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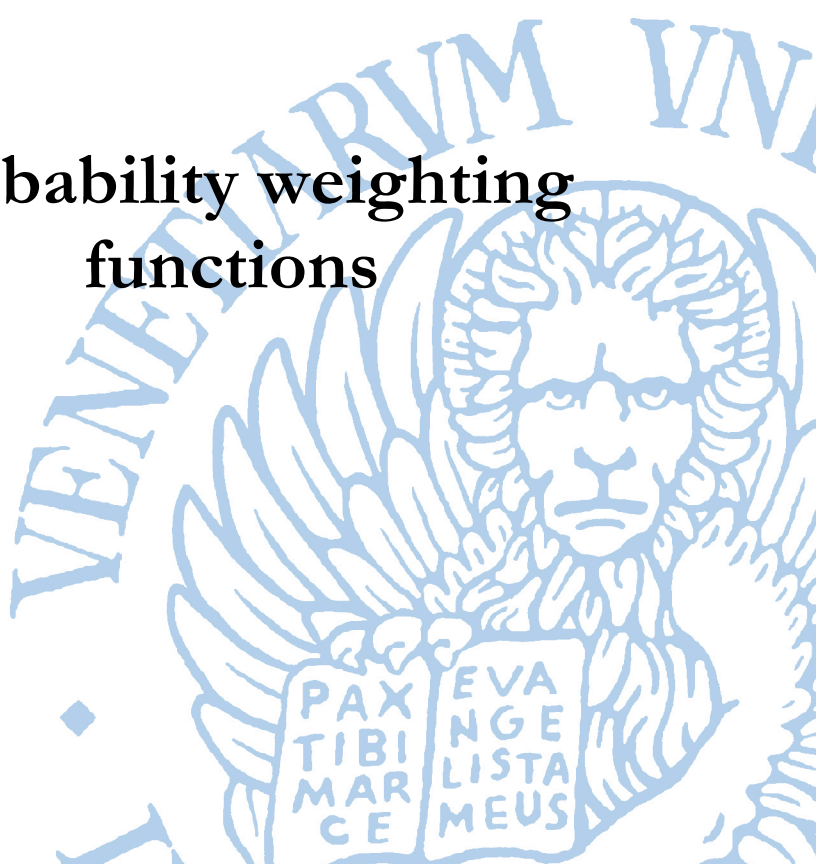
Department of Economics

Working Paper

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Probability weighting functions

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Abstract

Cumulative prospect theory (CPT) has been proposed as an alternative to expected utility theory to explain irregular behavior by economic agents. CPT comprises two key transformations: one of outcome values and the other of objective probabilities. Risk attitudes are derived from the shapes of these transformations as well as their interaction. The focus of this contribution is on the transformation of objective probability, which is commonly referred as *probability weighting function*. We review different families of weighting functions proposed in the literature and study their features.

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1 Introduction

Cumulative Prospect Theory (CPT) has been proposed as an alternative to Expected Utility (EU) theory with the aim of explaining the complexity of observed behaviors followed by the economic agents[1]. According to prospect theory, individuals do not always take their decisions in order to maximize expected utility; they are risk averse with respect to gains and risk-seeking for losses; people are much more sensitive to losses than they are to gains of comparable magnitude (loss aversion). Outcomes are evaluated based on potential gains and losses relative to a reference point, rather than in terms of final wealth (as in EU). Moreover, decision makers tend to underweight high probabilities and overweight low probabilities. The degree of risk aversion or risk seeking seems to depend not only on the value of the outcomes but also on the probability and ranking of outcome. CPT is based on two key transformations which try to capture all these behaviors: individuals evaluate outcomes through a value function and objective probabilities are replaced by decision weights. Risk attitudes are derived from the shapes of these transformations as well as their interaction.

Prospect theory allows to accomodate the classical paradoxes of decision making under risk, such as the common consequence effect (e.g. the Allais paradox), the common ratio effect, the four pattern of risk preferences, and the simultaneous attraction for buying lottery tickets and insurance policies.

The focus of this paper is on the transformation of objective probability, which is commonly referred as *probability weighting function* (pwf). The pwf is of particular interest because, along with gain-loss separability, it represents a distinguish feature between CPT and EU.

While there is a general consensus about the qualitative shape of the pwf (inverse sigmoid), numerous functional forms have been proposed in the literature: some forms are derived axiomatically, some are based on psychological factors, and others seem to have no normative justification at all. As a result, each functional form of pwf, embedded in CPT framework, yields a different model with potentially different implications for choice behavior. Thus, while the inclusion of subjective probabilities of any form allows CPT to outperform EU in describing actual choice patterns, despite the functional and theoretical differences between forms of weighting functions, attempts to identify the form that best describes human behaviors have yielded ambiguous results. Judging by a visual inspection of the shapes of probability weighting curves, it is not surprising that the forms are so difficult to discriminate; by appropriately choosing the values of the parameters, one can draw curves that appear virtually identical although they belong to different families of functions. As an example, Figure 1 shows two weighting functions, one suggested by Prelec [12] and the linear in log-odds (LinLog) function (both functions will be discussed later). The curves mimic one another so closely, hence one may wonder whether it really matters which functional form is used; if two or more forms are so similar such that it is impossible to empirically discriminate amongst them, then the debate over which one most closely approximates human decision making might appear not relevant. However, to the extent that the functions can be discriminated empirically with choice data, we should do our best to compare them and thereby sharpen our understanding of probability weighting in risk

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[1]Prospect Theory in its cumulative version has been introduced by Tversky and Kahneman [14]. For a thorough treatment on Prospect Theory, we refer to the book by Wakker [15].
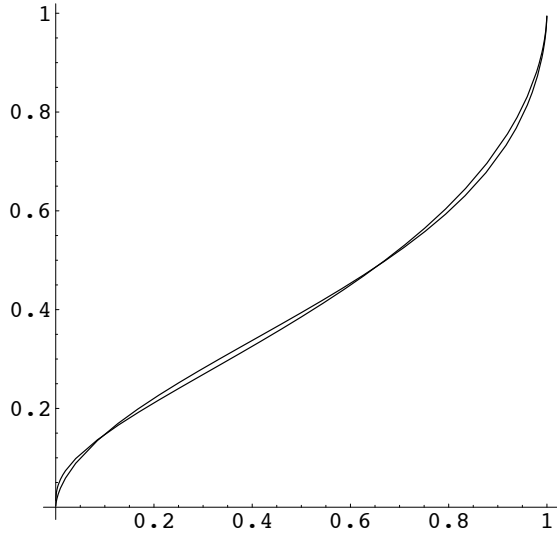
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Figure 1: A Prelec (two parameters) function and a LinLog function that closely match each other

choice.

The shape of the two functions in Figure 1 is an inverse-S. Such a shape seems to be supported by empirical studies, which have shown that decision makers do not usually treat probabilities linearly. Instead, people tend to overweight small probabilities and underweight hight probabilities. One way to model such distortions in decision making under risk and uncertainty is through a probability weighting function.

In the remainder of this paper, we review and discuss the main features of different probability weighting functions. Section 2 defines the notion of probability weighting function. Then we describe some families of pwf which depend on one parameter (Section 3), two or more parameters (Section 4), of polynomial form (Section 5). Finally, Section 6 concludes.

2 Probability weighting functions under prospect theory

A probability weighting function is a strictly increasing function $w(p) : [0,1] \rightarrow [0,1]$; such a function is not simply a subjective probability but rather a distortion of objective probabilities.

A pwf has the following properties:

i) $w(0) = 1$ and $w(1) = 1$;

ii) w has a unique inverse function w^{-1} which is strictly increasing from $[0,1]$ onto $[0,1]$;

iii) w and w^{-1} are continuous.

There is empirical and theoretical interest in discontinuous weighting functions at 0 and at 1.
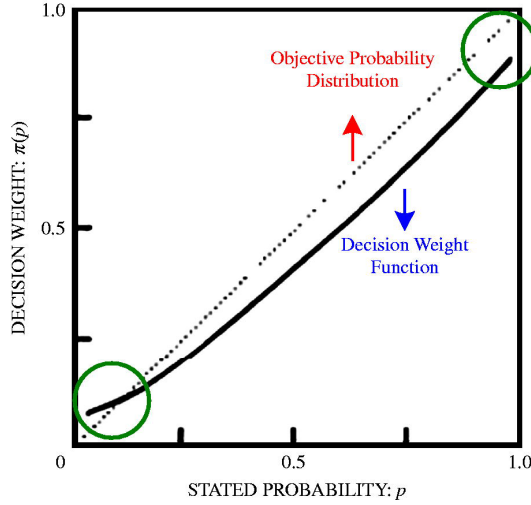
Figure 2: Transformation of probability proposed by Kahneman and Tversky (this picture is reported from the paper [6])

A *standard* pwf is characterized by infinitely overweighting of infinitesimal probabilities and infinitely underweighting of probabilities near 1, i.e.

$$\lim_{p \to 0} \frac{w(p)}{p} = \infty, \qquad \lim_{p \to 1} \frac{1 - w(p)}{1 - p} = \infty, \tag{1}$$

respectively.

The function w should exhibit a set of basic features which try to capture observed departures from classical expected utility theory. Kahneman and Tversky [6], in their seminal paper on Prospect Theory (PT), recognize the need to change the objective probabilities and introduce decision weights $\pi = w(p)$. The authors state some properties of such a function and consider a hypothetical weighting function which is shown in Figure 2; note that the function suggested by Kahneman and Tversky is not defined near the end points (for extreme probabilities).

Kahneman and Tversky [6] identify other properties of the weighting function: overweighting of small probability, underweighting of large probability, *subcertainty* (i.e. the sum of the weights for complementary probabilities is less than one, $w(p) + w(1-p) < 1$). The authors also observed that the probability weighting function may not be well behaved near the endpoints 0 and 1. The function shown in Figure 2 is consistent with these properties.

CPT developed by Tversky and Kahnemann [14] overcomes some drawbacks (such as violation of stochastic dominance) of the original PT. In CPT, decision weights π_i are differences in transformed (through a weighting function) cumulative probabilities of gains or losses. Formally,

$$\pi_i = \begin{cases} w^-(p_{-m}) & i = -m \\ w^- \left(\sum_{j=-m}^{i} p_j \right) - w^- \left(\sum_{j=-m}^{i-1} p_j \right) & i = -m+1, \ldots, -1 \\ w^+ \left(\sum_{j=i}^{n} p_j \right) - w^+ \left(\sum_{j=i+1}^{n} p_j \right) & i = 0, \ldots, n-1 \\ w^+(p_n) & i = n, \end{cases} \tag{2}$$
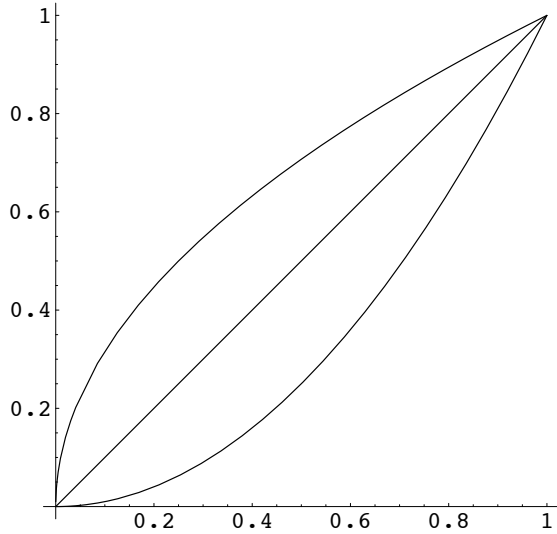
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Figure 3: Optimistic, neutral and pessimistic probability attitude

where w^- denotes the weighting function for losses and w^+ for gains, respectively.

In the literature, there is specific interest in weighting functions that are initially concave, say for low probabilities in an interval $(0, \delta)$, for $0 < \delta < 1$, and convex for medium and large probabilities, on $(\delta, 1)$. We call these functions *inverse-S shaped* weighting functions, reflecting the shape of the corresponding mapping. The function shown in Figure 2 cannot account for such pattern because it is not concave for low probabilities.

Related to the curvature of weighting functions is the notion of probabilistic risk aversion. A convex weighting function characterizes probabilistic risk aversion (or *pessimism*) whereas a concave weighting function characterizes risk proneness (or *optimism*). A linear weighting function is characterized by probabilistic risk neutrality. Figure 3 depicts examples of continuous weighting functions of the form $w(p) = p^\gamma$ corresponding to previous notions of optimism ($0 < \gamma < 1$), neutrality ($\gamma = 1$) and pessimism ($\gamma > 1$), respectively.

Empirical support is for a function which is inverse-S shaped. Observe that w with inverse-S shape need not cross the linear and continuous weighting function: it can be completely above or completely below it (except at 0 and 1). Concavity (convexity) of w is not necessarily associated with overweighting (underweighting). In some families of weighting functions, elevation of w, i.e. the value of p in $(0, 1)$ such that $w(p) = p$, coincides with the inflection point. Empirical findings indicate that the intersection between the weighting function and the linear function is for probability around one third. Elevation has also an interesting interpretation as a measure of *relative optimism* (see Abdellaoui et al. [1]).
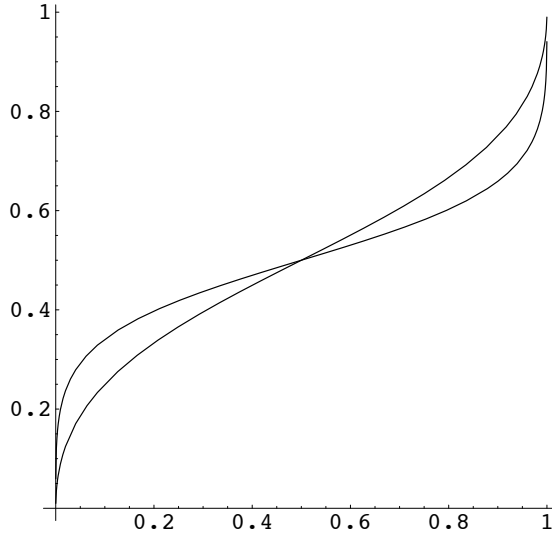
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Figure 4: Transformation of probability by function (4)

3 One parameter weighting functions

Many different parametric functional forms have been proposed in the literature. In this section, we analyze some families of pwf in which a single parameter determines the nature and the magnitude of the discrepancy between the transformed probabilities, $w(p)$, and the original ones, p, by capturing features such as the curvature and the elevation of the function and the position of the fixed point $(w(p) = p)$.

The single parameter of the weighting probability function may be different for gains and for losses and there are large variations in the estimates across the studies.

Karmarkar [7] proposed a weighting function defined implicitly by the relation

$$\log \frac{w_i}{1 - w_i} = \alpha \log \frac{p_i}{1 - p_i}, \tag{3}$$

where $0 < \alpha < \infty$; or

$$\frac{w_i}{1 - w_i} = \left(\frac{p_i}{1 - p_i} \right)^\alpha. \tag{4}$$

Therefore we have

$$w_i = \frac{(Odds_i)^\alpha}{1 + (Odds_i)^\alpha} \quad \text{with} \quad Odds_i = \frac{p_i}{1 - p_i}. \tag{5}$$

Note that for binary games where $p_1 + p_2 = 1$, it is easy to prove that $w_1 + w_2 = 1$. This property is not true for prospects with more than two outcomes. For any value of the parameter α, the function (5) has a fixed point in p=1/2. As α tends to zero, every outcome is seen as equally likely; for $\alpha = 1$, we obtain the true probabilities. For $\alpha \to \infty$, an event with probability less than 1/2 is impossible, whereas it is certain if its probability is greater than 1/2.
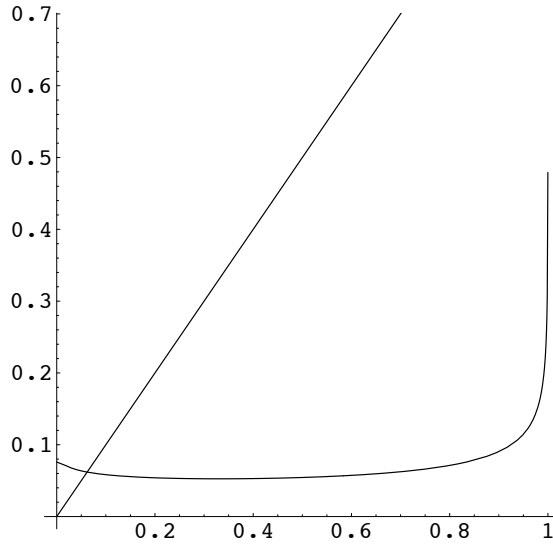
Figure 5: Probability weighting function proposed by [14] with $\gamma = 0.2$

Tversky and Kahneman [14] generalize prospect theory using a rank-dependent representation. The probability weighting function axiomatically proposed by the authors is given by

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}} \qquad \text{with} \quad \gamma > 0.278; \qquad (6)$$

numerical computation evidences that the function (6) is partially decreasing for $\gamma < 0.278$ (see Figure 5 and [5]).

This function has been the subject of several parametric studies and it is a special case of the following pwf (see [17]):

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^s} \qquad s > 0. \qquad (7)$$

The one parameter probability weighting function proposed by Prelec (see [12]) is

$$w(p) = \exp[-(-\log p)^\alpha] \qquad 0 < \alpha < 1. \qquad (8)$$

The function (8) has an invariant fixed and inflection point at $p = 1/e$. Different instances of one parameter Prelec function are represented in Figure 6; lower values of α correspond to higher curvature and departure from the 45° line.

4   Weighting probability functions with two or more parameters

The Karmarkar's one parameter weighting function can be generalized including the intercept parameter, then obtaining a two parameters weighting function based on the assumption of a
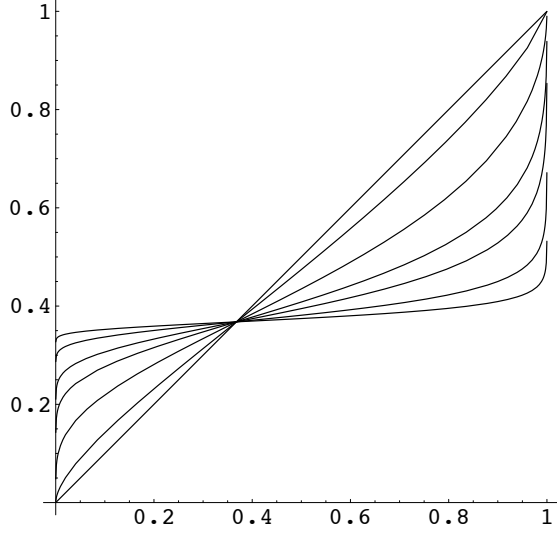
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Figure 6: Transformation of probability proposed in [12] for $\alpha = 0.05, 0.1, 0.2, 0.3, 0.5, 0.8, 1$

linear relation between the log of weighted odds and the log probability odds

$$\log \frac{w_i}{1 - w_i} = r \log \frac{p_i}{1 - p_i} + \log s. \tag{9}$$

By solving equation (9) with respect to w, we obtain the linear in log odds (LinLog) probability function

$$w(p) = \frac{s\, p^r}{s\, p^r + (1-p)^r} \qquad r > 0, \ s > 0. \tag{10}$$

This form was originally used by Goldstein and Einhorn [4], although not as a probability weighting function. Function (10) is a special case (obtained for two outcome gambles) of the form used by Lattimore et al. [8] given by

$$w(p_i) = \frac{s\, p_i^r}{s\, p_i^r + \sum_{k \neq i} p_i^r}. \tag{11}$$

In (10) the parameter r primary controls curvature and the parameter s elevation. Figures 7 and 8 show how the two parameters r and s control curvature and elevation almost independently.

Prelec [12] considers also a two parameters probability weighting function of the form

$$w(p) = \exp[-\beta(-\log p)^\alpha] \qquad \alpha > 0, \ \beta > 0. \tag{12}$$

This function has two nested cases: the one parameter probability weighting function (8) obtained by setting the parameter $\beta = 1$, and the power law obtained by setting the parameter $\alpha = 1$.
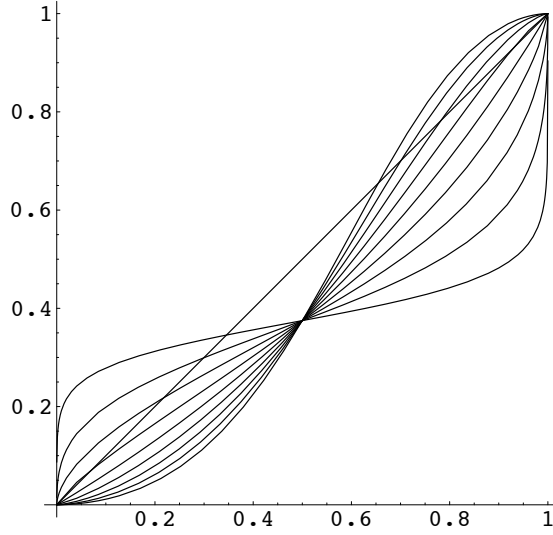
Figure 7: Instances of weighting function defined by (10) as r (curvature) varies, for $s = 0.6$ fixed and r between 0.2 and 1.8 (the lower r, the higher the curvature)
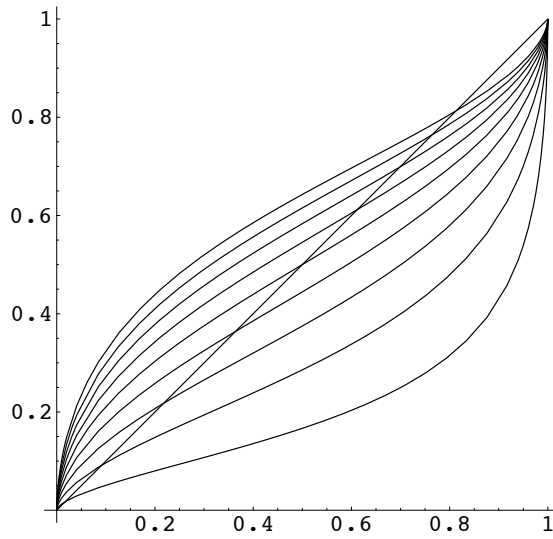

Figure 8: Instances of weighting function defined by (10) as s (elevation) varies, for $r = 0.6$ fixed and s between 0.2 and 1.8 (higher values of s correspond to more elevated functions)
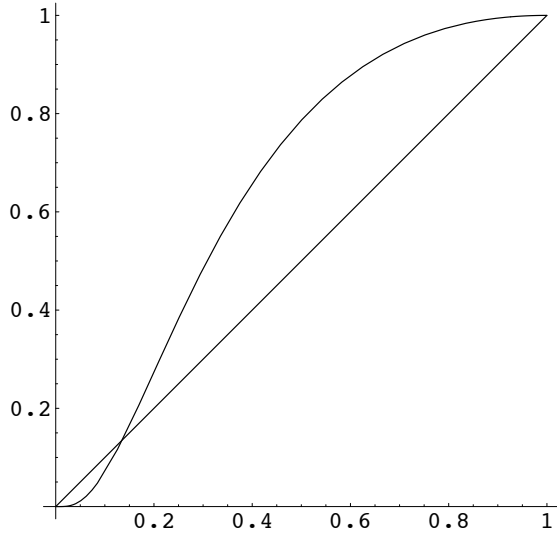
Figure 9: The two parameters probability weighting function proposed by Prelec for $\alpha = 2$ and $\beta = 0.5$

The parameter α controls the convexity/concavity of the Prelec function (12). If $\alpha < 1$ the function is strictly concave for low probabilities but strictly convex for high probabilities, i.e. it is *inverse-S shaped*. The converse holds if $\alpha > 1$, i.e. the function is *S-shaped*.

The parameter β controls the location of the inflexion point relative to the $45°$ line (elevation of the pwf). Thus for $\beta = 1$ the point of inflexion is at $p = e^{-1}$ and lies on the $45°$ line. However, if $\beta < 1$ then the point of inflexion lies above the $45°$ line. For example if $\alpha = 2$ and $\beta = 0.5$, the fixed point $w(p^*) = p^*$ is at $p^* \simeq 0.14$ but the point of inflexion $w''(p^+) = 0$ is at $p^+ \simeq= 0.20$ (see Figure 9).

In the same paper (see [12]), Prelec derives two other probability weighting functions: the *exponential-power* function

$$w(p) = \exp\left[-\frac{\eta}{\gamma}(1 - p^\gamma)\right],$$ (13)

and the *hyperbolic-logarithm* function

$$w(p) = (1 + \gamma \log p)^{-\eta/\gamma}.$$ (14)

In [10] Luce presents the following pwf

$$w(p) = \exp\left[-\beta\left(\frac{1-p}{p}\right)^\alpha\right] \qquad \alpha > 0, \ \beta > 0.$$ (15)

This family cannot include the power function as special case but it is very flexible, since, depending of the choice of parameters, it can be wholly above the main diagonal, wholly below it, or it crosses the $45°$ line from above to below. Figure 10 provides some illustrative examples.
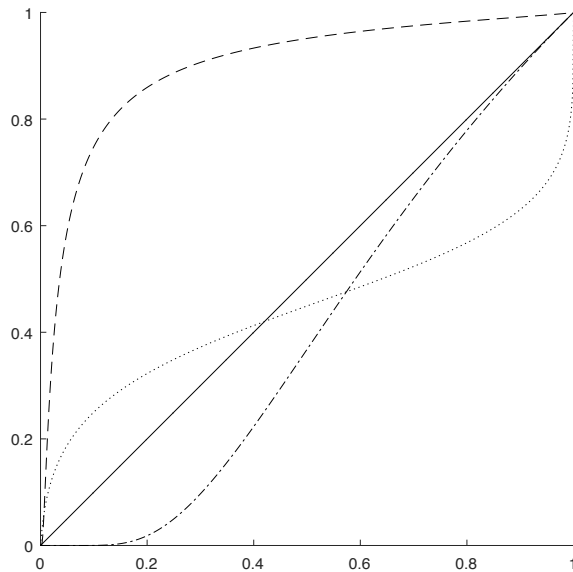
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Figure 10: Plots of the pwf defined by (15) for $\alpha = \beta = 1$ (dashed-dotted line), $\alpha = 0.25, \beta = 0.80$ (dotted line) and $\alpha = 0.8, \beta = 0.05$ (dashed line)

Diecidue et al. [3] derive axiomatically the class of *switch-power weighting functions* which are power functions for probabilities below a level $\widehat{p} \in (0,1)$, and dual power functions for probabilities grater than $\widehat{p}$, i.e.

$$w(p) = \begin{cases} c\,p^a & \text{if } p \leq \widehat{p} \\ 1 - d(1-p)^b & \text{if } p > \widehat{p}. \end{cases} \tag{16}$$

Note that functions (16) depend on five parameters; however, if one accepts the plausible hypothesis of continuity and differentiability in $\widehat{p}$, then the number of parameters can be reduced to three since with simple algebra we obtain

$$c = \frac{b\,\widehat{p}^{1-a}}{a(1-\widehat{p}) + b\,p} \qquad\qquad d = \frac{a(1-\widehat{p})^{1-b}}{b\,\widehat{p} + a(1-\widehat{p})}.$$

Continuity and monotonicity imply that all parameters must be positive. For $0 < a \leq 1$ the probability functions are concave on $(0, \widehat{p})$ and for $0 < b \leq 1$ they are convex on $(\widehat{p}, 1)$.

If we consider the particular case $a = b$, it is easy to show that the graphic of the weighting function (16) intersects the 45° line (the true probabilities) exactly at $\widehat{p}$; since in this case $w'(\widehat{p}) = a$, we conclude that parameter a controls the curvature of the weighting function. The parameter $\widehat{p}$, however, discriminates between the interval relating to overweighting probabilities and that related to the underweighting[2], and therefore governs the elevation of weighting function. Figure 11 shows two instances of the function defined by (16), when $a = b$ and $a < 1$; the function has the typical S-shape.
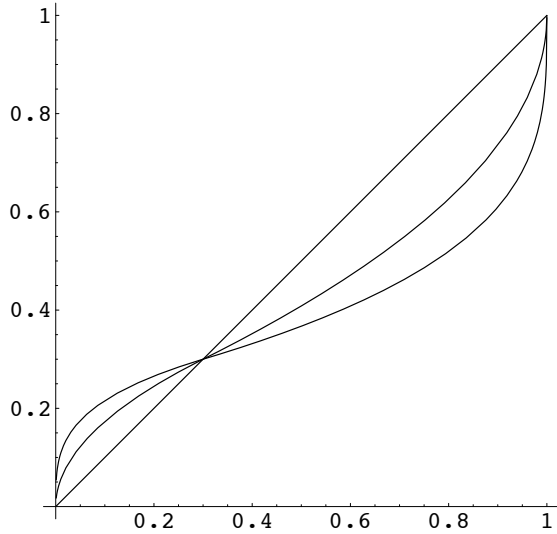
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[2]Empirically the latter interval is larger.

Figure 11: Transformation of probability by function (16) with $a = b$

In the general case ($a \neq b$) both parameters a and b control the curvature and the parameter $\widehat{p}$ will still influence the elevation, even if the value $p = \widehat{p}$ may not lie on the 45° line. Anyway the main role of $\widehat{p}$ is to separate the range of probabilistic risk aversion from the one of risk seeking (as it coincides with the inflection point); when $a = b$, $p = \widehat{p}$ belongs to the 45° line and also separates the interval of probability overweighting and underweighting.

Note that not all the probability weighting functions proposed in the literature allow for a clear separation between curvature and elevation, which is particularly the case for pwf's that involve only one parameter. Weighting functions with two or more parameters allow for more flexibility, but of course are less parsimonious in terms of parameters estimation.

Abdellaoni et al. [1] propose the constant relative sensitivity (CRS) weighting function defined by

$$w(p) = \begin{cases} \delta^{1-\gamma} p^{\gamma}, & 0 \leq p \leq \delta; \\ 1 - (1-\delta)^{1-\gamma}(1-p)^{\gamma}, & \delta < p \leq 1. \end{cases} \tag{17}$$

$0 \leq \delta \leq 1$, $0 < \gamma$. Function (17) corresponds to function defined in (16) when $a = b$ and after a redefinition of the parameters.

These functions exhibit an inverse-S shape if $0 < \delta < 1$, $\gamma < 1$, and S-shape if $0 < \delta < 1$ and $\gamma > 1$. The functions (17) are linear if $\gamma = 1$, concave if $\delta = 1$ and $\gamma < 1$ or $\delta = 0$ and $\gamma > 1$, and convex if $\delta = 1$ and $\gamma > 1$ or $\delta = 0$ and $\gamma < 1$. Moreover, these functions have a fixed point at δ (in addition to 0 and 1) and their derivative at δ is equal to γ (see Figure 12).

The CRS weighting functions are power function on the interval $[0, \delta]$ and dual power functions in the interval $[\delta, 1]$. This suggest an interpretation for the parameter γ as degree of curvature (see Figure 13).

Al-Nowaihi and Dhami [2] make the ambitious proposal of combining Prospect Theory and Cumulative Prospect Theory into a single theory, that they call *composite cumulative prospect*
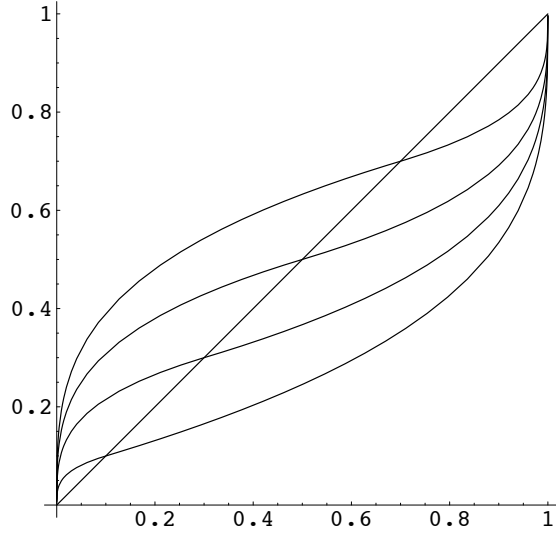
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Figure 12: Plot of wpf (17) for $\gamma = 0.3$ and $\delta = 0.1, 0.3, 0.5, 0.7$; for higher values of δ the function is more elevated
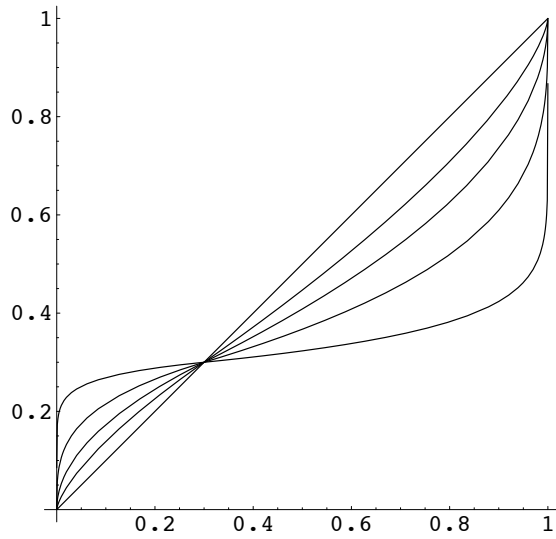

Figure 13: Plot of wpf (17) for $\delta = 0.3$ and $\gamma = 0.1, 0.3, 0.5, 0.7$; for lower values of γ the function exhibits higher curvature

theory. In order to implement such a theory, the authors introduce a modification of the Prelec weighting function. They call their suggested modification *composite Prelec weighting function*

$$w(p) = \begin{cases} 0 & p = 0 \\ e^{-\beta_0(-\log p)^{\alpha_0}} & 0 < p \le \underline{p} \\ e^{-\beta(-\log p)^{\alpha}} & \underline{p} < p \le \overline{p} \\ e^{-\beta_1(-\log p)^{\alpha_1}} & \overline{p} < p \le 1, \end{cases} \tag{18}$$

where:

$$\underline{p} = e^{(\frac{\beta}{\beta_0})^{1/(\alpha_0-\alpha)}} \qquad \overline{p} = e^{(\frac{\beta}{\beta_1})^{1/(\alpha_1-\alpha)}}$$

and

$$0 < \alpha < 1, \ \beta > 0, \ \alpha_0 > 0, \ \beta_0 > 0, \ \beta_1 > 0, \ \beta_0 < 1/\beta^{\frac{\alpha_0-1}{1-\alpha}}, \ \beta_1 > 1/\beta^{\frac{\alpha_1-1}{1-\alpha}}.$$

Note that the restrictions $\alpha > 0$, $\beta > 0$, $\alpha_0 > 0$ and $\beta_1 > 0$ are required by axiomatic derivation of the Prelec function (see [12]). The restriction $\beta_0 < 1/\beta^{\frac{\alpha_0-1}{1-\alpha}}$ guarantees that the first segment of the pwf (18), $exp[-\beta_0(-\log p)^{\alpha_0}]$, crosses the 45° to the left of $\underline{p}$ and the restriction $\beta_1 > 1/\beta^{\frac{\alpha_1-1}{1-\alpha}}$ guarantees that the third segment of the pwf (18), $\exp[-\beta_1(-\log p)^{\alpha_1}]$, crosses the 45° to the right of $\overline{p}$. These conditions jointly imply that the second segment of the curve, $exp[-\beta(-\log p)^{\alpha}]$, crosses the 45° between these two limits. It follows that the interval $[\underline{p},\overline{p}]$ is not empty. These interval limits are chosen so that the wpf (18) is continuous across them.

Figure 14 gives a numerical example of the pwf (18); such a figure is composed by the following three Prelec functions:

$$w(p) = \begin{cases} 0 & & p = 0 \\ e^{-0.61266(-\log p)^2} & \alpha_0 = 2, \beta_0 = 0.61266, & 0 < p \le 0.25 \\ e^{-(-\log p)^{0.5}} & \alpha = 0.5, \beta = 1, & 0.25 < p \le 0.75 \\ e^{-6.4808(-\log p)^2} & \alpha_1 = 2, \beta_1 = 6.4808, & 0.75 < p \le 1. \end{cases} \tag{19}$$

The three segments of the function (19) are described as follows:

i) for $0 \le p < 0.25$ the composite pwf is identical to the S-shaped Prelec function $e^{-\beta_0(-\log p)^{\alpha_0}}$, with $\alpha_0 = 2$ and $\beta_0 = 0.61266$. β_0 is chosen to make $w(p)$ continuous at $p = 0.25$;

ii) for $0.25 \le p < 0.75$ the composite pwf is identical to the inverse S-shaped Prelec function with $\alpha = 0.5$ and $\beta = 1$;

iii) for $0.75 \le p \le 1$ e composite pwf is identical to the S-shaped Prelec function $e^{-\beta_1(-\log p)^{\alpha_1}}$, with $\alpha_1 = 2$ and $\beta_1 = 6.4808$. β_1 is chosen to make $w(p)$ continuous at $p = 0.75$.

Observe that the function (19) has five fixed points: 0, 0.19549, $e^{-1} = 0.36788$, 0.85701 and 1. It is strictly convex in $0 < p < 0.25$ and in $e^{-1} < p < 0.75$, and it is strictly concave in $0.25 < p < e^{-1}$ and in $0.75 < p < 1$.
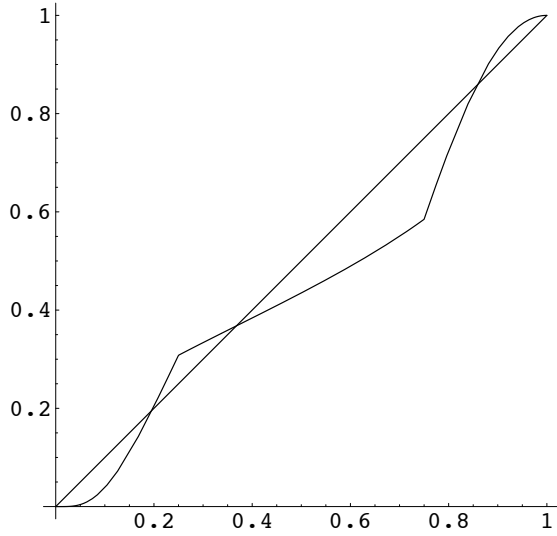
Figure 14: An example of the composite Prelec probability weighting function

5   Polynomial probability weighting functions and other forms

In the literature, other parametric families of weighting functions have been used. The following linear form with discontinuous end points has been proposed by Loomes et al. [9]

$$w(p) = \begin{cases} 0 & p = 0 \\ a + (1 - a - b)p & 0 < p < 1 \\ 1 & p = 1, \end{cases} \tag{20}$$

with $0 \le a, b < 1$.

To avoid the St. Petersburg paradox under cumulative prospect theory, Rieger and Wang [13] proposed the following polynomial of degree three as a weighting function

$$w(p) = \frac{3 - 3b}{a^2 - a + 1}(p^3 - (a + 1)p^2 + ap) + p \tag{21}$$

with $a \in (0, 1)$ and $b \in (0, 1)$. Figure 15 shows two examples of function (21).

The solution proposed in [13] has been improved by Pfiffelmann in [11] where the following weighting function is proposed

$$w(p) = a\, p + b\, p^{1.1} + c\, p^{1.15} + d\, p^{1.2} + e\, p^2 + f\, p^{2.5} + g\, p^6, \tag{22}$$

with $a + b + c + d + e + f = 1$, such as $w(1) = 1$.

Walther [16] introduces the following weighting function

$$w(p) = p\frac{1 + (1 - p)\mu}{1 + (1 - p)p(\gamma + \mu)}, \tag{23}$$

where the distortion weights $\gamma \ge 0$ and $\mu \ge 0$ are referred to as the "elation" and "disappointment" parameter, respectively.
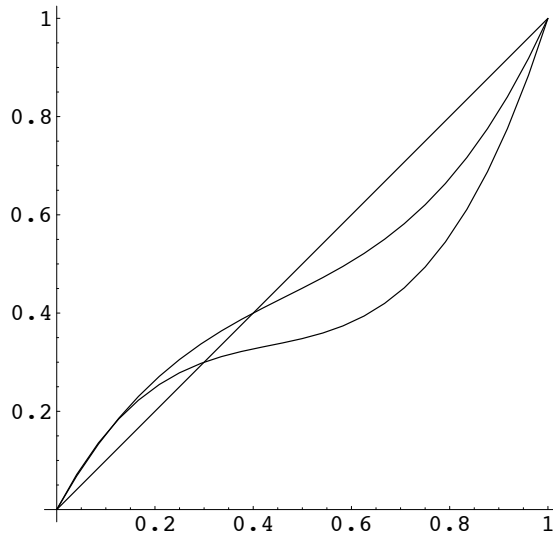
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Figure 15: Alternative weighting functions proposed in [13] with parameters: b=1, a=0.4 and b=0.5, a=0.3 and b=0.2

6 Summary

Weighting functions are a key element in modeling decisions under risk and uncertainty when one try to capture behavioral patterns which departure from Expected Utility theory. The literature related to Prospect Theory, in its original and cumulative versions, and Rank Dependent Utility is very large; several functional forms of probability weighting functions have been proposed and tested in many theoretical and empirical studies.

In this paper, after having introduced the main features and properties of a probability weighting function, we provide a review of different families of pwf which depend on one or more parameters and some polynomial forms. In particular, two parameters allow for separate control of curvature and elevation, even though only the constant relative sensitivity pwf proposed by Abdellaoui et al. [1] model distinctly these two features and is of particular interest.

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