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Bank Competition and Financial Stability: A General Equilibrium Exposition

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Bank Competition and Financial Stability:

A General Equilibrium Exposition

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Abstract

We study versions of a general equilibrium banking model with moral hazard under either constant or increasing returns to scale of the intermediation technology used by banks to screen and/or monitor borrowers. If the intermediation technology exhibits increasing returns to scale, or it is relatively efficient, then perfect competition is optimal and supports the lowest feasible level of bank risk. Conversely, if the intermediation technology exhibits constant returns to scale, or is relatively inefficient, then imperfect competition and intermediate levels of bank risks are optimal. These results are empirically relevant and carry significant implications for financial policy.

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[1] This paper is a revised and significantly expanded version of IMF WP 09/105. Marcella Lucchetta is Assistant Professor in the Department of Economics at the University Ca' Foscari of Venice.

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I. INTRODUCTION

The theoretical literature offers contrasting results on the relationship between bank competition and financial stability. Yet these results arise from models with three important limitations: they are partial equilibrium set-ups; there is no special role for banks as institutions endowed with some comparative advantage in screening and/or monitoring borrowers; and bank risk is not determined jointly by the borrower and the bank. This paper contributes to overcome these limitations. A more general assessment of the relationship between bank competition, financial stability and welfare is not only important *per se*, but it is also essential to evaluate whether "granting" banks the ability of earning rents may reduce their risk-taking incentives.

We study the relationship between bank competition, financial stability and welfare in versions of a general equilibrium banking model with moral hazard, where the choice of "systematic" risk by either banks or firms is unobservable. In our set-up, risk-neutral agents specialize in production at the start date, choosing to become entrepreneurs, bankers, or depositors, and at a later date they make their financing and investment decisions. In this risk-neutral world, the welfare criterion is total surplus, defined as total output net of effort costs. We consider two versions of the model. In the first version, called "basic", the bank is a coalition of entrepreneurs that are financed by depositors. In the second version, called "extended", the "firm" is a coalition of entrepreneurs that is financed by the "bank", which is a coalition of bankers financed by depositors. The firm, the bank and depositors can be also viewed as representing the business sector, the banking sector and the household sector.

In both versions, we consider two specifications of the bank's screening and/or monitoring technology, called the "intermediation technology". In the first specification, the intermediation technology exhibits constant returns to scale: the effort cost of screening and/or monitoring is proportional to the size of investment. In the second specification, this technology exhibits increasing returns to scale: the effort cost of screening and/or monitoring is independent of investment size. This second specification captures in a simple form the essential role of banks in economizing on monitoring and screening costs identified by a well-known literature briefly reviewed below.

In the basic model the bank chooses (systematic) risk, this choice is unobservable to outsiders, and there is competition in the deposit market only, indexed by the opportunity costs of depositors to invest in the bank. The results of this model differ strikingly depending on whether the intermediation technology exhibits constant or increasing returns to scale. Under constant returns to scale, as competition in the deposit market increases, bank risk increases, bank capital declines, and welfare is maximized for some intermediate degree of competition. Thus, perfect deposit market competition is sub-optimal, as it entails excessive bank risk-taking and sub-optimally low levels of bank capitalization. However, allocating large shares of surplus (or rents) to banks is not optimal either, as it results in sub-optimally low levels of bank-risk taking and excessive bank capitalization.

When the intermediation technology exhibits increasing returns to scale, however, results are totally reversed: as competition increases, bank risk declines, capitalization increases, perfect deposit market competition is optimal, and the lowest feasible level of bank risk is best. This reversal is simply explained as follows. As competition increases, a *ceteris paribus* increase in the cost of funding induces the bank to take on more risk. But at the same time the increase in the supply of funds to the bank reduces the costs of the intermediation technology owing to increasing returns to scale: this offsets the negative impact of higher funding costs on bank's expected profits, inducing the bank to take on less risk. This result is remarkable for two reasons: it is obtained under a standard assumption about the bank's intermediation technology, and without modeling loan market competition. Thus, introducing loan market competition, as in Boyd and De Nicolo' (2005), is not necessary—albeit it may be sufficient—to yield a positive relationship between bank competition and financial stability.

The extended model depicts the more realistic case in which there is competition in both lending and deposit markets, bank risk is jointly determined by borrowers and banks, and setting up the intermediation technology entails set-up costs. Here, bank competition is indexed by the opportunity costs of depositors to invest in the bank, and the opportunity costs of the firm to be financed by the bank. In this model, the relationship between bank competition, financial stability, and welfare becomes complex in a substantial economic sense, since double-sided competition determines how total surplus, whose size is endogenous, is shared by three sets of agents, rather than two, as in the basic model. When the degree of competition in lending and deposit markets differs, we illustrate several results suggestive of a rich comparative statics, which in some cases overturn simple conjectures on the relationship between bank risk, firm risk and capital.

Focusing on changes of competition in *both* loan and deposit markets, we obtain the following main results. If the bank intermediation technology is relatively *inefficient,* as defined as one that entails high monitoring and screening costs but relatively low set-up costs, then a level of competition *lower* than perfect competition is optimal, corresponding to an "intermediate" optimal levels of bank risk. However, if the bank intermediation technology is relatively *efficient*, defined as one that entails low monitoring and screening costs but relatively large set-up costs, then perfect competition is optimal, and the optimal level of bank risk turns out to be the lowest attainable. Notably, these results are independent of whether the intermediation technology exhibits constant or increasing returns to scale in screening and/or monitoring effort.

We discuss below the empirical relevance of some of our results. Furthermore, we believe these results throw a new light on the important policy question regarding the desirability of supporting bank profits, or banks' "charter value", with some "rents" in order to guarantee financial stability: what seems to matter are not necessarily rents *per se*, but what are their sources and how banks might exploit them.

The remainder of the paper is composed of five sections. Section II presents a brief literature review, pointing out the innovations introduced in our model. Section III describes the basic version of the model, and section IV derives the relevant comparative statics results. Section IV describes the extended model with firms, banks and depositors, and section V derives the main comparative statics results. Section VI concludes discussing the empirical relevance of our results and their importance for policy. Proofs of all propositions are in the Appendix.

II. LITERATURE REVIEW

As pointed out by Allen and Gale (2004a), the relationship between bank competition and financial stability has been primarily analyzed in the context of partial equilibrium modeling. Few general equilibrium models exist. Allen and Gale (2004b) consider a general equilibrium version of a Diamond and Dybvig (1983)-type economy, and demonstrate that perfect competition among intermediaries is Pareto optimal under complete markets, and constrained Pareto optimal under incomplete markets, with financial ”instability” as a necessary condition of optimality. Analogous results are obtained under low inflation in the general equilibrium monetary economy with aggregate liquidity risk analyzed by Boyd, De Nicolò and Smith (2004). However, these general equilibrium models do not feature moral hazard due to unobservable risk choices of banks and firms, as we do.

In partial equilibrium, the trade-off between competition and financial stability is typically derived through a standard risk shifting argument applied to a bank that raises funds from insured depositors and chooses the risk of its investment. Under limited liability, unobservable risk choices, risk-insensitive deposit demand, and constant return to scale in screening and monitoring, an increase in deposit market competition raises the deposit rate, reduces banks' expected profits and prompts banks to take on more risk. This implication has been illustrated by Allen and Gale (2000) in both static and simple dynamic settings, and it is the key thrust of work by Keeley (1990), Matutes and Vives (1996), Hellmann, Murdock and Stiglitz (2000), Cordella and Levi-Yeyati (2002), Repullo (2004), among many others.

However, when banks compete in both loan and deposit markets, the loan rate determines the level of risk-shifting undertaken by firms, as noted in Stiglitz and Weiss (1981). Boyd and De Nicolò (2005) showed that the trade-off between competition and financial stability can vanish when firms' risk choices are taken into account. An increase in loan market competition reduces bank loan rates, increasing firms' expected profits, inducing firms to choose safer investments, which translate into safer bank loan portfolios. In this more complex setting, the risk-shifting argument is applied to *two* entities, firms and banks, rather than one. Recent extensions of this type of model, including bank heterogeneity (De Nicolò and Loukoianova, 2007), the introduction of different assets (Boyd, De Nicolò and Jalal, 2009), or a different risk structure (Martinez-Miera and Repullo, 2010), have all aimed at establishing under what conditions the presence of two risk-shifting effects generates a trade-off between bank competition and financial stability.

Yet, all papers just mentioned display two features: bank screening and monitoring technologies exhibit constant return scale[2], and bank risk is not determined jointly by banks and borrowers. The constant returns to scale assumption contrasts with a large literature—including Diamond (1984), Boyd and Prescott (1986), Willliamson (1986), Krasa and Villamil (1992), and Cerasi and Daltung (2000)—that has identified economies of scale in

[2] The constant returns to scale assumption is also adopted in many other papers that do not focus on bank competition, such as Besanko and Kanatas (1993); Boot and Greenbaum (1993); Boot and Thakor (2000); Dell'Ariccia and Marquez (2006), and Allen et al. (2011).

screening and monitoring as an essential feature of intermediation.[3] This motivates our analysis of the models under both constant and increasing returns to scale in the intermediation technology. In addition, in these models there is either no distinction between banks' and borrowers' actions, so that risk is determined exclusively by the bank, or borrowers choose risk directly while banks choose risk only indirectly through their setting of loan rates. Differing from these models, bank risk is jointly determined by the bank and the borrower in our extended model.

Furthermore, as noted by Gale (2010), most partial equilibrium models assume that the supply of bank capital is perfectly elastic at a given exogenous rate and deliver contrasting results regarding the relationship between capital and bank risk-taking. Building on our previous work (De Nicolò and Lucchetta, 2009), in this study we introduce bank and firm capital in a simple way to capture bank and firm incentives to choose the entity of investment of internally generated funds jointly with their risk-taking decisions.

Lastly, most partial equilibrium models just reviewed assume the existence of deposit insurance. This assumption is necessary for the standard risk-shifting argument to hold, but non-existence of equilibria or multiple equilibria may arise when deposit insurance is fairly priced.[4] For this reason, and the fact that there is no rationale for deposit insurance in our risk neutral world, we do not assume deposit insurance.

III. The basic Model

There are three dates, 0, 1 and 2, and a continuum of agents on $[0,1]$ indexed by $q \in [0,1]$. Agents are risk neutral, have preferences over final date consumption, are endowed with effort (labor) at any date, and derive disutility from effort.

At date 0 agents choose to become either investors or entrepreneurs. If an agent chooses to become an investor, he/she uses effort at date 0 to obtain qW units of an intermediate good at date 1. This good can be reinvested at date 1 to obtain the date 2 consumption good in two alternative ways. It can be invested in an "autarkic technology" which yields an exogenously given return ρ per unit invested, or can be lent to entrepreneurs in exchange of promises to deliver date 2 consumption goods. The return of the "autarkic technology" is interpreted as the opportunity cost of investing in the bank.[5] If an agent chooses to become an entrepreneur, he/she forgoes the opportunity to produce the date 1 intermediate good qW, which is the opportunity cost of becoming an entrepreneur.

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[3] For surveys of this literature, see Gorton and Winton (2003) and Freixas and Rochet (2008).

[4] Examples of non-existence and multiplicity of equilibria under fairly priced deposit insurance are shown in Boyd and De Nicolò (2003) in a model by Allen and Gale (2000) with deposit market competition. In addition, standard implications of partial equilibrium modeling concerning the risk effects of deposit insurance may not necessarily hold in general equilibrium, as shown in Boyd, Chang and Smith (2002).

[5] Equivalently, this return may be associated with switching costs incurred by depositors. On switching costs, see Klemperer (1995), and for a recent application to banking, see Park and Pennacchi (2009).

Thus, at the initial date, agents with a smaller "labor" productivity (indexed by a lower $q \in [0,1]$) have a comparative advantage in becoming entrepreneurs. In equilibrium, there will be a cutoff level q^* such that agents with $q < q^*$ choose to become entrepreneurs, while those with $q > q^*$ choose to become investors.

A. The Bank

Entrepreneurs form a coalition called the bank. The bank collects funds from investors, called *depositors,* and distributes its profits to its members in equal shares. The bank has the ability to operate a risky project, an intermediation technology, and a capital technology.

The risky project is indexed by the probability of success $P \in [0,1]$. Using as input date 1 intermediate goods, the project yields date 2 consumption goods. A one unit investment in a risky project yields $X > W$ with probability P, and 0 otherwise.

The ability of the bank to choose P is interpreted as representing an *intermediation technology.* In transforming effort into a probability of project success, this technology can be viewed as embedding projects' screening and/or monitoring. Similarly to all papers we have reviewed, we assume that the bank does not incur any cost in setting up this technology. However, we consider two specifications. In the first specification, this technology exhibits constant returns to scale (CR), as the effort cost to implement P is linearly related to total investment in the bank, denoted by Z. In the second specification, the technology exhibits increasing returns to scale (IR), as the effort cost to implement P does not depend on Z. The relevant cost functions are:

$$C(P) = \frac{\alpha}{2} P^2 Z \quad \text{(CR)},$$

$$C(P) = \frac{\alpha}{2} P^2 \qquad \text{(IR)}.$$

The bank has also access to a *capital* technology that transforms date 1 entrepreneurs' collective effort *and* investment Z into an intermediate good that can be invested in the risky technology. Namely, by choice of $k \geq 0$, the bank generates total "capital" kZ at an effort cost $\frac{\beta}{2} k^2 Z$. The bank capital ratio is $K = k(1+k)^{-1}$. Note that bank capital is *endogenous:* it depends on agents' specialization choices through the endogenously determined amount of funds banks receive, as well as on the bank's choice of risk. The cost $\frac{\beta}{2} k^2 Z$ can be viewed as capturing in reduced form either a supply of capital that is not infinitely elastic due to limited resources, or the costs associated with the generation of internal funding.[6]

[6] This feature of our model is novel relative to many set-ups where the levels of internal funding by either firms or intermediaries are exogenously given (see e.g. Holmstrom and Tirole (1997) or Boot and Thakor (1997)).

B. Contracts, Information and Competition

Depositors finance the bank with simple debt contracts that pay a fixed amount R per unit invested if the outcome of the investment is positive, and 0 otherwise. *Moral hazard* is introduced by assuming that the bank choices of P and k are not observable by depositors. However, depositors take bank's optimal choices of P and k into account in their decision to accept the deposit terms offered by the bank.

The degree of competition in the deposit market is indexed by depositors' opportunity costs of investing in the bank, which is given by the return of the "autarkic technology" $\rho \in [0, \rho^{\max}]$ described previously. A higher opportunity costs will force the bank to raise the remuneration of deposits to ensure investors will become depositors. Thus, the degree of competition in the deposit market varies positively with ρ. The term $\rho^{\max}$ denotes the "maximal" level of deposit market competition (or "perfect" deposit market competition) which we derive below. In considering the bank as a coalition of all entrepreneurs, we focus on how total surplus is distributed between the banking sector and depositors, rather than on how surplus is distributed *within* the banking sector. Table 1 summarizes the sequence of events in the basic model.

Table 1. Sequence of Events in the Basic Model

Time	**Agents' decisions**	**Variables determined**
t=0	Agents choose to become entrepreneurs or investors Entrepreneurs form a coalition called the bank	q^*: measure (fraction) of entrepreneurs
t=1	Debt contract terms between the bank and depositors are determined. Depositors deliver funds to the bank The bank chooses the riskiness of projects and capital.	R, Z P, k
t=2	Project's output is realized and agents' consumption follows.	

IV. EQUILIBRIUM IN THE BASIC MODEL

Equilibrium and the associated welfare metrics are defined as follows.

Definition 1 (Equilibrium). Given $\rho \in [0, \rho^{\max}]$, an equilibrium is a level of bank risk $P^* \in (0,1]$, a capitalization rate k^*, a deposit rate R^*, total investment Z^*, and a cut-off level q^* such that:

1. **Date 1**

Given Z and q, R^* P^*, k^* maximize the profits of the bank:

$$\Pi^B(P,k,R,Z) \equiv [P(X - R + Xk) - \frac{\beta}{2}k^2]Z - C(P) \qquad (1)$$

subject to

$$P^* R \geq \rho \qquad (2)$$

Z^* satisfies:

$$Z^* = W\int_{q^*}^{1} q dq = (1 - q^{*2})\frac{W}{2} \qquad (3)$$

2. **Date 0**

q^* satisfies:

$$\frac{\Pi^B(P^*, k^*, R^*, Z^*)}{q^*} = P^* R^* q^* W \quad (4)$$

As of the end of date 1, the bank maximizes expected profits (Equation (1)) by choice of P, k. Given these choices, R is determined subject to depositors' participation constraint (Equation (2)). Equation (3) is the equilibrium condition in the deposit market: bank's demand for funds equals total funds supplied by depositors. Equation (4) determines the equilibrium "specialization" choices of agents: q^* is the agent who is indifferent between becoming entrepreneur or depositor. Hence, the fraction of agents becoming entrepreneurs (depositors) is q^* $(1 - q^*)$.

As all agents are risk neutral, the welfare metric of an equilibrium indexed by ρ is total surplus, defined as expected total output net of effort costs. Total output in the successful state is $X(1 + k^*)Z^*$, bank's effort in the choice of project risk is $C(P^*)$, and the total cost of capital is given by $\frac{\beta k^{*2}}{2}$. Hence, we can state the following

Definition 2 (Surplus) Given $\rho \in [0, \rho^{\max}]$, expected total output net of effort costs is:

$$Y(\rho) = \left(P^* X(1 + k^*) - \frac{\beta k^{*2}}{2} \right) Z^* - C(P^*) \qquad (5)$$

Solving backward, the characterization of the equilibrium values of bank risk, capitalization and the deposit rate is summarized by the following

Proposition 1

a) *Under (CR),* $P^* = \min\{\frac{\beta(X-R^*)}{\alpha\beta - X^2},1\}$ $\quad k^* = \min\{\frac{\beta(X-R^*)}{\alpha\beta - X^2},1\}\frac{X}{\beta}$;

$$R^*(\rho) = \frac{X}{2}\left(1-\sqrt{1-\frac{4\rho(\alpha\beta - X^2)}{\beta X^2}}\right) \quad \text{if } P^* < 1, \quad R^* = \rho \quad \text{if } P^* = 1$$

b) *Under (IR),* $P^* = \min\{\frac{\beta(X-R^*)Z}{\alpha\beta - X^2 Z},1\}$), $\quad k^* = \min\{\frac{\beta(X-R^*)Z}{\alpha\beta - X^2 Z},1\}\frac{X}{\beta}$.

$$R^*(\rho, Z) = \frac{X}{2}\left(1-\sqrt{1-\frac{4\rho(\alpha\beta / Z - X^2)}{\beta X^2}}\right) \quad \text{if } P^* < 1, \quad R^* = \rho \quad \text{if } P^* = 1$$

We illustrate the results of Proposition 1 focusing on the case in which it is too costly to implement $P^* = 1$. To this end, we assume $\alpha\beta > X^2 W$ and $0 < \frac{\beta X W}{\alpha\beta - X^2 W} < 1$ throughout. These assumptions, which are satisfied for a wide range of parameters, are sufficient to guarantee that $P^* \in (0,1)$ for all $\rho \in [0, \rho^{\max}]$.

Under (CR), as the deposit rate increases, bank risk increases (P^* declines) and capitalization declines. For a given level of Z, the same results hold under (IR). Turning to the deposit rate, observe that depositors' expected return $P^* R^*$ equals ρ, since depositors' participation constraint (2) is satisfied at equality. Under both (CR) and (IR), $P^* R^*$ is a strictly concave function of R^*, which is maximized at $R^{\max} = \frac{X}{2}$. This rate corresponds to the value $\rho^{\max}$ which makes the determinant associated with the quadratic equations defined by constraint (2) satisfied at equality. Thus, under (CR), and under (IR) for a given level of Z, the deposit rate R^* is increasing in ρ. The equilibrium corresponding to the maximization of depositors' expected returns ($\rho = \rho^{\max}$) denotes the "maximal", or "perfect", competition in the deposit market, while a value of ρ close to 0 is associated with "minimal" competition in the deposit markets, as almost the entire surplus is appropriated by the bank. Higher values of $\rho \in (0, \rho^{\max}]$ index increasing deposit market competition.

Proposition 1 illustrates the key difference between the (CR) and the (IR) cases. Under (CR), bank risk, capitalization and the deposit rate are independent of the total amount of funding Z, while under (IR) they do depend on Z.

We close the model by establishing existence of equilibriums.

Proposition 2 *Under both (CR) and (IR), an equilibrium exists for all* $\rho \in (0, \rho^{\max}]$. *The equilibrium functions* $\{P^*(\rho), k^*(\rho), R^*(\rho), Z^*(\rho), q^*(\rho)\}$ *are continuous and differentiable on* $\rho \in (0, \rho^{\max}]$.

The complete comparative statics of the model is summarized by the following:

Proposition 3.

a) *Under (CR),* $P^*_\rho < 0; k^*_\rho < 0$ *;* $q^*_\rho < 0$ *and* $Z^*_\rho > 0$.
b) *Under(IR),* $P^*_\rho > 0; k^*_\rho > 0$ *;* $q^*_\rho < 0$ *and* $Z^*_\rho > 0$.

Under (CR), as deposit market competition increases (ρ raises), bank risk increases and capital declines. Moreover, a larger fraction of agents become depositors (q^* declines) and, as a result, the total amount of funds available to the bank Z^* increases. Under (IR), as ρ raises, q^* declines and Z^* increases, as in the (CR) case. However, the results on risk and capital are reversed. As the deposit rate R^* increases, bank profits decline, *ceteris paribus*. However, in this case bank expected profits will on net *increase*, since the increase in Z offsets the decline in profits due to the higher cost of funds, owing to the increasing returns of the intermediation technology. Therefore, the bank will have an incentive to take on less risk (a higher P^*) and increase capitalization ((a higher k^*).[7] In sum, under (IR), the comparative statics of bank risk and capital is exactly the opposite of the (CR) case

The following proposition illustrates how these radically different implications translate into the welfare properties of the equilibriums.

Proposition 4.

a) *Under (CR), there exists a value* $\hat{\rho} \in (0, \rho^{\max})$ *such that* $Y(\hat{\rho}) \geq Y(\rho)$ *for all* $\rho \in [0, \rho^{\max}]$.
b) *Under (IR),* $Y(\rho^{\max}) > Y(\rho)$ *for all* $\rho \in (0, \rho^{\max}]$.

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[7] We can relax the parametric assumptions sufficient to guarantee $P^* < 1$ for all $\rho \in (0, \rho^{\max}]$ with no change in the qualitative results. Under (CR), since P^* is strictly decreasing in ρ, there can exist a range $[0, \hat{\rho}]$ with $\hat{\rho} < \rho^{\max}$ such that $P^* = 1$, and $P^* < 1$ for all $\rho \in [\hat{\rho}, \rho^{\max}]$. In this case Proposition 3(a) would hold with $P^*_\rho = 0$ for all $\rho \in [0, \hat{\rho}]$, and $P^*_\rho < 0$ for all $\rho \in [\hat{\rho}, \rho^{\max}]$. Under (IR), since P^* is strictly increasing in ρ, there can exist a range $[0, \hat{\rho}]$ with $\hat{\rho} < \rho^{\max}$ such that $P^* < 1$, and $P^* = 1$ for all $\rho \in [\hat{\rho}, \rho^{\max}]$. In this case Proposition 3(b) would hold with $P^*_\rho > 0$ for all $\rho \in [0, \hat{\rho}]$, and $P^*_\rho = 0$ for all $\rho \in [\hat{\rho}, \rho^{\max}]$.

Proposition 4 says that under (CR) a certain level of "imperfect" deposit market competition is optimal, while under (IR), perfect deposit market competition is optimal. These results can be simply explained as follows. Under (CR), the derivative of $Y(\rho)$ with respect to the competition parameter ρ can be written as:

$$Y_\rho = R^*_\rho[Z^*_{R^*}\left(P^*X - \frac{1}{2\beta}(\alpha\beta - X^2)P^{*2}\right) + Z^*P^*_{R^*}\left(X - \frac{1}{\beta}(\alpha\beta - X^2)P^*\right)] \quad (6)$$

The first term of (6) is positive, while the second term is negative. When Z^* is not too large, the first term dominates the second, as the marginal increase in expected output is larger than the marginal increase in the cost of the intermediation technology. However, when Z^* becomes sufficiently large, the second term dominates the first, since the increase in the cost of the intermediation technology becomes larger, being proportional to a higher level of investment Z^*. By contrast—as shown in the Appendix—under (IR) the derivative of $Y(\rho)$ with respect to the competition parameter ρ can be written as:

$$Y(\rho) = Z^*P^*X + \frac{P^{*2}}{2\beta}(Z^*X^2 - \alpha\beta) \qquad (7)$$

Both terms of (7) are positive. Since Z^* and P^* are strictly increasing in the competition parameter ρ, $Y(\rho)$ is strictly increasing in ρ. This happens because the cost of the intermediation technology per unit of investment declines owing to increasing returns to scale.

Summing up, the optimal level of deposit market competition is imperfect competition under (CR), with the optimal level of bank risk between the highest and lowest feasible levels. Under (IR), perfect deposit market competition is optimal and supports the lowest feasible level of bank risk.

V. THE EXTENDED MODEL

The basic model is extended by assuming that there are *two sets* of a continuum of agents on $[0,1]$, set F and set B, both indexed by $q \in [0,1]$, endowed with labor (effort) at any date. As before, agents are risk neutral, have preferences over date 2 consumption only, and derive disutility from effort.

At date 0, an agent in set F chooses to become either an investor or an entrepreneur, while an agent in set B chooses to become either an investor or a banker. An agent q in both sets F and B who decides to become an investor uses effort at date 0 to obtain qW units of an intermediate good at date 1, which can be reinvested at date 1 either in an "autarkic technology" which yields a return $\rho^D \geq 0$ at date 2, or can be lent to the bank.

A. The Firm

Agents in set F who have chosen to become entrepreneurs can become *successful* entrepreneurs with probability $P^B \in (0,1)$, or *unsuccessful* otherwise, with this event being realized at date 1. Agents who turn out unsuccessful entrepreneurs cannot operate any project, and employ effort to produce a given amount of the date 2 consumption good standardized to zero. Thus, differing from the previous set-up, becoming a successful entrepreneur is risky. We assume that successful entrepreneurs can be identified by all agents.

Successful entrepreneurs form a coalition called the firm. The firm has access to two mutually exclusive investment opportunities. It can operate risky projects indexed by the probability of success $P^F \in [0,1]$, whose returns are identical to the ones defined previously, the choice of this investment is observable, but the realization of the outcome can be observed only by the bank. This assumption prevents the firm to be financed directly by investors when it chooses to operate risky projects.

The firm employs a *managerial* technology to choose P^F, which transforms effort into a probability of project success. The effort cost function to implement P^F exhibits constant returns to scale, as the effort cost of choosing P^F is linearly related to the external funding obtained at date 1, denoted by Z^F:

$$C^F(P^F) = \frac{\alpha}{2} P^{F2} Z^F \quad \text{(M)}$$

The firm has also access to a *capital* technology that transforms date 1 efforts *and* external funding into the date 1 intermediate good of the type already described in the context of the base model. By choice of $k^F \geq 0$, the firm generates total "capital" $k^F Z^F$ at an effort cost $\frac{\beta}{2} k^{F2} Z^F$.

The second investment opportunity available to the firm is a risk-free technology that transforms any date 1 financing obtained into date 2 consumption goods with return $\rho^F > \rho^D$ for all (ρ^F, ρ^D). Thus, the firm can raise funds directly from investors remunerating them their opportunity cost ρ^D, thus obtaining a return $\rho^F - \rho^D$ per unit invested. The rate ρ^F indexes the degree of loan market competition, since its level limits the capacity of the bank to extract surplus from the firm.

B. The Bank

Similarly to the previous set-up, agents in set B who have chosen to be bankers form a coalition called the bank, whose proceeds are distributed to members in equal shares. Becoming a bank entails access to an *intermediation* technology, which is set up and implemented at the initial date, as well as a *capital* technology used at date 1.

At date 0 the bank selects the probability of entrepreneurs' success P^B employing effort, which can be interpreted as an information production technology embedding projects' screening. Differing from the basic model, however, the bank incurs a fixed (effort) cost ρ^B to set up this intermediation technology, Let Z^B denote a bank's external funding. As before, we assume that the effort cost function to implement P^B is either constant returns to scale (CR) or increasing returns to scale (IR):

$$C(P^B) = \frac{\delta}{2} P^{B2} Z^B - \rho^B \qquad \text{(CR)}$$

$$C(P^B) = \frac{\delta}{2} P^{B2} - \rho^B \qquad \text{(IR)}$$

Once the random variable "success" for entrepreneurs has been realized, the bank finances the risky projects of the firm, i.e. the coalition of successful entrepreneurs. Note that bank risk is different from firm risk (P^F). For simplicity, we assume that the probabilities of being a successful entrepreneur, and that of a successful realization of the technology selected by the firm, are independent. Therefore, bank risk, given by $\mathrm{P} = P^B P^F$, is determined jointly by the firm through its managerial technology, and by the bank through its intermediation technology.

Finally, as in the previous set-up, for any given external funding Z^B obtained at date 1, the bank has access to a *capital* technology identical to that described previously: by choice of $k^B \geq 0$, a bank generates total capital $k^B Z^B$ at an effort cost $\frac{\beta}{2} k^{B2} Z^B$.

C. Contracts, Information and Competition

Depositors finance banks, and banks finance firms with simple debt contracts *Firm and bank moral hazard* is introduced by assuming that the choices of (P^F, k^F) and (P^B, k^B) are not observed by outsiders. However, outsiders (depositors vs. the bank, and the bank vs. the firm) take the optimal choices of the bank and the firm into account in their decisions. Competition in the *loan market* is indexed by the opportunity cost of successful firms not to invest in the risk-free technology financed by borrowing directly from investors, which yields $\rho^F - \rho^D$. Competition in the *deposit market* is indexed by the opportunity costs for depositors to give up investing in their "autarkic technology" obtaining ρ^D per unit invested. The sequence of events in the extended model is summarized in Table 2.

Table 2. Sequence of Events in the Extended Model

Time	**Agents' decisions**	**Variables determined**
t=0	Agents choose to become entrepreneurs, bankers or investors/depositors Bankers form a coalition called the bank The bank chooses the probability of success for entrepreneurs	q_F : measure of entrepreneurs q_B : measure of bankers $2-q_F-q_B$: measure of depositors P^B
t=1	The event "entrepreneur success" is realized. Successful entrepreneurs form a coalition called the firm, which contracts with the bank The terms of deposit and lending contracts among the firm, the bank and depositors are determined. Depositors deliver funds to banks Banks choose capital and deliver funds to firms Firms choose the riskiness of projects and capital	R^D, R^L Z^B k^B, Z^F P^F, k^F
t=2	Project's output is realized and agents' consumption follows.	

VI. EQUILIBRIUMS IN THE EXTENDED MODEL

The equilibrium in the extended model is defined as follows.

Definition 3 (Equilibrium). Given competition parameters (ρ^F, ρ^D) □ 0, an equilibrium is a set of non-negative vectors of firm's choices of risk and capitalization (P^{F*}, k^{F*}), of bank's choices of capitalization, loan and deposit rates and bank portfolio risk $(k^{B*}, R^{L*}, R^{D*}, P^{B*})$, firm's and bank's investment (Z^{F*}, Z^{B*}), and fractions of entrepreneurs and bankers (q_F^*, q_B^*) that satisfy:

1. **Date 1**

P^{F*}, k^{F*} maximize firm profits:

$$\Pi^F \equiv \pi^F Z^F \equiv [P^F (X - R^L + Xk^F) - \frac{\alpha P^{F2}}{2} - \frac{\beta}{2} k^{F2}] Z^F \qquad (8)$$

Given (P^{F*}, k^{F*}), the bank chooses k^{B*}, R^{L*}, R^{D*} to maximize

$$\pi^B Z^B \equiv [P^{F*}(R^L - R^D + R^L k^B) - \frac{\beta}{2} k^{B2}] Z^B \qquad (9)$$

subject to:

$$P^{F*} R^D \geq \rho^D \qquad (10)$$

$$\pi^F (P^{F*}, k^{F*}) \geq \rho^F - \rho^D \qquad (11)$$

2. **Date 0**

Given $(P^{F*}, k^{F*}, k^{B*}, R^{L*}, R^{D*})$, the bank chooses $P^B \in [0,1]$ to maximize:

$$\Pi^B \equiv P^B \pi^B Z^B - C(P^B) \qquad (12)$$

subject to

$$\Pi^B (k^{B*}, R^{L*}, R^{D*}) \geq 0 \qquad (13)$$

$(Z^{F*}, Z^{B*}, q_F^*, q_B^*)$ solve

$$Z^B = (1 - q_F^2)\frac{W}{2} + (1 - q_B^2)\frac{W}{2} \qquad (14)$$

$$Z^F = (1 + k^{B*}) Z^B \qquad (15)$$

$$\frac{P^B \Pi^F}{q_F} = P^F R^D q_F W \qquad (16)$$

$$\frac{\Pi^B}{q_B} = P^F R^D q_B W \qquad (17)$$

As of the end of date 1, the firm maximizes expected profits (Equation (8)) by choice of P^F and k^F. Given these choices, the bank chooses capitalization and rates k^B, R^L, R^D to maximize expected date 1 profits (Equation (9)), subject to depositors' participation constraints (Equations (10)), and firm's participation constraints (Equations (11)), which states that the bank cannot grant a return to the firm lower than what the firm could obtain by borrowing directly from investors. At date 0, given $(P^{F*}, k^{F*}, k^{B*}, R^{L*}, R^{D*})$, the bank chooses P^B to maximize date 0 expected profits (Equation (12)), subject to its participation constraint (Equation (13)). The last set of conditions defines the general equilibrium. Equation (14) is the equilibrium in the deposit market: bank's demand for funds equals total funds supplied by depositors. Equation (15) is the equilibrium in the loan market: the supply of bank funds equals the firm's demand for funds. Finally, Equations (16) and (17) determine the equilibrium "specialization" choices of agents, that is, the proportions of agents becoming depositors, bankers and entrepreneurs.

In this set-up, total expected output net of effort costs (surplus) is constructed as follows. Total output of the investment in the technology in the good state is $X(1+k^F)Z^F = X(1+k^F)(1+k^B)Z^B$, with firm's effort employed in project choice given by $\frac{\alpha P^{F2}}{2}(1+k^B)Z^B$, and effort spent in firm's capital given by $\frac{\beta k^{F2}}{2}(1+k^B)Z^B$. The bank chooses capital after having chosen P^B, but before the realization of the firm technology, incurring an effort cost $\frac{\beta k^{B2}}{2}Z^B$. Since firm investment is successful with probability P^F, expected output (at date 1) net of all the effort costs above is:

$$\left(P^F X(1+k^F)(1+k^B) - \frac{\alpha P^{F2}}{2}(1+k^B) - \frac{\beta k^{F2}}{2}(1+k^B) - \frac{\beta k^{B2}}{2}\right)Z^B \quad (18)$$

Considering bank's choice of P^B, we arrive at the following definition:

Definition 4. (Surplus) Given an equilibrium indexed by (ρ^D, ρ^F), the expected total output net of effort costs is:

$$Y(\rho^D, \rho^F) \equiv$$
$$P^B\left(P^F X(1+k^F)(1+k^B) - \frac{\alpha P^{F2}}{2}(1+k^B) - \frac{\beta k^{F2}}{2}(1+k^B) - \frac{\beta k^{B2}}{2}\right)Z^B - C(P^B) \quad (19)$$

The characterization of the equilibrium values of firm risk, firm capitalization, bank capitalization, as well as loan and deposit rates, are summarized by the following

Proposition 5. Let $\bar{\rho} \equiv \frac{\beta X^2}{4(\alpha\beta - X^2)}$.

a) Firm risk P^{*F}, firm capital k^{F*}, and bank capital k^{B*} are given by:

$$P^{F*} = \frac{\beta(X - R^L)}{\alpha\beta - X^2}; \quad k^{F*} = P^{F*}\frac{X}{\beta}; \quad k^{B*} = \frac{(X - R^L)R^L}{\alpha\beta - X^2}.$$

b) The equilibrium loan rate R^L and the deposit rate R^D are:

$$R^L = \frac{X}{2} \;;\; R^D = 2\rho^D\frac{\alpha\beta - X^2}{\beta X} \qquad \text{if } \rho^F - \rho^D \le \bar{\rho}$$

$$R^L(\rho^F, \rho^D) = X - \sqrt{(\rho^F - \rho^D)\frac{2(\alpha\beta - X^2)}{\beta}}\;;$$

$$R^D(\rho^F, \rho^D) = \frac{\rho^D}{\sqrt{\rho^F - \rho^D}}\sqrt{\frac{\alpha\beta - X^2}{2\beta}} \qquad \text{if } \rho^F - \rho^D > \bar{\rho}$$

According to part (a) of Proposition 5, firm risk-taking increases and capital declines with a higher loan rate. By contrast, bank capitalization is a strictly concave function of the loan rate, which can be easily explained as follows. Replacing the firm optimal choice of P^F into date 1 bank profits (Equation (9)), the bank chooses the value of k^B maximizes $[\frac{\beta(X-R^L)}{\alpha\beta-X^2}(R^L-R^D+R^Lk^B)-\frac{\beta}{2}k^{B2}]Z^B$. As the loan rate increases, the bank obtains a high return in the good state on its' own funds, but at the cost of a lower probability of getting repaid. It turns out that charging $R^L=\frac{X}{2}$ maximizes date 1 bank profits if the firm participation constraint (15) is not binding. When the constraint (15) is binding, however, the term $\frac{\beta(X-R^L)}{\alpha\beta-X^2}R^L$—which represent the marginal revenue accruing from capital investment—is strictly decreasing in the loan rate for all $R^L>\frac{X}{2}$. Therefore, it is optimal for the bank to choose lower levels of capital as the loan rate is increasing.

According to part (b) of Proposition 5, loan and deposit rates differ depending on whether the firm participation constraint turns out to be binding. Constraint (11) is not binding when $\rho^F-\rho^D\le\bar{\rho}$. In this case, the bank extracts the maximum surplus in the loan market. By contrast, when the firm participation constraint (11) is binding ($\rho^F-\rho^D>\bar{\rho}$), the degree of competition in both deposit and loan markets affect loan and deposit rates simultaneously.

Interestingly, for a given level of competition in the deposit market ρ^D, an increase in competition in the loan market ρ^F corresponding to a move from the region where $\rho^F-\rho^D\le\bar{\rho}$ to the region where $\rho^F-\rho^D>\bar{\rho}$ results in a downward jump in both the lending and deposit rates. The lending rate declines, since the binding participation constraint of the firm forces the bank to lower the lending rate. But the bank can also pay a lower deposit rate, since depositors take into account the decline in the firm's probability of default as the lending rate declines, and require a lower "risk premium".

These results indicate the existence of "low" and "high" *relative* loan market competition regimes. In the "low" regime ($\rho^F-\rho^D\le\bar{\rho}$), changes in deposit market competition do not affect lending rates and vice versa. In the "high" regime ($\rho^F-\rho^D>\bar{\rho}$), changes in competition in both lending and deposit markets affect loan and deposit rates simultaneously. This implies that $\bar{\rho}$ is the threshold level below or above which the comparative statics results for firm risk, firm capitalization and bank capitalization differ, as shown in the following proposition.

Proposition 6

a) $P^F_\rho = K^F_\rho = K^B_\rho = 0$, $\rho = \rho^F, \rho^D$, for all (ρ^F, ρ^D) such that $\rho^F - \rho^D < \dfrac{\beta X^2}{4(\alpha\beta - X^2)}$;

b) $P^F_{\rho^F} > 0$ $K^F_{\rho^F} > 0$; $K^B_{\rho^F} < 0$, and $P^F_{\rho^D} < 0$ $K^F_{\rho^D} < 0$; $K^B_{\rho^D} > 0$ for all (ρ^F, ρ^D) such that $\rho^F - \rho^D > \dfrac{\beta X^2}{4(\alpha\beta - X^2)}$.

Proposition 6 says that when loan market competition (relative to deposit market competition) is "low" ($\rho^F - \rho^D \le \bar{\rho}$), risk and capital of firms, and capital of banks are constant, since the bank extracts the maximum rent on the loan market by charging the loan rate that maximizes its profits. The firm responds by choosing constant levels of project risk and capitalization. By contrast, when loan market competition is relatively "high" ($\rho^F - \rho^D > \bar{\rho}$), an increase in loan market competition, given deposit market competition, prompts firms to reduce risk and increase capital, while banks respond by decreasing capital. Conversely, an increase in deposit market competition, given loan market competition, produces the opposite results: firms increase risk and reduce capital, while banks increase capital. Note that in this case firm risk is affected directly by changes in deposit market competition, and in all cases, capital of firms and banks move in opposite directions.

To complete the characterization of the equilibrium, we solve the bank problem with respect to P^B (Equations (12) and (13)) considering both the (CR) and (IR) cases. Given the supply of funds Z^B, the bank chooses P^B to maximize $P^B \pi^B Z^B - \frac{\delta}{2} P^{B2} Z^B$ under (CR), and $P^B \pi^B Z^B - \frac{\delta}{2} P^{B2}$ under (IR). The solutions are respectively::

$$P^B = \min\{\delta^{-1}\pi^B, 1\} \qquad (22) \text{ if (CR)}$$

$$P^B = \min\{\delta^{-1}\pi^B Z^B, 1\} \qquad (23) \text{ if (IR)}$$

The complete characterization of equilibriums is summarized by the following

Proposition 7. *The equilibrium four-tuple* $(Z^{B*}, Z^{F*}, q^*_F, q^*_B)$ *satisfies* $Z^{F*} = (1 + k^{B*})Z^{B*}$ *and*:

a. *Under (CR)*:

$$Z^{B*} = \frac{4P^F R^D}{4P^F R^D + \delta^{-1}\pi^B(2\pi^F(1+k^B) + \pi^B)} W \quad ;$$

$$q^*_F = \sqrt{\frac{4\delta^{-1}\pi^B\pi^F(1+k^B)}{4P^F R^D + \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)}} \; ; \; q^*_B = \sqrt{\frac{2\delta^{-1}\pi^{B2}}{4P^F R^D + \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)}} \; .$$

b. Under (IR):

$$Z^{B*} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

with $A = \delta^{-1}\pi^B(2\pi^F(1+k^B) + \pi^B)$; $B = 4P^F R^D$; $C = -4P^F R^D W$;

$$q_F^* = \sqrt{\frac{4\delta^{-1}\pi^B Z^B \pi^F(1+k^B)}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)}}\ ;\ q_B^* = \sqrt{\frac{2\delta^{-1}\pi^{B2} Z^B}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)}}\ .$$

We focus on the impact of changes of bank competition in *both* markets. In this case, defining *perfect* competition requires specifying how the surplus that is not accruing to the bank is distributed among the firm and depositors. We assume that the surplus is distributed so that the firm and depositors get the same return. Therefore, we set $\rho^F - \rho^D = \rho^D$, and let $\rho \equiv \rho^D$. Accordingly, "perfect competition" in both markets is the equilibrium corresponding to the value of ρ, denoted by $\rho^{\max}(\rho^B)$, that satisfies the bank participation constraint (13) at equality. Thus, $\rho \in [0, \rho^{\max}(\rho^B)]$ indexes the degree of competition in both the loan and deposit markets. Clearly, $\rho^{\max}(\rho^B)$ is strictly decreasing in ρ^B, as the surplus that can be appropriated by depositors and the firms is bounded above by the requirement to cover the bank's (fixed) costs of the intermediation technology.

Despite the complicated appearance of the expressions of the four-tuple $(Z^{B*}, Z^{F*}, q_F^*, q_B^*)$ in Proposition 7, equilibriums can be easily computed. We report results for two polar representative configurations of parameters related to the fixed costs and efficiency of the intermediation technology under both the (CR) and (IR) assumptions. These two configurations are denoted by $(\delta_1, \rho^B{}_1)$ and $(\delta_2, \rho^B{}_2)$, with $\delta_1 < \delta_2$ and $\rho^B{}_1 > \rho^B{}_2$. The first configuration differs from the second because it represents a relatively more efficient, but more costly, intermediation technology.

We are primarily interested in assessing how bank risk $\mathrm{P}(\rho)$ and surplus $Y(\rho)$ vary with competition. Recall that $\mathrm{P}(\rho) = \min\{\delta^{-1}\pi^B, 1\}\dfrac{\beta(X - R^L)}{\alpha\beta - X^2}$ under (CR), and $\mathrm{P}(\rho) = \min\{\delta^{-1}\pi^B Z^B, 1\}\dfrac{\beta(X - R^L)}{\alpha\beta - X^2}$ under (IR). Therefore, how $\mathrm{P}(\rho)$ varies with ρ depends on whether P^B is constant or decreasing, and when decreasing, whether the decline in P^B is, or is not, offset by an increase in P^F. One effect would dominate the other depending on whether a decline in unit profits π^B, prompting the bank to choose a lower P^B, is offset by a decline in firm risk P^F. With regards to welfare, the impact of changes in ρ will depend primarily on $\mathrm{P}(\rho)$, on the evolution of aggregate funding Z^B, and on the combination of firm and capital choices and associated effort costs.

Figure 1 illustrates the case of an economy where the bank uses a relatively more efficient intermediation technology. As competition increases, bank risk declines under both the (CR) and (IR) assumptions, since $\mathrm{P}(\rho)$ increases. Note the jump in $\mathrm{P}(\rho)$ at a given $\bar{\rho}$, resulting from the switch from low to high relative loan market competition discussed previously. Furthermore, under both the (CR) and (IR) assumptions, perfect competition is optimal, achieving the lowest feasible level of bank risk. Finally, except for the discontinuity given by the jump in rates identified by Proposition 5, an increase in competition leads to an increase in the supply of funds to the bank Z^{B*} and the firm Z^{F*}. It is worthwhile to stress an interesting result regarding the interplay between bank capitalization, firm capitalization and bank risk. When competition increases from not too low levels, the firm increases capital, since lower loan rates increase the profitability of investing internally generated funds. By contrast, the bank capital declines, since the return to capital investment is reduced by a decline in loan rates. However, the decline in bank capital does *not necessarily imply that bank risk increases.*

Figure 2 illustrates the case where the bank intermediation technology is relatively inefficient. Under (CR), P^B declines while P^F remains constant when competition in the loan market, relative to that in the deposit market, is “low”. As competition rises and the threshold $\bar{\rho}$ is reached, $\mathrm{P}(\rho)$ jumps up owing to the jump up of P^F, but then it starts to decline again, as P^B declines at a rate higher than the rate of increase of P^F. As a result, the highest welfare is attained for “intermediate” values of ρ. Under (IR) we obtain essentially the same results. Therefore, i*mperfect* competition is optimal, corresponding to an “intermediate” level of bank risk.

Summing up, when the intermediation technology is relatively efficient, perfect competition is optimal and supports the lowest level of bank risk. Conversely, when the intermediation technology is relatively inefficient, a level of competition *lower* than perfect competition is optimal. These results are independent of whether the intermediation technology exhibits constant or increasing returns to scale.

VII. CONCLUSIONS

We studied versions of a general equilibrium banking model with moral hazard in which the bank’s intermediation technology exhibits either constant or increasing returns to scale. In the basic version of the model under constant returns of the intermediation technology we showed that as deposit market competition increases, bank risk increases, capitalization declines, and “intermediate” degreed of deposit market competition and bank risk are best..The result that the lowest attainable level of bank risk is not optimal echoes Allen and Gale’s (2004b) result that a positive degree of financial ”instability” can be a necessary condition for optimality. Yet, the efficiency of the intermediation technology matters. If this technology exhibits increasing returns to scale, then the implications of this model for bank risk, capitalization and welfare are totally reversed: as competition increases, bank risk declines, capitalization increases, perfect deposit market competition and the lowest attainable level of bank risk are optimal.

Subsequently, we studied the more realistic version of the model where there is competition in both lending and deposit markets and bank risk is determined jointly by the bank and the firm. The key results of the extended model pertain to the role of the efficiency of the intermediation technology in relationship to the level of competition in both lending and deposit markets. We showed that independently of whether the intermediation technology exhibits constant or increasing returns, perfect competition and the lowest attainable level of bank risk are optimal if the bank intermediation technology is relatively efficient. When such technology is relatively inefficient, however, perfect competition is suboptimal, and intermediates levels of competition and bank risk are best.

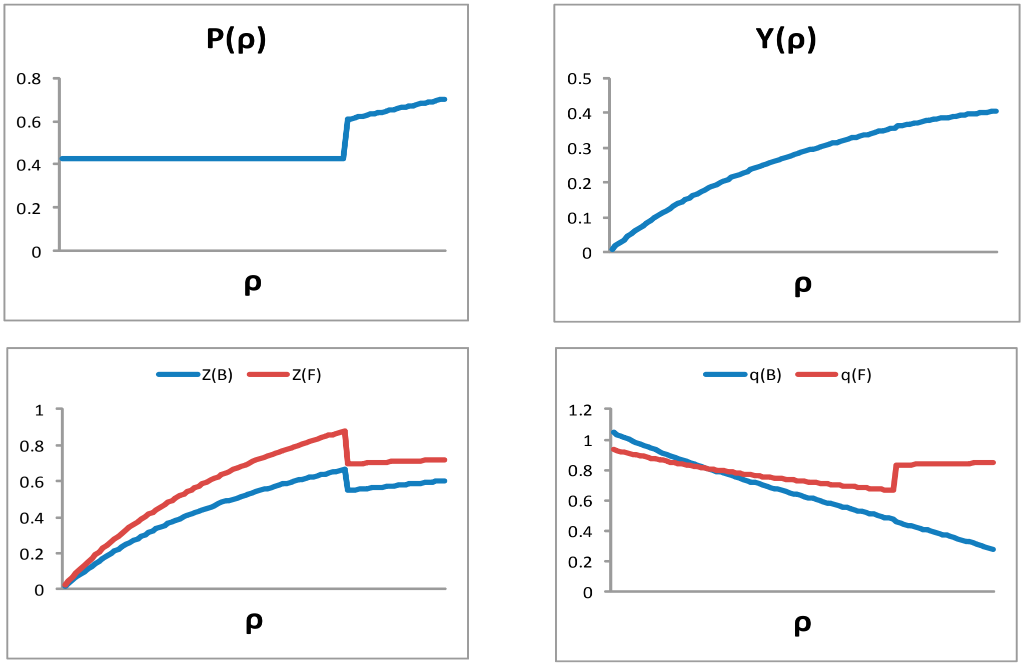
The theoretical results or our study are empirically relevant. Several studies present evidence consistent with a positive relationship between bank competition and financial stability. Jayaratne and Strahan (1998) find that branch deregulation resulted in a sharp decrease in loan losses. Restrictions on banks' entry and activity have been found to be negatively associated with some measures of bank stability by Barth, Caprio and Levine (2004), Beck (2006a and 2006b), and Schaeck et al. (2009). Furthermore, Cetorelli and Gambera (2001) and Cetorelli and Strahan (2006) find that banks with market power erect an important financial barrier to entry to the detriment of the entrepreneurial sector of the economy, leading to long-term declines in a country's growth prospect. Lastly, Corbae and D'Erasmo (2011) present a detailed quantitative study of the U.S. banking industry based on a dynamic calibrated version of Boyd and De Nicolo' (2005) model, finding evidence of a positive association between competition and financial stability. It is apparent that these results are consistent with the predictions of the basic model with increasing returns, and those of the extended model in which banks use relatively efficient intermediation technologies.

Under a policy viewpoint, we believe that our results provide an important insight with regard to the question of whether supporting bank profits with some rents—or, in a dynamic context, supporting banks' charter values—is a desirable public policy option. A substantial portion of the literature and the policy debate maintains that preserving bank profitability through rents enhancing bank profitability—or banks' charter values—may be desirable, as it induces banks to take on less risk. As we have shown, however, this argument ignores how these rents are generated, or how they may be eventually used once granted.

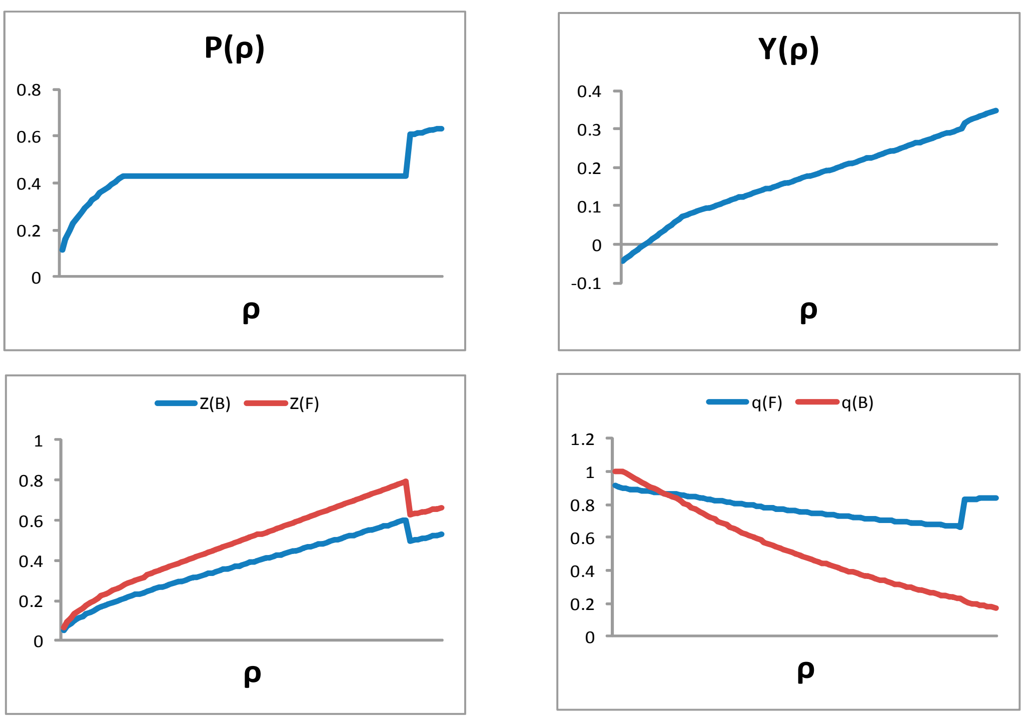
Our results suggest that supporting bank profitability (or charter values) with rents that are independent of bank's actions aimed at improving efficiency may be unwarranted. If rents accrue independently of banks' efforts to adopt more efficient intermediation technologies and, more generally, to provide better intermediation services, then rents are suboptimal and do not guarantee banking system stability. In this light, competitive pressures may be an effective incentive for banks to adopt more efficient intermediation technologies. In a competitive environment, rents would need to be earned by investing in technologies that provide banks a comparative advantage in providing intermediation services, rather than been derived from some market power enjoyed "freely".

Figure 1. Bank Risk and Welfare (Relatively *Efficient* Intermediation Technology)

Panel A. CR Intermediation technology



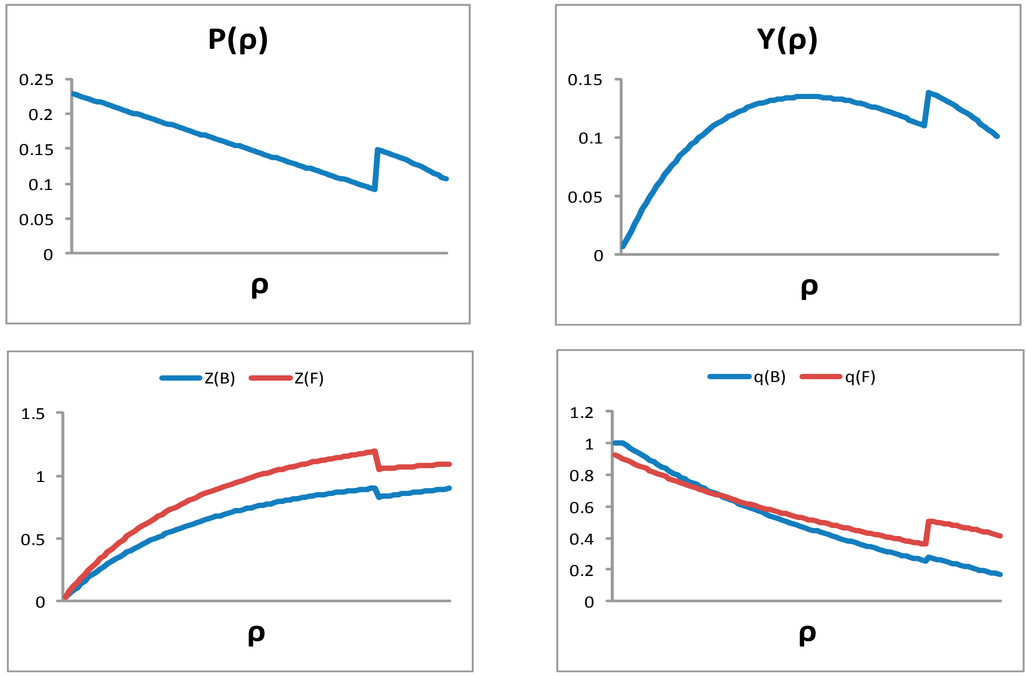
Panel B. IR Intermediation technology



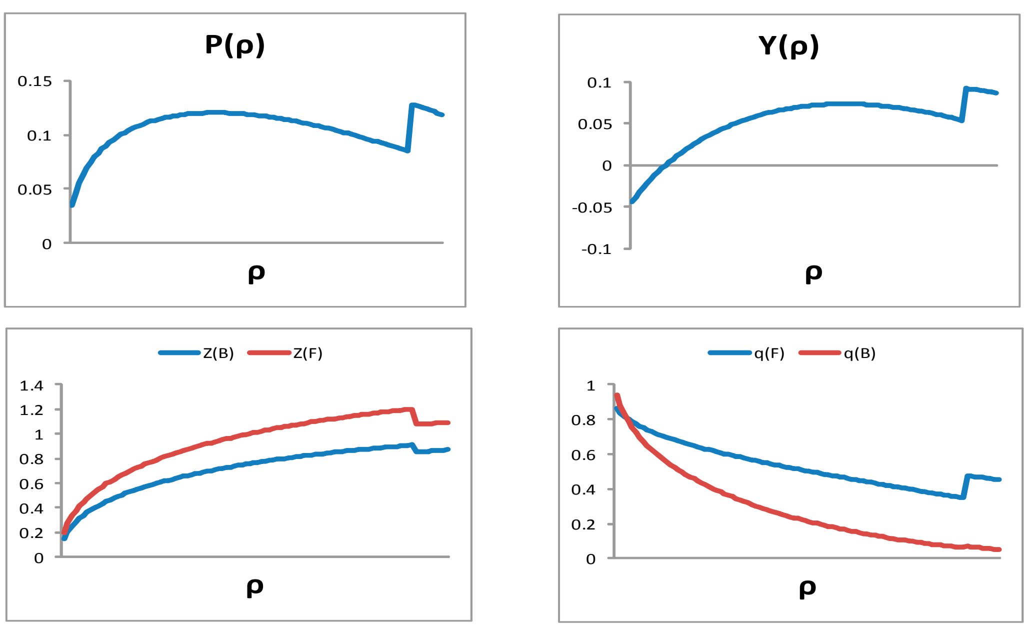
Parameters: $\delta = 0.1$, $\rho^B = 0.03$, $X = 1.5$, $W = 1$, $\alpha = 4$, $\beta = 1$

Figure 2. Bank Risk and Welfare (Relatively *Inefficient* Intermediation Technology)

Panel A. CR Intermediation technology



Panel B. IR Intermediation technology



Parameters: $\delta = 1$, $\rho^B = 0.01$, $X = 1.5$, $W = 1$, $\alpha = 4$, $\beta = 1$

APPENDIX

Proposition 1

c) *Under (CR),* $P^* = \min\{\frac{\beta(X-R^*)}{\alpha\beta - X^2}, 1\}$ $\quad k^* = \min\{\frac{\beta(X-R^*)}{\alpha\beta - X^2}, 1\}\frac{X}{\beta}$ *;*

$$R^*(\rho) = \frac{X}{2}(1-\sqrt{1-\frac{4\rho(\alpha\beta - X^2)}{\beta X^2}}) \quad \text{if } P^* < 1, \quad R^* = \rho \quad \text{if } P^* = 1$$

d) *Under (IR),* $P^* = \min\{\frac{\beta(X-R^*)Z}{\alpha\beta - X^2 Z}, 1\}$ *),* $k^* = \min\{\frac{\beta(X-R^*)Z}{\alpha\beta - X^2 Z}, 1\}\frac{X}{\beta}$.

$$R^*(\rho, Z) = \frac{X}{2}(1-\sqrt{1-\frac{4\rho(\alpha\beta / Z - X^2)}{\beta X^2}}) \quad \text{if } P^* < 1, \quad R^* = \rho \quad \text{if } P^* = 1$$

Proof:

a) (P^*, k^*, R^*) maximizes:

$$\Pi^B \equiv [P(X - R + Xk) - \frac{\beta}{2}k^2]Z - \frac{\alpha}{2}P^2 Z \quad \text{(A.1)}$$

subject to

$$P^* R \geq \rho \qquad \text{(A.2)}$$

The optimality conditions with respect to (P^*, k^*) are:

$$P^* = \min\{\frac{\beta(X-R)}{\alpha\beta - X^2}, 1\} \text{ and } k^* = P^*\frac{X}{\beta}$$

Let $P^* < 1$. Substituting (6) and (7) into (A.1), bank profits are:

$$\Pi^B = \frac{\beta(X-R)}{\alpha\beta - X^2}\frac{(X-R)Z}{2} \qquad \text{(A.3)}$$

Since bank profits (A.3) are strictly decreasing in *R,* the depositor participation constraint (A.2) is satisfied at equality:

$$\frac{\beta(X-R)}{\alpha\beta - X^2}R = \rho \quad \text{(A.4)}$$

The right hand side of equation (A.4) is a strictly concave function of *R.* Thus, the optimal value of R for the bank is the *minimum* value that satisfies the quadratic equation:

$$\beta R^2 - \beta X R + \rho(\alpha\beta - X^2) = 0 \quad \text{(A.5)}$$

Solving (A.5) yields:

$$R^* = \frac{\beta X - \sqrt{(\beta X)^2 - 4\rho(\alpha\beta - X^2)\beta}}{2\beta} = \frac{X}{2}(1-\sqrt{1-\frac{4\rho(\alpha\beta - X^2)}{\beta X^2}}) \text{ (A.6)}$$

If $P^*=1$, then (A.2) implies $R^*=\rho$.

b) (P^*,k^*,R^*) maximizes:

$$\Pi^B \equiv [P(X-R+Xk)-\frac{\beta}{2}k^2]Z-\frac{\alpha}{2}P^2 \quad \text{(A.7)}$$

subject to $$P^*R \geq \rho \qquad \text{(A.8)}$$

The optimality conditions with respect to (P^*,k^*) are:

$$P^*=\min\{\frac{\beta(X-R)Z}{\alpha\beta-X^2Z},1\} \text{ and } k^*=P^*\frac{X}{\beta}.$$

Let $P^*<1$. Substituting (P^*,k^*) into (A.7), bank profits are:

$$\Pi^B=\frac{\beta(X-R)Z}{\alpha\beta-X^2Z}\frac{(X-R)Z}{2} \qquad \text{(A.9)}$$

Since bank profits (A.7) are strictly decreasing in *R,* the depositor participation constraint (A.4) is satisfied at equality:

$$\frac{\beta(X-R)Z}{\alpha\beta-X^2Z}R=\rho \qquad \text{(A.10)}$$

The right hand side of equation (A.10) is a strictly concave function of *R.* Thus, the optimal value of R for the bank is the *minimum* value that satisfies the quadratic equation:

$$\beta R^2-\beta XR+\rho(\alpha\beta/Z-X^2)=0 \quad \text{(A.11)}$$

Solving (A.11) yields:

$$R(\rho,Z)=\frac{1}{2\beta}(\beta X-\sqrt{(\beta X)^2-4\beta\rho(\alpha\beta/Z-X^2)})=\frac{X}{2}(1-\sqrt{1-\frac{4\rho(\alpha\beta/Z-X^2)}{\beta X^2}})$$

If $P^*=1$, then (A.2) implies $R^*=\rho$. QED

Proposition 2 *Under both (CR) and (IR), an equilibrium exists for all* $\rho\in(0,\rho^{\max}]$. *The equilibrium functions* $\{P^*(\rho),k^*(\rho),R^*(\rho),Z^*(\rho),q^*(\rho)\}$ *are continuous and differentiable on* $\rho\in(0,\rho^{\max})$.

Proof

a. (CR).
Using the expressions of bank risk, capitalization and the deposit rate derived in Proposition 1, the equilibrium conditions (3) and (4) become:

$$\frac{P^*}{2q}(X-R^*)Z = P^*R^*qW \quad \text{(A.12)}$$

$$2Z = (1-q^2)W \qquad \text{(A.13)}$$

Solving (A.2) and (A.13), we obtain:

$$q^*(\rho) = \sqrt{\frac{X-R(\rho)}{X+3R(\rho)}} \quad \text{(A.14)}$$

$$Z^*(\rho) = \frac{2R(\rho)W}{X+3R(\rho)} \quad \text{(A.15)}$$

(A.14) and (A.15) imply that a) $q^*(\rho)\in(0,1]$, and b) $q^*(\rho)$ and $Z^*(\rho)$ are continuous and differentiable on $\rho\in[0,\rho^{\max}]$. At $\rho^{\max}$, $R(\rho^{\max}) = X/2$. Therefore:

$$Z^*(\rho^{\max}) = \frac{2W}{5} \quad \text{(A.16)}$$

$$q^*(\rho^{\max}) = \sqrt{\frac{1}{5}} \quad \text{(A.17)}$$

b. (IR)

Rearranging (3) and (4), we get

$$q_1(Z) \equiv q^2 = \frac{Z}{2W}\left(\frac{X}{R(Z,\rho)}-1\right) \quad \text{(A.18)}$$

$$q_2(Z) \equiv 1-\frac{2}{W}Z \qquad \text{(A.19)}$$

Z is an equilibrium when $Q(Z)\equiv q_1(Z)-q_2(Z)=0$. Observe that $Q(0)\equiv q_1(0)-q_2(0)<0$, whereas $Q(\frac{W}{2}) = q_1(\frac{W}{2})-q_2(\frac{W}{2}) = \frac{Z}{2W}\left(\frac{X}{R(\frac{W}{2},\rho)}-1\right)-0 \to +\infty$, since $R(\frac{W}{2},\rho)\to 0$.

Moreover, $Q'(Z)>0$. Therefore, there exists a unique value of Z that satisfies $Q(Z)=0$. for any given ρ. Since $Q(.)$ is continuous and differentiable in ρ, there exists a unique value of Z that satisfies $Q(Z)=0$ for every ρ. The implicit equilibrium functions Z and q are also continuous and differentiable in ρ. QED

Proposition 3.

a) Under (CR), $P_\rho^*<0; k_\rho^*<0$; $q_\rho^*<0$ *and* $Z_\rho^*>0$.

b) Under (IR), $P_\rho^*>0; k_\rho^*>0$; $q_\rho^*<0$ *and* $Z_\rho^*>0$.

Proof.

a) By Proposition 1, $P^* = \dfrac{\beta(X - R^*(\rho))}{\alpha\beta - X^2}$, and $k^* = P^* \dfrac{X}{\beta}$. Clearly, $P^*_\rho < 0$, hence $k^*_\rho < 0$, since . $R^*_\rho > 0$. By (A.11), $q^*(\rho) = \sqrt{\dfrac{X - R(\rho)}{X + 3R(\rho)}}$, and $q^*_\rho < 0$. By Proposition 2, $Z^*_\rho = Z_R R^*_\rho = \dfrac{2WX}{(3R + X)^2} R^*_\rho > 0$

b) By Proposition 2, $Z^*(\rho)$ and $q^*(\rho)$ are continuous and differentiable functions. The equilibrium conditions (3) and (4) can be written as:

$$(X - R^*(Z^*(\rho),\rho))Z^*(\rho) - 2R^*(Z^*(\rho),\rho)q^*(\rho)^2 W = 0 \quad \text{(A.20)}$$
$$2Z^*(\rho) = (1 - q^*(\rho)^2)W \quad \text{(A.18)}$$

Differentiating (A.17) and (A.18) with respect to ρ, we get

$$(X - R - R_Z(Z^* + 2q^2W))Z^*_\rho - 4Rq^*Wq^*_\rho = R_\rho(Z^* + 2Wq^{*2}) \qquad \text{(A.21)}$$

$$2Z^*_\rho + 2Wq^*q^*_\rho = 0 \qquad \text{(A.22)}$$

Plugging (A.22) in (A.21) and rearranging, we obtain:

$$Z^*_\rho = \frac{R_\rho(Z^* + 2q^{*2}W)}{(X + 3R - R_Z(Z^* + 2q^{*2}W))} \qquad \text{(A.23)}$$

$$q^*_\rho = -\frac{Z^*_\rho}{Wq^*} \qquad \text{(A.24)}$$

From Proposition 1, differentiating R^* with respect to Z and ρ we obtain:

$$\begin{aligned} R^*_Z &= -\frac{X}{4}\Big(1 - \frac{4\rho(\alpha\beta/Z - X^2)}{\beta X^2}\Big)^{-1/2} \frac{4\rho\alpha\beta/Z^2}{\beta X^2} < 0 \\ R^*_\rho &= \frac{X}{4}\Big(1 - \frac{4\rho(\alpha\beta/Z - X^2)}{\beta X^2}\Big)^{-1/2} \frac{4(\alpha\beta/Z - X^2)}{\beta X^2} > 0 \end{aligned} \qquad \text{(A.25)}$$

By (A.25) $R_\rho(Z^* + 2q^{*2}W) > 0$ and $X + 3R - R_Z(Z^* + 2q^{*2}W) > 0$. Hence, $Z_\rho > 0$, and by (A.24), $q_\rho < 0$. The derivative of $P = \dfrac{\beta(X - R(Z,\rho))Z}{\alpha\beta - X^2 Z}$ with respect to ρ is:

$$P_\rho = \frac{\beta}{(\alpha\beta - X^2Z)^2}[[(X - R)Z_\rho - (R_Z Z_\rho + R_\rho)Z](\alpha\beta - X^2Z) + (X - R)ZX^2Z_\rho] \quad \text{(A.26)}$$

Using (A.25) and rearranging, the term $\frac{dR(Z(\rho),\rho)}{d\rho} = R_Z Z_\rho + R_\rho$ is:

$$R_Z Z_\rho + R_\rho = R_Z \frac{R_\rho (Z+2q^2W)}{(X+3R-R_Z(Z+2q^2W))} + R_\rho = \frac{R_\rho (X+3R)}{X+3R-R_Z(Z+2q^2W)} = Z_\rho \quad (A.27)$$

The last expression in (A.27) derives from (A.23). Substituting (A.27) in (A.26), we get:

$$\begin{aligned} P_\rho &= \frac{\beta}{(\alpha\beta - X^2Z)^2}[[(X-R)Z_\rho - Z_\rho Z](\alpha\beta - X^2Z) + (X-R)ZX^2Z_\rho] = \\ &\frac{\beta Z_\rho (X-R)(\alpha\beta - X^2Z)}{(\alpha\beta - X^2Z)^2}[\frac{\alpha\beta}{(\alpha\beta - X^2Z)} - \frac{Z}{X-R}] \end{aligned} \quad (A.28)$$

Note that $\frac{\alpha\beta}{(\alpha\beta - X^2Z)} > 1$ for any $Z > 0$. The highest value of the term $\frac{Z}{X-R}$ obtains when $Z = Z(\rho^{\max}) = \frac{2}{5}W$ and $R = R(\rho^{\max}) = \frac{X}{2}$. Thus, $(\frac{Z}{X-R})^{\max} = \frac{4W}{5X} < 1$, since $X > W$.

Thus:

$$\begin{aligned} P_\rho &= \frac{\beta Z_\rho (X-R)(\alpha\beta - X^2Z)}{(\alpha\beta - X^2Z)^2}[\frac{\alpha\beta}{(\alpha\beta - X^2Z)} - \frac{Z}{X-R}] > \\ &\frac{\beta Z_\rho (X-R)(\alpha\beta - X^2Z)}{(\alpha\beta - X^2Z)^2}[1 - \frac{4W}{5X}] > 0. \end{aligned} \quad (A.29)$$

Moreover, $k^*_\rho = P^*_\rho \frac{X}{\beta} > 0$. QED

Proposition 4.

c) *Under (CR), there exists a value* $\hat{\rho} \in (0, \rho^{\max})$ *such that* $Y(\hat{\rho}) \geq Y(\rho)$ *for all* $\rho \in [0, \rho^{\max}]$.

d) *Under (IR),* $Y(\rho^{\max}) > Y(\rho)$ *for all* $\rho \in (0, \rho^{\max}]$.

Proof:

a) Replacing $k^* = P^* \frac{X}{\beta}$ in (5), we obtain

$$Y(R^*(\rho)) = Z^* \left(P^*X + P^*Xk^* - \frac{\alpha P^{*2}}{2} - \frac{\beta k^{*2}}{2} \right) = Z^* \left(P^*X - \frac{1}{2\beta}(\alpha\beta - X^2)P^{*2} \right) \quad (A.30)$$

Differentiating (A.30) with respect to ρ, we get:

$$Y_\rho = Y_{R^*}R^*_\rho = R^*_\rho [Z^*_{R^*} \left(P^*X - \frac{1}{2\beta}(\alpha\beta - X^2)P^{*2} \right) + Z^*P^*_{R^*} \left(X - \frac{1}{\beta}(\alpha\beta - X^2)P^* \right)] \quad (A.31)$$

Plugging $P^* = \frac{\beta(X-R^*)}{\alpha\beta - X^2}$, $P^*_{R^*} = -\frac{\beta}{\alpha\beta - X^2}$, $Z^* = \frac{2R^*W}{X+3R^*}$ and $Z^*_{R^*} = \frac{2WX}{(3R^*+X)^2}$ in (A.31), and rearranging, we obtain:

$$Y_\rho = R^*_\rho [Z^*_{R^*}\left(\frac{\beta(X-R^*)}{\alpha\beta - X^2}X - \frac{1}{2\beta}(\frac{(\beta(X-R^*))^2}{\alpha\beta - X^2}\right) + Z^*P^*_{R^*}\left(X - \frac{1}{\beta}(\alpha\beta - X^2)\frac{\beta(X-R^*)}{\alpha\beta - X^2}\right)] =$$
$$R^*_\rho \frac{\beta 2W}{(\alpha\beta - X^2)(3R^*+X)}[\frac{X}{(3R^*+X)}(X-R^*)\left(\frac{X}{2}+R^*\right) - R^{*2}]$$

Evaluating Y_ρ with respect to $\rho = 0$ ($R^*(0)=0$) we get:

$$Y_\rho(R^*=0) = R^*_\rho \frac{\beta 2W}{(\alpha\beta - X^2)X}[X\left(\frac{X}{2}\right)] > 0 \quad \text{(A.32)},$$

Evaluating Y_ρ with respect to $\rho = \rho^{\max}$ ($R^*(\rho^{\max}) = \frac{X}{2}$), we get:

$$\begin{aligned} Y_\rho(R^* = \frac{X}{2}) &= R^*_\rho \frac{\beta 2W}{(\alpha\beta - X^2)(3(X/2)+X)}[\frac{2X}{(3X+2X)}(\frac{X}{2})X - (\frac{X}{2})^2] = \\ & R^*_\rho \frac{\beta 2WX^2}{(\alpha\beta - X^2)(3(X/2)+X)}[\frac{X}{(3X+2X)} - \frac{1}{4}] = R^*_\rho \frac{4}{5X}(-\frac{\beta W X^3}{(\alpha\beta - X^2)}) < 0 \end{aligned} \quad \text{(A.33)}$$

Thus, neither $\rho = 0$ nor $\rho = \rho^{\max}$ are maximums of $Y(\rho)$. Since $Y(\rho)$ is a continuous function defined on the compact set $[0, \rho^{\max}]$, it has a maximum and a minimum. Therefore, there exists a value $\hat{\rho} \in (0, \rho^{\max})$ such that $Y(\hat{\rho}) \geq Y(\rho)$ for all $\rho \in (0, \rho^{\max})$.

b) Using $k^* = P^*\frac{X}{\beta}$ and $\alpha\beta - X^2 Z = \frac{\beta(X-R)Z}{P^*}$, the expected output net of effort costs evaluated at an equilibrium indexed by ρ is:

$$\begin{aligned} Y(R^*(\rho)) &= Z^*\left(P^*X + P^*Xk^* - \frac{\beta k^{*2}}{2}\right) - \frac{\alpha P^{*2}}{2} = \\ & P^*(Z^*X - \frac{1}{2}XZ^* + R^*Z^*) = P^*(\frac{1}{2}XZ^* + R^*Z^*) \end{aligned} \quad \text{(A.34)}$$

By Proposition 1, $Z^*_\rho > 0$; $P^*_\rho > 0$. Therefore, $Y(R^*(\rho))$ is strictly increasing in ρ. QED

Proposition 5. Let $\bar{\rho} \equiv \frac{\beta X^2}{4(\alpha\beta - X^2)}$.

a) Firm risk P^{*F}, firm capital k^{F*}, and bank capital k^{B*} are given by:

$$P^{F*} = \frac{\beta(X - R^L)}{\alpha\beta - X^2}; \quad k^{F*} = P^{F*}\frac{X}{\beta}; \; k^{B*} = \frac{(X - R^L)R^L}{\alpha\beta - X^2}.$$

b) The equilibrium loan rate R^L and the deposit rate R^D are:

$$R^L = \frac{X}{2} \;\; ; \;\; R^D = 2\rho^D \frac{\alpha\beta - X^2}{\beta X} \qquad \text{if } \rho^F - \rho^D \leq \bar{\rho}$$

$$R^L(\rho^F, \rho^D) = X - \sqrt{(\rho^F - \rho^D)\frac{2(\alpha\beta - X^2)}{\beta}};$$

$$R^D(\rho^F, \rho^D) = \frac{\rho^D}{\sqrt{\rho^F - \rho^D}}\sqrt{\frac{\alpha\beta - X^2}{2\beta}} \qquad \text{if } \rho^F - \rho^D > \bar{\rho}$$

Proof:

a) (P^{*F}, k^{*F}) maximize the firm expected profits (8). The first order conditions are:

$$P^{F*} = \min\{\frac{\beta(X - R^L)}{\alpha\beta - X^2}, 1\} \text{ (A.35)}; \;\; k^{F*} = P^{F*}\frac{X}{\beta} \quad \text{(A.36)}$$

Under the maintained assumptions, $P^{F*} \in (0,1)$. Turning to the bank problem, given (P^{F*}, k^{F*}), a bank chooses k^B, R^L, R^D to maximize (9), subject to (10) and (11). The first order condition for the optimal bank capital yields:

$$k^{B*} = \frac{(X - R^L)R^L}{\alpha\beta - X^2} \qquad \text{(A.37)}$$

b) Substituting (A.37) in the profit function (9) yields:

$$\pi^B Z^B = [\frac{\beta(X - R^L)}{\alpha\beta - X^2}(R^L - R^D + \frac{(X - R^L)}{(\alpha\beta - X^2)}R^{L2}) - \frac{\beta}{2}(\frac{(X - R^L)}{(\alpha\beta - X^2)}R^L)^2]Z^B =$$
$$\frac{\beta}{\alpha\beta - X^2}[(X - R^L)(R^L - R^D) + \frac{(X - R^L)^2}{2(\alpha\beta - X^2)}R^{L2}]Z^B \qquad \text{(A.38)}$$

Bank profits (A.38) are strictly decreasing in the deposit rate for any value of the loan rate. Therefore, the deposit participation constraint (10) is satisfied at equality, which implies:

$$R^D = \rho^D \frac{\alpha\beta - X^2}{\beta(X - R^L)} \qquad \text{(A.39)}$$

Substituting (A.39) in (A.38) and using $P^F = \frac{\beta(X - R^L)}{\alpha\beta - X^2}$, we get:

$$\pi^B Z^B = \frac{\beta}{\alpha\beta - X^2}[(X - R^L)(R^L - \rho^D \frac{\alpha\beta^F - X^2}{\beta(X - R^L)}) + \frac{(X - R^L)^2}{2(\alpha\beta - X^2)} R^{L2}]Z^B =$$
$$[P^F R^L + P^F \frac{1}{2\beta} - \rho^D]Z^B \qquad \text{(A.40)}$$

Let $y = (X - R^L)R^L$. Choosing the loan rate that maximizes (A.40) is equivalent to choosing $y = (X - R^L)R^L$ that maximizes:

$$\pi^B Z^B = [\frac{\beta}{\alpha\beta - X^2}(y + \frac{y^2}{2\beta(\alpha\beta - X^2)}) - \rho^D]Z^B \quad \text{(A.41)}$$

Both $y = (X - R^L)R^L$ and $y^2 = [(X - R^L)R^L]^2$ are maximized at $R^L = \frac{X}{2}$, which results in $y = X^2 / 4$. Therefore, the bank would choose to charge $R^L = \frac{X}{2}$ if (11) is not binding.

Firm profits at an optimum are:

$$\Pi^F(P^{F*}, k^{F*}) \equiv [P^F((X - R^L) + X\frac{P^F X}{\beta}) - \frac{\alpha P^{F2}}{2} - \frac{\beta}{2}\frac{P^{F2}X^2}{\beta^2}]Z^F =$$
$$[\frac{\beta[(X - R^L)]^2}{2(\alpha\beta - X^2)}]Z^F = [P^F \frac{(X - R^L)}{2}]Z^F \qquad \text{(A.42)}$$

Define profit per unit of investment as:

$$\pi^F \equiv \frac{\beta[(X - R^L)]^2}{2(\alpha\beta - X^2)} = P^F \frac{(X - R^L)}{2} \qquad \text{(A.43)}$$

If $R^L = \frac{X}{2}$, then (11) is not binding if:

$$\pi^F \equiv \frac{\beta[(X - X/2)]^2}{2(\alpha\beta - X^2)} = \frac{\beta X^2}{4(\alpha\beta - X^2)} \geq \rho^F - \rho^D \quad \text{(A.44)}$$

Clearly, for all ρ^F such that $\rho^F - \rho^D > \frac{\beta X^2}{4(\alpha\beta - X^2)}$, (11) is binding. Since the loan rate satisfies $\frac{\beta[(X - R^L)]^2}{2(\alpha\beta - X^2)} = \rho^F - \rho^D$, and the deposit rate satisfies (A.39), both imply

$$R^L(\rho^F,\rho^D) = X - \sqrt{(\rho^F-\rho^D)\frac{2(\alpha\beta - X^2)}{\beta}}$$
$$R^D(\rho^F,\rho^D) = \frac{\rho^D}{\sqrt{\rho^F-\rho^D}}\sqrt{\frac{\alpha\beta-X^2}{2\beta}} \qquad \text{(A.45).}$$

QED

Proposition 6

a) $P_\rho^F = K_\rho^F = K_\rho^B = 0$, $\rho = \rho^F, \rho^D$, for all (ρ^F,ρ^D) such that $\rho^F - \rho^D < \frac{\beta X^2}{4(\alpha\beta - X^2)}$;

b) $P_{\rho^F}^F > 0$ $K_{\rho^F}^F > 0$; $K_{\rho^F}^B < 0$, and $P_{\rho^F}^F < 0$ $K_{\rho^D}^F < 0$; $K_{\rho^D}^B > 0$ for all (ρ^F,ρ^D) such that $\rho^F - \rho^D > \frac{\beta X^2}{4(\alpha\beta - X^2)}$.

Proof:

a) If $\rho^F - \rho^D < \frac{\beta X^2}{4(\alpha\beta - X^2)}$, by Proposition 3, $R^L = \frac{X}{2}$. Substituting this loan rate in P^F, k^F, k^B , we obtain:

$$P^F = \frac{\beta X}{2(\alpha\beta - X^2)} \; ; \; k^F = \frac{X^2}{2\beta(\alpha\beta - X^2)} \; ; \; k^B = \frac{X^2}{4\beta(\alpha\beta - X^2)} \quad \text{(A.46)}$$

As the above expressions do not depend on either ρ^F or ρ^D, we have $P_\rho^F = K_\rho^F = K_\rho^B = 0$, $\rho = \rho^F, \rho^D$ for all (ρ^F,ρ^D) such that $\rho^F - \rho^D < \frac{\beta X^2}{4(\alpha\beta - X^2)}$;

a) If $\rho^F - \rho^D > \frac{\beta X^2}{4(\alpha\beta - X^2)}$, by Proposition 3 ,

$R^L(\rho^F,\rho^D) = X - \sqrt{(\rho^F-\rho^D)\frac{2(\alpha\beta - X^2)}{\beta}}$, hence $R_{\rho^F}^L < 0$ and $R_{\rho^D}^L > 0$

Differentiating $P^F = \frac{\beta(X - R^L(\rho^F,\rho^D))}{\alpha\beta - X^2}$ and $k^F = \frac{X(X - R^L(\rho^F,\rho^D))}{\alpha\beta - X^2}$ we get:

$$P_{\rho^F}^F = -\frac{\beta}{\alpha\beta - X^2}R_{\rho^F}^L > 0 \; ; \; P_{\rho^D}^F = -\frac{\beta}{\alpha\beta - X^2}R_{\rho^D}^L < 0; \quad \text{(A.47)}$$
$$k_{\rho^F}^F = -\frac{X}{\alpha\beta - X^2}R_{\rho^F}^L > 0 \; ;. \; k_{\rho^D}^F = -\frac{X}{\alpha\beta - X^2}R_{\rho^D}^L < 0 \quad \text{(A.48)}$$

Bank capital is given by $k^B = \frac{(X - R^L(\rho^F,\rho^D))R^L(\rho^F,\rho^D)}{\alpha\beta - X^2}$, which implies.

$$k^B_{\rho^F} = \frac{R^L_{\rho^F}}{\alpha\beta - X^2}(X - 2R^L(\rho^F,\rho^D)) < 0; k^B_{\rho^D} = \frac{R^L_{\rho^D}}{\alpha\beta - X^2}(X - 2R^L(\rho^F,\rho^D)) > 0$$

since $R^L(\rho^F,\rho^D) < X/2$ and $R^L_{\rho^F} < 0$ and $R^L_{\rho^D} > 0$ QED

Proposition 7. *The equilibrium four-tuple* $(Z^{B*}, Z^{F*}, q_F^*, q_B^*)$ *satisfies* $Z^{F*} = (1+k^{B*})Z^{B*}$ *and*:

a. Under (CR):

$$Z^{B*} = \frac{4P^F R^D}{4P^F R^D + \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)}W \quad ;$$

$$q_F^* = \sqrt{\frac{4\delta^{-1}\pi^B\pi^F(1+k^B)}{4P^F R^D + \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)}}\ ;\ q_B^* = \sqrt{\frac{2\delta^{-1}\pi^{B2}}{4P^F R^D + \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)}}\ .$$

b. Under (IR):

$$Z^{B*} = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

With $A = \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)$; $B = 4P^F R^D$; $C = -4P^F R^D W$;

$$q_F^2 = \sqrt{\frac{4\delta^{-1}\pi^B Z^B\pi^F(1+k^B)}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)}}\ ;\ q_B^* = \sqrt{\frac{2\delta^{-1}\pi^{B2}Z^B}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)}}\ .$$

Proof:

a) $(Z^{F*}, Z^{B*}, q_F^*, q_B^*)$ solve the system:

$$2Z^B = W(2 - q_F^2 - q_B^2) \quad \text{(A.49)}$$

$$Z^F = (1+k^B)Z^B \quad \text{(A.50)}$$

$$\frac{P^B\pi^F Z^F}{q_F} = P^F R^D q_F W \quad \text{(A.51)}$$

$$\frac{P^B\pi^B Z^B}{2q_B} = P^F R^D q_B W \quad \text{(A.52)}$$

The system (A.49)-(A.52) is a linear in (Z^B, Z^F, q_F^2, q_B^2), and can be solved by substitution. Plugging (A.55) in (A.56), we obtain:

$$2Z^B = W(2 - q_F^2 - q_B^2) \quad \text{(A.53)}$$

$$q_F^2 = \frac{P^B\pi^F(1+k^B)Z^B}{P^F R^D W} \quad \text{(A.54)}$$

$$q_B^2 = \frac{P^B\pi^B Z^B}{2P^F R^D W} \quad \text{(A.55)}$$

Plugging (A.55) and (A.54) in (A.49), we get:

$$Z^B = \frac{4P^F R^D}{4P^F R^D + P^B(2\pi^F(1+k^B)+\pi^B)} W \qquad \text{(A.56)}$$

Plugging (A.56) in (A.55) and (A.54), we get:

$$q_F^2 = \frac{4P^B \pi^F (1+k^B)}{4P^F R^D + P^B(2\pi^F(1+k^B)+\pi^B)}\ ;\ q_B^2 = \frac{2P^B \pi^B}{4P^F R^D + P^B(2\pi^F(1+k^B)+\pi^B)} \text{(A.57)}$$

b) Since $P^B = \delta^{-1}\pi^B Z^B$, (14) implies

$$Z^B = \frac{4P^F R^D}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)} W \Leftrightarrow \qquad \text{(A.58)}$$

$$\delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)Z^{B2} + 4P^F R^D Z^B - 4P^F R^D W = 0$$

Therefore:

$$Z^B = \frac{-B+\sqrt{B^2-4AC}}{2A} \quad \text{(A.59)}$$

where: $A = \delta^{-1}\pi^B(2\pi^F(1+k^B)+\pi^B)$; $B = 4P^F R^D$; $C = -4P^F R^D W$.

Accordingly, $Z^F = (1+k^B)Z^B$ and

$$q_F^2 = \frac{4\delta^{-1}\pi^B Z^B \pi^F(1+k^B)}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)} ; q_B^2 = \frac{2\delta^{-1}\pi^{B2} Z^B}{4P^F R^D + \delta^{-1}\pi^B Z^B(2\pi^F(1+k^B)+\pi^B)} \text{(A.60)}$$

QED

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