DEA IN MUTUAL FUND EVALUATION

Stefania FUNARI
E-mail: funari@unive.it

Dipartimento di Matematica Applicata
Università Ca' Foscari di Venezia

ABSTRACT - In this contribution we illustrate the recent use of Data Envelopment Analysis (DEA) approach in the evaluation of the performance of mutual funds. The DEA approach allows to build performance indicators which jointly take into account the different elements that contribute to determine the overall performance of mutual funds: not only the return and risk of the investment, but also the subscription and redemption costs which burden the investors. Moreover, in the evaluation of ethical mutual funds, the use of the DEA methodology permits to consider also the ethical aim which motivates the socially responsible investment. The adoption of DEA allows to overcome some of the shortcomings arising when using numerical traditional indicators to evaluate mutual fund performance. We focus mainly on the way DEA can be used to extend the well-known Sharpe index by considering the investment costs and the ethical component of the investment.

KEYWORDS - Data envelopment analysis, mutual fund performance, ethical funds, Sharpe index.

1. INTRODUCTION

Mutual funds are very popular investment instruments; they pool financial resources of many persons and invest them in portfolio of securities.

In the literature various methodologies have been adopted in order to evaluate the performance of mutual funds. Most of these techniques are based on the assumption that individuals who purchase mutual funds take their investment decisions on the ground of two dimensions, the profitability of the investment and the risk involved.

In particular some traditional performance indicators are widely used, which reduce the two (return-risk) dimensions of the investment performance to a single index that measures the performance of mutual funds by considering a return measure per unit of risk. Among them we recall for example, the well known

Sharpe, Treynor and reward to half-variance indexes ([17], [18], [1]).

One of the advantages of using numerical performance measures is that they provide a complete ordering of mutual funds, thus allowing a definitive comparison of the performance of mutual funds with different returns and risks.

On the other hand the traditional performance measures present some shortcomings that have been highlighted in the literature ([8], [13]).

For example they are based on strong assumptions on investor behavior, so that it is not clear which of them represents the best performance measure, as each indicator may be valid under some assumptions but may be overcome by one of the other performance measures in a different context.

We may also mention another issue in portfolio performance that is of particular interest for our purposes, i.e. the investment costs.

When deciding the purchase of mutual funds, individuals include investment costs in their decision process. Mutual funds usually charge management expenses that are reflected in the rate of return of the investment; in addition most funds also charge initial fees and/or redemption costs, and the annual rates of return are not net of such costs (see [17]).

In the financial literature some measures have been developed which link the analysis of the mutual fund performance to some characteristics of the funds, including fund costs (see for example [9] and [19]).

The traditional performance indicators, instead, do not allow to directly take into account the subscription costs and redemption fees, even if the overall return on the investment is indeed influenced by these costs.

Recently the use of data envelopment analysis (DEA) approach has been suggested to compare the performance of mutual funds.

Data envelopment analysis is an operational research methodology that allows to measure the relative efficiency of organizations which are characterized by a multiple input and multiple output structure.

There is interest in using the DEA approach for the appraisal of mutual funds, since DEA allows to consider simultaneously various elements that contribute to determine the overall performance of mutual funds, in addition to the return and risk dimensions.

Thus using DEA one can build performance indicators which consider, for example, the return and risk of the investment, together with the subscription and redemption costs which burden the investors and, in case, the ethical aim which motivates the purchase of ethical (or socially responsible) mutual funds.

Moreover, one of the features of DEA is that it synthesizes the various dimensions of the mutual fund performance in a unique numerical value, the so-called efficiency score, which can be used to obtain a ranking of mutual funds.

Various DEA performance measures for mutual funds have recently been proposed in the literature. Some of them constitute a direct generalization of the traditional performance indicators.

Murthi, Choi and Desai [15], who first used DEA in mutual fund performance, proposed a DEA portfolio efficiency index, called DPEI, that can be viewed as an extension of the Sharpe ratio with the inclusion of mutual fund transaction costs.

One of the advantages of using DEA in mutual fund evaluation is certainly its ability to consider alternative risk measures simultaneously and also its ability to include in the evaluation process additional elements which are useful for evaluating the performance of mutual funds.

Basso and Funari [2] proposed two DEA performance measures for mutual funds. One of them may be viewed as an extension of the DPEI index which allows to consider simultaneously the standard deviation of the return, the beta of the portfolio and the square root of half-variance as risk measures. The DEA measure proposed provides a generalization of the traditional Sharpe, Treynor and reward to half-variance indexes. Moreover, a second DEA index for mutual funds has been defined which includes a stochastic dominance indicator that reflects both the investors'preference structure and the time occurrence of the returns.

In Joro and Na [11] the DEA approach has been used to include in the analysis also the third moment of the fund's return distribution; the DEA performance measure that has been formed is thus based on the mean-variance-skewness framework.

The ability of DEA to consider various objectives simultaneously may be useful when evaluating ethical funds.

In fact, when investing in ethical mutual funds, savers aim to satisfy an ethical need and at the same time to obtain a satisfactory return; hence both objectives have to be considered in the appraisal of ethical funds. Nevertheless, the traditional performance indicators are not able to take into account the ethical aim of investors.

In Basso and Funari [3], [4], the ethical component of the investment has been considered, together with the expected return, the investment risk and the subscription and redemption costs, in order to define a DEA performance measure for socially responsible funds.

Also in Morey and Morey [14] the DEA methodology has been used to evaluate the performance of mutual funds. Nevertheless, this contribution differs from the aforementioned applications of DEA technique to mutual fund appraisal, because it uses the philosophy of the DEA approach in a different context. In fact, the authors use DEA in order to evaluate the performance of mutual funds over different horizons; the DEA approach allows, in this case, to obtain an efficiency score which summarizes the various measures of the risk and return over different time horizons.

The remainder of presentation proceeds as follows. In section 2 we will present the basic DEA model which in most cases has been used in the applications of DEA to mutual fund performance, that is the so-called CCR (Charnes, Cooper and Rhodes) model. In section 3 we will show how to use the CCR model in order to obtain DEA mutual fund performance indexes that can be seen, in a sense, as a generalization of the Sharpe ratio. In section 4 we illustrate further extensions of DEA models for the appraisal of mutual funds. Finally, we conclude the presentation with some remarks on the use of DEA as a mutual fund performance device.

2. THE BASIC CCR MODEL

Data envelopment analysis (DEA) is a mathematical programming technique widely cited in the operations research literature; it has been traditionally adopted to evaluate the relative performance of organizations (called “decision making units” in the DEA language) which are characterized by a multiple input-multiple output structure, such as schools, hospitals, banks, non profit institutions, and so on.

In this section we present the basic DEA model, the CCR model, which was originally proposed in [6].

Let us consider n decision making units whose efficiency has to be evaluated and suppose that they use m inputs to obtain t outputs.

Let us denote by

x_{ij} the amount of input i used by unit j $(i = 1, \ldots, m,\ j = 1, \ldots, n)$
y_{rj} the amount of output r used by unit j $(r = 1, \ldots, t,\ j = 1, \ldots, n)$
v_i weight assigned to input i $(i = 1, \ldots, m)$
u_r weight assigned to output r $(r = 1, \ldots, t)$

Let us assume to evaluate the efficiency of the “target unit” j_0 ($j_0 \in \{1, 2, \ldots, n\}$). The DEA efficiency measure is defined as the ratio of a weighted sum of outputs to a weighted sum of inputs:

$$\frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}. \tag{2.1}$$

The distinctive feature of DEA is that the weights which allow to aggregate the inputs and outputs do not reflect the preference structure of the decision maker, but they are obtained by solving optimization problems which change with the decision making units under evaluation. That is the weights are peculiar to each unit under evaluation and constitute the most favorable weights for that unit.

More precisely, the weights $\{v_i, u_r\}$ in (2.1) are computed by maximizing the efficiency ratio of the target unit, provided that the efficiency ratios of all units, computed with the same weights, have an upper bound, usually set equal to 1. Formally, the DEA efficiency measure for target unit j_0 is defined as the optimal value of the following optimization problem:

$$\max_{\{v_i, u_r\}} \quad z = \frac{\sum_{r=1}^{t} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}} \tag{2.2}$$

subject to

$$\frac{\sum_{r=1}^{t} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \qquad j = 1, 2, \ldots, n \tag{2.3}$$

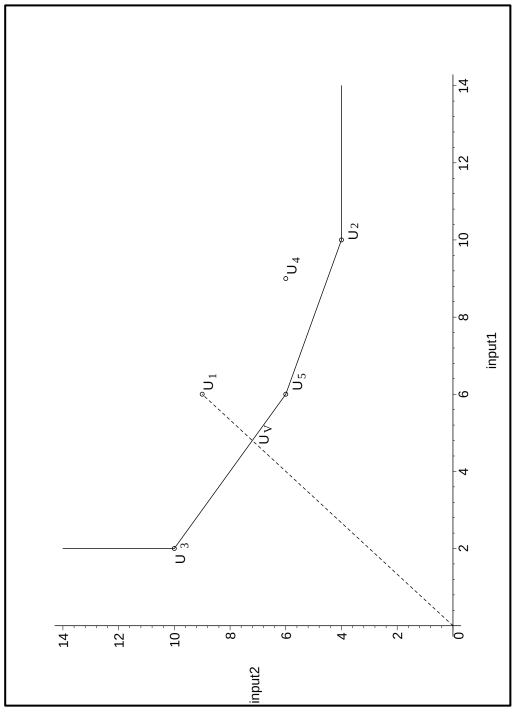
$$u_r \geq \varepsilon \qquad r = 1, 2, \ldots, t \tag{2.4}$$

$$v_i \geq \varepsilon \qquad i = 1, 2, \ldots, m, \tag{2.5}$$

where ε is a convenient small positive number that prevents the weights from vanishing.

Problem (2.2)-(2.5) has to be executed n times, once for each target decision making unit j ($j = 1, \ldots, n$), in order to find the optimal efficiency measures for all the decision making units.

Figure 2.1. *An example of the production frontier of the CCR model.*



The solution of DEA model (2.2)-(2.5) provides, first of all, an optimal efficiency measure which is assigned to unit j_0.

Notice that the efficiency measures have an upper bound of 1; if the efficiency measure of unit j_0 is less than 1 then the unit is considered inefficient relative to the other units; otherwise an efficiency measure equal to 1 characterizes the efficient decision making units, that is those units which lie on the efficient frontier and are not dominated by the other units in the set.

Figure 2.1 illustrates an example of the use of the DEA approach; it considers five decision making units $U_1 - U_5$, each producing one output, using two kinds of inputs. The units are represented as points in the Cartesian plane, where each axis shows one of the inputs used, divided by the quantity of the single output. The solid line represents the production frontier of the CCR model.

Units U_2, U_3 and U_5 are relatively efficient, whereas both U_1 and U_4, are evaluated as inefficient. We can measure the efficiency of the decision making units by considering their distance from the frontier line.

For example, the efficiency of unit U_1 is evaluated by computing the ratio of the distance from the origin to U_V and the distance from the origin and U_1. Unit U_1 can improve its efficiency by decreasing both inputs, maintaining the same level of output.

From a mathematical point of view, problem (2.2)-(2.5) is a linear fractional programming problem. It is possible to prove that the fractional program (2.2)-(2.5) is equivalent to the following linear programming problem (see [7]):

$$\max \quad z = \sum_{r=1}^{t} u_r y_{rj_0} \tag{2.6}$$

subject to

$$\sum_{i=1}^{m} v_i x_{ij_0} = 1 \tag{2.7}$$

$$\sum_{r=1}^{t} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \qquad j = 1, 2, \ldots, n \tag{2.8}$$

$$- u_r \leq -\varepsilon \qquad r = 1, 2, \ldots, t \tag{2.9}$$

$$- v_i \leq -\varepsilon \qquad i = 1, 2, \ldots, m. \tag{2.10}$$

One key information of CCR model is the measure of the relative efficiency obtained for each decision making unit under evaluation.

However, the DEA approach provides another kind of information that could be very useful in the operational practice. It identifies, for each inefficient unit j_0, a set of corresponding units, the so called "peer units", which are efficient with the weights of unit j_0.

In fact, we may observe that if a decision making unit j_0 has an efficiency score $z^* < 1$ with optimal weights u_r^*, v_i^*, then there must be at least one constraint in (2.8) for which the weights u_r^*, v_i^* produce equality between the left and right hand sides, otherwise z^* could be enlarged. The set of the efficient units associated to such equality constraints constitutes the peer group for the inefficient unit j_0.

Hence, the DEA approach suggests a "virtual (composite) unit" that the inefficient unit j_0 could imitate in order to improve its efficiency.

Technically, the inputs and outputs of the composite unit are linear combinations of the inputs and outputs of the peer units; the coefficients of the linear combination are the non null optimal variables of the dual of the linear programming problem (2.6)-(2.10) (see [5], [7]).

In Figure 2.1 the peer units for the inefficient unit U_1 are the units U_3 and U_5. The virtual unit is U_V and it is obtained from a linear combination of units U_3 and U_5; the virtual unit provides a target for the inefficient unit U_1, since it represents a unit having the U_1's input-output orientation, producing the same quantity of the single output, but using a lower quantity of both inputs, even if their proportions are unchanged.

In the applications of DEA to mutual funds this kind of information has proved to be useful; in fact, from a financial point of view, this composite unit could be considered as a benchmark for the inefficient mutual fund j_0. Fund j_0 could improve its performance by trying to imitate the behavior of the efficient composite unit, which has an input/output orientation which is similar to that of fund j_0 (see [2]).

3. THE SHARPE RATIO IN A DEA ENVIRONMENT

Some applications of data envelopment analysis approach to mutual fund evaluation aim at building DEA performance measures that generalize the traditional performance indicators (see [15], [2], [4]).

In this section we first show how to define a suitable DEA indicator that may be viewed as a generalization of the Sharpe index, which considers the subscription and redemption costs that burden the investors. Then we extend the analysis by defining a more general DEA performance index that includes also the ethical component of the investment.

Let us consider n mutual funds whose efficiency has to be evaluated. We remind that the Sharpe index (called "reward-to-variability" index in [17]), computed for fund j $(j = 1, \dots, n)$, is defined as the ratio between the expected excess return (in excess of the riskless rate) and the standard deviation of the return, that is:

$$I_{j,Sharpe} = \frac{E(R_j) - r_f}{\sigma_j} \tag{3.1}$$

where R_j is the return of fund j, $E(R_j)$ is the expected return of fund j, r_f denotes the riskless rate of return and $\sigma_j = \sqrt{Var(R_j)}$ is the standard deviation of the return.

The Sharpe ratio allows to rank a set of mutual funds by suggesting to prefer the mutual fund that presents the higher expected excess return per unit of risk. The standard deviation of the return has been chosen as a measure of risk.

Notice that the standard deviation of the returns may be a proper risk measure when the investor holds a single risky asset and the returns probability distribution is symmetric. As noted in [13], the performance indexes that assume a single asset portfolio seem to be appropriate for mutual fund evaluation, since individuals mostly hold the fund shares without attempting any further diversification on their own.

In order to extend Sharpe's analysis and define a performance measure that includes also the initial and final investment costs, one can adopt the DEA approach.

We have seen that the DEA technique allows to measure the performance of decision making units in presence of multiple inputs and multiple outputs. We can thus apply this technique to the evaluation of mutual funds: it is sufficient to consider the funds as decision making units which require a set of inputs to provide some outputs. In effect the concepts of input and output in financial applications of DEA have to be clarified: the outputs represent desirable objectives that we want to maximize, whereas the inputs represent undesirable objectives that we want to minimize.

Let us consider the basic CCR model presented in the previous section and let us take as output the expected excess return $(E(R_j) - r_f)$ and as inputs the standard deviation of the return (σ_j) together with the subscription and redemption costs, which we denote by $c_{1j}, \dots, c_{kj}$.

A DEA performance measure for a target mutual fund j_0 can thus be defined as the optimal value of the following problem:

$$\max_{\{u,v,w_i\}} \quad \frac{u[E(R_{j_0}) - r_f]}{\sum_{i=1}^{k} w_i c_{ij_0} + v\sigma_{j_0}} \tag{3.2}$$

subject to

$$\frac{u[E(R_j) - r_f]}{\sum_{i=1}^{k} w_i c_{ij} + v\sigma_j} \leq 1 \qquad j = 1, \ldots, n \tag{3.3}$$

$$u \geq \varepsilon \tag{3.4}$$

$$v \geq \varepsilon \tag{3.5}$$

$$w_i \geq \varepsilon \qquad i = 1, \ldots, k. \tag{3.6}$$

It can be proved that the DEA performance measure so obtained may be considered as a generalization of the Sharpe ratio, in the sense that when the subscription and redemption costs are omitted, the DEA measure coincides with the Sharpe ratio re-scaled in the interval [0,1].

The following proposition holds:

Proposition 3.1. *Let us denote by $I_{j_0,DEA-S}$ the optimal value of the optimization problem obtained by letting $c_{ij} = 0 \quad (i = 1, \ldots, k; j = 1, \ldots, n)$ in problem (3.2)-(3.6). Then $I_{j_0,DEA-S}$, coincides with the Sharpe ratio computed for fund j_0, multiplied by a normalization constant which scales it off in the interval [0,1], that is:*

$$I_{j_0,DEA-S} = \frac{I_{j_0,Sharpe}}{\max_j I_{j,Sharpe}}. \tag{3.7}$$

The proof of Proposition 3.1 can be found in the Appendix.

The Sharpe ratio does not allow to consider different elements other than the return and risk dimensions of the performance. On the other hand, when evaluating ethical mutual funds, the ethical aim which motivates the purchase of socially responsible funds have to be considered. In this situation, the DEA approach can naturally be used to include in the performance analysis the ethical objective besides the investment return considered by the Sharpe ratio.

Ethical mutual funds are financial tools that enable investors to combine investment decisions with socially responsible objectives, such as peace, defence of the environment, social justice, economic development. The investment in ethical mutual funds guarantees individuals an ethical use of their savings, by means of a proper selection of the investments carried out by the fund managers; the selection can be carried out either by including in the portfolio the assets of the companies which ethically behave (in the respect of human rights, the defence of the environment and so on) or by excluding from the portfolios the assets of the companies with a profile that is bad for socially responsible criteria (for example the companies involved in the weapon industry or in polluting activities).

If we have at our disposal an indicator that allows to measure the ethical level of mutual funds, it is possible to compute a DEA performance measure that considers also the ethical component of the investment by simply adding a second output in problem (3.2)-(3.3).

Let e_j denote the ethical measure (social responsibility level) for fund j ($j = 1, 2, \dots, n$) defined on the ground of the ethical criteria adopted in the investment selection; let us assume that the ethical level is a non negative number and that an higher e_j is associated to the funds which have an higher degree of ethicality (that is whose ethical constraints in the investment selection are stronger). A natural choice is to associate a strictly positive ethical measure for the ethical funds and suppose that if a fund is a non-ethical fund its ethical level is equal to zero.

The performance measure that considers also the ethical component of the investment may be computed by solving the following optimization problem:

$$\max_{\{u_r, v, w_i\}} \quad \frac{u_1[E(R_{j_0}) - r_f] + u_2 e_{j_0}}{\sum_{i=1}^{k} w_i c_{ij_0} + v\sigma_{j_0}} \tag{3.8}$$

subject to

$$\frac{u_1[E(R_j) - r_f] + u_2 e_j}{\sum_{i=1}^{k} w_i c_{ij} + v\sigma_j} \leq 1 \qquad j = 1, \dots, n \tag{3.9}$$

$$u_r \geq \varepsilon \qquad r = 1, 2 \tag{3.10}$$

$$v \geq \varepsilon \tag{3.11}$$

$$w_i \geq \varepsilon \qquad i = 1, \dots, k. \tag{3.12}$$

We may observe that a result similar to that of Proposition 3.1 holds; that is, when the subscription and redemption costs are omitted and the ethical level is equal to zero for all funds under evaluation, then the DEA measure obtained as the optimal value of the objective function (3.8), coincides with the Sharpe ratio re-scaled in the interval [0,1].

Table 3.1 presents an example of the application of the previously described DEA models to a set of 46 mutual funds; among them there are seven ethical funds, which are marked in the table with one or more asterisks. One asterisk denotes the ethical funds that present a low degree of ethicality (in this case we set $e_{j_0} = 1$), two asterisks denote the funds with a medium degree of ethicality ($e_{j_0} = 2$) and three asterisks mark the funds with an higher degree of ethicality ($e_{j_0} = 3$).

In the example we have considered among the outputs the expected excess return and the ethical level of the investment; as inputs, in addition to the portfolio standard deviation, we have chosen one subscription cost and one redemption cost.

Table 3.1 compares the results obtained by solving the various DEA models.

We call $I_{j_0,DEA-S}$ the normalized Sharpe ratio for fund j_0; as previously seen, it coincides with the "one output–one input" DEA measure obtained by solving a reduced version of problem (3.2)-(3.6), in which the investment costs are omitted.

$I_{j_0,DEA-S1}$ denotes the "one output–multiple inputs" DEA measure that considers the subscription and redemption costs; it is computed as the optimal value of the objective function of problem (3.2)-(3.6).

Tabella 3.1. *Comparison among the various performance measures. $I_{j_0,DEA-S}$ is the normalized Sharpe ratio, $I_{j_0,DEA-S1}$ is the DEA performance measure which considers the investment costs and $I_{j_0,DEA-S2}$ is the DEA performance measure which considers also the ethical component of the investments. One or more asterisks mark the ethical mutual funds. The relative fund ranking is given in italics*

Fund j_0	$I_{j_0,DEA-S}$		$I_{j_0,DEA-S1}$		$I_{j_0,DEA-S2}$	
F_1	0.1803	*31*	0.4369	*25*	0.4369	*28*
F_2	0.4126	*7*	1.0000	*1*	1.0000	*1*
F_3	0.2363	*21*	0.5348	*18*	0.5348	*21*
F_4	0.3260	*13*	0.7436	*11*	0.7436	*15*
F_5	0.2064	*25*	0.4428	*24*	0.4428	*27*
F_6**	0.0009	*46*	0.0009	*46*	1.0000	*1*
F_7	0.1498	*38*	0.3214	*36*	0.3214	*39*
F_8	0.3931	*10*	0.8432	*6*	0.8432	*10*
F_9	0.0642	*44*	0.1440	*44*	0.1440	*45*
F_{10}	0.4274	*5*	0.8265	*8*	0.8265	*12*
F_{11}	0.1827	*30*	0.3276	*35*	0.3276	*38*
F_{12}	0.1637	*35*	0.1942	*41*	0.1942	*44*
F_{13}	0.2604	*18*	0.5587	*17*	0.5587	*20*
F_{14}	0.3399	*12*	0.7726	*10*	0.7726	*14*
F_{15}	0.1637	*36*	0.3512	*34*	0.3512	*37*
F_{16}	0.0933	*41*	0.2095	*39*	0.2095	*42*
F_{17}	0.2968	*15*	0.6366	*15*	0.6366	*18*
F_{18}***	0.2491	*19*	0.5344	*19*	0.9598	*7*
F_{19}	0.1936	*28*	0.4153	*28*	0.4153	*31*
F_{20}*	1.0000	*1*	1.0000	*1*	1.0000	*1*
F_{21}	0.2143	*23*	0.4597	*23*	0.4597	*26*
F_{22}	0.1840	*29*	0.3946	*29*	0.3946	*32*
F_{23}	0.4662	*2*	1.0000	*1*	1.0000	*1*
F_{24}	0.4041	*8*	0.8669	*4*	0.8669	*8*
F_{25}	0.1938	*27*	0.4168	*27*	0.4168	*30*
F_{26}*	0.0748	*43*	0.1814	*43*	0.3733	*34*
F_{27}	0.2086	*24*	0.2086	*40*	0.2086	*43*
F_{28}	0.3052	*14*	0.6546	*13*	0.6546	*16*
F_{29}	0.2468	*20*	0.5294	*20*	0.5294	*22*
F_{30}	0.3885	*11*	0.8334	*7*	0.8334	*11*
F_{31}	0.4377	*3*	0.8615	*5*	0.8615	*9*
F_{32}	0.1151	*40*	0.2182	*38*	0.2182	*41*
F_{33}	0.4260	*6*	0.5051	*21*	0.5051	*24*
F_{34}	0.1341	*39*	0.2542	*37*	0.2542	*40*
F_{35}	0.4012	*9*	0.8142	*9*	0.8142	*13*
F_{36}	0.4278	*4*	0.6091	*16*	0.6091	*19*
F_{37}	0.2701	*17*	0.6546	*14*	0.6546	*17*
F_{38}	0.0177	*45*	0.0379	*45*	0.0379	*46*
F_{39}	0.1622	*37*	0.3684	*32*	0.3684	*35*
F_{40}	0.2333	*22*	0.5004	*22*	0.5004	*25*
F_{41}***	0.2824	*16*	0.6844	*12*	1.0000	*1*
F_{42}	0.2028	*26*	0.4350	*26*	0.4350	*29*
F_{43}	0.1752	*33*	0.3758	*31*	0.3758	*33*
F_{44}	0.1637	*34*	0.3513	*33*	0.3513	*36*
F_{45} **	0.0796	*42*	0.1930	*42*	1.0000	*1*
F_{46}*	0.1771	*32*	0.3800	*30*	0.5226	*23*

$I_{j_0,DEA-S2}$ represents the "two outputs–multiple inputs" DEA measure that includes also the ethical component, obtained by solving problem (3.8)-(3.12).

Table 3.1 shows that the inclusion of both the investment costs and the ethical component of the investment in the mutual fund performance analysis changes the efficiency measures and the relative ranking of mutual funds.

For example, fund F_2, which is evaluated as inefficient according to the Sharpe ratio, becomes efficient when the investment costs are considered. The ethical fund F_6, on the other hand, becomes relatively efficient only when the ethical component of the investment is included in the performance analysis.

It could also happen that a fund (for example F_{10}) worse its position in the ranking when either the costs or the ethical element are considered, even if the value of its performance measure does not decrease.

In fact it is possible to prove that the following relations hold among the performance measures previously defined:

$$I_{j_0,DEA-S} \leq I_{j_0,DEA-S1} \leq I_{j_0,DEA-S2}. \tag{3.13}$$

This means that when using more output (or input) indicators instead of one, the number of efficient funds increases, since we have more criteria with respect to which some funds can be considered as efficient.

Moreover it is possible to prove that, under the assumptions previously stated on the ethical level, the value of the DEA measure $I_{j_0,DEA-S2}$, computed for the non-ethical funds, coincides with the value of the DEA measure $I_{j_0,DEA-S1}$:

$$I_{j_0,DEA-S1} = I_{j_0,DEA-S2} \quad \text{if} \quad e_{j_0} = 0 \quad \text{and} \quad e_j \geq 0 \quad \forall j. \tag{3.14}$$

Table 3.1 highlights these properties.

4. EXTENSIONS OF DEA MODELS FOR MUTUAL FUND APPRAISAL

We have seen in the previous section that DEA allows to build performance indexes that simultaneously consider the return and risk dimensions, together with the subscription and redemption costs and, in case, the ethical component of the investment. The standard deviation of the return was taken as a measure of risk, as in the Sharpe's analysis.

Nevertheless, there is no complete agreement regarding the best risk indicator to be used.

In fact, in the financial literature there have been proposed performance measures that are similar to the Sharpe index, in the sense that they consider the expected excess return per unit of risk, but they differ from the Sharpe index in their choice of the risk indicator. Among them we recall the reward to half-variance index ([1]), which considers as risk indicator the square root of the half-variance (i.e. the average of the squared negative deviations from the mean) and the Treynor performance index ([18]) that measures the risk with the β of the portfolio (we remind that the β of the portfolio is the ratio of the covariance between the portfolio return and the market portfolio return, to the variance of the market portfolio return).

In this context, the DEA approach can be used to build performance indexes that consider alternative risk measures simultaneously. In this way DEA allows to overcome the problem of choosing only one particular risk indicator at a time, which is relevant only under special assumptions on the investors' behavior.

In [2] a DEA performance measure is proposed that jointly takes into account a return measure and several risk measures, together with the subscription and redemption costs.

By denoting with o_j a return measure of fund j and assuming that there are k subscription and redemption costs c_{ij} $(i = 1, \dots, k)$ and h risk measures q_{ij} $(i = 1, \dots, h)$, the DEA performance measure proposed involves the solution of the following optimization problem:

$$\max_{\{u, v_i, w_i\}} \quad \frac{u o_{j_0}}{\sum_{i=1}^{h} v_i q_{ij_0} + \sum_{i=1}^{k} w_i c_{ij_0}} \tag{4.1}$$

subject to

$$\frac{u o_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1 \qquad j = 1, \dots, n \tag{4.2}$$

$$u \geq \varepsilon \tag{4.3}$$

$$v_i \geq \varepsilon \qquad i = 1, \dots, h \tag{4.4}$$

$$w_i \geq \varepsilon \qquad i = 1, \dots, k. \tag{4.5}$$

In problem (4.1)-(4.5) the output o_j represents a return measure of fund j; one can choose as output the expected return $E(R_j)$ or the expected excess return $E(R_j) - \delta$; in many empirical DEA applications to mutual funds the expected return $E(R_j)$ is often used instead of the expected excess return in order to reduce the occurrence of negative values in the output.

As risk measures one can take the standard deviation of the return, the root of the half-variance of the return and the β of the portfolio, but nothing prevents from choosing other risk measures.

It is possible to prove that the DEA performance measure obtained by solving problem (4.1)-(4.5) generalizes the traditional performance Sharpe, Treynor and reward-to-half-variance indexes.

In fact, if we choose the excess return as output and the standard deviation of the return as the only input, thus omitting the entrance and exit investment costs (i.e., $h = 1$, $k = 0$), then the DEA performance measure obtained by solving problem (4.1)-(4.5) coincides with the normalized Sharpe ratio, similarly to the result obtained in (3.7). Analogously, by taking as risk measure either the root of the half-variance or the β coefficient we obtain the reward-to-half-variance and the Treynor indexes, respectively.

Model (4.1)-(4.5) can also be extended in order to include other output indicators besides the mean return, which could shed light on different aspects of the portfolio returns.

In [2] a stochastic dominance indicator d_j, which reflects both the investors' preference structure and the time occurrence of the returns, is included among the outputs; A DEA portfolio performance measure may therefore be defined by solving the following optimization problem:

$$\max_{\{u_r,v_i,w_i\}} \quad \frac{u_1 o_{j_0} + u_2 d_{j_0}}{\sum_{i=1}^{h} v_i q_{ij_0} + \sum_{i=1}^{k} w_i c_{ij_0}} \tag{4.6}$$

subject to

$$\frac{u_1 o_j + u_2 d_j}{\sum_{i=1}^{h} v_i q_{ij} + \sum_{i=1}^{k} w_i c_{ij}} \leq 1 \qquad j = 1, \ldots, n \tag{4.7}$$

$$u_r \geq \varepsilon \qquad r = 1, 2 \tag{4.8}$$

$$v_i \geq \varepsilon \qquad i = 1, \ldots, h \tag{4.9}$$

$$w_i \geq \varepsilon \qquad i = 1, \ldots, k. \tag{4.10}$$

The stochastic dominance indicator d_j is defined by using stochastic dominance relations between the mutual fund returns; it is computed as the relative number of subperiods in which fund j is not dominated by other funds. The idea is to assign a higher score to the mutual funds which are not dominated by other funds in the higher number of subperiods (see [2] for more details).

We conclude this section by noting that in many DEA applications to mutual funds the basic CCR model is used. Nevertheless in some cases the use of the basic DEA model might be not appropriate. For example, this could happen when the performance of ethical mutual funds have to be evaluated.

We have seen in section 3 how to include considerations on the ethical aim of investors into the performance analysis. However the DEA performance measure obtained by solving problem (3.8)-(3.12) is appropriate when an indicator e_j which measures the ethical level of mutual funds is available.

Nevertheless, many consultancy agencies and research institutes document only the ethical nature of mutual funds, so in most cases we have at our disposal only the binary information on the ethical/non ethical nature of a fund.

In addition, some agencies (for example the Belgian agency Ethibel and the Italian institute Axia Financial Research) offer a classification of mutual funds into categories of increasing ethical level; thus sometimes we have at our disposal a rating of mutual funds into categories of different ethical levels.

Moreover, it is reasonable to assume that investors choose the ethical level of an investment a priori, so that the ethical measure have to be considered exogenously fixed.

In all these cases the CCR model is not appropriate, since it assumes that the variables are not exogenously fixed; moreover the virtual unit suggested by the basic CCR model as efficient benchmark could have an ethical level that has no correspondence with any of the existing categories of mutual funds.

In [4] various extensions of the basic DEA model are presented, according to the nature of the ethical indicator that characterizes the ethical funds. In particular, the use of a DEA categorical model with exogenously fixed variables has been suggested for the appraisal of socially responsible mutual funds.

5. FINAL REMARKS

Recently the use of the data envelopment analysis approach has been suggested to evaluate the performance of mutual funds. We think that the DEA methodology can be considered as a useful device which complement (does not substitute) the traditional performance indicators for the appraisal of mutual funds.

There are some advantages. The DEA methodology allows to define performance measures that are able to treat the multidimensional nature of the mutual fund performance. Besides the risk-return dimensions, one can consider the investment costs, the ethical aim, the occurrence of stochastic dominance relations between the fund return and so on. Moreover DEA is able to include in the analysis various risk measures simultaneously.

At the same time, DEA synthesizes the various dimensions of mutual fund performance in a numerical efficiency measure that allows to rank mutual funds.

The DEA technique is a non parametric approach that does not require to specify a functional form for the correspondence between outputs and inputs.

Moreover, the DEA performance measure does not require the specification of a benchmark. Instead, each mutual fund that has been evaluated as inefficient, can be compared with a composite unit that acts as a benchmark for that fund.

Nevertheless, caution is required in using DEA. One could be induced to expand the number of input and output variables that describe the performance aspect of mutual funds. Instead, it is important to select only the essential variables; the number of inputs and outputs must not be excessive in comparison to the number of units being assessed, otherwise the methodology loses its discriminatory power.

Caution is required also when the results are interpreted. The DEA technique provides a relative performance measure. Unlike the traditional mutual funds performance indexes, the value of which don't change when the set of mutual funds to be compared is modified, the values of the DEA efficiency indexes depend on the choice of the mutual funds that are compared.

Moreover, in the basic DEA model the weights used in defining the efficiency measure are not set in advance, in a subjective manner, but are the most favorable weights for each fund. On the one hand this is an element in favor of DEA models, respect to multiple criteria approaches. On the other hand there may be situations where investors could make assumptions on the various criteria. For example, if investors prefer a lower risk of the investment rather than lower costs, some additional constraints on the weights associated to these input variables have to be added.

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APPENDIX

This Appendix reports the proof of Proposition 3.1.

Let us consider problem (3.2)-(3.6). By letting $c_{ij} = 0$, $(i = 1, \ldots, k; j = 1, \ldots, n)$ problem (3.2)-(3.6) becomes:

$$\max_{\{u,v\}} \quad \frac{u[E(R_{j_0}) - r_f]}{v\sigma_{j_0}} \tag{A.1}$$

subject to

$$\frac{u[E(R_j) - r_f]}{v\sigma_j} \leq 1 \qquad j = 1, \ldots, n \tag{A.2}$$

$$u \geq \varepsilon \tag{A.3}$$

$$v \geq \varepsilon. \tag{A.4}$$

Let us denote by $I_{j_0,DEA-S}$ the optimal value of the objective function (A.1). The linear programming problem equivalent to problem (A.1)-(A.4) may be written as follows:

$$\max \quad u[E(R_{j_0}) - r_f] \tag{A.5}$$

subject to

$$v\sigma_{j_0} = 1 \tag{A.6}$$

$$u[E(R_j) - r_f] - v\sigma_j \leq 0 \qquad j = 1, \ldots, n \tag{A.7}$$

$$u \geq \varepsilon \tag{A.8}$$

$$v \geq \varepsilon. \tag{A.9}$$

From the equality constraint (A.6) we obtain $v^* = 1/\sigma_{j_0}$; by substituting this value for v in constraints (A.7) we obtain the inequalities:

$$u \leq \left[\sigma_{j_0} \frac{E(R_j) - \delta}{\sigma_j}\right]^{-1} \qquad j = 1, \ldots, n. \tag{A.10}$$

The value of u which maximizes the objective function value while satisfying the constraints is

$$u^* = \left[\sigma_{j_0} \max_j \frac{E(R_j) - r_f}{\sigma_j}\right]^{-1}. \tag{A.11}$$

Therefore the optimal value of the objective function (A.5), which coincides with $I_{j_0,DEA-S}$, is equal to

$$\left[\max_j \frac{E(R_j) - r_f}{\sigma_j}\right]^{-1} \frac{E(R_{j_0}) - r_f}{\sigma_{j_0}} = \frac{I_{j_0,Sharpe}}{\max_j I_{j,Sharpe}}. \tag{A.12}$$

■