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Capital Regulation, Liquidity Requirements and Taxation in a Dynamic Model of Banking

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**Capital Regulation, Liquidity Requirements and
Taxation in a Dynamic Model of Banking[1]**

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Abstract

This paper studies the impact of bank regulation and taxation in a dynamic model with banks exposed to credit and liquidity risk. We find an inverted U-shaped relationship between capital requirements and bank lending, efficiency, and welfare, with their benefits turning into costs beyond a certain requirement threshold. By contrast, liquidity requirements reduce lending, efficiency and welfare significantly. The costs of high capital and liquidity requirements represent a lower bound on the benefits of these regulations in abating systemic risks. On taxation, corporate income taxes generate higher government revenues and entail lower efficiency and welfare costs than taxes on non-deposit liabilities.

I. Introduction

The 2007–2008 financial crisis has been a catalyst for significant bank regulation reforms, as the pre-crisis regulatory framework has been judged inadequate to cope with large financial shocks. The new Basel III framework envisions a raise in bank capital requirements and the introduction of new liquidity requirements, while several proposals have been recently advanced to use forms of taxation with the twin objectives of raising funding to pay for resolution costs in stressed times, as well as a way to control bank risk–taking behavior.[1] To date, however, the relatively large literature on bank regulation offers no formal analysis where a *joint* assessment of these policies can be made in a *dynamic* model of banking where banks play a role and are exposed to multiple sources of risk. The formulation of such a dynamic banking model is the main contribution of this paper.

Our model is novel in three important dimensions. First, we analyze a bank that dynamically transforms short term liabilities into longer–term partially illiquid assets whose returns are uncertain. This feature is consistent with banks' special role in liquidity transformation emphasized in the literature (see e.g. Diamond and Dybvig (1983) and Allen and Gale (2007)).

Second, we model bank's financial distress explicitly. This allows us to examine optimal banks' choices on whether, when, and how to continue operations in the face of financial distress. The bank in our model invests in risky loans and risk–less bonds financed by (random) government-insured deposits and short–term debt. Financial distress occurs when the bank is unable to honor part or all of its debt and tax obligations for given realizations of credit and liquidity shocks. The bank has the option to resolve distress in three costly forms: by liquidating assets at a cost, by issuing fully collateralized bonds, or by issuing equity. The liquidation costs of assets are interpreted as fire sale costs, and modeled introducing asymmetric costs of adjustment of the bank's risky asset portfolio. The importance of fire sale costs in amplifying banks financial distress has been brought to the fore in the recent crisis (see e.g. Acharya, Shin, and Yorulmazer (2010) and Hanson, Kashyap, and Stein (2011)).

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[1]The new Basel III framework is detailed in *Basel III: A global regulatory framework for more resilient banks and banking systems*, Bank for International Settlements, Basel, June 2011. On taxation proposals, see Acharya, Pedersen, Philippon, and Richardson (2010) and *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010.

Third, we evaluate the impact of bank regulations and taxation not only on bank optimal policies, but also in terms of metrics of bank efficiency and welfare. The first metric is the enterprise value of the bank, which can be interpreted as the efficiency with which the bank carries out its maturity transformation function. The second one, called "social value", proxies welfare in our risk-neutral world, as it summarizes the total expected value of bank activities to all bank stakeholders and the government. To our knowledge, this is the first study that evaluates the joint welfare implications of bank regulation and taxation.

Our benchmark bank is unregulated, but its deposits are fully insured. We consider this bank as the appropriate benchmark, since one of the asserted roles of bank regulation is the abatement of the excessive bank risk–taking arising from moral hazard under partial or total insurance of its liabilities. We use a standard calibration of the parameters of the model —with regulatory and tax parameters mimicking current capital regulation, liquidity requirement and tax proposals— to solve for the optimal policies and the metrics of efficiency and welfare.

We obtain three sets of results. First, if capital requirements are mild, a bank subject only to capital regulation invests more in lending and its probability of default is lower than its unregulated counterpart. This additional lending is financed by higher levels of retained earnings or equity issuance. Importantly, under mild capital regulation bank efficiency and social values are *higher* than under no regulation, and their benefits are larger the higher are fire sale costs. However, if capital requirements become too stringent, then the efficiency and welfare benefits of capital regulation disappear and turn into costs, even though default risk remains subdued: lending declines, and the metrics of bank efficiency and social value drop below those of the unregulated bank. Thus, there exists an inverted-U-shaped relationship between bank lending, efficiency, welfare and the stringency of capital requirements. These novel findings suggest the existence of an optimal level of bank-specific regulatory capital under deposit insurance.

Second, the introduction of liquidity requirements reduces bank lending, efficiency and social value significantly, since these requirements hamper bank maturity transformation. In addition, the reduction in lending, efficiency, and social values increases monotonically with their stringency. When liquidity requirements are added to capital requirements, they also eliminate the benefits of mild capital requirements, since bank lending, efficiency and social values are reduced relative to the bank subject to capital regulation only. We should stress

that these results do not have to be necessarily interpreted as an indictment of liquidity requirements. If liquidity requirements were found to be optimal regulations to correct some negative externalities arising from excessive bank's reliance on short term debt—which we do not model—then our results indicate how large the costs associated with these negative externalities should be to rationalize the need of liquidity requirements.

On taxation, an increase in corporate income taxes reduces lending, bank efficiency and social values due to standard negative income effects. However, tax receipts increase, generating higher government revenues. With the introduction of a tax on non-deposit liabilities, which in our model is short–term debt, the decline in bank lending, efficiency and social values is larger than that under an increase in corporate taxation, while the increase in government tax receipts is lower. Therefore, in our model corporate taxation is preferable to a tax on non–deposit liabilities, although both forms of taxation reduce lending, efficiency and social value.

The remainder of this paper comprises five sections. Section II presents a brief review of the literature. Section III describes the benchmark model of an unregulated bank. Section IV introduces bank regulation and illustrates some basic trade-offs on optimal policies. Section V details the impact of bank regulation and taxation. Section VI concludes. The Appendix describes some properties of the bank's dynamic program, a comparison of a bank closure rule with capital requirements, and the computational procedures used to solve the model and run simulations on the optimal solution.

II. A brief literature review

The literature on bank regulation is large, but it offers few formal analyses of the impact of regulatory constraints on bank optimal policies in a dynamic framework.

The great majority of studies have focused on capital regulation. Capital requirements has been typically justified by their role in curbing excessive risk–taking (risk–shifting or asset substitution) induced by moral hazard of banks whose deposits are insured (for a review, Freixas and Rochet (2008) and Lucas and Stokey (2011)). However, whether an increase in capital requirements unambiguously reduces banks' incentives to take on more risk appears

an unsettled issue even in the context of static partial equilibrium models of banking. Several studies, as for instance Besanko and Kanatas (1996), Hellmann, Murdock, and Stiglitz (2000), and Repullo (2004), using partial equilibrium set–ups show that an increase in capital results in reduced risk taking. However, in similar models like Blum (1999), and Calem and Rob (1999), just to mention a few of them, this conclusion can be reversed. Such reversal can also occur in general equilibrium set–ups (see e.g. Gale and Özgür (2005), and De Nicolò and Lucchetta (2009) and Gale (2010)), with capital regulation potentially entailing high welfare costs, as in Van den Heuvel (2008).

The relatively sparse literature of dynamic models of banking has mainly focused on the pro-cyclicality aspects of bank capital regulation.[2] Based on the experience of the 2007-2008 financial crisis, Brunnermeier, Crockett, Goodhart, Persaud, and Shin (2009) have stressed the role of pro–cyclicality as an important factor underlying the amplification of credit cycles, and likely to increase the probability of so–called illiquidity spirals, in which the entire banking system liquidates its assets at fire sale prices in a downturn, with adverse systemic consequences. Even in this case, available results are mixed depending on the way the model is specified: Estrella (2004) and Repullo and Suarez (2008) find that capital requirements are indeed pro-cyclical, while Peura and Keppo (2006) and Zhu (2008) find that this may not necessarily be the case.

A recent line of research has addressed the potential bank vulnerabilities arising from reliance on wholesale funding, as in Acharya, Gale, and Yorulmazer (2011), or on the use of short-term debt and maturity choice (Brunnermeier and Oehmke (2010), and Gale and Yorulmazer, 2010). While these contributions have provided important insights on the costs and benefits of market–based funding for banks, their implications have been only loosely linked to the necessity of bank liquidity requirements. The only exception is the simple static model of Perotti and Suarez (2011), where a liquidity requirement arises through an exogenously imposed "externality" in the bank revenue function. To date, we know of no model that assesses the impact of liquidity requirements in the context of well-defined banking optimal policies where systemic liquidity risk arises endogenously. Finally, we are unaware of dynamic banking models that analyze the impact of taxation on bank optimal policies, efficiency and welfare.

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[2]For a review of the pre-crisis literature on pro–cyclicality and some empirical evidence, see Zhu (2008) and Panetta and Angelini (2009)).

Zhu (2008) presents a modeling approach similar to ours. Extending the model by Cooley and Quadrini (2001), he considers a bank that invests in a risky decreasing return to scale technology, its sole source of financing are uninsured (and fairly priced) deposits, it faces linear equity issuance costs, and is subject to minimum capital requirements. However, Zhu (2008) considers the effect of neither maturity transformation, nor liquidity requirements or taxation, which are important aspects of our results.

III. The model

Time is discrete and the horizon is infinite. We consider a bank that receives a random stream of short term deposits, can issue risk–free short term debt, and invests in longer term assets and short term bonds. The bank manager maximizes shareholders' value, so there are no managerial agency conflicts, and bank's shareholders are risk–neutral.

A. Bank's balance sheet

On the asset side, the bank can invest in a liquid, one–period bond (a T-bill), which yields a constant risk–free rate r_f, and in a portfolio of risky assets, called loans. We denote with B_t the face value of the risk–free bond, and with $L_t \geq 0$ the nominal value of the stock of loans outstanding in period t (i.e., in the time interval $(t-1, t]$). Similarly to Zhu (2008), we make the following

Assumption 1 (Revenue function). The total revenue from loan investment is given by $Z_t \pi(L_t)$, where $\pi(L_t)$ satisfies conditions $\pi(0) = 0$, $\pi > 0$, $\pi' > 0$, and $\pi'' < 0$.

This assumption is empirically supported, as there is evidence of decreasing return to scale of bank investments.[3] Loans may be viewed as including traditional loans as well as risky securities. Z_t is a random *credit shock* realized on loans in the same time period, which captures variations in banks' total revenues as determined, for example, by business cycle conditions. Note that the choice variables B_t and L_t are set at the beginning of the period, while Z_t is realized only at the end of the period.

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[3]See for instance, Berger, Miller, Petersen, Rajan, and Stein (2005), Carter and McNulty (2005), Cole, Goldberg, and White (2004).

The maturity of deposits is set to one period. Bank maturity transformation is introduced with the following

Assumption 2. (Loan reimbursement) A constant proportion $\delta \in (0, 1/2)$ of the existing stock of loans at t, L_t, becomes due at $t + 1$.

The parameter $\delta < 1/2$ gauges the average maturity of the existing stock of loans, which is $1/\delta - 1 > 1$.[4] Thus, the bank is engaging in maturity transformation of short term liabilities into longer term investments. Under Assumption 2, the law of motion of L_t is

$$L_t = L_{t-1}(1 - \delta) + I_t, \tag{1}$$

where I_t is the investment in new loans if it is positive, or the amount of cash obtained by liquidating loans if it is negative.

To capture bank's monitoring and liquidation costs, we introduce convex asymmetric adjustment costs as in the Q-theory of investment (see e.g. Abel and Eberly (1994)) with the following

Assumption 3 (Loan Adjustment Costs). The adjustment costs function for loans is quadratic:

$$m(I_t) = |I_t|^2 \left(\chi_{\{I_t > 0\}} \cdot m^+ + \chi_{\{I_t < 0\}} \cdot m^- \right), \tag{2}$$

where $\chi_{\{\mathcal{A}\}}$ is the indicator of event $\mathcal{A}$, and $m^+ > m^- > 0$ are the unit cost parameters.

In increasing its investment in loans, the bank incurs monitoring costs, whereas bank incur liquidation costs when the loans are reduced. Adjustment costs are deducted from profit. The asymmetry in the adjustment costs $(m^- > m^+)$ captures costly reversibility: the bank faces higher costs to liquidate investments rather then expanding them. The higher costs of reducing the stock of loans can be interpreted as capturing fire sales costs incurred in financial distress.

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[4]The (weighted) average maturity of existing loans at date t, assuming the bank does not default nor it makes any adjustments on the current investment in loans, is

$$M_t = \sum_{s=0}^{\infty} s \frac{\delta L_{t+s}}{L_t} = \sum_{s=0}^{\infty} s\delta(1 - \delta)^s = \frac{1}{\delta} - 1,$$

as the residual loans outstanding at date $t + s$, $s \geq 0$, is $L_{t+s} = L_t(1 - \delta)^s$.

On the liability side, the bank receives a random amount of one-period deposits D_t at the beginning of period t, and this amount remains outstanding during the period. The interest rate on deposits is $r_d \leq r_f$, where the difference between the risk–free rate and the remuneration of deposits captures implicit costs of payment services possibly associated with deposits. The stochastic process followed by D_t is detailed below. Deposits are insured according to the following

Assumption 4 (Deposit insurance). The deposit insurance agency insures all deposits. In the event the bank defaults on deposits and on the related interest payments, depositors are paid interest and principal by the deposit insurance agency, which absorbs the relevant loss.

Under this assumption, with no change in the model, the depositor can be viewed as the deposit insurance agency itself, and its claims are risky, while deposits are effectively risk–free from depositors' standpoint. Note that the difference between the ex–ante yield on deposits and the risk–free rate also includes a subsidy that the agency provides to the bank, as the cost of this insurance is not charged to either banks or depositors.

To fund operations, the bank can issue one–period bonds and equity. For tractability, we follow Hennessy and Whited (2005) by assuming that the bank is constrained to issue fully collateralized bonds, so that their yield is the risk–free rate. We denote $B_t < 0$ the notional amount of the bond issued at $t-1$ and outstanding until t. The collateral constraint is described below.

To summarize, at $t-1$ (i.e., at the beginning of period t), after the investment and financing decisions have been made, the balance sheet equation is

$$L_t + B_t = D_t + K_t, \tag{3}$$

where K denotes the ex–ante book value of equity, or *bank capital*. In this equation, B denotes the face value of a risk–free investment when $B > 0$, and the face value of issued bond when $B < 0$.

B. Bank's cash flow

Once Z_t and D_{t+1} are realized at t, the current state (before a decision is made) is summarized by the vector $x_t = (L_t, B_t, D_t, Z_t, D_{t+1})$ as the bank enters date t with loans, bonds and deposits in amounts L_t, B_t, and D_t, respectively. *Prior* to its investment, financing and cash distribution decisions, the total internal cash available to the bank is

$$w_t = w(x_t) = y_t - \tau(y_t) + B_t + \delta L_t + (D_{t+1} - D_t). \tag{4}$$

Equation (4) says that total internal cash w_t equals bank's earnings before taxes (EBT),

$$y_t = y(x_t) = \pi(L_t)Z_t + r_f B_t - r_d D_t, \tag{5}$$

minus corporate taxes $\tau(y_t)$, plus the principal of one–year investment in bond maturing at t, $B_t > 0$ (or alternatively the amount of maturing one–year debt, $B_t < 0$) and from loans that are repaid, δL_t, plus the net change in deposits, $D_{t+1} - D_t$.

Consistently with current dynamic models of a non–financial firm (see e.g. Hennessy and Whited (2007)), corporate taxation is introduced with the following

Assumption 5 (Corporate Taxation). Corporate taxes are paid according to the following convex function of EBT:

$$\tau(y_t) = \tau^+ \max\{y_t, 0\} + \tau^- \min\{y_t, 0\}, \tag{6}$$

where τ^- and τ^+, $0 \leq \tau^- \leq \tau^+ < 1$, are the marginal corporate tax rates in case of negative and positive EBT, respectively.

The assumption $\tau^- \leq \tau^+$ is standard in the literature, as it captures a reduced tax benefit from loss carryforward or carrybacks. Note that convexity of the corporate tax function creates an incentive to manage cash flow risk, as noted by Stulz (1984).

Given the available cash w_t as defined in Equation (4) and the residual loans, $L_t(1 - \delta)$, bank's managers choose the new level of investment in loans, L_{t+1} and the amount of risk–free bonds B_{t+1} (purchased if positive, issued if negative). As a result, Equation (3) applies to B_{t+1}, L_{t+1}, D_{t+1}, and both L_{t+1} and B_{t+1} remain constant until the next decision date, $t+1$.

However, these choices may differ according to whether the bank is or is not in financial distress. If total internal cash w_t is *positive*, it can be retained or paid out to shareholders. If w_t is *negative*, the bank is in *financial distress*, since absent any action, it would be unable to honor part, or all, of its obligations towards either the tax authority, or depositors, or bondholders. When in financial distress, the bank can finance the shortfall w_t either by selling loans at fire sale prices, or by issuing bonds ($B_{t+1} < 0$), or by injecting equity capital. However, overcoming this shortage of liquidity is expensive, because all these transactions generate (either explicit of implicit) costs. In a fire sale, the bank incurs the downward adjustment cost defined in Equation (2), bond issuance is subject to a collateral restriction, and floatation costs are paid when seasoned equity is offered. We now present these latter two restrictions on the banks' financing channels.

Bank's issuance of bonds is constrained by the following:

Assumption 6 (Collateral constraint). If $B_t < 0$, the amount of bond issued by the bank must be fully collateralized. In particular, the constraint is

$$L_t - m(-L_t(1 - \delta)) + \pi(L_t)Z_d - \tau(y_t^{\min}) + B_t(1 + r_f) - D_t(1 + r_d) + D_d \geq 0, \qquad (7)$$

where Z_d is the worst possible credit shock (i.e., the lower bound of the support of Z), D_d is the worst case scenario flow of deposits, and $y_t^{\min} = \pi(L_t)Z_d + r_f B_t - r_d D_t$ is the EBT in the worst case end–of–period scenario for current L_t, B_t and D_t.

The constraint in (7) reads as follows: the end–of–period amount $B_t(1 + r_f) < 0$ that the bank has to repay must not be higher than the after–tax operating income, $\pi(L_t)Z_d - r_d D_t - \tau(y_t^{\min})$ in the worst case scenario, plus the total available cash obtained by liquidating the loans, $L_t - m(-L_t(1 - \delta))$, plus the flow of new deposits in the worst case, D_d, net of the claim of current depositors, D_t. The proceeds from loans liquidation are the sum of the loans that will become due, $L_t \delta$, plus the amount that can be obtained by a forced liquidation of the loans, $L_t(1 - \delta)$ net of the adjustment cost $m(-L_t(1 - \delta))$, as from Equation (2).[5]

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[5]As per Assumption 9 introduced below, the support for deposits and credit shock processes is compact, implying that the collateral constraint is well defined.

We denote with $\Gamma(D_t)$ the feasible set for the bank when the current deposit is D_t; i.e., the set of (L_t, B_t) such that condition (7) is satisfied, if $B_t < 0$, no restrictions being imposed when $B_t \geq 0$:

$$\Gamma(D_t) = \{(L_t, B_t) \mid$$
$$\frac{L_t - m(-L_t(1-\delta)) + D_d + \pi(L_t)Z_d(1 - \tau(y_t^{\min}))}{1 + r_d(1 - \tau(y_t^{\min}))} + B_t \frac{1 + r_f(1 - \tau(y_t^{\min}))}{1 + r_d(1 - \tau(y_t^{\min}))} \geq D_t,$$
$$B_t < 0\} \cup \{B_t \geq 0\}. \quad (8)$$

In the plane (L_t, B_t), the lower boundary of $\Gamma(D_t)$, when $B_t < 0$, is convex due to concavity of the revenue function. This means that a bank can fund more investment in risky loans by issuing more risk–free one period bonds. However, this investment has decreasing return to scale, and at some point the net return of a dollar raised by issuing bonds and invested in loans becomes negative.

Bank's costs on equity issuance are introduced to capture information asymmetries and underwriting fees and are modeled in a standard fashion (see e.g. Cooley and Quadrini (2001)) with the following

Assumption 7 (Equity floatation costs). The bank raises capital by issuing seasoned shares incurring a proportional floatation cost $\lambda > 0$ on new equity issued.

As a result of the choice of (L_{t+1}, B_{t+1}), the residual cash flow to shareholders at date t is

$$u_t = u(x_t, L_{t+1}, B_{t+1}) = w_t - B_{t+1} - L_{t+1} + L_t(1 - \delta) - m(I_{t+1}). \quad (9)$$

If u_t is positive, it is distributed to shareholders (either as dividends or stock repurchases). If u_t is negative, it equals the amount of newly issued equity inclusive of the higher cost due to underpricing. Hence, the actual cash flow to equity holders is

$$e_t = e(x_t, L_{t+1}, B_{t+1}) = \max\{u_t, 0\} + \min\{u_t, 0\}(1 + \lambda). \quad (10)$$

Figure 1 depicts the evolution of the state variables and the related bank's decisions when the bank is solvent.

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Lastly, bank's insolvency occurs according to the following

Assumption 8 (Insolvency). In the case of default, bank shareholders exercise the limited liability option (i.e., equity value is zero), and the assets are transferred to the deposit insurance agency, net of verification and reorganization costs in proportion $\gamma > 0$ of the face value of deposits, D_t. Right after default the bank is reorganized as a new entity endowed with deposits D_{t+1} and new capital $K_{t+1} = D_u - D_{t+1} \geq 0$, where D_u is the upper bound of deposit process. The restructured bank invests initially only in risk–free bonds, $B_{t+1} = D_u$, so that $L_{t+1} = 0$.

This assumption embeds three features. First, restructuring costs are proportional to the size of the bank, proxied by D_t. Second, since default is irreversible, a new bank financed with initial public capital is formed to replace the defaulted bank in order to preserve intermediation services in the economy. Third, the capital injected by the government in the new bank is assumed to be financed with tax proceeds. Since no new shares are issued in the open market, no floatation cost is incurred.

The probabilistic assumptions of our model are as follows. There are two exogenous sources of uncertainty: the credit shock on the loan portfolio, Z, and the funding available from deposits, D. Denote with $s = (Z, D)$ the pair of state variables, and with $\mathcal{S}$ the state space.

Assumption 9. The state space $\mathcal{S}$, is compact. The random vector s evolves according to a stationary and monotone (risk–neutral) Markov transition function $Q(s_{t+1} \mid s_t)$ defined as

$$Z_t - Z_{t-1} = (1 - \kappa_Z)\left(\overline{Z} - Z_{t-1}\right) + \sigma_Z \varepsilon_t^Z \tag{11}$$

$$\log D_t - \log D_{t-1} = (1 - \kappa_D)\left(\log \overline{D} - \log D_{t-1}\right) + \sigma_D \varepsilon_t^D. \tag{12}$$

The error terms ε_t^Z and ε_t^D are i.i.d and have jointly normal truncated distribution with correlation coefficient ρ.[6]

In the above equations, κ_Z is the persistence parameter, σ_Z is the conditional volatility, and $\overline{Z}$ is the long term average of the credit shock; κ_D is the persistence parameter for the deposit process, σ_D is the conditional volatility, and $\overline{D}$ is the long term level of deposits.

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[6]As detailed in Appendix C, in the simulations the support of each state variable is within three times the unconditional standard deviation of each marginal distribution around the long term average.

C. The unregulated bank program and the valuation of securities

Let E denote the market value of bank's equity. Given the state, $x_t = (L_t, B_t, D_t, Z_t, D_{t+1})$, bank's equity value is the result of the following program

$$E_t = E(x_t) = \max_{\{(L_{i+1}, B_{i+1}) \in \Gamma(D_{i+1}), i=t,\dots,T\}} \mathbb{E}_t \left[\sum_{i=t}^{T} \beta^{i-t} e(x_i, L_{i+1}, B_{i+1}) \right], \tag{13}$$

where $\mathbb{E}_t[\cdot]$ is the expectation operator conditional on D_t, on the state variables at t, (Z_t, D_{t+1}), and on the decision (L_{t+1}, B_{t+1}); β is a discount factor (assumed constant for simplicity); (L_{i+1}, B_{i+1}) is the decision at date i, for $i = t, \dots$, and T is the default date. Because the model is stationary and the Bellman equation involves only two dates (the current, t, and the next one, $t+1$), we can drop the time index t and use the notation without a prime for the current value of the variables, and with a prime to denote end–of–period value of the variables. The value of equity satisfies the following Bellman equation

$$E(x) = \max \left\{ 0, \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \beta \mathbb{E} \left[E(x') \right] \right\} \right\}. \tag{14}$$

Compactness of the feasible set of the bank and standard properties of the value function are described in Appendix A.

When the bank is solvent, the value of equity satisfies the following Bellman equation:

$$E(x) = \max_{(L', B') \in \Gamma(D')} \left\{ e(x, L', B') + \beta \mathbb{E} \left[E(x') \right] \right\}. \tag{15}$$

We denote with $(L^*(x), B^*(x))$ the optimal policy when the bank is solvent. When it is insolvent, shareholders exercise the limited liability option, which puts a lower bound on E at zero. The default indicator function is denoted $\Delta(x)$.[7]

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[7]Given the collateral constraint, in the model default of the non–regulated bank may occur only when $B_t > 0$.

We solve equation (14) to determine the value of equity, the optimal policy including the optimal default policy, Δ, as a function of the current state, x. We denote φ, the state transition function based on the optimal policy:

$$\varphi(x) = \begin{pmatrix} L^* \\ B^* \\ D' \end{pmatrix} (1 - \Delta) + \begin{pmatrix} 0 \\ D_u \\ D' \end{pmatrix} \Delta, \tag{16}$$

meaning that the new state is (L^*, B^*, D') if the bank is solvent, and $(0, D_u, D')$ if the bank defaults and a new bank is started endowed with seed capital $D_u - D'$ and deposits, D', and a cash balance D_u, and no loans. This bank will revise its investment (together with the financing) policy in the following decision dates.

The end–of–year cash flow from current deposits, D_{t+1}, for a given realization of the exogenous state variables, (Z_{t+1}, D_{t+2}), and on the related optimal policy, is

$$f(x_{t+1} \mid \varphi(x_{t+1})) = D_{t+1}(1 + r_d)(1 - \gamma \Delta(x_{t+1})). \tag{17}$$

Hence, the ex–ante fair value of newly issued deposits at t, from the viewpoint of the deposit insurance agency (i.e, incorporating the risk of bank's default), is

$$F(x_t) = \beta \mathbb{E}_t \left[f(x_{t+1} \mid \varphi(x_{t+1})) \right] = \beta D_{t+1}(1 + r_d) \left(1 - \gamma P(x_t) \right), \tag{18}$$

where $P(x_t) = \mathbb{E}_t \left[\Delta(x_{t+1}) \right]$ is the conditional default probability. Dropping the dependence on the calendar date,

$$F(x) = \beta D'(1 + r_d) \left(1 - \gamma P(x) \right). \tag{19}$$

D. Efficiency and welfare metrics

A standard valuation concept is the market value of bank assets $E(x) + F(x)$, which includes current cash holdings, B. Yet, the market value of bank's assets does not necessarily capture the role of banks as maturity transformers of liquid liabilities into longer term *productive* assets (loans). One of the key economic contributions of banks identified in the literature is their role in efficiently intermediating funds toward their best productive use (see e.g. Diamond (1984)

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and Boyd and Prescott (1986)). But banks play no such role if they just raise funds to acquire risk–free (cash–equivalent) bonds. While bonds investment helps reducing the costs triggered by high cash flows volatility, it is not providing necessarily efficient intermediation. Thus, the *enterprise value* of the bank, defined as $V(x) = E(x) + F(x) - B$, is a suitable metric of bank efficiency, as it captures banks ability to create "productive" intermediation.[8]

In our risk neutral world, a metric proxying welfare is "social value", defined as the sum of the values to the government and to the bank's stakeholders, and which is constructed as follows. A first component of social value is the value of the net payoff to the government, defined by the recursive equation

$$G(x) = (1 - \Delta(x)) \left(\tau(y') + \beta \mathbb{E}[G(x')] \right) - \Delta(x) \left(\gamma D + K_{t+1} \right). \tag{20}$$

with Δ denoting the default indicator at x. Equation (20) reads as follows: so long as the bank is solvent ($\Delta = 0$), taxes are collected, where $\mathbb{E}[G(x')]$ is the present value of future tax proceeds. If the bank is insolvent ($\Delta = 1$), then the government incurs direct bankruptcy costs γD, and injects new equity capital $K_{t+1} = D_u - D_{t+1}$.

A second component of social value is the present value of expected equity floatation costs:

$$FC(x) = -\lambda \min \{u, 0\} + \beta \mathbb{E} \left[FC(x') \right], \tag{21}$$

where u is defined in equation (9). While underpricing of newly issued equity affects current shareholders, it benefits new shareholders because they can buy a share in the bank's capital for a lower price. Therefore, equity floatation costs are not deadweight costs but just a wealth transfer from old to new shareholders.

Finally, the social value of the bank is the sum of values to all the stakeholders in the model

$$SV(x) = E(x) + D - B + G(x) + FC(x), \tag{22}$$

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[8]For the use of enterprise value as a metric of efficiency in the context of dynamic models of non–financial firms, see e.g. Gamba and Triantis (2008) and Bolton, Chen, and Wang (2011).

where D is the book value of current deposits. In essence, $SV(x)$ captures the net impact on welfare of stricter constraints on bank policies, which in general may reduce the value of the bank's loans and the flow of corporate taxes, but can also abate expected bailout costs.

IV. Bank regulation

In this section we define capital and liquidity requirements, illustrate their implications for banks feasible choice sets, and examine some trade–offs on optimal policies they impose in the context of a highly simplified version of our model.[9]

A. Capital requirement

In our model, Basel–type capital regulation establishes a lower bound K_d on the book value of equity, set by the regulator as a function on bank's risk exposure at the beginning of the period. In particular, this requirement is a weighted average of banks risks. Since our model has just one composite risky asset, we set the weight applied to loans equal to 100%. Thus, in our setting the *required* capital K_d is at least a proportion k of the principal of the loans at the beginning of the period, L, or $K_d = kL$. This requirement is equivalent to constraining net worth to be positive ex–ante. Given the definition of bank capital in (3), under the capital requirement, the bank's feasible choice set is

$$\Theta(D) = \{(L, B) \mid (1 - k)L + B \geq D\}. \tag{23}$$

When we compare the feasible choice set under the collateral constraint in Equation (8) with the feasible set under the capital requirement, in general neither $\Gamma(D) \subset \Theta(D)$ nor $\Theta(D) \subset \Gamma(D)$ in a proper sense. Hence, the capital requirement may (or may not) restrict the bank's feasible policies, depending on the values of the parameters. Figure 2 shows how the capital requirement is related to the collateral constraint for a specific set of parameters.

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[9]There is a literature examining the impact of bank closure rules that may augment standard regulation. In Appendix B we examine one such rule, and show that it may complement capital regulation under some assumptions about bank reorganization costs.

If the bank is short term borrowing, $B < 0$, for a given D the capital constraint results in a restriction of the bank's choice set if $\Theta(D) \subset \Gamma(D)$. This is equivalent to

$$\frac{L - m(-L(1-\delta)) + D_d + \pi(L)Z_d(1 - \tau(y^{\min}))}{1 + r_f(1 - \tau(y^{\min}))} \geq (1-k)L. \qquad (24)$$

Since the inequality above is independent of D, what follows holds for any current D. If we assume a constant corporate tax rate (in place of two tax rates: τ^+ and τ^-) for the sake of simplicity, for a large range of values of the model parameters and of L, the above inequality is satisfied. This means that the capital requirement restricts the bank's policy. Alternatively, if the bank is short term lending, $B \geq 0$, then the capital requirement restricts the choice set if $L < D/(1-k)$, because it forces the bank to have a fairly large cash balance B, while the constraint is not binding if $L \geq D/(1-k)$.

The Bellman equation for the equity value of a currently solvent bank under a capital requirement is given by Equation (15), the only difference being a feasible set $\Gamma(D') \cap \Theta(D')$ in place of $\Gamma(D')$, since the bank is forced to comply ex–ante with the capital requirement. However, at the end of the period, when the credit shock on existing loans Z' and the new deposit D' are realized, the bank may still face default risk if the innovations of the state variables are particularly unfavorable.

B. Liquidity requirement

Current Basel III regulation introduces a mandatory *liquidity coverage ratio*: banks would be prescribed to hold a stock of high quality liquid assets such that the ratio of this stock over what is defined as a net cash outflows over a 30-day time period is not lower than a certain percentage threshold. In turn, the net cash outflow is expected to be determined by what would be required to face an acute short term stress scenario specified by supervisors. Banks would need to meet this requirement continuously as a defense against the potential onset of severe liquidity stress.

In our model, the stock of high quality liquid assets over the net cash outflows over a period is given by the total cash available at the end of the period over the total net cash flow in the

worst case scenario for both credit shocks and deposit flows. Formally, this liquidity coverage ratio should be not lower than a level ℓ defined by the regulator, or

$$\frac{\delta L + Z_d \pi(L) - \tau(y^{\min}) + B(1 + r_f)}{D(1 + r_d) - D_d} \geq \ell. \tag{25}$$

Hence, the feasible set for a bank complying with the liquidity requirement is

$$\Lambda(D) = \left\{ (L, B) \mid \frac{\delta L + \ell D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{\ell(1 + r_d) - \tau(y^{\min}) r_d} + B \frac{(1 + r_f(1 - \tau(y^{\min})))}{\ell(1 + r_d) - \tau(y^{\min}) r_d} \geq D \right\}. \tag{26}$$

For a bank that is short term borrowing, $B < 0$, the liquidity constraint restricts the feasible choice set, or $\Lambda(D) \subset \Gamma(D)$, if

$$\frac{L - m(-L(1 - \delta)) + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{1 + r_f(1 - \tau(y^{\min}))} \geq \frac{\delta L + D_d + Z_d \pi(L)(1 - \tau(y^{\min}))}{(1 + r_f(1 - \tau(y^{\min})))},$$

or equivalently, if $L(1 - \delta) \geq (L(1 - \delta))^2 m^-$. This is indeed the case for a wide range of parameters and a large set of values of L. Moreover, the liquidity constraint always restricts the feasible choice set when the bank is short term lending, $B > 0$.

Figure 2 shows a comparison of the liquidity requirement to the collateral constraint for a specific choice of parameters. The liquidity constraint may turn out to restrict the bank's feasible choice set relative to the collateral constraint for a wide range of parameter values.

In sum, when considered together, capital and liquidity constraints may create considerable restrictions on bank's feasible choices, as we show momentarily.

C. Bank regulation in a simplified version of the model

To illustrate some trade–offs on bank optimal policies implied by regulatory restrictions in the simplest way, we collapse our model to two periods. Now t is the decision date, $t + 1$ is the final date, and the bank initial conditions are determined at $t - 1$.

We make the following simplifying assumptions. There are no taxes, no adjustment costs, deposits are deterministic and constant ($D_t = D_{t+1} = D > 0$, and $D_{t+2} = 0$ since $t + 1$ is the last period), $\delta = 0$, $\beta \leq (1 + r_f)^{-1}$, and $r_d) = r_f$. Furthermore, we assume a simple two–

point credit shock distribution: Z^H with probability $p \in (0,1)$, and Z^L otherwise, where Z^L is such that $Z_d = \frac{Z^L L_{t+1}}{\pi(L_{t+1})}$, with $Z^H > 0 \geq Z^L \geq -1$. Under these assumptions, the collateral constraint for $B_{t+1} < 0$, denoted with (C), the capital constraint, denoted with (K), and the liquidity constraint, denoted with (L), are:

$$B_{t+1} \geq \frac{r_d}{1+r_f}D - \frac{1+Z^L}{1+r_f}L_{t+1} \tag{C}$$

$$B_{t+1} \geq D - (1-k)L_{t+1} \tag{K}$$

$$B_{t+1} \geq \frac{\ell r_d}{1+r_f}D - \frac{Z^L}{1+r_f}L_{t+1} \tag{L}$$

Recall that as per equation (10), the cash flow to shareholders is $u_t = w_t + L_t - B_{t+1} - L_{t+1}$ if $u_t > 0$, and $u_t(1+\lambda)$ if $u_t < 0$, as the bank issues new equity at a cost $\lambda > 0$. We discuss both cases at the same time by setting λ to either zero or to a positive value.

The bank chooses (L_{t+1}, B_{t+1}) to maximize

$$
\begin{aligned}
e_t + \beta \mathbb{E}_t\left[e_{t+1}\right] = {} & (w_t + L_t)(1+\lambda) - (1+\lambda-\beta p(1+r_f))B_{t+1} - (1+\lambda)L_{t+1} \\
& + \beta \left[p\left(Z^H \pi(L_{t+1}) - (1+r_d)D + L_{t+1}\right) \right. \\
& \left. + (1-p)\max\left\{0, (1+Z^L)L_{t+1} + (1+r_f)B_{t+1} - (1+r_d)D)\right\} \right] \quad (27)
\end{aligned}
$$

Since $1+\lambda > \beta p(1+r_f)$, it is optimal to maximize debt ($B_{t+1} < 0$), because in the good state profits are increasing in debt, while in the bad state losses are bounded to be non-negative by limited liability. This implies that at most one of the constraints (C), (K), and (L) will be binding.

The unregulated bank maximizes (27) subject to constraint (C). Inserting (C) into (27), the term $\max\{\cdot\}$ in the third line of (27) vanishes, and the optimal loan level L_{t+1}^c satisfies the (necessary and sufficient) first order condition

$$\beta p Z^H \pi'(L_{t+1}^c) = 1 + \lambda - \beta p - (1+\lambda-\beta p(1+r_f))\frac{1+Z^L}{1+r_f}. \tag{28}$$

Suppose now that the capital constraint (K) is tighter than (C), that is, (K) is binding. The third line of (27) is $\max\{0, (1+Z^L - (1+r_f)(1-k))L_{t+1}\}$.

The optimal loan investment when (K) is binding, defined by L_{t+1}^k, satisfies:

$$\beta p Z^H \pi'(L_{t+1}^k) = 1 + \lambda - \beta p - (1 + \lambda - \beta p(1 + r_f))(1 - k). \tag{29}$$

By comparing the right–hand–sides of (28) and (29), it is straightforward to verify that $L_{t+1}^k > L_{t+1}^c$ when $(1 + Z^L) < (1 + r_f)(1 - k)$, due to the strict concavity of function π.

Observe that the inequality $(1 + Z^L) < (1 + r_f)(1 - k)$ may hold for relatively low levels of k, but it is reversed for values of k close to one. Thus, there may exist a threshold value $\hat{k}$ such that $L_{t+1}^k < L_{t+1}^c$ for all $k > \hat{k}$. In other words, under a sufficiently "mild" capital constraint (or $k < \hat{k}$), lending can be higher than in the unregulated case, and this is true whether $\lambda = 0$ or $\lambda > 0$.

Thus, when (K) is binding, depending on parameters, lending could be higher than in the unregulated case under mild capital requirements even though borrowing is lower ($B_{t+1}^k > B_{t+1}^c$ holds when constraint (K) is more stringent than (C)). This is because the capital requirement lowers the return of holding cash relative to the expected return on loan investment.

In sum, there may exist parameter configurations such that the relationship between loans and capital requirements is inverted U-shaped. Interestingly, this result may hold for any $\lambda \geq 0$.

Consider now the addition of a liquidity requirement to the capital requirement. Inspecting constraints (L) and (C), it can be seen that the liquidity constraint is always tighter than the collateral constraint when $\ell \leq 1$. Moreover, suppose that the liquidity constraint (L) is tighter than (K) at the optimal choice L_{t+1}^k, that is, (L) is binding. Replacing (L) in (27), the $\max\{\cdot\}$ term turns into $\max\{0, L_{t+1} + (r_f(\ell - 1) - 1)D\}$.

If at the optimal solution $L_{t+1} + (r_f(\ell - 1) - 1)D \leq 0$, then L_{t+1}^ℓ satisfies

$$pZ^H \pi'(L_{t+1}^\ell) = r_f - (1 - p)Z^L. \tag{30}$$

Otherwise, L_{t+1}^ℓ satisfies

$$pZ^H \pi'(L_{t+1}^\ell) = r_f - Z^L. \tag{31}$$

Comparing (28) with (30), it is easy to verify that the right hand side of (29) is always strictly lower than that of (30) and (31). By strict concavity of the revenue function, this implies that $L_{t+1}^{\ell} < L_{t+1}^{k}$: the liquidity constraint unambiguously reduces lending relative to the bank subject to a (binding) mild capital constraint. Comparing (28) with (31), the same result is obtained if p is close to 1. Thus, there exist parameter configurations such that the liquidity constraint reduces lending relative to the capital constraint. Note again that this result holds for any $\lambda \geq 0$.

Summing up, we have used a radical simplification of our model to illustrate cases — depending on parameters— in which lending may increase under a capital requirement, and may exhibit an inverted U-shaped relationship with the stringency of capital regulation, while liquidity requirements may reduce lending. These conclusions may or may not hold under complex dynamic trade-offs arising from multiple sources of risk, asymmetric adjustment costs, and endogenous choices of default that characterize the full version of our model, whose solutions are analyzed next.

V. The impact of bank regulation and taxation

Our evaluation of the impact of bank regulation and taxation on banks' optimal policies and the two metrics defined previously proceeds as follows. First, we describe a set of benchmark parameters calibrated using selected statistics from U.S. banking data, some previous studies, and current regulatory and tax parameters under which we carry out our simulation exercise (Subsection A). Then, we present the results of the optimal bank policies and relevant metrics for given realized states (Subsection B). Subsection C reports the results a steady state analysis, where we report statistics of the steady state distributions of optimal policies and our metrics under different sets of bank regulation and taxation parameters, and examine the differential impact of bank regulations for different parameters indexing equity issuance costs, fire sale costs and the degree of bank maturity transformation.

A. Calibration

Our calibration of the model is based on three sets of parameters, summarized in Table I. The first set comprises parameters of the two exogenous state variables. We estimated the VAR of equations (11) and (12) using U.S. yearly aggregate time series for the period 1983-2009 for the entire universe of banks included in the Federal Reserve Call Reports constructed by Corbae and D'Erasmo (2011). The shock process was proxied by the return on bank investments before taxes, given by the ratio of interest and non-interest revenues to total lagged assets. As can be seen in Table I, the shock process exhibits high persistence and the correlation with the process of (log)deposit is negative. Estimates of the autocorrelation process for (log) deposit produced estimates closed to unity, indicating the possibility that such process has a unit root. To guarantee convergence of the fixed point algorithm, we set this parameter equal to 0.95.

The second set of parameters is taken from previous research. The annual discount factor β is 0.95, equal to that used by Zhu (2008) and Cooley and Quadrini (2001). The risk–free rate, r_f, is set to 2.5% and the deposit rate, r_d, is set to 0. These values are consistent with the average effective cost of funds documented in Corbae and D'Erasmo (2011). With regard to corporate taxation, recall that the tax function is defined by the marginal tax rates, τ^+ and τ^-, for positive and negative income, respectively. Since we do not explicitly consider dividend and capital gain taxation for shareholders or interest taxation for depositors and bond holders, the two marginal rates for corporate taxes are to be considered net of the effect of personal taxes. For this reason we choose $\tau^+ = 15\%$, which is close to the values determined by Graham (2000) for the marginal tax rate. The marginal tax rate for negative income is $\tau^- = 5\%$ to allow for convexity in the corporate tax schedule.

Furthermore, the proportional bankruptcy cost is $\gamma = 0.10$, This is a value close to the (structural) estimate of 0.104 for this cost based on U.S. non–financial firms found by Hennessy and Whited (2007). Since this estimate is based on nonfinancial firms, it can be viewed as a lower bound for bankruptcy costs incurred in the financial sector. The annual percentage of reimbursed loan is 20%, so that the average maturity of outstanding loans is 4 years, in line with the assumption made by Van den Heuvel (2009). The floatation cost for seasoned equity issuance is 10%. This means the bank incurs a significant transaction cost to tap the equity capital market when in financial distress.

We specify the revenue function from loan investment as $\pi(L) = L^\alpha$, as in Zhu (2008), and set our base case value for α to 0.90, which is in line with the one used in other papers. Lastly, we set $m^+ = 0.03$ and $m^- = 0.04$ by matching two moments from empirical data. The first moment is the average Bank Credit over Deposit ratio, in which bank credit is loans and other financial investments. From our dataset, this is 1.271. The second moment we match is bank's book leverage, or deposits plus other financing liabilities over loans and other financial investments. In the data, the average book leverage is 0.89. The corresponding unconditional moments from a Monte Carlo simulation of the model with the selected parameters are respectively 1.1098 and 0.9031.

The third set of parameters is based on regulatory prescriptions. In our case, these are the ratio of capital to risk–weighted assets and the liquidity coverage ratio. The benchmark capital ratio k is set to 4%, while the benchmark liquidity ratio is set to $\ell = 0.2$.

B. State-dependent analysis

In this section we illustrate the impact of bank regulations on optimal policies and our metrics of bank efficiency and welfare for given realizations of the states. Although based on a specific realization of the state variables, this analysis is instrumental in evaluating the impact of bank regulations along the business and liquidity cycles. Three cases are considered: the unregulated bank, the bank subject to capital regulation only, and that subject to both capital regulation and liquidity requirements.

While many states can be possibly chosen, we set our analysis at the steady state for both deposits ($D = 2$) and credit shock ($Z = 0.0717$), while choosing $B = 0$ to avoid the impact of current liquidity, and $L = 4.1$, which is very close to the unconditional median of L for several versions of the model. As a result, bank's capital is $K = 2.1$. Figure 3 depicts the impact of regulatory restrictions on loan investment and short term investment and financing. Tables II and III report average capital ratios and liquidity coverage ratios for a solvent bank under the three cases as functions of different levels of credit shocks and different levels of bank capital at a point in time.

Consider first the unregulated bank. Figure 3 shows that when D' is low (liquidity shock) or Z is low (credit shock), the bank reduces short term debt and liquidates loans. Conversely,

with an expansion both in liquidity (high D') or more favorable credit shocks (high Z), there is an increase in loan investment. It is important to note that the loan policy of an unregulated bank is *pro-cyclical*, since loans vary positively with credit and liquidity conditions. Thus, pro-cyclicality of lending is a feature of an optimal policy *independently of capital requirements*. As shown in Table II and Table III, the resulting average capital and liquidity coverage ratios are *negative* in all cases. This means that the unregulated bank takes an exposure in loans so that, in the worst case scenario for the credit and the liquidity shocks, a forced liquidation of loans is likely needed. In essence, the bank is trading off liquidation costs with the benefits of a larger investment in loans.

Consider now the bank subject to capital regulation only. Relative to the unregulated bank, Figure 3 shows that this bank takes less debt This is one possible scenario identified in the simplified model under a mild capital requirement. Under the benchmark parameterization, equation (23) implies that the bank satisfies the capital requirement by increasing loans at a rate proportionally higher than the capital ratio coefficient. In essence, a "mild" capital requirements induces a reduction of the rate of return of holding cash relative to the expected returns on loans, prompting a higher investment in loans. With regard to lending pro-cyclicality, Figure 3 also shows that loans vary positively with credit and liquidity conditions, as in the case of the unregulated bank. However, under the benchmark parameterization, the pro-cyclicality of lending relative to the unregulated case does not appear to be significantly enhanced by capital requirements, as the slope of the relevant loan policies are almost parallel. This result is similar to those obtained by Peura and Keppo (2006) and Zhu (2008). As shown in Table II, the capital ratio is constantly higher than the prescribed level of 4%, due to the (shadow) cost of the capital requirement, which prompts the bank to use retained earnings as a precautionary tool to avoid hitting the constraint. Moreover, this ratio is increasing with respect to the credit shock, and in general it is lower the higher the current K. As shown in Table III, the average liquidity coverage ratio of this bank is higher than its unregulated counterpart, owing to the higher level of loan investment and a less than proportional increase in short term financing, which raises the numerator of Equation (25).

By adding a liquidity requirement to a capital requirement, Figure 3 shows that both debt and investment levels shrink substantially. As we have seen with our example of a simplified version of the model, and as is apparent in Figure 2, the liquidity requirement turns out

to be far more restrictive than the capital requirement for large enough L, correspondingly forcing the bank to reduce both debt and loan investment. The dominant tightness of the liquidity requirement is also reflected in the average capital ratios and the liquidity coverage ratios reported in Tables II and III respectively. The average capital ratio under a liquidity requirement becomes inflated relative to the previous case, and is pushed up by a relatively large net bond holding (the numerator) and a lower investment in loans (the denominator). Note that this mechanism is totally different from that induced by capital regulation: in that case, the capital ratio is ultimately pushed up by retained earnings and possibly equity issuance. Not surprisingly, the average liquidity coverage ratio is higher than the prescribed level ($\ell = 20\%$), since the (shadow) cost associated with the liquidity constraint forces the bank to hold precautionary cash to avoid hitting that constraint.

Turning to our efficiency and welfare metrics, Figure 4 shows enterprise and social values divided by the corresponding values for the unregulated case for the bank subject to capital regulation, as well as that subject to both capital regulation and liquidity requirements. With regard to capital regulation, there is a value loss in both metrics, since the relevant ratios are all below one. The value losses associated with capital regulation are more severe the lower are the levels of new deposits and the credit shock. This is because in a downturn, during which credit quality deteriorates, the bank is forced to liquidate more loans, thereby incurring in significant liquidation (fire sale) costs. With regard to the bank subject to capital regulation and liquidity requirements, Figure 4 also shows that the losses of efficiency and social value are significantly larger than those under capital regulation only, reflecting the significant stringency of the liquidity requirement.

C. Steady state analysis

We now turn to the analysis of the steady state distribution of optimal policies and metrics of efficiency and welfare, obtained through numerical results based on Monte Carlo simulation.

Recall that a bank is represented by the set of parameters governing credit and liquidity shocks, its revenue function, the maturity of its loan portfolio, and loan adjustment costs. We subject this bank to a large number of shocks, and compute statistics of its optimal policies and our metrics. These statistics can be interpreted in two complementary ways: they represent

the steady state behavior of a "theoretical" bank subject to all possible shock realizations, or they can be viewed as representing the average optimal steady state policies and metrics of a banking industry in which each bank is represented by a particular path of shocks.

Specifically, we simulate a random sample of 10,000 paths of the exogenous shocks (or 10,000 possible scenarios this bank may face) of 100 annual periods each, and apply the optimal policy to these random paths. To better approximate the steady state distribution of policies and metrics and reduce the dependence on the initial state, we drop the first 50 periods. Hence, we obtain a panel of 50 years for the 10,000 scenarios of a bank, or 10,000 banks in the industry, and report the mean and standard deviation of the relevant policies and metrics. Here and in the sequel, in illustrating our results, all differences in means we note are also statistically significant according to standard tests, unless otherwise indicated.

C.1. The impact of bank regulation

Table IV presents these statistics when the bank is unregulated, when it is subject to capital requirements only, or when it is subject to both capital and liquidity requirements for benchmark parameters and deviations from benchmarks.

Compared to the unregulated bank, the bank operating under a "mild" capital requirement (base case, or $k = 4\%$) invests more in loans and holds more debt, with the increase in debt significantly smaller than the increase in loans. Since deposits are not a control variable and follow the same exogenous process of the unregulated case, the regulated bank can fund this additional investment increasing retained earnings and equity issuance. Specifically, from equations (9) and (10), given the choice of L_{t+1} and B_{t+1}, more earnings are retained from w_t or shares (incurring floatation costs λ) are issued if w_t is negative.

As a result of these optimal policies, the bank holds a higher capital ratio than that prescribed by regulation. This is because the positive shadow price of the capital constraint forces the bank to manage its earnings and investments so as to maintain a capital buffer to minimize the risk that the constraint is hit. When such constraint is hit, it can become too expensive for the bank to inject new equity capital to comply with the regulatory restriction. These results are consistent with the empirical evidence regarding banks holding (ex–ante) capital larger than required by regulations, as in Flannery and Kasturi (2008). Importantly, capital

regulation results in a bank with a lower probability of default than in the unregulated case. Thus, a capital requirement is successful in abating default risk under deposit insurance.

Remarkably, mild capital regulation implies an *increase* in the efficiency of intermediation, since the enterprise value is larger than the one attained by the unregulated bank. This suggests that the unregulated bank, relative to the bank subject to capital regulation, is inefficiently under-investing in loans, as it prefers to payout earnings rather than retaining them to support loan investment. The social value of the bank is also larger than in the unregulated case, due to both higher enterprise and government values. The higher government value stems from higher tax receipts accruing from a larger taxable profit base, as well as from a lower probability of bank default, which reduces expected bailout costs.

However, capital regulation needs to be "mild" for the efficiency and welfare benefits of capital regulation to materialize. An increase in the capital requirement (from $k = 4\%$ to $k = 12\%$) results in a significant *reduction* in loans as well as indebtedness. Recall that the bank was satisfying a "mild" capital requirement by an increase in loans financed in small part with debt, and in larger part with higher retained earnings or equity issuance. However, when the capital requirement becomes too stringent, such a strategy becomes too costly: payouts need to be significantly reduced, and it becomes too expensive to raise new equity capital due to equity issuance costs. Thus, the bank is compelled to reduce both loan investments and further reduce debt. Furthermore, the increase in the capital requirement impacts negatively on bank's efficiency and social value. The bank enterprise value declines significantly, and such decline accounts for the bulk of the decline in bank's social value. This means that the benefits of a mild capital regulation (relative to the unregulated case) disappear as the cost of capital regulation increases more than proportionally with its stringency beyond a certain threshold. Equivalently, this result suggests the existence of an optimal level of regulatory capital as a function of banks' characteristics.

Turning to the case of a bank subject to both capital regulation and liquidity requirement, results are significantly different from the previous ones. Relative to bank subject only to a "mild" capital requirements (the base case), lending is significantly reduced, and enterprise, government and social values are all significantly lower in the base case with capital and liquidity restrictions. As already noted, the liquidity requirement results in an over–bloated

book capital ratio. Such a ratio may be viewed as an indication of a safe but a very inefficient bank or banking industry.

Furthermore, an increase in the capital requirement (from $k = 4\%$ to $k = 12\%$) for the bank subject to a liquidity requirement ($\ell = 20\%$) implies negligible changes in both loans and in indebtedness, as well as in the efficiency and welfare metrics. This is because the dominance of the liquidity requirement in constraining bank's decisions allows the bank to respond to an increase in the capital requirement only in a limited way. On the other hand, an increase in the liquidity requirement (from $\ell = 20\%$ to $\ell = 50\%$, with constant $k = 4\%$) again reduces loans and short term debt due to the mechanisms already discussed. In all these cases, both the efficiency and welfare metrics are significantly reduced.

Two key results emerge from this analysis. First, capital requirements can achieve the twin objectives of abating banks' incentives to take on excessive risk induced by deposit insurance and limited liability, and increase efficiency and welfare. However, if these requirements are too strict, then the benefits of capital regulation disappear, and the associated efficiency and social costs may be significant.

Second, liquidity requirements are associated with significant costs in terms of reductions in lending, bank efficiency and welfare. As noted earlier, these costs might be justified if negative externalities due to excessive bank shortterm indebtedness may give rise to systemic risk consequences. However, measuring the benefits arising from a role of liquidity requirements in preventing systemic financial crises is difficult and an open research area. In this regard, our contribution is to provide an estimate of the costs associated with these requirements: the magnitude of the benefits associated with liquidity requirements arising from systemic concern should at least compensate for the costs we have identified.

C.2. The impact of taxation

A variety of taxes on financial institutions have been recently proposed or enacted.[10] The justifications of these taxes are: a) to address the budgetary costs of the crisis (ex-post), b) to create resolution funds to address future distress (ex-ante), c) to better align bank managers' incentives to target levels of bank risks, and d) to control systemic risk in the banking system.

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[10]See *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010.

Pigouvian taxation has also been proposed to internalize the negative externalities arising from collective bank failures.[11] Particular emphasis has been placed on taxes on bank liabilities, with some stressing their potential role as a complement to bank regulation.

Here we examine the impact of *changes* in taxation on optimal policies and the efficiency and welfare metrics of a bank subject to both capital regulation and liquidity requirements. Specifically, we compare the effects of an increase of corporate income taxes relative to the benchmark, with those resulting from the imposition of a tax of non–deposit liabilities.

We consider an increase in taxation of bank's income (from $\tau^+ = 15\%$ and $\tau^- = 5\%$ to $\tau^+ = 20\%$ and $\tau^- = 7.5\%$), and compare this increase with the imposition of a flat tax rate τ_B on banks' non-deposit liabilities. We set $\tau_B = 0.005$, which is a value in the range of taxes currently in place or proposed.[12] Under this scheme, the tax revenue is $-\tau_B \min\{0, B\}$, and we assume that these taxes are deductible from earnings for income taxation purposes. The results are in Table V.

For the bank subject to capital regulation only, higher corporate income taxes reduce loans and indebtedness, owing to a negative income effect. Moreover, both the enterprise value and the social value of the bank are reduced, although higher tax receipts increase government value. When we consider the bank subject to both capital regulation and liquidity requirements, the results are similar: loans, enterprise and social values are all reduced, while government value increases.

Turning on the impact of a tax on non-deposit liabilities, consider first a bank subject to capital regulation. The introduction of this tax results in a decline in lending and indebtedness sharper than under the increase in income taxation. This is because the bank reduces debt—hence, lending—as debt becomes more expensive, but the bank cannot substitute debt with deposits, since the latter are exogenous. Both enterprise and social values decrease relative to the base case, indicating (percent–wise) non-trivial efficiency and social costs associated with this form of taxation, despite the fact that government value increases as total tax revenues rise.

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[11]Current proposals include systemic risk levies designed to mimic such Pigouvian levies. See, for example, Acharya and Richardson (2009), Perotti and Suarez (2011), and for a critical evaluation see Shackelford, Shaviro, and Slemrod (2010).

[12]See the relevant tables in *Financial Sector Taxation*, International Monetary Fund, Washington D.C., September 2010. Note that our metrics, as well as the collateral constraint and the liquidity constraints, are reformulated to incorporate these taxes

When we consider a bank subject to both capital regulation and liquidity requirements, the effects of this taxation on lending, and the efficiency and welfare metrics are essentially muted, since on average the bank holds no debt (net bond holdings are positive) owing to a stringent liquidity requirement. In practice, the presence of a liquidity requirement dampens the effects of this tax.

In sum, the negative impact on lending, efficiency and social value associated with income taxes is significantly lower than that associated with a tax on deposit liabilities. In terms of government value, income taxes dominate non-deposit taxation under our benchmark parameterization.

C.3. The role of equity issuance costs, fire sales and maturity transformation

In this section we assess the differential impact of bank regulation on bank steady state optimal policies, and metrics of efficiency and welfare under different parameters of costs of equity issuance and fire sales, and the degree of banks maturity transformation. Namely, we consider: no cost of equity issuance, and an increase of λ from the benchmark value 0.1 to 0.2; a doubling of "fire sale" costs, m^-, from 0.04 to 0.08; a reduction of δ, the parameter gauging maturity transformation, from 20% to 10%, indicating a longer loan maturity (from 4 years to 9 years) and correspondingly a higher maturity mismatch. Table VI reports the results for a bank subject to capital requirements only, and for one subject to both capital and liquidity requirements.

To what extent the cost of equity issuance contributes to determine the inverted U-shaped relationship between lending, efficiency, welfare, and the stringency of capital requirements? On the one hand, with no equity issuance costs the U-shaped relationship is strengthened. This is not surprising, since distress can be faced at a lower cost by the bank, allowing it to increase its lending under a even higher capital requirement. On the other hand, doubling equity issuance costs relative to the already significantly high benchmark generates a decline in lending, enterprise and social values, but this decline is relatively small. Note that these results hold for a bank subject to capital requirements only, as well as for one subject to both capital and liquidity requirements. Thus, the role of the cost of equity issuance in determining

the costs of more stringent capital requirements appears relatively less important than the management of retained earnings.

Downward adjustment costs are proxies of fires sales costs. As noted, fire sales have been identified as one of the key sources of systemic risk in the financial crisis of 2007-2009 (see e.g. Kashyap, Brener, and Goodhart (2011)) When we double the relevant parameter in our simulation, we find that lending, enterprise and social values all *increase*. Note that such increase is found not only for the bank subject to capital requirements, but also for that subject to both capital and liquidity requirements. This result reveals again the key role of retaining earnings in supporting bank optimal choices: when facing higher fire sales costs, the bank will respond by increasing lending and retained earnings in such a way the latter would minimize the probability of incurring in fire sales costs in the event of distress. However, note that the addition of liquidity requirements to capital requirements still results in a significant worsening of lending and the efficiency and welfare metrics, as in all previous simulations. Therefore, the efficiency and welfare improving role of "mild" capital requirements may be even more important when fire sale costs are high.

Lastly, we have stressed the role of liquidity requirements in hampering the maturity transformation function of a bank. This is starkly illustrated by the case in which a bank lengthens the maturity of its loans. Under capital requirements only, the bank with a larger maturity mismatch undertakes a more intense maturity transformation, as witnessed by higher levels of lending and indebtedness relative to the bank with a milder maturity mismatch. When liquidity requirements are added to capital requirements, however, the reduction in lending, enterprise value and social value is significantly greater than that witnessed by the bank whose loan maturity is shorter. Again, liquidity requirements turn out to be most detrimental to lending, efficiency and welfare, the more intense is the transformation of short term liabilities into longer term assets.

VI. Conclusions

This paper has formulated a dynamic model of a bank exposed to credit and liquidity risk that can face financial distress by reducing loans, issuing secured debt, or issuing equity at a

cost. We evaluated the joint impact of capital regulation, liquidity requirements and taxation on banks' optimal policies and metrics of bank efficiency of and welfare.

We have uncovered an important inverted U-shaped relationship between bank lending, bank efficiency, social value and regulatory capital ratios. This result suggests the existence of optimal levels of regulatory capital, which are likely to be highly bank-specific, depending crucially on the configuration of risks a bank is exposed to as a function of the chosen business strategies. Similarly, our results on the high costs of liquidity requirements point out the adverse consequences of the repression of the key maturity transformation role of bank intermediation. Given our finding of the adverse effects of liquidity requirements, the argument by Admati, DeMarzo, Hellwig, and Pfleiderer (2011) that capital requirements can be designed to substitute for liquidity requirements is reinforced. Finally, for the purpose of rising tax revenues, corporate income taxation seems preferable to taxation of non-deposit liabilities, since the former generates higher revenues and lower efficiency and welfare costs.

Overall, our results suggest that implementing non-trivial increases in capital requirements, liquidity requirements and taxation may be associated with costs significantly larger than what proponents of these policies may have thought. This implies that the benefits of these requirements in terms of their ability to abate systemic risk should at least offset the costs we have identified.

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Appendix

A. Properties of the unregulated bank program

Compactness of the feasible set of the bank can be shown as follows. Given the strict concavity of $\pi(L)$, there exists a level L_u such that $\pi(L_u)Z_u - rL_u = 0$, where r is the cost of capital of the marginal dollar raised either through deposits or short term financing.[13] Thus, any investment $L > L_u$ would be unprofitable. This establishes an upper bound on the feasible set of L, given by $[0, L_u]$ for some L_u. With an upper bound on L, and because the stochastic process D has compact support, the collateral constraint sets a lower bound B_d (i.e., an upper bound on bond issuance). Specifically, this is obtained by putting D_d in place of D_t and L_u in place of L_t in equation (7).

Lastly, an upper bound on B can be obtained assuming that the proceeds from risk–free investments made by the bank are taxed at a higher rate than the personal tax rate and that floatation costs are positive. Specifically, assume that the current deposits D are all invested in short term bonds, B, with no investments in loans. To further increase the investment in bonds of one dollar, the bank must raise equity capital. A shareholder thus incurs a cost $1 + \lambda$, where λ is the floatation cost. This additional dollar is invested at the risk–free rate, so that at the end of the year, the proceeds of this investment that can be distributed are $(1 + r_f(1 - \tau^+))$. Alternatively, the shareholder can invest $1 + \lambda$ in a risk–free bond, obtaining $(1 + \lambda)(1 + r_f)$. Because $\tau^+ \geq 0$ and $\lambda \geq 0$, then $(1 + \lambda)(1 + r_f) \geq (1 + r_f(1 - \tau^+))$, there is no incentive of the bank to have a cash balance that is larger than D as long as either λ or τ^+ are strictly positive. The foregoing argument is made for simplicity. If the shareholders are taxed on their investment proceeds at a rate τ_p, they obtain $(1 + \lambda)(1 + r_f(1 - \tau_p))$ from their investment in the risk–free asset. If $\tau_p \leq \tau^+$, then $(1 + \lambda)(1 + r_f(1 - \tau_p)) > (1 + r_f(1 - \tau^+))$, and the bank has no incentive to increase the investment in risk–free bonds beyond D. Moreover, if floatation costs associated with equity issuance are strictly increasing in the amount issued, no assumption about differential tax rates are needed to establish an upper bound on B. In conclusion, the feasible set of the bank can be assumed to be $[0, L_u] \times [B_d, B_u]$.

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[13]Deposits and short term bonds are the cheapest form financing. If the same dollar were raised by issuing equity, the cost would be higher due to both the higher cost of equity capital and to floatation costs. In this case the upper bound would be even lower.

Furthermore, standard arguments establish the existence of a unique value function $E(x) = E(L, B, D, Z, D')$ that satisfies (15) and is continuous, increasing, and differentiable in all its arguments, and concave in L. The existence and uniqueness of the value function E follow from the Contraction Mapping Theorem (Theorem 3.2 in Stokey and Lucas (1989)). The continuity, monotonicity, and concavity of the value function E in the argument L follow from Theorem 3.2 and Lemma 9.5 in Stokey and Lucas (1989). The continuity and monotonicity of E in B, D, Z, and D' follow from the continuity and monotonicity of e in Z and D' and the Monotonicity of the Markov transition function of the process (Z, D).

B. A bank closure rule

The literature examining the impact of bank closure rules has typically analyzed setups under some form of asymmetric information, obtaining results highly dependent on model specification (see e.g. Kasa and Spiegel (2008) for a review of these models and empirical evidence). While examining the optimality of bank closure rules is outside the scope of the paper, here we illustrate how one such a rule can be viewed as a partial substitute of capital regulation under specific assumptions about bank reorganization costs.

Consider a bank closure rule according to which a bank that has a negative accounting net worth is taken over by the government and restructured. A similar rule has been conisdered for example by Elizalde and Repullo (2007) and Zhu (2008)).

Formally, the bank is closed at time t if the ex–post net asset value (i.e., ex–post bank capital, as opposed to K_t, which is the ex–ante bank capital) is negative:

$$v_t = L_t + B_t - D_t + y_t - \tau(y_t) = K_t + y_t - \tau(y_t) < 0. \tag{32}$$

The closure threshold is higher than the insolvency threshold, so this provision rules out bank's insolvency. As a consequence, current equity value is positive when this closure rule applies. The equity value $E(x) > 0$ is expropriated from shareholders and is tranferred to the government. New capital $K_{t+1} = D_u - D_{t+1}$ is injected. A key assumption is that the government incurs direct restructuring costs, $\gamma = 0$. Under these assumptions, the closure rule is superior to bank's insolvency from the government's view point, as a positive equity

value is collected, while saving direct bankruptcy costs. Equation (20) is still valid in this case, provided that Δ is the indicator of the event $\{v < 0\}$ and γ is set to zero.

When the bank is taken over by the government and $B_t < 0$ is fully collateralized, bond holders, (old) depositors, and the government are paid in full.[14] In this case, the financing shortfall (net obligations minus available liquid funds) of the bank is $(1 + r_d)D_t - (1 + r_f)B_t + \tau(y_t) - (D_{t+1} + \pi(L_t)Z_t)$. A portion of loan portfolio is liquidated in order to match (net of fire sales costs) the shortfall, or

$$L_t - L_{t+1} - m(L_t(1 - \delta) - L_{t+1}) = (1 + r_d)D_t - (1 + r_f)B_t + \tau(y_t) - (D_{t+1} + \pi(L_t)Z_t).$$

From this equation, a new level of loans, L_{t+1}^*, is determined. L_{t+1}^* is positive because the bank satifies the collateral constraint. Clearly, in this reorganization procedure, an indirect social cost is incurred because of loans fire sales costs, $m(L_t(1 - \delta) - L_{t+1}^*)$.

For the bank subject to this closure provision, the Bellman equation of the solution of the bank's program, when the bank is solvent (i.e., v_t as defined in (32) is positive), is as in equation (15). Note that under this closure rule, the value of the net payoff to the government is defined by the recursive equation

$$G(x) = (1 - \Delta(x)) \left(\tau(y') + \beta \mathbb{E}[G(x')] \right) + \Delta(x) \left(E(x) - \gamma D - D_u + D_{t+1} \right). \qquad (33)$$

with Δ denoting the default indicator at x. If the bank is insolvent ($\Delta = 1$), then the government incurs direct bankruptcy costs γD, expropriates the shareholders getting $E(x)$, and it injects new equity capital $K_{t+1} = D_u - D_{t+1}$.

In Table VII, we compare the unregulated bank with a bank subject to the closure rule, and the one just constrained to hold non–negative book equity ($k = 0$). It is apparent that the bank closure rule has an impact similar to the requirement to hold non–negative book equity.

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[14]This is because the collateral constraint in (7) is

$$L_t + \pi(L_t)Z_d + (1 + r_f)B_t - (1 + r_d)D_t - \tau(y_t^{\min}) \geq m(-L_t(1 - \delta)) - D_d,$$

the closure rule in (32) is

$$L_t + \pi(L_t)Z_t + (1 + r_f)B_t - (1 + r_d)D_t - \tau(y_t) < 0,$$

and the left–hand–side of the second inequality is higher than the corresponding side of the first inequality. Therefore, all stakeholders can be paid by liquidating the assets.

Relative to the unregulated bank, the closure rule results in an increase in loans, enterprise and social values, the latter crucially depending on the assumption of no bank reorganization costs. Note, however, that the rate of government intervention in closing and reopening banks is very high, which suggests this rule might be more costly than the imposition of capital requirements under the more realistic assumption of positive bank reorganization costs.

C. Algorithm

The solution of the Bellman equation in (15) is obtained numerically by a value iteration algorithm. The valuation model for bank's equity is a continuous–decision and infinite–horizon Markov Decision Processes. The solution method is based on successive approximations of the fixed point solution of the Bellman equation. Numerically, we apply this method to an approximate discrete state-space and discrete decision valuation operator.[15]

Given the dynamics of Z_t in (11) and of $\log D_t$ in (12), using a vector notation $\xi(t) = (Z(t), \log D(t))$, we have a VAR of the form

$$\xi(t) = c + \mathcal{K}\xi(t-1) + \varepsilon(t) \tag{34}$$

where $\varepsilon = (\varepsilon^Z, \varepsilon^D)$ is a bivariate Normal variate with zero mean and covariance matrix Σ,

$$c = \begin{pmatrix} (1-\kappa_1)\bar{\xi}_1 \\ (1-\kappa_2)\bar{\xi}_2 \end{pmatrix}, \qquad \mathcal{K} = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho & \sigma_2^2 \end{pmatrix},$$

where, differently from the approach proposed in Tauchen (1986), Σ may be non–diagonal and singular (when $|\rho| = 1$). We restrict the support of each stochastic process within three times the unconditional standard deviation of the marginal distribution around the long term average.

Given this alternative assumption for the error covariance matrix, we follow an efficient approach first proposed by Knotek and Terry (2011) based on numerical integration of the multivariate Normal distribution. In particular, the continuous–state process $\xi(t)$ is approximated by a discrete–state process $\widehat{\xi}(t)$, which varies on the finite grid $\{X_1, X_2, \ldots, X_n\} \subset \mathbb{R}^2$.

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[15]See Rust (1996) or Burnside (1999) for a survey on numerical methods for continuous decision infinite horizon Markov Decision Processes.

Define a partition of $\mathbb{R}^2$ made of n non–overlapping 2–dimensional intervals $\{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_n\}$ such that $X_i \in \mathcal{X}_i$ for all $i = 1, \ldots, n$ and $\cup_{i=1}^n \mathcal{X}_i = \mathbb{R}^2$. The $n \times n$ transition matrix Π is defined as

$$\begin{aligned} \Pi_{i,j} &= \text{Prob}\left\{\widehat{\xi}(t+1) \in \mathcal{X}_j \mid \widehat{\xi}(t) = X_i\right\} \\ &= \text{Prob}\left\{c + \mathcal{K}\xi(t) + \varepsilon(t+1) \in \mathcal{X}_j\right\} \\ &= \text{Prob}\left\{\varepsilon(t+1) \in \mathcal{X}_j'\right\} = \int_{\mathcal{X}_j'} \phi(\xi, 0, \Sigma)d\xi \end{aligned}$$

where $\mathcal{X}_j' = \mathcal{X}_j - c - \mathcal{K}\xi(t)$, and $\phi(\xi, 0, \Sigma)$ is the density of the bivariate Normal with mean zero and covariance matrix Σ. The integral on the last line is computed numerically by Monte–Carlo integration. We select the grid points for each variable based on approximately equal weighting from the univariate normal cumulative distribution function.

The feasible interval for loans, $[0, L_u]$, and for the face value of bonds, $[B_d, B_u]$ (with $B_d < 0 < B_u$), is set so that they are never binding for the equity maximizing program. We discretize $[L_d, L_u]$, to obtain a grid of N_L points

$$\widetilde{L} = \left\{\widetilde{L}_j = L_u(1-\delta)^j \mid j = 1, \ldots, N_L - 1\right\} \cup \{L_{N_L} = 0\}$$

such that, if the bank choose inaction, the loan's level is what remains after the portion δL has been repaid. The interval $[B_d, B_u]$ is discretized into N_b equally–spaced values, making up the set $\widetilde{B}$. To keep the notation simple, we denote $x = (\xi, L, B)$ the generic element of the discretized state.

We solve the problem

$$E(x) = \max\left\{0, \max_{(L',B')\in\mathcal{A}(D)} \left\{e(x, L', B') + \beta\mathbb{E}\left[E(x')\right]\right\}\right\},$$

where function $e(x, L, B)$, is defined in equation (10), and $\mathcal{A}(D)$ is the case specific feasible set defined differently for the unregulated and the regulated case, in all points of the discrete state space. The solution is found by successive approximations, starting from a guess function $E_0(\cdot)$, putting it on the right–hand–side of the Bellman equation obtaining $E_1(\cdot)$ and than by iterating on the same procedure obtaining the sequence $\{E_n(\cdot), n = 0, 1, \ldots\}$. The procedure is terminated when the error $\|E_{j+1} - E_j\|$ is lower than the desired tolerance.

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For the set of parameters in Table I, we use $L_u = 10$, $B_d = -5$ and $B_u = 5$. Given the properties of the quadrature scheme, we solve the model using only 9 points for Z, 9 points for D_t. However, we need to allow for many more points when discretizing the control variables, so we choose $N_L = 27$, and $N_B = 36$. The tolerance for termination of the iterative procedure is set at 10^{-5}.

Given the optimal solution, we can determine the optimal policy and the transition function $\varphi(x)$ in (16) based on the arg-max of equity value at the discrete states x. We use Monte Carlo simulation to generate a sample of Ω possible future paths (or scenarios) for the bank. In particular, we obtain the simulated dynamics of the state variable $\xi = (Z, D)$ by application of the recursive formula in (34), starting from $Z(0) = \overline{Z}$ and $D(0) = D_d$. Then, setting a feasible initial choice $L(0) = 0$ and $B(0) = D_u$ (so that the initial bank capital is $D_u - D_d$), we apply the transition function φ along each simulated path recursively. If a bank defaults at a given step, then the current depositors receive the full value of their claim, while the deposit insurance agency pays the bankruptcy cost. Afterwards, a seed capital $D_u - D_d$ is injected in the bank. Together with deposit D_d, the total amount D_u is momentarily invested in bonds, $B = D_u$, while $L = 0$. Then the "new" bank follows on the same path by applying the optimal policy. In our numerical experiments, we generate simulated samples with $\Omega = 10,000$ paths and $T = 100$ years (steps). To limit the dependence of our results on the initial conditions, we drop the first 50 steps.

$$
\begin{array}{cccc}
Z_{t-1} & \longrightarrow Z_t & \longrightarrow Z_{t+1} \\
D_t & D_{t+1} & D_{t+2} \\
(L_t, B_t) & (L_{t+1}, B_{t+1}) & (L_{t+2}, B_{t+2})
\end{array}
$$

$\cdots \quad t-1 \qquad\qquad\qquad t \qquad\qquad\qquad t+1 \quad \cdots$
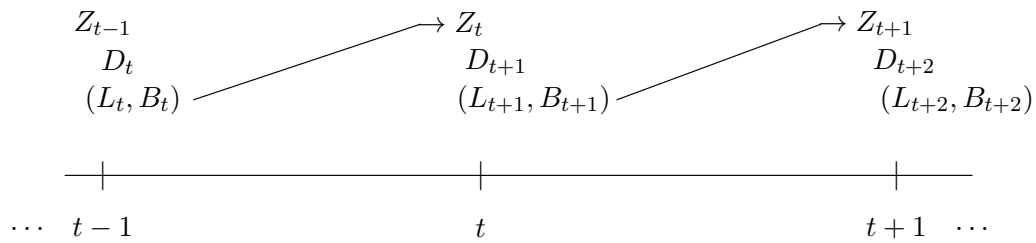
Figure 1: **Bank's dynamic**. Evolution of the state variables (credit shock, Z, and deposits, D) and of the bank's control variables (cash and liquid investments, B, and loans, L) assuming the bank is solvent at each date.
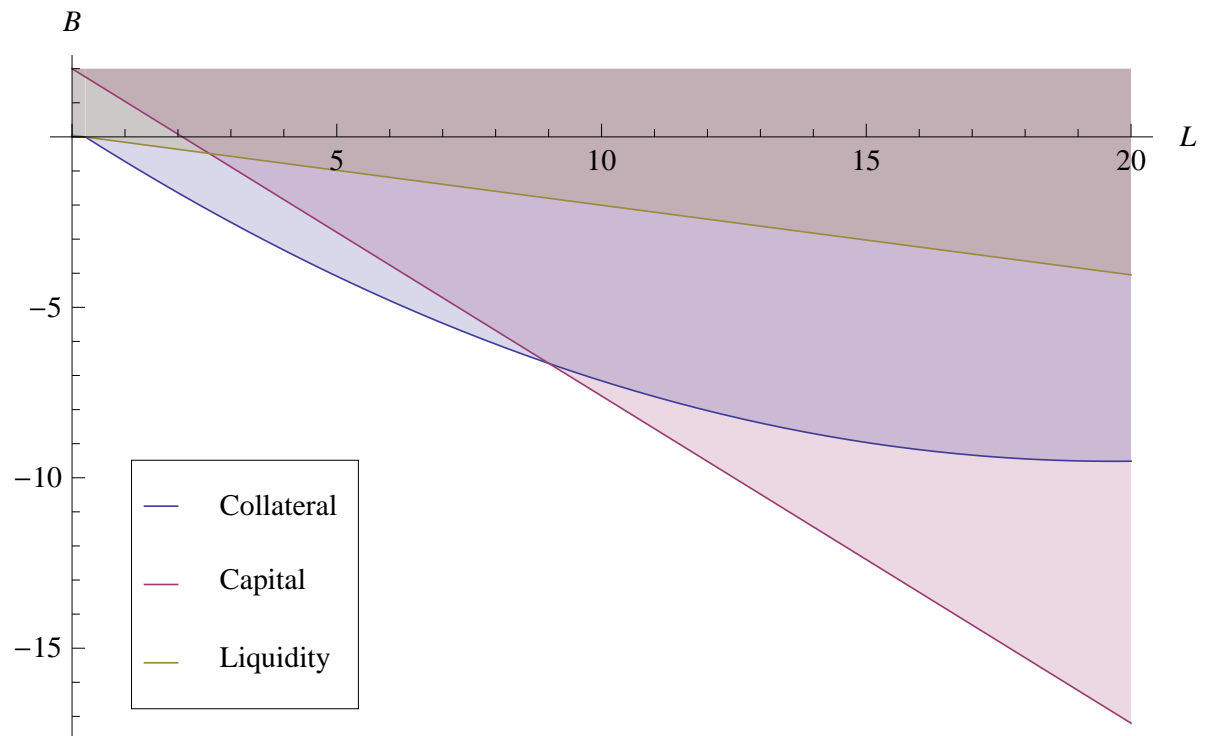
Figure 2: **Comparison of constraints**. This figure presents the three feasible regions of (L, B) defined by the collateral constraint, $\Gamma(D)$ in Equation (8), the capital requirement, $\Theta(D)$ from (23), and by the liquidity requirement, $\Lambda(D)$ from Equation (26). The plot is based on the parameter values in Table I, for a current $D = 2$.
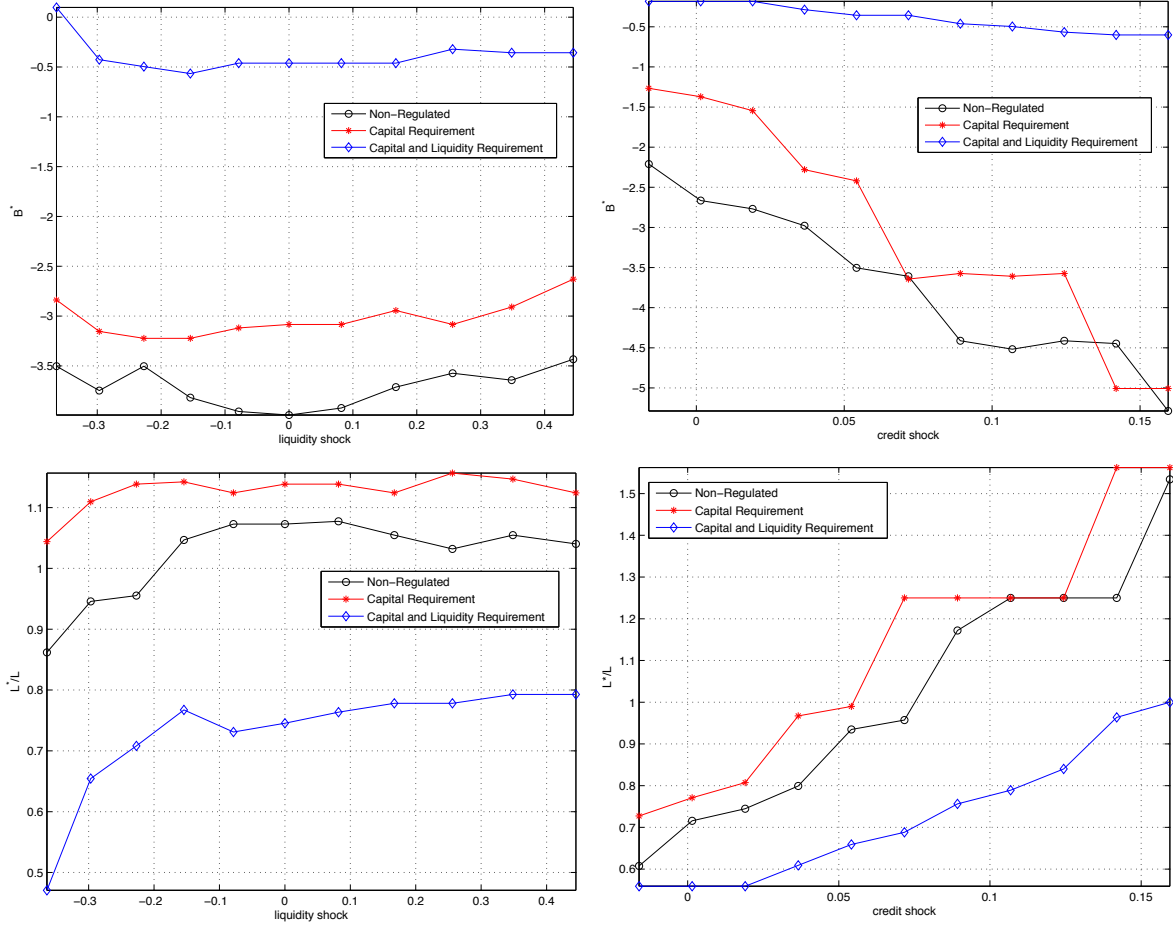
Figure 3: **Bank's policy**. This figure illustrates the impact of regulatory restrictions on the bank's policy related to loan investment and to short term investment and financing with bonds, for the non–regulated case, for the cases with capital constraint, and with both capital and liquidity constraints altogether. The short term investment/financing policy is given by the optimal B^* given the current state, averaged across all possible Z in the left panel and averaged across all possible D' in the right panel. The loan investment policy is represented by the ratio L^*/L given the current state and averaged across Z in the left panel and across D' in the right panel. These values are plotted against the liquidity shock, $D' - D$, in the left panels and the credit shock, Z, on loans in the right panels, and are obtained assuming that the bank is currently at the steady state (so that the credit shock is 0.0717, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.1$ so that bank capital is $K = 2.1$. The values are from the numerical solution of the model using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, based on the parameter values in Table I.

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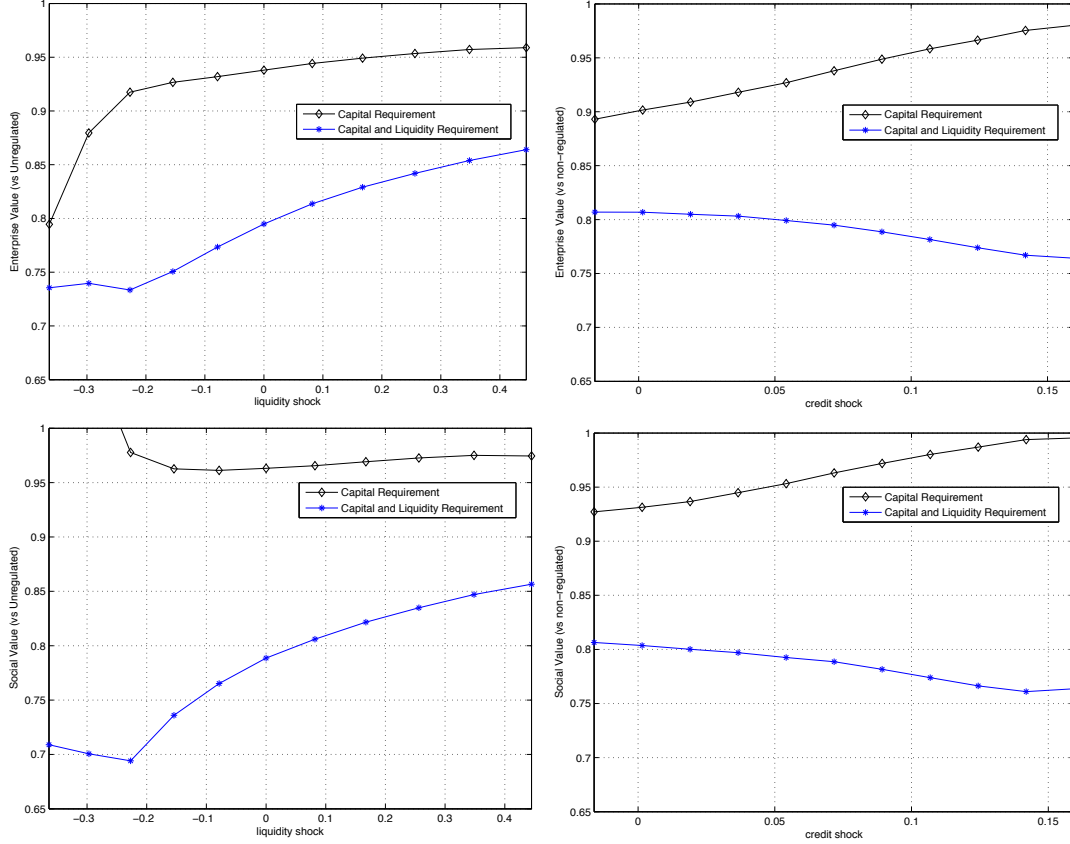
Figure 4: **Value loss associated with regulatory restrictions**. This figure illustrates the impact of regulatory restrictions by comparing the enterprise value (i.e., market value of deposits plus market value of equity net of cash balance, or plus short term debt), and the social value (i.e., book value of deposits plus market value of equity plus the value to the government, plus the present value of issuance costs) of the bank, for the case with capital constraint, and with both capital and liquidity constraints as a proportion of the value from the non–regulated case. These values are plotted against the shock on deposits, $D' - D$, in the left panels and the credit shock, Z, on loans in the right panels, and are obtained assuming that the bank is currently at the steady state (so that the credit shock is 0.0717, and the deposits from the previous date are $D = 2$, respectively), while $B = 0$, and $L = 4.1$ so that bank capital is $K = 2.1$. The values are from the numerical solution of the model using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, based on the parameter values in Table I.

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κ_Z	annual persistence of the credit shock	0.88
σ_Z	annual conditional std. dev. of the credit shock	0.0139
$\overline{Z}$	unconditional average of the credit shock	0.0717
κ_D	annual persistence of the log of deposits	0.95
σ_D	annual conditional std. dev. of the log of deposits	0.0209
$\overline{D}$	unconditional average of deposits	\$2
ρ	correlation between log–deposit and credit shock	-0.85
β	annual discount factor	0.95
r_f	annual risk–free rate on bonds	2%
r_d	annual rate on deposits	0%
τ^+	corporate tax rate for positive earnings	15%
τ^-	corporate tax rate for negative earnings	5%
δ	annual percentage of reimbursed loan	20%
γ	bankruptcy costs	0.1
λ	flotation cost for equity	0.1
α	return to scale for loan investment	0.90
m^+	unit price for loan investment	0.03
m^-	unit price for loan fire sales	0.04
k	percentage of loans for capital regulation	4%
ℓ	liquidity coverage ratio	20%
τ_B	tax rate on uninsured liabilities	0

Table I: **Base case model parameters**

Z	0.019	0.037	0.054	0.072	0.089	0.107	0.124
				Unregulated			
$K = 2.22$	-0.270	-0.228	-0.173	-0.173	-0.106	-0.090	-0.081
$K = 2.18$	-0.270	-0.228	-0.173	-0.173	-0.106	-0.090	-0.063
$K = 2.14$	-0.270	-0.228	-0.173	-0.167	-0.106	-0.072	-0.055
$K = 2.10$	-0.306	-0.262	-0.173	-0.167	-0.106	-0.046	-0.037
$K = 2.05$	-0.407	-0.326	-0.287	-0.230	-0.161	-0.074	-0.028
				Capital			
$K = 2.22$	0.068	0.073	0.073	0.071	0.071	0.071	0.071
$K = 2.18$	0.082	0.073	0.073	0.071	0.071	0.071	0.071
$K = 2.14$	0.082	0.073	0.073	0.071	0.071	0.080	0.080
$K = 2.10$	0.115	0.084	0.084	0.071	0.080	0.080	0.089
$K = 2.05$	0.074	0.088	0.091	0.067	0.089	0.098	0.098
				Capital & liquidity			
$K = 2.22$	0.240	0.264	0.309	0.372	0.372	0.377	0.383
$K = 2.18$	0.233	0.240	0.309	0.352	0.372	0.377	0.383
$K = 2.14$	0.233	0.240	0.287	0.331	0.372	0.377	0.383
$K = 2.10$	0.203	0.233	0.264	0.309	0.352	0.377	0.383
$K = 2.05$	0.171	0.171	0.179	0.257	0.301	0.352	0.358

Table II: **Capital ratio**. Average capital ratio (i.e., bank capital over loans, or $(L^* + B^* - D')/L^*$, where (L^*, B^*) is the optimal solution for a solvent bank and D' is the new possible level of deposits) as a function of the credit shock, Z, at different levels of current bank capital, $K = L + B - D$. The average is computed over the different possible levels of D'. The average capital ratio is computed at $B = 0$. The current level of L is 4.1. The different levels of K are obtained by changing D. These results are based on the numerical solution of the valuation problem in (15) using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, based on the parameter values in Table I.

Z	0.019	0.037	0.054	0.072	0.089	0.107	0.124
	Unregulated						
$K = 2.22$	-8.208	-9.542	-10.268	-10.268	-12.680	-12.808	-12.398
$K = 2.18$	-8.208	-9.542	-10.268	-10.268	-12.680	-12.808	-11.722
$K = 2.14$	-8.208	-9.542	-10.268	-10.570	-12.680	-12.344	-11.526
$K = 2.10$	-7.258	-8.201	-10.268	-10.570	-12.680	-11.372	-11.266
$K = 2.05$	-5.870	-6.618	-7.363	-8.587	-9.870	-10.453	-11.141
	Capital						
$K = 2.22$	-4.751	-6.070	-6.070	-9.318	-9.318	-9.318	-9.318
$K = 2.18$	-4.342	-6.070	-6.070	-9.318	-9.318	-9.318	-9.318
$K = 2.14$	-4.342	-6.070	-6.070	-9.318	-9.318	-9.121	-9.121
$K = 2.10$	-3.169	-5.661	-5.661	-9.318	-9.121	-9.121	-8.854
$K = 2.05$	-2.755	-3.908	-4.317	-7.042	-8.712	-8.445	-8.445
	Capital & liquidity						
$K = 2.22$	0.433	0.473	0.578	0.907	0.907	0.691	0.550
$K = 2.18$	0.489	0.433	0.578	0.753	0.907	0.691	0.550
$K = 2.14$	0.489	0.433	0.520	0.652	0.907	0.691	0.550
$K = 2.10$	0.677	0.489	0.473	0.578	0.753	0.691	0.550
$K = 2.05$	0.799	0.799	0.743	0.707	0.839	0.753	0.612

Table III: **Liquidity coverage ratio**. Average liquidity coverage ratio (i.e., end–of–period total cash available in the worst case scenario over the end–of–period net cash outflows due to a variation in deposits, or $(\delta L^* + \pi(L^*)Z_d - \tau(y^{\min}) + B^*(1 + r_f))/(D'(1 + r_d) - D_d)$, where (L^*, B^*) is the optimal solution for a solvent bank and D' is the new possible level of deposits) as a function of the credit shock, Z, at different levels of current bank capital, $K = L + B - D$. The average is computed over the different possible levels of D'. The average capital ratio is computed at $B = 0$. The current level of L is 4.1. The different levels of K are obtained by changing D. These results are based on the numerical solution of the valuation problem in (15) using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, based on the parameter values in Table I.

	Unregulated	Capital		Capital & Liquidity		
	base	base	k = 12%	base	k = 12% ℓ = 20%	k = 4% ℓ = 50%
Loan (book)	4.78	6.37	5.90	2.68	2.67	2.65
Net Bond Holdings (book)	-3.48	-3.89	-3.05	0.16	0.22	0.26
Bank Capital (book)	-0.70	0.47	0.84	0.82	0.87	0.90
Equity (mkt)	4.49	4.87	4.90	3.69	3.72	3.74
Deposits (mkt)	1.89	1.91	1.91	1.91	1.91	1.91
Enterprise Value (mkt)	9.88	10.71	9.90	5.43	5.40	5.38
Government Value (mkt)	0.54	0.88	0.85	0.37	0.37	0.37
Social value (mkt)	10.56	11.72	10.87	5.91	5.89	5.86
Default/Closure Rate (pct)	5.34	0.00	0.00	0.00	0.00	0.00

Table IV: **The impact of bank regulation**. These results are obtained from the solution of the bank valuation problem using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, and the parameters in Table I. The optimal policy is then applied to 10,000 random paths of 50 periods (years) length for the credit and liquidity shocks. The table presents the median of the time series averages (computed on non–defaulted instances) of the different metrics. The columns represent different choices of parameters: the column denoted "base", is the base case, with the parameters in Table I. The others are obtained by changing only the parameter(s) we use to denominate the column. The results are presented for the unregulated case, the case with capital ratio restrictions, and the case with both capital and liquidity restrictions.

	Capital			Capital & liquidity		
	base	τ	τ_B	base	τ	τ_B
Loan (book)	6.37	6.09	5.89	2.68	2.48	2.63
Net Bond Holdings (book)	-3.89	-3.62	-3.44	0.16	0.24	0.21
Bank Capital (book)	0.47	0.45	0.45	0.82	0.70	0.81
Equity (mkt)	4.87	4.49	4.46	3.69	3.38	3.65
Deposits (mkt)	1.91	1.91	1.91	1.91	1.91	1.91
Enterprise Value (mkt)	10.71	10.06	9.84	5.43	5.05	5.34
Government Value (mkt)	0.88	1.14	1.10	0.37	0.48	0.38
Social value (mkt)	11.72	11.31	11.05	5.91	5.64	5.83

Table V: **The impact of taxation**. These results are obtained from the solution of the bank valuation problem using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, and the parameters in Table I. The optimal policy is then applied to 10,000 random paths of 50 periods (years) length for the credit and liquidity shocks. The table shows, respectively, from left to right a regulated bank with capital requirement, and a regulated bank subject to both capital and liquidity restrictions. The table presents the median of the time series averages (computed on non–defaulted instances) of the different metrics. The columns represent different choices of parameters: the column denoted "base", is the base case, with the parameters in Table I. The others are obtained by changing only the parameter(s) we use to denominate the column: "τ" is with $\tau^+ = 20\%$ and $\tau^- = 7.5\%$; in "τ_B" we set $\tau_B = 0.005$.

	Unregulated	base	$\lambda = 0$	$\lambda = .2$	$m^- = .08$	$\delta = .1$
			Capital			
Loan (book)	4.78	6.37	6.45	6.29	6.46	8.55
Net Bond Holdings (book)	-3.48	-3.89	-3.95	-3.80	-3.97	-5.51
Bank Capital (book)	-0.70	0.47	0.48	0.47	0.48	1.04
Equity (mkt)	4.49	4.87	4.90	4.85	4.87	6.70
Enterprise Value (mkt)	9.88	10.71	10.84	10.60	10.79	14.09
Government Value (mkt)	0.54	0.88	0.90	0.87	0.89	1.14
Social value (mkt)	10.56	11.72	11.85	11.60	11.80	15.34
			Capital & liquidity			
Loan (book)	4.78	2.68	3.15	2.47	2.72	3.47
Net Bond Holdings (book)	-3.48	0.16	0.05	0.23	0.13	0.19
Bank Capital (book)	-0.70	0.82	1.21	0.70	0.83	1.68
Equity (mkt)	4.49	3.69	4.18	3.51	3.70	4.70
Enterprise Value (mkt)	9.88	5.43	6.02	5.19	5.47	6.40
Government Value (mkt)	0.54	0.37	0.44	0.34	0.37	0.49
Social value (mkt)	10.56	5.91	6.58	5.64	5.96	7.02

Table VI: **The role of equity issuance costs, adjustment costs and maturity transformation**. These results are obtained from the solution of the bank valuation problem using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, and the parameters in Table I. The optimal policy is then applied to 10,000 random paths of 50 periods (years) length for the credit and liquidity shocks. The table shows two panels: above the case of a bank with capital requirement, and below the case of a bank subject to both capital and liquidity restrictions. The table presents the median of the time series averages (computed on non–defaulted instances) of the different metrics. The columns represent different choices of parameters: the column denoted "base", is the base case, with the parameters in Table I. The others are obtained by changing only the parameter used to denominate the column (e.g., in "$\lambda = 0$" all the parameters are at the base case value, but λ, which is set to zero).

	Unregulated	$k = 0$	*Closure*
Loan (book)	4.78	6.14	5.60
Net Bond Holdings (book)	-3.48	-3.82	-3.37
Bank Capital (book)	-0.70	0.31	0.25
Equity (mkt)	4.49	4.82	5.48
Enterprise Value (mkt)	9.88	10.58	10.76
Government Value (mkt)	0.54	0.86	0.76
Social value (mkt)	10.56	11.56	11.63
Default/Closure Rate (pct)	5.34	0	21.75

Table VII: **A bank closure rule**. Comparison between a bank closure rule, a bank subject to hold nonnegative book equity, and the unregulated bank. These results are obtained from the solution of the bank valuation problem using 9 points for Z, 9 points for D, 27 points for L, and 36 points for B, and the parameters in Table I. The optimal policy is then applied to 10,000 random paths of 50 periods (years) length for the credit and liquidity shocks. The table shows the median of the time series averages (computed on non–defaulted instances) of the different metrics. The columns represent the case with unregulated banks, the case with nonnegative book equity requirement $k = 0$, and the case with banks subject to the closure rule.