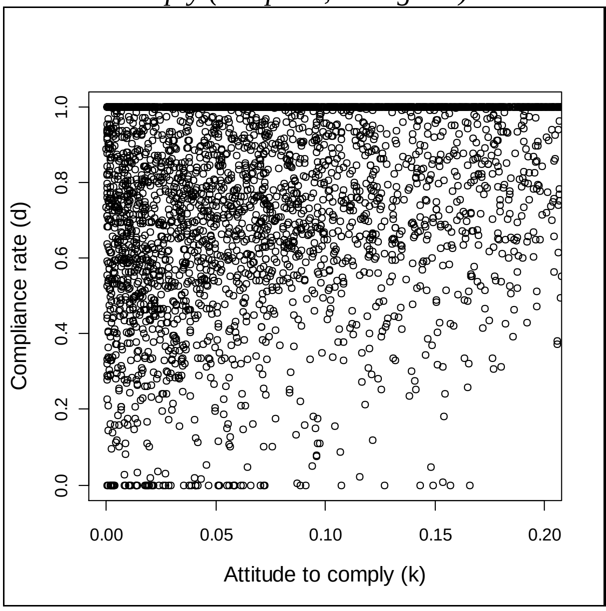
a) Compliance rates



b) Per capita public expenditure



c) Individual compliance rates and attitude to comply (sample 5,000 agents)



d) Individual compliance rates and conjecture (sample 5,000 agents)

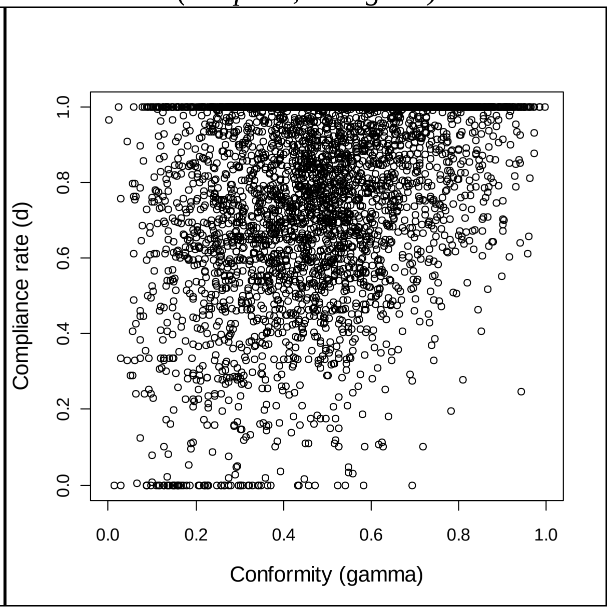


Table 3 – Tobit regressions of individual compliance rates

Dependent variable: d_i			Number of obs = 250,000	
	Linear model		*Quadratic model*	
	Log likelihood = 4,152.20		Log likelihood = 11,212.94	
	Coeff.	*t*	*Coeff.*	*t*
I_i	2.43E-08	1.31	4.21E-08	1.33
ρ_i	0.447	176.13	-0.161	-15.60
α_i	0.801	267.89	1.159	100.73
γ_i	0.794	254.71	1.201	98.09
k_i	1.552	279.47	1.830	134.49
p_i	3.285	284.32	3.000	100.86
$\tilde{I}_i$	5.62E-06	87.13	1.18E-05	35.00
g_{-1}	3.14E-06	8.80	3.09E-05	15.18
τ	-1.610	-207.70	-5.074	-82.64
f	0.078	156.71	0.088	38.58
I_i^2			-9.00E-14	-0.46
ρ_i^2			0.631	60.56
α_i^2			-0.400	-35.56
γ_i^2			-0.433	-39.27
k_i^2			-0.821	-22.19
p_i^2			1.972	9.20
$\tilde{I}_i^2$			-9.50E-11	-19.07
g_{-1}^2			-1.58E-09	-14.53
τ^2			4.856	59.02
f^2			-1.09E-03	-2.99
constant	-0.296	-62.34	0.0257	1.94
sigma (st.err.)	0.17266 (0.000469)		0.16564 (0.000456)	
	2,627 left-censored observations at d=0 131,584 uncensored observations 115,789 right-censored observations at d=1			

Note: robust estimates

In a tobit regression, marginal effects are not simply the estimated coefficients; they must be computed, as they depend on the chosen values of dependent variables. In order to get an idea of agents' behavior, we investigate three typical taxpayers called *low, average* and *high citizenship agents*. A low citizenship agent has parameters a, g and k in the lower range of possible values and, hence, has relatively low preference for public expenditure, and little conformity and attitude to comply. We expect a greater amount of underreporting from such a taxpayer, all other variables being equal. Similarly, we consider an average agent with parameters set at the average of the possible ranges, and a high citizenship individual with relatively large values for a, g and k, who is likely to be very compliant in a variety of situations.

To assess the effects in different institutional frameworks, it is convenient to evaluate the reactions of the typical previously defined agents when they are embedded in three societies distinguished by low, average and high enforcing power on the part of the Government. Hence, we consider a *low* (respectively, *average* or *high) perceived power* society where the subjective auditing probability p_i and penalty rate f are low (average or high). All in all, we consider 9 cases (3 types times 3 societies) in our analysis; Table 4 summarizes the relevant values of the parameters. As an example, the average taxpayer in an average power society, corresponding to the middle column of the second panel in Table 4, believes there is a 5% audit probability per year and fines are equal to three times the tax evaded.

Table 4 – Simulated typical taxpayers

	Low citizenship			*Average citizenship*			*High citizenship*		
	Low power	*Average power*	*High power*	*Low power*	*Average power*	*High power*	*Low power*	*Average power*	*High power*
α_i	0.25	0.25	0.25	0.50	0.50	0.50	.75	.75	.75
γ_i	0.25	0.25	0.25	0.50	0.50	0.50	.75	.75	.75
k_i	0.05	0.05	0.05	0.125	0.125	0.125	.20	.20	.20
p_i	0.01	0.05	0.10	0.01	0.05	0.10	0.01	.05	.10
f	1	3	5	1	3	5	1	3	5

Note: all other variables are set at the sample average

The elasticities of compliance rate with respect to individual parameters, computed using estimated marginal effects, are reported in Table 5. Take again the average agent in an average power society: compliance is about 90% and all parameters positively affect the contribution with the exception of tax rate t. An increase of 1%, say, in the risk aversion parameter ρ results in an additional compliance of 0.152% on average. The same increment in α (+1%) produces an almost two-fold effect on compliance (0.272%).

Table 5 – Elasticities of compliance rates for some typical taxpayers

	Low citizenship			*Average citizenship*			*High citizenship*		
	Low power	*Average power*	*High power*	*Low power*	*Average power*	*High power*	*Low power*	*Average power*	*High power*
d	16.6%	43.5%	74.9%	66.1%	90.3%	99.6%	98.6%	99.9%	100%
I_i	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0
ρ_i	1.079	0.510	0.275	0.329	0.152	0.013	0.034	0.001	0
α_i	0.967	0.457	0.246	0.590	0.272	0.024	0.092	0.002	0
γ_i	0.959	0.453	0.244	0.585	0.270	0.023	0.091	0.002	0
k_i	0.375	0.177	0.095	0.286	0.132	0.011	0.048	0.001	0
p_i	0.159	0.375	0.404	0.048	0.112	0.019	0.005	0.001	0
$\tilde{I}_i$	0.815	0.385	0.207	0.249	0.115	0.010	0.026	0.001	0
g_{-1}	0.130	0.062	0.033	0.040	0.018	0.002	0.004	0.000	0
τ	-2.722	-1.287	-0.694	-0.831	-0.383	-0.033	-0.087	-0.002	0
f	0.377	0.534	0.480	0.115	0.159	0.023	0.012	0.001	0

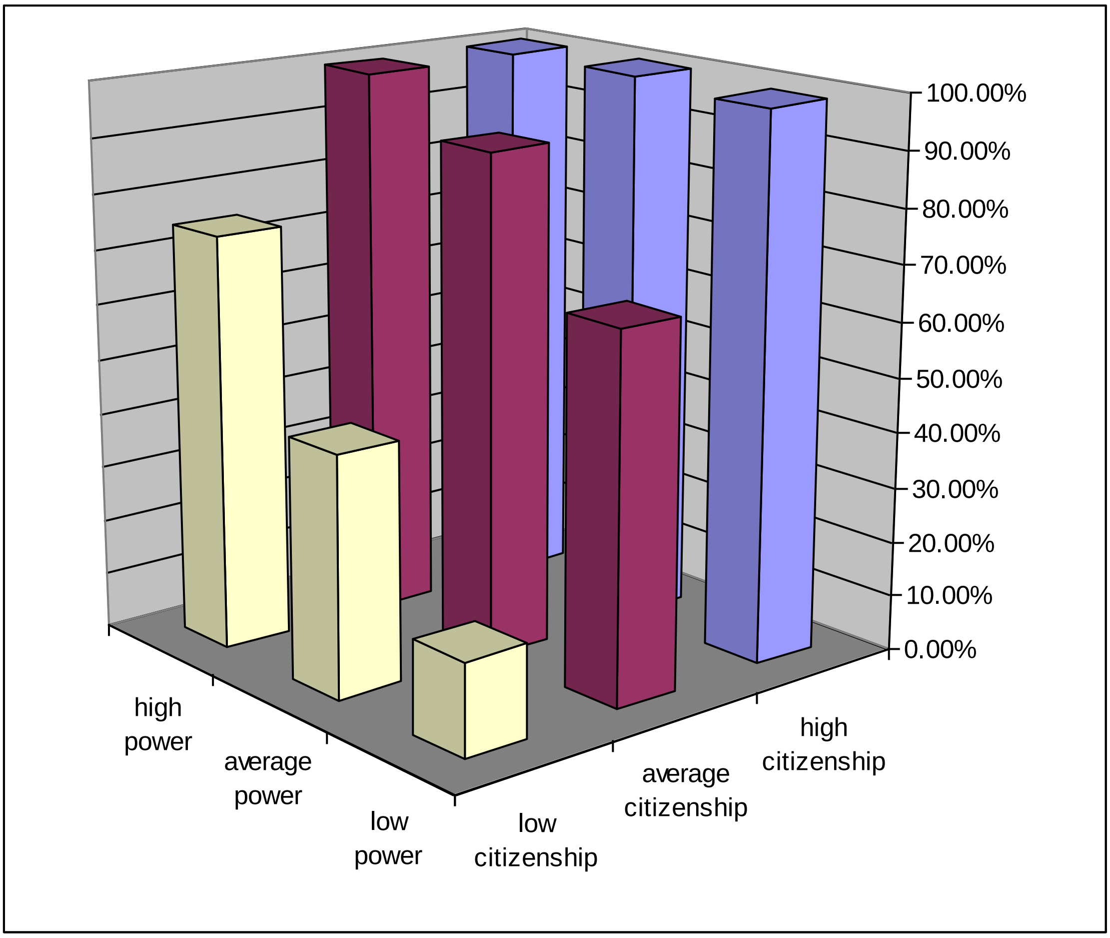
Compliance of the average agent decreases by about 0.383% when the tax rate is increased by 1%. The strong negative sensitivity to hikes in t is a robust result of our simulations that correlates well with run-of-the-mill intuition.

Table 5 shows that low citizenship individuals will massively underreport income when little power is exerted. Their compliance grows from 16.6% to 43.5% and 74.9% when the probability of audit and the penalty rates increase. Agents with average citizenship share this behavior to a different degree; they both need the presence of a certain degree of power to approach full compliance.

Figure 2 represents the average compliance of our typical taxpayers under the three institutional arrangements. A kind of slippery slope can be discerned from it where both power and citizenship contribute at different degrees to obtaining high compliance. A more thorough investigation of this topic is deferred to the next section.

Table 5 offers at least one additional insight related to the strength of different agents' reactions. Indeed, the elasticities of low citizenship taxpayers are always larger than the average, which in turn are larger than those of high citizenship agents. At the two extremes, while the sensitivities of low citizenship individuals in a low power society are remarkably pronounced, high citizenship makes agents virtually insensitive to changes in the parameters in any power situation (last columns of Table 5).

Figure 2 – Estimated compliance rates for some typical taxpayers



5 Societal compliance

In this section we analyze how meta-parameters (i.e., features of society) affect average compliance. We will focus on the different outcomes that can be observed in various social contexts and support the finding that estimates based on simulated societies produce a slippery slope that depends jointly on measures of citizenship and power.

The estimates from the previous section implicitly describe individual decisions (based on personal conjectures) as if agents were unaware of the characteristics of the community to which they belong and, hence, the results do not enable us to assess the effects of a specific mix of features that distinguishes a given society.

For instance, an agent with particular characteristics (e.g., $\alpha = 0.5$) can belong to very different societies (where $0 < \alpha \leq 0.6$ or $0.4 < \alpha \leq 1$), so her tax compliance decision cannot be explained by simply looking at her individual parameters.

Compared with estimates from the previous section, the results presented here depend on average and macro-values representative of the whole society that are unavailable to individuals.

We are aware that there are obvious limitations in mapping societies to sets of values for the parameters, but the final picture is coherent and, as far as we know, this is the first case in which a full slippery slope has been estimated using simulated data arising from "artificial" societies.

The average compliance obtained from each of our 250 simulated societies is regressed against the mean of the parameters' distributions, which can be derived from the meta-parameters listed in Table 1. Since the rate of compliance is now an average value for all individuals in each society, the dependent variable is no longer censored and lies inside the range [0,1]. However, a tobit regression is used again given the nature of the variable. Table 6 shows the estimated coefficients of the following model for average compliance:

[22] $$\mu_d = a + a_1 \mu_\rho + a_2 \tau + \left[c_1 \mu_\alpha + c_2 \mu_\gamma + c_3 \mu_k \right] + \left[p_1 q + p_2 f \right] + \varepsilon \,,$$

where μ_X denotes the sample mean of individual variable *X* (index *s*=1,…,250 is omitted for simplicity). We denote by $c_j, j = 1,...,3$ the coefficients related to *citizenship* and by $p_j, j = 1,2$ the estimates linked to *power*.

Table 6 also shows the estimated coefficients of a model with additional squared variables; only the four significant squared variables were retained.

Table 6 – Tobit regression results: average compliance

Dependent variable: average compliance rate μ_d			Number of obs = 250	
	Linear model		*Quadratic model**	
	Log-likelihood = 384.21		Log-likelihood = 393.82	
	Coeff.	*t-value*	*Coeff.*	*t-value*
μ_α	0.449	12.56	0.448	13.40
μ_ρ	0.176	5.29		
μ_γ	0.370	10.82	0.364	11.04
μ_k	0.633	12.52	1.252	6.12
q	1.551	12.69	1.574	13.33
f	0.050	14.51	0.082	4.30
τ	-0.703	-15.33	-1.498	-4.28
μ_ρ^2			0.173	5.63
μ_k^2			-2.432	-3.22
f^2			-0.005	-1.71
τ^2			1.131	2.27
constant	0.258	7.23	0.366	4.99
sigma (st.err.)	0.052 (0.00264)		0.050 (0.00259)	

Note: * selected model that keeps coefficients with p-value<0.10, robust estimates

The coefficients are strongly significant, they have the same sign as those shown in Table 3, and the elasticities computed at the sample means, not reported here for brevity, have comparable magnitude.

Compliance depends on variables that can be broadly related to the "power" of the Government and to the agents' degree of "citizenship". In particular, expression [22], where each term in square brackets has been divided by its maximum value, can be rewritten as:

$$[23] \qquad \mu_d = A + \left(c_1\bar{\mu}_\alpha + c_2\bar{\mu}_\gamma + c_3\bar{\mu}_k\right)\frac{c_1\mu_\alpha + c_2\mu_\gamma + c_3\mu_k}{c_1\bar{\mu}_\alpha + c_2\bar{\mu}_\gamma + c_3\bar{\mu}_k} + \left(p_1\bar{q} + p_2\bar{f}\right)\frac{p_1 q + p_2 f}{p_1\bar{q} + p_2\bar{f}}$$

in which $A = a + a_1\bar{\mu}_\rho + a_2\tau$ and $\bar{\mu}_\alpha, \bar{\mu}_\gamma, \bar{\mu}_k, \bar{q}, \bar{f}$ are the maxima of respective meta-parameters (see Table 1). Defining $A_C = c_1\bar{\mu}_\alpha + c_2\bar{\mu}_\gamma + c_3\bar{\mu}_k$ and $A_P = p_1\bar{q} + p_2\bar{f}$, the average compliance becomes:

[24] $$d = A + A_C\delta_C + A_P\delta_P\,,$$

in which

[25] $$\delta_C = \frac{c_1\mu_\alpha + c_2\mu_\gamma + c_3\mu_k}{c_1\overline{\mu}_\alpha + c_2\overline{\mu}_\gamma + c_3\overline{\mu}_k}$$

[26] $$\delta_P = \frac{p_1 q + p_2 f}{p_1\overline{q} + p_2\overline{f}}$$

can be interpreted as normalized indices of citizenship and power, which are in the range $[0,1]$ by construction. This formalizes the intuition that citizenship is a mixture of individual parameters, whereas, in a similar way, power blends institutional policy variables.

It is also possible to express compliance from the estimate of the quadratic specification as[12]:

[27] $$d = A + A_C\delta_C + A_P\delta_P + B_C\delta_C{}^2 + B_P\delta_P{}^2\,.$$

Equations [24] and [27] are intuitive expressions that relate citizenship and power indices to aggregate compliance. This can be analyzed in different frameworks where attributes of citizenship and power change and all other relevant parameters are assumed to be fixed. Based on the estimates of Table 6, we obtain the values for A, A_C, A_P, B_C, B_P given in Table 7, in which coefficients suggest that citizenship contributes to raising average compliance to a greater extent than power.

[12] This holds under the assumption that $\delta_C = \frac{\mu_\alpha}{\overline{\mu}_\alpha} = \frac{\mu_\gamma}{\overline{\mu}_\gamma} = \frac{\mu_k}{\overline{\mu}_k}$ and $\delta_P = \frac{q}{\overline{q}} = \frac{f}{\overline{f}}$.

Table 7 – Coefficients of simulated slippery slopes for a number of tax rates

Linear model	*Average compliance* $d = A + A_C\delta_C + A_P\delta_P$
Constant A, τ =20%	0.20538
Constant A, τ =35%	0.09997
Constant A, τ =50%	-0.00544
Citizenship: A_C	1.13552
Power: A_P	0.40375
Quadratic model	*Average compliance* $d = A + A_C\delta_C + A_P\delta_P + B_C\delta_C{}^2 + B_P\delta_P{}^2$
Constant A, τ =20%	-0.12807
Constant A, τ =35%	-0.25940
Constant A, τ =50%	-0.33984
Citizenship: A_C	1.43827
Citizenship^2: B_C	-0.60791
Power: A_P	0.56667
Power^2: B_P	-0.13203

In other words, in our model, which includes several more supplementary variables than in the classic setup where only risk aversion plays a role, *voluntary compliance* seems more relevant than *enforced compliance* in determining a society's average rate of evasion. Note that our citizenship index encompasses more than trust in the Government's ability to provide valuable public services in a fair manner (mainly captured by preference a for public expenditure vis-à-vis private income) as it depends also on the belief that other taxpayers will react in the same way (conformity g), together with an innate attitude to comply (the role of k). While these components are not new and are often referred to in the literature in isolation, we feel that their joint presence, which we termed citizenship, is a novelty that captures compliance behavior better than trust in authority alone.

Our power index is more typical and is based on the two most natural attributes of enforcement, namely audit probability and penalty rate.

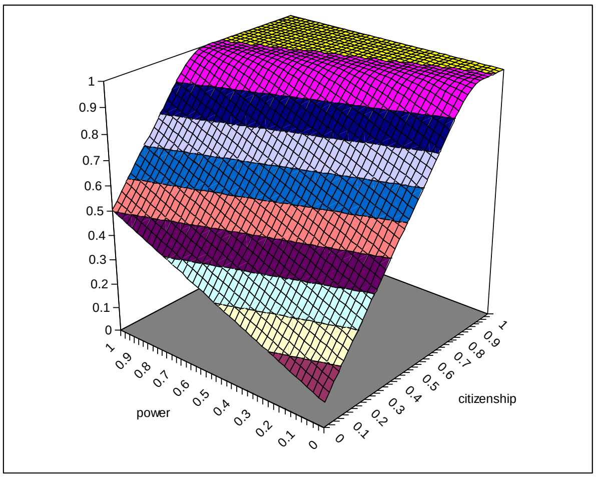
Figure 3 shows slippery slopes of compliance in societies where different degrees of citizenship and enforcement are in place. Panel *a)* depicts the surface obtained by using the linear model when the tax rate is equal to 35%; the quadratic surface relative to the same tax rate is shown in panel *b)*. Although the two pictures look similar, there are some differences: the area of full compliance at the top of the surface is larger when the linear model is used and, in particular, in the quadratic specification some evasion is present at the highest level of citizenship when power is low.

The surface is also shown for other tax rate levels, see panels *c)* and *d)*. A higher (lower) tax rate leads to a lower (higher) constant *A* and, therefore, to less (more) average compliance. In the case of 50% tax rate, full compliance is never reached (at least for the ranges of the indices arising from reasonable values of the meta-parameters shown in Table 1).

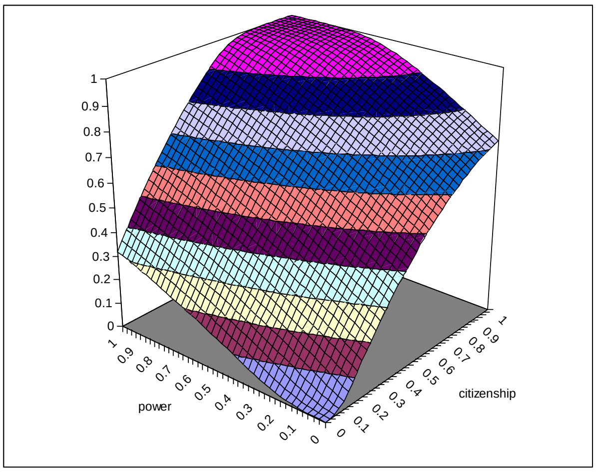
We stress that the surface is the by-product of the estimate of statistical models from data gathered by simulating agent-based societies of heterogeneous utility maximizing taxpayers, whereas other researchers, Kirchler et al. (2008), have discussed in depth the merits of the slippery slope as a conceptual model of tax behavior and have found some empirical support for its plausibility, Muehlbacher et al. (2011). Similarly, our average compliance depends on both the citizenship and power dimensions and, interestingly, high values of both indices are required to obtain high compliance. By contrast, neither maximum citizenship nor maximum power alone would, under most circumstances, suffice to reach the full compliance visible in the upper plateau of the surface. This outcome correlates well with the compliance of the illustrative types of taxpayers portrayed in the previous section; indeed, Figure 2 can be thought of as a caricature of these more detailed slippery slopes.

Figure 3 – Simulated slippery slope: societal compliance rate

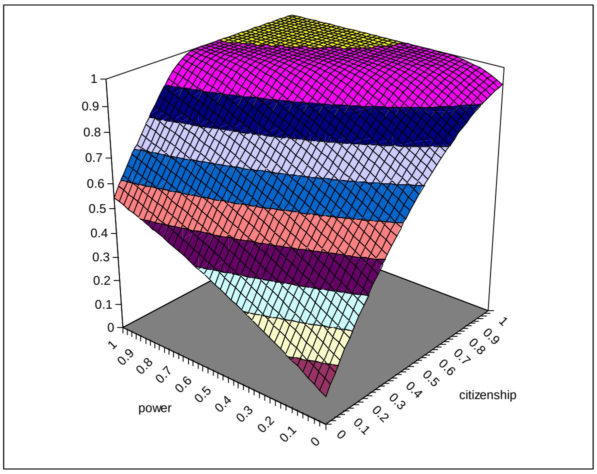
a) Linear, tax rate = 35%



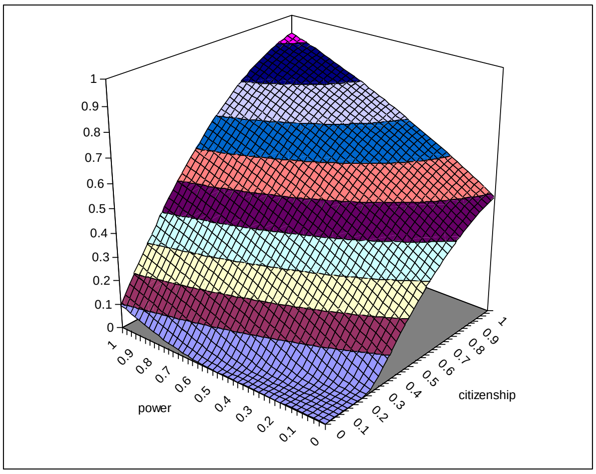
b) Quadratic, tax rate = 35%



c) Quadratic, tax rate = 20%



d) Quadratic, tax rate = 50%



6. Conclusion

We described an agent-based model of individual tax compliance, where heterogeneous agents maximize utility under risk, based on several individual features, such as their preference for public expenditure, their personal attitudes to comply, conformity and risk aversion. Decisions obviously depend not only on individual traits, but also on the perceived quality and quantity of per capita public expenditure and on the recognition that the bulk of revenues comes from other taxpayers. Simulations show a wide range of conducts, spanning from full honesty to massive underreporting, as expected from the agents' large degree of heterogeneity. As a whole, however, each society of taxpayers has an aggregate behavior shaped by meta-parameters that describe distributions of individual features. From our artificial cross-section data we estimate a model that shows how individual compliance depends positively on citizenship-related attributes and on the strength of the enforcement system. By contrast, the tax rate negatively affects compliance, other things being fixed.

We then described and built indices of citizenship and power based on meta-parameters in several societies, and estimated a slippery slope fitting the artificial data we generated. This exercise produced a surface that is remarkably similar, but adds significant insight to previous works by other scholars, stressing the role of habits and beliefs that contributes to the success of a civil community.

Estimates show that, in general, both citizenship and power are required to achieve high compliance, where the former has a greater impact than the latter, even though this may be caused to some extent by our exploratory choice of meta-parameters, which are not calibrated to any specific situation.

Acknowledgements

This paper is based on a previous version presented at the SHADOW 2011 conference, Münster, 28-30/7/2011. The authors would like to thank Michele Bernasconi, Kim Bloomquist, Sasha Hokamp, Giacomo Pasini and three referees for their useful comments and suggestions. All errors remain with the authors.

References

Allingham, M. G., & Sandmo, A. (1972). Income Taxation: A Theoretical Analysis, *Journal of Public Economics*, 1, 323-338.

Alm, J., & Torgler, B. (2006). Culture differences and tax morale in the United States and in Europe, *Journal of Economic Psychology*, 27(2), 224-246.

Andreoni, J., Erard, B., & Feinstein, J. (1998). Tax Compliance, *Journal of Economic Literature*, 36(2), 818-860.

Antunes L., Balsa J., Respicio A., Coelho H. (2007) Tactical Exploration of Tax Compliance Decisions in Multi-agent Based Simulation, in Antunes L., Takadama K. (eds.) *Proceeding MABS'06 Proceedings of the 2006 international conference on Multi-agent-based simulation VII*, 80-95, Springer-Verlag Berlin, Heidelberg.

Bloomquist, K. M. (2006). A Comparison of Agent-Based Models of Income Tax Evasion, *Social Science Computer Review*, 24(4), 411-425.

Bloomquist, K. M. (2011). *Agent-Based Simulation of Tax Reporting Compliance*, mimeo, Shadow 2011, Münster.

Bernasconi, M. (1998). Tax Evasion and Orders of Risk Aversion, *Journal of Public Economics*, 67, 123-134.

Bernasconi, M., & Zanardi A. (2004). Tax Evasion, Tax Rates, and Reference Dependence, *FinanzArchiv*, 60(3), 422-45.

Bordignon, M. (1993). A fairness approach to income tax evasion, *Journal of Public Economics*, 52(3), 345-362.

Cornes, R., & Sandler, T., (1986) *The Theory of Externalities, Public Goods and Club goods*, Cambridge University Press, Cambridge.

Cowell, F. A. (1992). Tax evasion and inequity, *Journal of Economic Psychology,* 13(4), 521-543.

Cowell, F. A., & Gordon J.P.F. (1988). “Unwillingness to Pay”, *Journal of Public Economics*, 36, pp. 305-321.

Davis, Jon S., Hecht, Gary & Perkins, Jon D. (2003), “Social Behaviors, Enforcement and Tax Compliance Dynamics”, *Accounting Review*, Vol. 78(1), pp. 39-69.

Epstein, Joshua M. (2002) *Modeling civil violence: An agent-based computational approach*, PNAS Vol. 99, May 14, 2002, p. 7243-7250.

Frank, B., & Kirchler E. (2006). (selected and introduced) *Günter Schmölders. The Psychology of Money and Public Finance*. New York: Palgrave.

Gordon, J. F. (1989). “Individual Morality and Reputation Costs as Deterrents to Tax Evasion”, *European Economic Review*, 33(4), 797-805.

Hokamp, S., & Pickhardt M. (2010), Income Tax Evasion in a Society of Heterogeneous Agents – Evidence from an Agent-Based Model, *International Economic Journal*, 24(4), 541-553.

Hommes, C. H. (2006). Heterogeneous Agent Models in Economics and Finance, in Tesfatsion, L., & Judd K. L. (eds) *Handbook of Computational Economics*, Vol. 2, 1109-1186, Elsevier B.V.

Kirchler, E. (2007). *The Economic Psychology of Tax Behaviour*, Cambridge, UK: Cambridge University Press.

Kirchler, E., Hoelzl E., & Wahl I. (2008). Enforced versus voluntary tax compliance: The slippery slope framework, *Journal of Economic Psychology*, 29, 210-225.

Korobow, A., Johnson, C., & Axtell, R. (2007). An Agent-based Model of Tax Compliance with Social Networks, *National Tax Journal*, Vol. 60(3), 589-610.

Mittone, L., & Patelli P. (2000). “Imitative behaviour in tax evasion”, in Stefansson, B., & Luna, F., (eds.), *Economic simulations in swarm: Agent-based modelling and object oriented programming*, 133-158, Amsterdam: Kluwer.

Muehlbacher, S., Kirchler, E., & Schwarzenberger, H. (2011). Voluntary versus enforced tax compliance: empirical evidence for the ‘‘slippery slope’’ framework, *European Journal of Law and Economics*, 32(1), 89-97.

Myles, G. D., & Naylor, R. A. (1996). “A Model of Tax Evasion with Group Conformity and Social Customs”, *European Journal of Political Economy*, 12(1), 49-66.

Pickhardt, M., & Seibold, G. (2011). *Income tax evasion dynamics: Evidence from an agent-based econophysics model*, CAWM Discussion Papers 53, Center of Applied Economic Research Münster (CAWM), University of Münster.

Pyle, D. J. (1991). The Economics of Taxpayer Compliance, *Journal of Economic Surveys*, 5(2), 163-198.

Rablen, M. D. (2010). Tax Evasion and Exchange Equity: A Reference-Dependent Approach, *Public Finance Review*, Vol. 38, pp. 282-305.

Sandmo, Agnar (2005). The Theory of Tax Evasion: A Retrospective View, *National Tax Journal*, 58(4), 643-663.

Sims, C. A. (1980). Macroeconomics and Reality, *Econometrica*, 48(1), 1-48.

Torgler, B. (2007). Tax compliance and tax morale: A theoretical and empirical analysis. Cheltenham, UK: Edward Elgar.

Yitzhaki, S. (1974). A Note on Income Tax Evasion, *Public Finance Quarterly*, 15(2), 123-137.

Zaklan, G., Westerhoff F., & Stauffer, D. (2009). Analysing Tax Evasion Dynamics via the Ising Model, *Journal of Economic Interaction and Coordination*, 4(1), 1-14.