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Elastic Stress Distribution in and Stability of Rectangularly Notched Plates Axially Loaded by Uniform Compression

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OF RECTANGULARLY NOTCHED PLATES AXIALLY

LOADED BY UNIFORM COMPRESSION

This thesis is approved as a creditable and independent investigation by a candidate for the degree, Master of Science, and is acceptable as meeting the thesis requirements for this degree, but without implying that the conclusions reached by the candidate are necessarily the conclusions of the major department.

Taher. Shoukry   May 23, 67
Thesis Adviser   Date

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The author wishes to express his deep gratitude to Dr. Zaher Shoukry, Associate Professor, Department of Civil Engineering, for his great assistance, encouragement, and the sacrifice of much of his own time throughout this work. To him the author is truly indebted.

This thesis is dedicated to the author's mother and father for their deep encouragement and long sacrifice.

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CHAPTER I

INTRODUCTION

A. General

In recent years, the design of notched members has been a challenge to structural, mechanical and aeronautical engineers. Notched members are frequently used in the aircraft, missiles, automobile, and ship-building industries where available space is limited to a minimum. More often notches are needed for various other reasons as the passage of electrical wire bundles, hydraulic and air conduits or other structural members. Sometimes a notch serves as a lightening measure or as an inspection access opening.

More precise solutions in determining the critical buckling loads are needed to replace guess and approximate solutions. This is due to the fact that stress concentrations in plates become large in magnitude especially in the notch vicinity, even for very simple loading conditions.

An analysis of the elastic stress distribution is the first step leading to the theoretical determination of the elastic buckling load of a plate member.

B. Historical Background

The problem of notched plates has been totally avoided in the past due to its complexity. The nature of the problem prohibits an exact solution in a closed form but with the availability of electronic computers, and by applying numerical techniques, investigators recently worked out some problems of this nature.

Shoukry[1] investigated with a similar procedure the stress distribution in and the buckling characteristics of the webs of castellated steel beams. The results indicated that the elastic stress distribution was accurate for the relatively fine mesh employed (18 x 10).

Hoffman[2] discussed the stress distribution in a plate with a side notch. The results showed that the finite difference method employed for stress distribution analysis had given relatively accurate data, considering the complexities of the problem treated.

Several previous investigators employed the method of finite differences successfully when investigating stress distributions in deep beams.[3,4,5] They concluded that the results are excellent as long as a relatively fine mesh is employed.

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[1] Z. Shoukry, "Elastic Flexural Stress Distribution in and Buckling Characteristics of the Webs of Castellated Steel Beams" (unpublished Doctor of Philosophy Thesis, University of Missouri, Columbia, Missouri, 1964).

[2] P. Hoffman, "Elastic Stress Distribution in Rectangularly Notched Members" (unpublished Master of Science Thesis, South Dakota State University, Brookings, South Dakota, 1965).

[3] I. Chow, H. D. Conway and G. Winter, "Stress in Deep Beams," Transactions, American Society of Civil Engineers, Vol. 118 (1963), p. 686.

[4] F. Geer, "Stresses in Deep Beams," Journal of the American Concrete Institute, Vol. 56 (1960), p. 151.

[5] H. D. Conway, L. Chow, and G. W. Morgan, "Analysis of Deep Beams," Journal of Applied Mechanics, Vol. 18 (1951), p. 686.

White and Cottingham[6] investigated critical buckling loads and the stability of solid plates partially loaded at the ends. They concluded that use of the finite difference method for determining elastic buckling loads is useful and satisfactorily accurate. They found that increased accuracy can be obtained by using a relatively fine mesh subdivision. The error introduced by the use of such mesh can be significantly reduced by utilizing a finer mesh than the one they used.

C. Object and Scope of Investigation

The main objective of this investigation is to obtain the critical buckling load of a thin elastic rectangularly notched plate simply supported along two edges and free along the other two edges, and subjected to uniform longitudinal compression. Four plates with the same overall dimensions but different notch dimensions are under consideration. One side of the rectangular notches is held constant while the other side is varied. A graph can then be plotted for the critical buckling coefficient versus the aspect ratios of the notch.

The method of solution followed here is to determine first the elastic stress distribution in the notched plate for the different notch sizes. The results are applied into the finite-differences

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[6] R. N. White and W. S. Cottingham, "Stability of Plates Under Partial Edge Loading," Proceedings, American Society of Civil Engineers, Engineering Mechanics Division, No. 3297, Vol. 88 (October, 1962).

equations for the plate-deflection and then the elastic buckling loads are obtained.

The finite-difference numerical approach to determine both the elastic stress distribution and critical buckling load appears to give excellent results. This is true provided the mesh subdivision is sufficiently fine. In this investigation the mesh subdivision is limited by the storage capacity of the computer available at South Dakota State University Experiment Station.

The critical buckling coefficients and deflected shapes of the different notched plates are obtained as eigenvalues and eigenvectors for some assumed deformations of the plates. The entire process is programmed in Fortran II language for calculation on a digital computer. The programs are shown in Appendix A.

The computer output for the stress distribution is printed out using a special board so that the results are shown on the actual plan shape of the particular plate section under consideration. This permits stress contours to be drawn within the outlines of the section.

The solution presented herein will serve as a source of information for future problems of similar nature. The results are independent of plate material, but their normal use would be for relatively thin metal sections or plate elements.

CHAPTER II

ELASTIC STRESS ANALYSIS

A. Choice of Analytical Method

The ideal method of determining the stress distribution in a plate is to obtain an exact solution of the differential equation[7]:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\,\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^2} = 0 \tag{1}$$

where ϕ is called the stress function or Airy's stress function.

Equation (1) must be satisfied for any point within the domain of the plate. The stresses in the plate can then be computed from the stress function ϕ according to the expressions[8]:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} \tag{2}$$

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} \tag{3}$$

$$\tau_{xy} = -\,\frac{\partial^2 \phi}{\partial x \partial y} \tag{4}$$

where

σ_x = stress in the x-direction,

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[7] S. Timoshenko and J. N. Goodier, Theory of Elasticity (McGraw-Hill Book Company, Incorporated, New York, New York, 1951), p. 26.

[8] Ibid., p. 26.

σ_y = stress in the y-direction,

τ_{xy} = shearing stress at any point.

Since the nature of the problem and the introduction of the notch in the plate prohibit an exact solution for equation (1), an approximate method of analysis is employed and is chosen to be the method of finite differences. This method is employed because of its simplicity, flexibility to handle general boundary conditions, and adaptability to electronic computations.

The method of finite differences[9] is based upon the use of approximating expressions for the derivatives appearing in the governing differential equations and boundary conditions of the problem. Here the structural element is subdivided into a network of points and a finite-difference equation valid for each point is derived to replace the differential equation. This reduces the problem to the solution of a number of algebraic equations, which can be solved by some iterative technique.

B. Plate Stress Distribution

The finite-difference method essentially approximates partial differential equations by finite-difference equations in the form of an "operator molecule".[10] Assuming a square network, the operator

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[9]G. Gerard, Introduction to Structural Stability Theory (McGraw-Hill Book Company, Incorporated, New York, New York, 1962), p. 55.

[10]P. C. Wang, Numerical and Matrix Methods in Structural Mechanics (John Wiley and Sons, Incorporated, New York, New York, 1966), pp. 50-52.

molecule for the biharmonic equation is[11]

$$\nabla^4 \phi = \frac{1}{h^4}\left[\begin{array}{ccccc} & & 1 & & \\ & 2 & -8 & 2 & \\ 1 & -8 & 20 & -8 & 1 \\ & 2 & -8 & 2 & \\ & & 1 & & \end{array}\right] \qquad (5)$$

in which "h" is the mesh spacing.

In the process of determining the stress function ϕ by this method, the domain of the plate is replaced by a mesh of individual nodal points of square spacing. The stress function values are then calculated for each discrete nodal point.

When the final value of the stress function at each point of the mesh system is known, the stresses can be determined. The expressions for the stress components and their finite difference operator molecules are as follows[12]:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} \simeq + \frac{1}{h^2}\left[\begin{array}{c} 1 \\ -2 \\ 1 \end{array}\right] \qquad (6)$$

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[11] S. H. Crandall, Engineering Analysis (McGraw-Hill Book Company, Incorporated, New York, New York, 1956), pp. 243-247.

[12] S. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, Second Edition (McGraw-Hill Book Company, Inc., New York, New York, 1959), p. 360.

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} \simeq + \frac{1}{h^2} \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \qquad (7)$$

$$\tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} \simeq + \frac{1}{4h^2} \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \qquad (8)$$

The above molecules are employed by first placing each molecule's central value over each nodal point. The next step is to sum algebraically the terms resulting from the multiplication of each molecule's component by its corresponding stress function value. This summation gives the stress at each nodal point.

C. Boundary Value Problems

1. General

Structural sections investigated by the finite difference method are referred to in the literature as "boundary value problems." This is due to the fact that the boundary values of the stress function depend upon the loading conditions applied at the boundary of the section. Such values, which can be determined mathematically, are the controlling values for the stress function throughout the domain of the plate and henceforth are stabilized along the boundary.

Since the biharmonic molecule must be satisfied when applied to each interior nodal point within the domain, initial stress function values are assigned for each nodal point and an iterative technique is employed to modify these values so that the biharmonic molecule is satisfied. When the molecule is applied to interior points adjacent to the boundary, values of the stress function immediately outside the boundary are required. For this reason the stress function values for points outside the boundary are ascertained by extrapolating the stress function values on the boundary.

2. General Stress Function Boundary Equations

Consider a thin elastic rectangular plate hinged along the short edges and free along the other edges, and subjected to a uniform longitudinal compression, as shown in Figure 1.

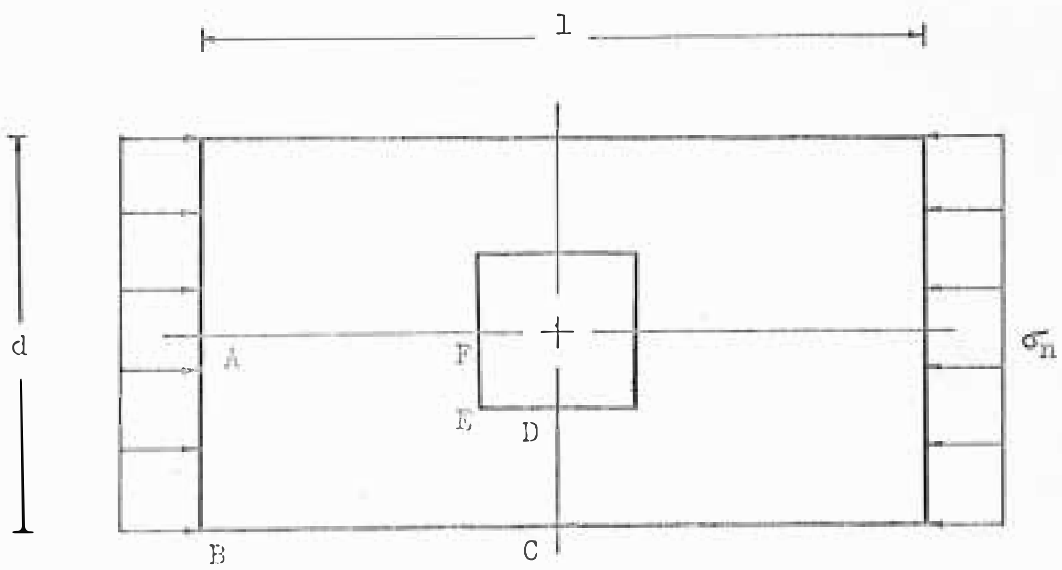

Figure 1. The Full Plate

Due to symmetry about the vertical and the horizontal axes, the solution to the boundary value problem can therefore be obtained by considering only one quarter of the plate as shown in Figure 2. This consideration allows the same number of nodal points dictated by the storage capacity of the computer to be distributed over a smaller area. Henceforth, the accuracy of the solution will be increased to a great extent.

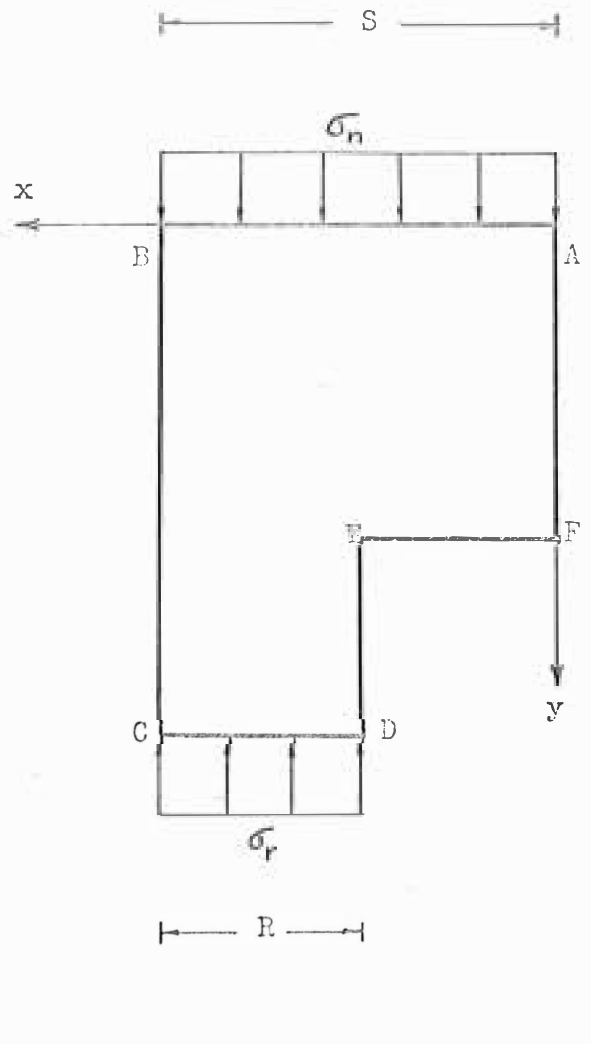

Figure 2. The Quarter Plate

With the coordinate axes selected as shown in Figure 2, the stress function ϕ on the boundary is evaluated due to the external stresses σ_n and σ_r, where

$$\sigma_r = \frac{\sigma_n S}{R} \tag{9}$$

and

σ_n = axial compressive stress along the side A-B ,

σ_r = axial resulting compressive stress on the throat section C-D ,

S = half width of the plate (distance A-B in Figure 2) ,

R = throat width at the centerline of the notch (distance C-D in Figure 2) .

Starting from the origin along A-B, there acts the uniformly distributed compressive stress of intensity σ_n. Therefore,

$$\sigma_y = \frac{\partial^2\phi}{\partial x^2} = \sigma_n \tag{10}$$

Integrating twice with respect to x,

$$\frac{\partial\phi}{\partial x} = \sigma_n x + c_1 \tag{11}$$

and

$$\phi = \sigma_n \frac{x^2}{2} + c_1 x + c_2 \tag{12}$$

where c_1 and c_2 are constants of integration.

At point A,

$$x = 0$$

Hence

$$\frac{\partial \phi}{\partial x} = c_1$$

and

$$\phi = c_2$$

The terms c_1 and c_2 can be taken arbitrarily as zero values since point A is a reference point for ϕ and $\frac{\partial \phi}{\partial x}$.

Therefore, along the boundary A-B,

$$\frac{\partial \phi}{\partial x} = \sigma_n x \tag{11a}$$

and

$$\phi = \sigma_n \frac{x^2}{2} \tag{12a}$$

Equation (11a) shows that the slope of the stress function varies linearly with the distance from the reference point along the x-axis. Equation (12a) shows that the stress function is a second degree parabola.

The stress function value can be determined at point B by substituting x = S in equations (11a) and (12a). Therefore, at point B,

$$\phi = \sigma_n \frac{S^2}{2} \tag{12b}$$

and

$$\frac{\partial \phi}{\partial x} = \sigma_n S \tag{11b}$$

The boundary B-C is subject to pure compressive stress in the y-direction. Therefore,

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad (13)$$

Integrating twice with respect to y, and knowing that

$$\frac{\partial^2 \phi}{\partial x \partial y} = 0$$

then

$$\phi = c_3 y + c_4 \; .$$

But at point B, there are only normal stresses along the side A-B in the y-direction. Hence,

$$\frac{\partial \phi}{\partial y} = 0$$

and

$$c_3 = 0$$

The stress function value at B is reduced to

$$\phi = c_4 \qquad (14)$$

where c_4 is a constant of integration and equals the value for ϕ obtained at point B on the side A-B. Therefore, at point B,

$$\phi = \sigma_n \frac{S^2}{2} \; . \qquad (12b)$$

Hence, along the boundary B-C,

$$\phi = \text{constant} = \sigma_n \frac{S^2}{2} \; , \qquad (14a)$$

$$\frac{\partial \phi}{\partial y} = 0 , \tag{15}$$

and

$$\frac{\partial \phi}{\partial x} = \sigma_n S \tag{16}$$

This value is the same as for B obtained from equation (11b). This indicates that the stress function is constant along the side C-B and its value is equal to the stress function value calculated for point B.

Along the boundary C-D, there acts a uniformly distributed compressive stress of intensity σ_r. Therefore,

$$\sigma_y = \frac{\partial^2 \phi}{\partial x^2} = \sigma_r .$$

Integrating twice with respect to x,

$$\frac{\partial \phi}{\partial x} = \sigma_r x + c_5 \tag{17}$$

and

$$\phi = \sigma_r \frac{x^2}{2} + c_5 x + c_6 \tag{18}$$

The terms c_5 and c_6 are constants of integration which can be evaluated by matching ϕ and $\frac{\partial \phi}{\partial x}$ at point C with those given for point B in equations (12b) and (11b) respectively.

Along the vertical line D-E there are no external forces applied. Therefore, the values $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ along D-E must be the same as those for point D. Hence,

$$\frac{\partial \phi}{\partial y} = 0 \tag{19}$$

and

$$\frac{\partial \phi}{\partial x} = K_1 \tag{20}$$

where K_1 is a constant that can be obtained from equation (17) with x as the distance from the y-axis to point D.

Integrating equation (20) with respect to x,

$$\phi = K_1 x + c_7$$

At point D, ϕ must have the same value calculated from the side C-D. Therefore, the stress function along the side D-E is constant and equal to the value obtained at point D from C-D. This value is true because the side D-E is at a constant distance x from the y-axis. Therefore,

$$\phi = \text{constant} = c_7$$

and

$$K_1 = 0 \text{ at point D and along D-E} \tag{21}$$

Along the side E-L there are no forces applied. Therefore,

$$\frac{\partial^2 \phi}{\partial x^2} = 0,$$

$$\frac{\partial \phi}{\partial x} = c_8,$$

and

$$\phi = c_8 x + c_9$$

where c_8 and c_9 are constants of integration which can be evaluated by matching ϕ and $\frac{\partial \phi}{\partial x}$ at point E with those given for point D in equations (17) and (18).

It is realized that the line F-A is a line of symmetry. Due to this fact the operator molecule can be applied to points on the boundary line F-A extending to points along two lines outside this boundary. Therefore, the stress function values along F-A are not controlling values and they are modified with iteration.

It has also been noticed that the line C-D is a line of symmetry and the same procedure as for line F-A could have been adopted. However, if this procedure had been followed, the problem would have been difficult. This is true due to the fact that the assumed uniform stress distribution σ_r gives controlling values of the stress function along the boundary C-D. A uniform normal distribution will be accurate enough for large size notches, which is the case in this investigation. However, the amount of error will be slightly increased if the notches are smaller.

3. Stress Function Values Outside the Boundary

The amount of information to evaluate points outside the boundary is negligible. Nevertheless, in this investigation, values of the stress function outside the boundary are obtained either by extrapolation or, where it applies, from conditions of symmetry.

The values of the stress function immediately beyond the boundary A-B are taken identical to those on the boundary itself. This assumption is valid since the member can be considered as a continuous plate.

The value beyond B along A-B is determined by extrapolation of the second degree parobola for the function along A-B. For the line D-C, the value beyond C is determined by extrapolation of the second degree parabola for the function along D-C.

Values for the exterior points beyond the line B-C are computed on the assumption that the values of the stress function vary linearly between the exterior points beyond points B and C. This assumption is valid since the variation between the values at the exterior points beyond B and C is not very large and the rate of change of warping along the boundary A-B does not vary considerably.

Values for the exterior points beyond D-E and E-F are assumed to be identical to those on the boundary itself. This is justified since there are no normal or shearing stresses applied to these boundaries.

Since the line F-A is a line of symmetry, the stress function values of points along two lines outside the boundary line A-F are equal to the corresponding opposite values inside the boundary.

4. Iterative Technique for Determining Interior Values of the Stress Function

After the equations for the boundary stress function values are obtained, the surface of the plate is replaced by a lattice system of square spacing. The stress function values are then calculated for each discrete point on the fixed boundaries and immediately outside these boundaries. Initial stress function values are then assigned for each discrete point within the domain and along the vertical centerline F-A. At this stage each nodal point has either a stress function value or an assigned value. This information is taken as computer input.

An iterative technique is employed to satisfy the biharmonic operator molecule whenever this is applied to each interior nodal point and to points along the vertical centerline. By this technique the molecule is applied to every point on the plate, and the existing stress function value is replaced each time by a modified computed value. By this process, the stress function values on the iterated points are "forced" to converge to suitable values dictated by the stress function values on and immediately outside the controlling fixed boundaries.

As the convergence of the system continues, the biharmonic molecule becomes more nearly satisfied at each nodal point within the domain, and the residual for each nodal point iterated approaches zero.

The entire process has been programmed in Fortan II language for calculation on the IBM 1620 electronic computer. The program is given in Appendix A. It has been so constructed that two values are printed by the typewriter after each iteration. The first number is the largest difference between any two values obtained at any point in two successive iterations, while the second number is the largest percentage of change between two successive iterations. These two numbers supply a visual running check on the convergence of the system.

5. Determination of Stresses

After the final value of the stress function ϕ has been determined at each point, the stresses σ_x, σ_y, and τ_{xy} are determined by their finite difference operator molecules as shown in equations (6), (7), and (8) respectively. Determination of these stresses is obtained by utilizing the iterated stress function values and is included as a part of program I in Appendix A. The stresses are punched out on cards for every point in the plate.

CHAPTER III

CRITICAL BUCKLING ANALYSIS

Chapter II illustrates the adaptability of the finite differences method to the computations of elastic stresses at different points within the domain of the plate. With the stresses known, and using the same procedure, it is possible to replace the plate-deflection equation by finite-difference equations[13] involving the stresses and the critical multiple of stresses that will cause elastic buckling.

The theoretical determination of the elastic buckling load of a notched plate such as that shown in Figure 1 will be presented herein. Although the true plate strength requires investigation of buckling into the plastic range[14], the elastic buckling load is physically important in design because it is actually the initial step in changing the plate configuration that will eventually lead to complete failure.

A. Choice of Analytical Method

The ideal method of solving a plate-buckling problem is to obtain an exact solution in a closed form of the differential equation for the buckled shape[15]:

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[13]R. N. White and W. S. Cottingham, op. cit., p. 67.

[14]Ibid., p. 68.

[15]S. Timoshenko and J. M. Gere, Theory of Elastic Stability (McGraw-Hill Book Company, Incorporated, New York, New York, 1961), p. 348.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D}\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y}\right) \qquad (22)$$

where

w = Deflection perpendicular to middle surface of plate.

N_x = Normal force per unit edge length acting in the x-direction.

N_y = Normal force per unit edge length acting in the y-direction.

N_{xy} = Shear force per unit length in the xy-plane.

D = Flexural rigidity of plate = $\frac{Et^3}{12(1-\nu^2)}$

E = Young's modulus.

t = Plate thickness.

ν = Poisson's ratio.

Although an exact solution to the governing equilibrium equations and specified boundary conditions can in principle be found[16] for many buckling problems, certain practical computational difficulties usually arise. Yet the existence of the notch in the plate makes the problem more involved, and an exact solution for equation (22) becomes impossible. Consequently, the solution has been obtained by the use of an approximate method. The method of finite-differences is chosen again for determining the buckling loads. Equation (22) can be generated in finite-difference form from the stresses at the nodal points and then all calculations are performed by the use of an electronic computer.

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[16] G. Gerard, Introduction to Structural Stability Theory (McGraw-Hill Book Company, Incorporated, New York, New York, 1961), p. 348.

B. The Eigenvalue Problem

As the buckling characteristics of a notched plate are highly influencing its load carrying capacity, then the smallest compressive stress for which the unbent configuration ceases to be stable, is highly valuable. The following assumptions[17] are made to obtain some comparable results.

1. The plate is made of perfectly elastic, homogeneous material.
2. The plate is perfectly flat before loads are applied.
3. The loads are applied in the plane of the middle surface of the plate.

If the plate is assumed to be in equilibrium in a slightly bent state under the critical compressive load, P per unit length, then the transverse deflection, w(x,y), must satisfy Equation (22). As a matter of fact there exist a large number of compressive loads, P per unit length, that satisfy this equation. Then the eigenvalue problem[18] consists of determining the deflected configuration w and the smallest load at the verge of plate buckling. The critical load depends upon "λ" and it is known as the "eigenvalue".

By finite differences method, the differential equation for the deflected shape of the plate can be reduced to the form[19]:

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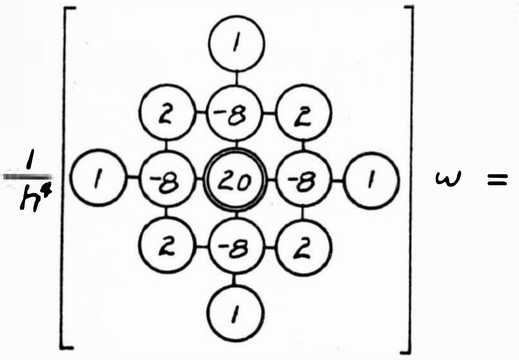
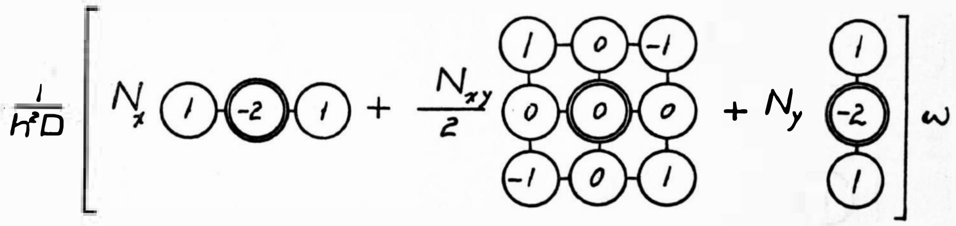
[17]Ibid., p. 39.

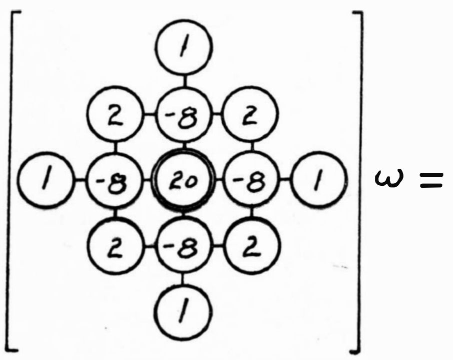
[18]S. H. Crandall, op. cit., p. 282.

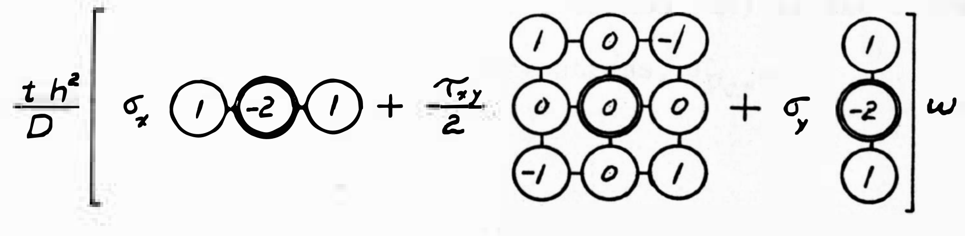
[19]Z. Shoukry, op. cit., p. 42.



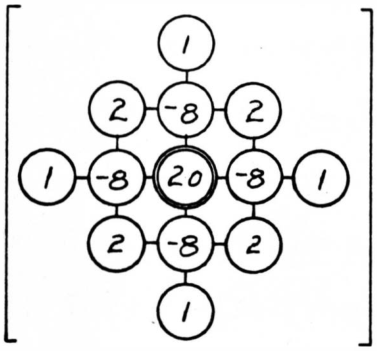


Substituting $N_x = \sigma_x t$, $N_y = \sigma_y t$ and $N_{xy} = \tau_{xy} t$ and multiplying all terms by h^4, the equation above then takes the form

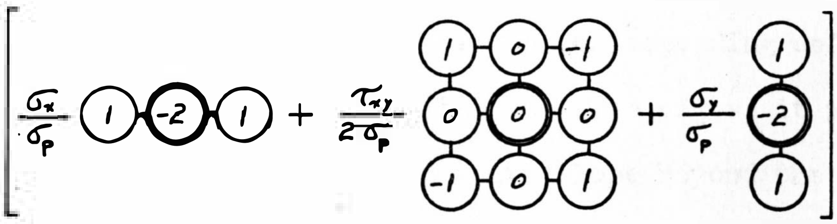




When all the stresses are normalized to a unit stress at the top ($\sigma_p = 1$), then σ_p can be factored out of the bracketed portion and the equation reduces to



$\omega =$



$\lambda \left[\frac{\sigma_x}{\sigma_p} (\ldots) + \frac{\tau_{xy}}{2\sigma_p} (\ldots) + \frac{\sigma_y}{\sigma_p} (\ldots) \right] \omega$ (23)

where

$$\lambda = \frac{\sigma_p th^2}{D}$$

The stresses σ_x, σ_y and τ_{xy} are non-dimensional multiples of a unit stress at the top of the plate.

The critical buckling stress, $\sigma_{p_{cr}}$, is then given by the equation

$$\sigma_{p_{cr}} = \frac{D}{th^2} \lambda \qquad (24)$$

The only unknown on the right side of equation (24) is the eigenvalue λ, which defines the critical buckling stress, $\sigma_{p_{cr}}$.

C. Assumed Boundary Conditions

When the center of the biharmonic molecule in equation (23) is applied to nodal points immediately inside the boundary, the operator will extend to points immediately beyond the boundary. Consequently, plate deflections must be known at such points. When the center of the molecule is applied to points on the boundary itself, the molecule will extend to two points immediately outside the boundary and hence the plate deflections must also be known at these points.

For the simply supported edges of the plate in Figure 1, the deflections at points immediately outside the boundary are assumed equal in value and opposite in sign to the corresponding deflections immediately inside the boundary[20]. For the free edges, it is assumed that when they deflect they maintain their slope beyond the free boundary[21]. This assumption is preferred over any other method because it results in a convergent matrix which will yield a critical loading constant (see Appendix C).

D. Method of Analysis

When the plate is replaced with a set of nodal points of number n, it is possible to apply the difference equation (23) to each point, and this will result in a set of n equations with n unknowns. In matrix form these equations can be represented as follows:

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[20] S. Timoshenko and J. M. Gere, op. cit., p. 329.

[21] Z. Shoukry, op. cit., pp. 46-47.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & & & & \\ a_{31} & & & & \\ \vdots & & & & \\ a_{n1} & & & & a_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} = \lambda \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & & & & \\ b_{31} & & & & \\ \vdots & & & & \\ b_{n1} & & & & b_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{bmatrix} \tag{25}$$

Equation (25) can be expressed in the matrix form:

$$[A][w] = \lambda [B][w] \tag{26}$$

where $[A]$ and $[B]$ are square matrices of order n, $[w]$ is a column matrix, and λ is a scalar quantity.

If equation (26) is pre-multiplied by the inverse of the matrix $[A]$, $[A]^{-1}$, this gives

$$[A]^{-1} [A][w] = \lambda [A]^{-1} [B][w]$$

or

$$[w] = \lambda [G][w] \tag{27}$$

where $[G]$ is a square matrix of order n and equals $[A]^{-1} [B]$.

Equation (27) can be reduced to the form

$$\frac{1}{\lambda} [w] = [G][w]$$

$$k [w] = [G][w] \tag{28}$$

where

$$k = \frac{1}{\lambda}.$$

In the method to be described in the succeeding section, generally the largest eigenvalue is found first.[22] Then the smallest eigenvalue λ, which is the one of concern in this investigation, is the reciprocal of the largest eigenvalue k. $[w]$ is known as the eigenvector and it gives the relative deflections at the nodal points of the plate when normalized to w_1.

E. Iterative Technique for Computing the Critical Buckling Stress

The iterative technique[23] followed in computing the critical buckling stress from equation (28) is based upon finding a matrix $[w']$ which is proportional to $[w]$; that is, each element of $[w]$ is a scalar multiple of the corresponding element in $[w']$. The scalar multiple, then, is the eigenvalue k.

In the iteration process (see Appendix D), a trial vector $[w']$ is assumed for the eigenvalue problem given in equation (31). The trial vector is chosen so that its first element is unity, although this is not a necessity. The matrix $[G]$ is then multiplied by $[w']$ and a column matrix $[w'']$ results.

$$[G][w'] = [w''] \tag{29}$$

22P. C. Wang, Numerical and Matrix Methods in Structural Mechanics (John Wiley and Sons, Incorporated, New York, New York, 1966), pp. 182-185.

23Ibid., p. 186.

The column matrix $[w'']$ is normalized by factoring out (w_1''). Then (w_1'') is the scalar multiple. The matrix $[w'']$ replaces $[w']$ in equation (29) and a new matrix $[w''']$ results. After $[w''']$ is normalized by factoring out (w_1'''), the two scalars (w_1'') and (w_1''') are checked if they are equal. If not, $[w''']$ replaces $[w']$ in equation (29) and the process continues. If the same scalar results in several successive interations, this scalar is then the largest eigenvalue, and the last $[w^n]$ is the eigenvector which is proportional to $[w]$.

The process has been programmed for the computer and it is given in Appendix A. The program instructs the typewriter to type the scalar which has been factored out after each iteration. If the same scalar results in several successive iterations, the computer is instructed to punch out the scalar as well as the elements of the eigenvector which are the relative deflections.

Other auxiliary programs needed for the determination of the eigenvalue iteration are a matrix inversion program and a program for multiplying an inverted square matrix by another matrix of the same order. The two programs are shown in Appendix A.

CHAPTER IV

THE GENERAL COMPRESSION PLATE PROBLEM

A. Stress Function Values

Chapters II and III deal with the theory involved in the determination of the critical buckling load for a rectangularly notched plate under uniform compression. Four plates with the same overall dimensions but different size notches are considered to illustrate the effect of notch size on the buckling load. The overall height-to-width "aspect ratio" of all the plates is taken as 1/2, 1, 3/2, and 2 for a constant notch width. The plates will be referred to by the letters A, B, C, and D for the aspect ratios 1/2, 1, 3/2, and 2 respectively.

As it was pointed out in Chapter II, the quarter plate will be considered in calculating both the stress distribution and the critical buckling load in the plate. The deflection of the plate will be assumed symmetrical about both the horizontal and the vertical axes of the full plate. A square system with a spacing "h" will be employed. The dimensions of both the plate and the notch will be expressed in terms of this spacing as shown in Figure 3.

The general boundary stress function equations in Appendix E will be expressed in terms of a varying distance x along the x and y-axes as shown in Figure 3. The value "ah" indicates half the width of the plate at the throat section. The only value changing is "bh" which is equal to one-half the length of the notch.

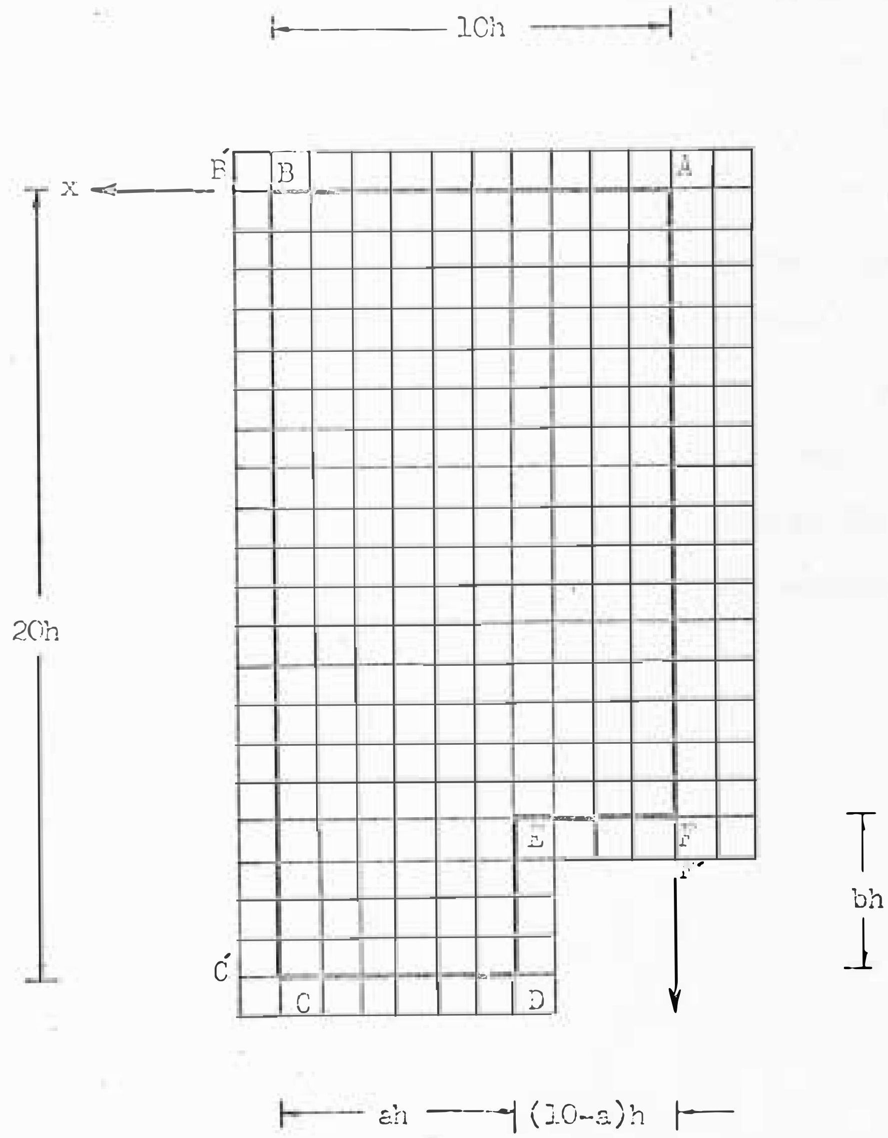

Figure 3. Quarter Plate - Mesh Subdivision

Figure 4 shows the smooth surface formed by the stress function values at the boundary line for plate B. An extension of these surfaces, to include the stress function values within the boundaries, yields a smooth undulating surface on the plane of the plate.

Figures 5, 9, 13, and 17 give the computer outputs for the stress function after iteration for the four quarter plates A, B, C, and D respectively. Figures 6, 10, 14, and 18 give the stress values in the x-direction for plates A, B, C, and D respectively. Figures 7, 11, 15, and 19 give the stress values in the y-direction for plates A, B, C, and D respectively. Figures 8, 12, 16, and 20 give the shearing stress values for plates A, B, C, and D respectively. For all the plates the top width of the notch is taken equal to "8h" while the length is taken equal to 4h, 8h, 12h, and 16h according to plates A, B, C, and D respectively.

Contours for the stress function values, stresses σ_x, σ_y, and τ_{xy} for plate A are given in Appendix F (Figures 23, 24, 25, and 26 respectively).

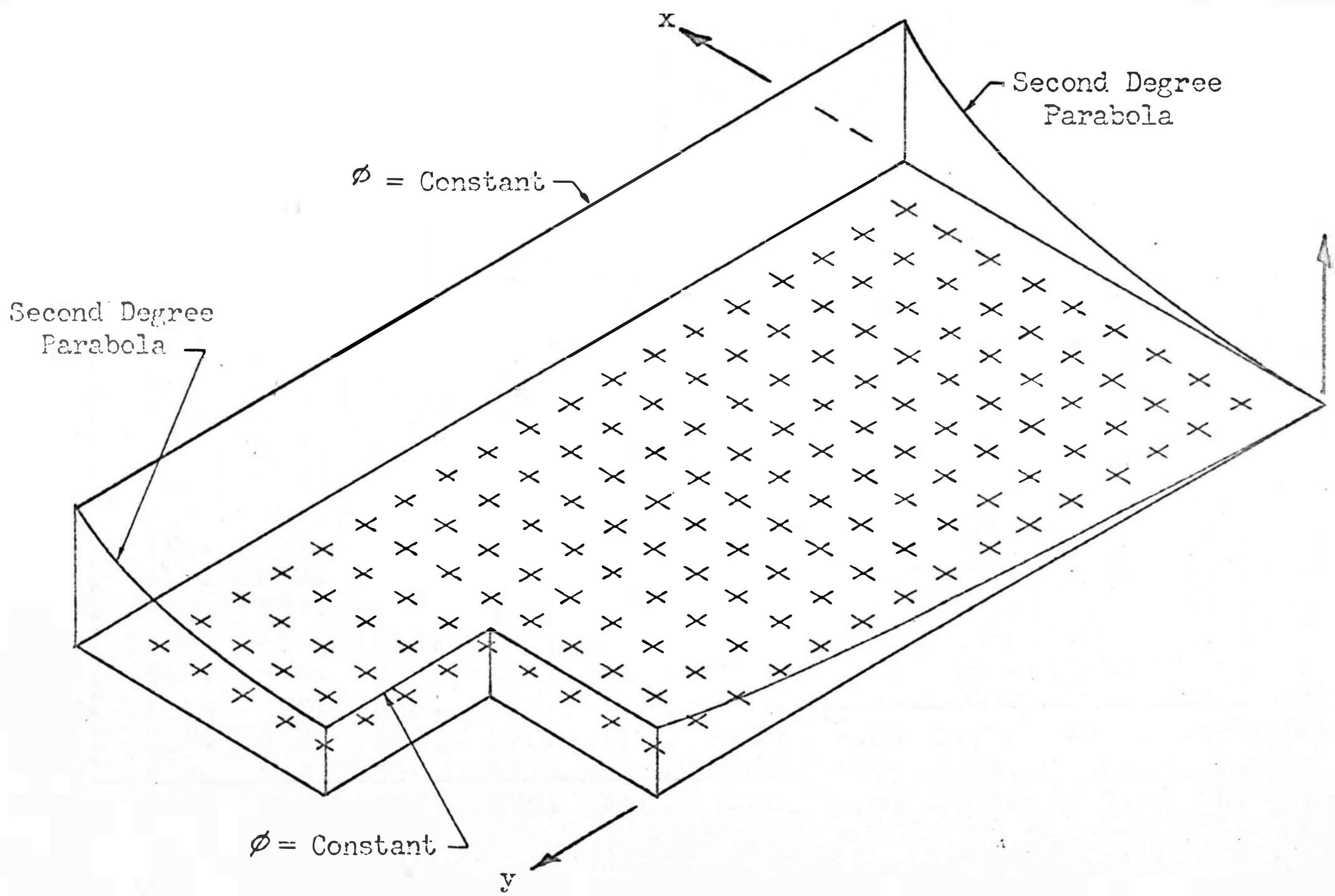

Figure 4. Boundary Stress Function Surface, Plate B

1210.	1000.	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000	809.	639.	490.	360.	250.	161.	91.	42.	12.	2	12.	42.
1211.	1000	809.	639.	490.	361.	252.	164.	95.	46.	16.	7	16.	46.
1211.	1000	809.	640.	491.	363.	256.	168.	101.	52.	24.	14	24.	52.
1211.	1000	809.	641.	493.	367.	261.	175.	109.	62.	34.	25	34.	62.
1212.	1000	810.	642.	496.	372.	268.	184.	120.	75.	48.	39	48.	75.
1212.	1000	810.	644.	500.	378.	277.	197.	135.	92.	66.	57	66.	92.
1212.	1000	811.	647.	506.	387.	289.	212.	153.	112.	88.	80	88.	112.
1213.	1000	812.	650.	512.	396.	303.	229.	174.	136.	113.	106	113.	136.
1213.	1000	813.	654.	519.	408.	318.	249.	198.	163.	143.	136	143.	163.
1214.	1000	815.	657.	526.	420.	335.	271.	224.	192.	174.	168	174.	192.
1214.	1000	816.	661.	534.	432.	353.	293.	251.	223.	207.	202	207.	223.
1214.	1000	817.	665.	542.	444.	370.	316.	278.	254.	241.	236	241.	254.
1215.	1000	818.	669.	549.	455.	386.	337.	305.	285.	274.	270	274.	285.
1215.	1000	819.	672.	554.	465.	401.	357.	330.	314.	306.	303	306.	314.
1215.	1000	820.	673.	558.	472.	412.	374.	353.	341.	336.	334	336.	341.
1216.	1000	820.	674.	560.	477.	421.	388.	372.	366.	363.	362	363.	366.
1216.	1000	819.	673.	559.	477.	424.	397.	388.	386.	385.	385	385.	386.
1217.	1000	818.	670.	556.	474.	423.	400	400.	400.	400.	400	400.	400.
1217.	1000	817.	668.	552.	470.	420.	400	400.	400.	400.	400.	400.	400.
1217.	1000	817.	667.	550.	467.	417.	400	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	668.	552.	470.	420.	400.	400.	0.	0.	0.	0.	0.

Figure 5. Stress Function, ϕ, for Plate A

1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.99	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.99	.99
1.03	1.02	1.02	1.01	1.01	1.00	.99	.98	.98	.97	.97
1.03	1.04	1.04	1.03	1.01	1.00	.99	.97	.96	.96	.95
1.04	1.06	1.06	1.04	1.02	1.00	.98	.96	.94	.93	.93
1.11	1.10	1.08	1.06	1.03	1.00	.97	.94	.91	.90	.89
1.14	1.14	1.12	1.09	1.05	1.00	.95	.91	.88	.85	.85
1.18	1.18	1.16	1.11	1.06	1.00	.94	.88	.83	.80	.79
1.28	1.24	1.20	1.14	1.07	1.00	.92	.84	.78	.75	.73
1.34	1.31	1.25	1.18	1.09	.99	.90	.80	.73	.68	.66
1.45	1.38	1.30	1.21	1.11	.99	.87	.76	.67	.61	.59
1.52	1.44	1.36	1.26	1.13	.99	.85	.71	.60	.53	.51
1.59	1.51	1.41	1.30	1.16	1.00	.83	.66	.53	.45	.42
1.69	1.57	1.46	1.34	1.19	1.01	.81	.62	.46	.36	.34
1.73	1.62	1.52	1.39	1.24	1.04	.81	.57	.38	.28	.24
1.76	1.66	1.57	1.45	1.29	1.08	.82	.51	.30	.18	.15
1.80	1.69	1.61	1.51	1.37	1.16	.86	.43	.20	.09	.07
1.77	1.70	1.64	1.57	1.45	1.26	.97	.28	.08	.02	.01
1.77	1.70	1.66	1.62	1.53	1.37	1.18	0.00	0.00	0.00	0.00
1.72	1.67	1.66	1.67	1.61	1.48	1.01	0.00	0.00	0.00	0.00
1.70	1.65	1.65	1.70	1.65	1.65	.85	0.00	0.00	0.00	0.00

Figure 6. Stress σ_x for Plate A

0.00	0.00	0.00	0.00	.01	.04	.06	.08	.10	.11	.11
0.00	0.00	0.00	.02	.03	.05	.07	.09	.10	.11	.12
0.00	0.00	.01	.03	.05	.07	.09	.11	.12	.13	.13
0.00	0.00	.02	.04	.06	.08	.11	.13	.14	.15	.16
0.00	0.00	.02	.04	.07	.10	.12	.14	.16	.17	.18
0.00	.01	.02	.05	.08	.10	.13	.16	.18	.19	.19
0.00	.01	.02	.05	.08	.11	.14	.16	.18	.20	.20
0.00	0.00	.02	.04	.07	.10	.13	.15	.17	.19	.19
0.00	0.00	.02	.04	.06	.08	.11	.13	.15	.16	.16
0.00	0.00	.01	.02	.04	.06	.08	.10	.11	.12	.12
0.00	0.00	0.00	.01	.02	.03	.04	.05	.06	.07	.07
0.00	0.00	0.00	0.00	-.01	0.00	0.00	0.00	.01	.02	.02
0.00	-.01	-.02	-.03	-.04	-.04	-.04	-.03	-.02	-.02	-.01
0.00	-.01	-.03	-.06	-.08	-.09	-.08	-.07	-.06	-.06	-.05
0.00	-.02	-.05	-.08	-.11	-.13	-.13	-.11	-.10	-.09	-.09
0.00	-.02	-.06	-.11	-.15	-.18	-.18	-.14	-.13	-.14	-.14
0.00	-.02	-.07	-.12	-.17	-.22	-.23	-.18	-.20	-.23	-.24
0.00	-.02	-.05	-.10	-.16	-.22	-.29	-.25	-.34	-.41	-.43
0.00	0.00	-.01	-.04	-.08	-.12	-.14	-.55	-.67	-.71	-.71
0.00	.03	.05	.06	.05	0.00	0.00	0.00	0.00	0.00	0.00
0.00	.04	.14	.27	.34	.33	0.00	0.00	0.00	0.00	0.00

Figure 7. Stress σ_y for Plate A

0.00	0.00	0.00	0.00	0.00	.01	.01	0.00	0.00	0.00	0.00
-.01	0.00	.01	.02	.02	.03	.03	.02	.01	.01	0.00
-.01	0.00	.02	.03	.04	.05	.04	.04	.03	.01	0.00
0.00	.01	.03	.05	.06	.07	.06	.05	.04	.02	0.00
0.00	.02	.05	.07	.09	.09	.09	.07	.05	.03	0.00
0.00	.04	.07	.10	.11	.12	.11	.10	.07	.03	0.00
.01	.05	.09	.12	.14	.15	.14	.12	.08	.04	0.00
.01	.07	.11	.15	.17	.18	.17	.14	.10	.05	0.00
.01	.08	.13	.17	.20	.20	.19	.16	.12	.06	0.00
.01	.09	.15	.19	.22	.23	.21	.18	.13	.07	0.00
.02	.09	.15	.20	.23	.24	.23	.19	.14	.07	0.00
.03	.09	.15	.20	.24	.25	.24	.21	.15	.08	0.00
.01	.09	.15	.20	.23	.25	.25	.22	.16	.08	0.00
.01	.07	.13	.18	.22	.25	.25	.23	.16	.08	0.00
.01	.05	.10	.15	.20	.25	.26	.24	.17	.09	0.00
0.00	.02	.06	.11	.17	.24	.28	.26	.18	.08	0.00
-.02	0.00	.02	.06	.13	.21	.30	.28	.16	.07	0.00
-.03	-.04	-.02	.01	.08	.17	.30	.27	.11	.04	0.00
-.03	-.05	-.06	-.03	.03	.12	.19	.13	.03	.01	0.00
-.01	-.04	-.06	-.04	0.00	.09	.08	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 8. Shearing Stress τ_{xy} for Plate A

1210.	1000.	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000.	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000.	809.	639.	490.	360.	250.	161.	91.	42.	12.	2.	12.	42.
1211.	1000.	809.	639.	490.	361.	252.	164.	95.	46.	16.	6.	16.	46.
1211.	1000.	809.	640.	491.	363.	256.	168.	100.	52.	23.	14.	23.	52.
1211.	1000.	809.	641.	493.	367.	261.	175.	108.	61.	33.	24.	33.	61.
1212.	1000.	810.	642.	496.	372.	268.	184.	119.	73.	46.	37.	46.	73.
1212.	1000.	810.	644.	500.	378.	277.	195.	133.	89.	63.	54.	63.	89.
1212.	1000.	811.	647.	505.	386.	288.	210.	150.	108.	84.	76.	84.	108.
1213.	1000.	812.	650.	511.	396.	301.	227.	171.	132.	109.	102.	109.	132.
1213.	1000.	814.	654.	519.	407.	317.	247.	196.	160.	139.	132.	139.	160.
1214.	1000.	815.	658.	527.	420.	335.	271.	224.	192.	174.	168.	174.	192.
1214.	1000.	816.	662.	536.	434.	355.	296.	255.	228.	213.	208.	213.	228.
1214.	1000.	818.	667.	544.	448.	375.	323.	288.	267.	255.	251.	255.	267.
1215.	1000.	819.	671.	552.	460.	394.	350.	322.	306.	298.	296.	298.	306.
1215.	1000.	820.	673.	558.	471.	410.	373.	354.	345.	340.	339.	340.	345.
1215.	1000.	820.	675.	561.	477.	421.	391.	381.	378.	376.	376.	376.	378.
1216.	1000.	820.	674.	561.	479.	426.	400.	400.	400.	400.	400.	400.	400.
1216.	1000.	819.	673.	559.	477.	425.	400.	400.	400.	400.	400.	400.	400.
1217.	1000.	818.	670.	556.	474.	423.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	668.	552.	470.	420.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	667.	550.	467.	417.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	668.	552.	470.	420.	400.	400.	0.	0.	0.	0.	0.

Figure 9. Stress Function, ∅, for Plate B

1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.99	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.99	.99
1.03	1.02	1.02	1.01	1.00	1.00	.99	.98	.98	.98	.97
1.03	1.04	1.04	1.03	1.01	1.00	.98	.97	.96	.96	.96
1.04	1.06	1.06	1.04	1.02	1.00	.98	.96	.94	.93	.93
1.11	1.10	1.08	1.06	1.03	1.00	.97	.94	.92	.90	.90
1.14	1.14	1.11	1.08	1.04	.99	.95	.91	.89	.87	.86
1.18	1.18	1.15	1.10	1.05	.99	.94	.89	.85	.82	.81
1.28	1.24	1.19	1.13	1.06	.99	.92	.85	.80	.76	.75
1.35	1.30	1.24	1.16	1.08	.99	.90	.81	.74	.69	.68
1.46	1.38	1.29	1.20	1.10	.99	.88	.76	.67	.61	.59
1.54	1.45	1.35	1.25	1.14	1.01	.86	.71	.59	.51	.48
1.61	1.52	1.42	1.31	1.18	1.03	.85	.65	.49	.40	.37
1.73	1.59	1.48	1.37	1.25	1.08	.85	.58	.38	.28	.24
1.77	1.64	1.54	1.44	1.32	1.16	.90	.48	.25	.15	.12
1.78	1.68	1.59	1.50	1.40	1.26	1.03	.30	.10	.04	.03
1.81	1.70	1.62	1.54	1.45	1.35	1.30	0.00	0.00	0.00	0.00
1.77	1.70	1.64	1.57	1.47	1.34	1.26	0.00	0.00	0.00	0.00
1.76	1.69	1.65	1.62	1.54	1.40	1.15	0.00	0.00	0.00	0.00
1.71	1.67	1.66	1.67	1.61	1.49	1.00	0.00	0.00	0.00	0.00
1.70	1.65	1.65	1.70	1.65	1.65	.85	0.00	0.00	0.00	0.00

Figure 10. Stress σ_x for Plate B

0.00	0.00	0.00	0.00	.01	.04	.06	.08	.10	.11	.11
0.00	0.00	0.00	.02	.03	.05	.07	.09	.10	.11	.11
0.00	0.00	.01	.03	.05	.07	.08	.10	.11	.12	.12
0.00	0.00	.02	.04	.06	.08	.10	.11	.13	.14	.14
0.00	0.00	.02	.04	.07	.09	.11	.13	.15	.16	.16
0.00	.01	.02	.05	.07	.10	.13	.15	.17	.18	.18
0.00	.01	.03	.05	.08	.11	.14	.17	.19	.20	.21
0.00	0.00	.02	.05	.08	.11	.15	.18	.20	.22	.22
0.00	.01	.02	.05	.08	.11	.15	.18	.21	.23	.23
0.00	0.00	.02	.04	.07	.10	.14	.17	.21	.23	.23
0.00	0.00	.01	.02	.04	.07	.11	.15	.18	.21	.21
0.00	0.00	0.00	0.00	0.00	.02	.06	.10	.13	.16	.16
0.00	-.01	-.02	-.04	-.05	-.04	-.02	.02	.05	.07	.07
0.00	-.01	-.05	-.08	-.12	-.14	-.13	-.09	-.07	-.07	-.07
0.00	-.02	-.07	-.12	-.19	-.25	-.29	-.24	-.26	-.29	-.31
0.00	-.03	-.08	-.14	-.22	-.33	-.46	-.44	-.56	-.64	-.67
0.00	-.02	-.06	-.11	-.17	-.26	-.42	-.91	-1.08	-1.15	-1.17
0.00	-.01	-.03	-.06	-.08	-.07	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	-.01	-.03	-.04	0.00	0.00	0.00	0.00	0.00
0.00	.03	.05	.06	.05	0.00	0.00	0.00	0.00	0.00	0.00
0.00	.03	.12	.24	.30	.30	0.00	0.00	0.00	0.00	0.00

Figure 11. Stress σ_y for Plate B

0.00	0.00	0.00	0.00	0.00	.01	.01	0.00	0.00	0.00	0.00
-.01	0.00	.01	.02	.02	.03	.03	.02	.01	0.00	0.00
-.01	0.00	.02	.03	.04	.04	.04	.04	.02	.01	0.00
0.00	.01	.03	.05	.06	.06	.06	.05	.04	.02	0.00
0.00	.02	.05	.07	.08	.09	.08	.07	.05	.02	0.00
0.00	.04	.07	.09	.11	.11	.10	.09	.06	.03	0.00
.01	.05	.09	.12	.13	.14	.13	.11	.08	.04	0.00
.01	.07	.11	.15	.16	.17	.16	.14	.10	.05	0.00
.01	.08	.14	.17	.20	.20	.19	.16	.12	.06	0.00
.02	.09	.15	.20	.23	.24	.23	.20	.14	.07	0.00
.02	.10	.17	.22	.26	.27	.27	.23	.17	.09	0.00
.03	.11	.17	.23	.28	.30	.31	.27	.20	.10	0.00
.02	.10	.16	.23	.28	.33	.34	.31	.23	.11	0.00
.01	.08	.14	.20	.27	.33	.38	.35	.24	.12	0.00
.01	.05	.09	.15	.22	.30	.40	.37	.23	.11	0.00
-.01	.01	.04	.09	.14	.22	.37	.35	.17	.07	0.00
-.02	-.02	0.00	.02	.06	.10	.18	.16	.06	.02	0.00
-.03	-.05	-.04	-.01	.02	.06	.03	0.00	0.00	0.00	0.00
-.03	-.06	-.06	-.03	.02	.09	.06	0.00	0.00	0.00	0.00
-.01	-.04	-.06	-.04	0.00	.09	.07	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 12. Shearing Stress τ_{xy} for Plate B

1210.	1000.	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000	809.	639.	489.	359.	250.	160.	90.	41.	11.	1	11.	41.
1211.	1000	809.	639.	489.	360.	251.	162.	93.	44.	15.	5	15.	44.
1211.	1000	809.	639.	490.	361.	254.	166.	98.	50.	21.	12	21.	50.
1211.	1000	809.	640.	492.	365.	258.	173.	106.	60.	32.	23	32.	60.
1212.	1000	809.	641.	495.	370.	266.	183.	119.	74.	47.	39	47.	74.
1212.	1000	810.	643.	499.	378.	277.	197.	137.	94.	69.	61	69.	94.
1212.	1000	811.	647.	506.	388.	292.	217.	160.	121.	98.	90	98.	121.
1213.	1000	812.	651.	514.	401.	311.	241.	189.	154.	134.	127	134.	154.
1213.	1000	814.	656.	524.	417.	333.	269.	224.	194.	177.	171	177.	194.
1214.	1000	816.	662.	535.	434.	357.	301.	263.	239.	225.	221	225.	239.
1214.	1000	818.	667.	545.	450.	381.	333.	304.	286.	277.	274	277.	286.
1214.	1000	819.	671.	554.	465.	402.	363.	343.	333.	328.	326	328.	333.
1215.	1000	820.	674.	560.	475.	418.	387.	377.	373.	372.	371	372.	373.
1215.	1000	821.	676.	563.	480.	426.	400	400.	400.	400.	400	400.	400.
1215.	1000	820.	675.	563.	480.	427.	400	400.	400.	400.	400.	400.	400.
1216.	1000	820.	674.	561.	479.	426.	400	400.	0.	0.	0.	0.	0.
1216.	1000	819.	672.	558.	476.	424.	400	400.	0.	0.	0.	0.	0.
1217.	1000	818.	670.	555.	473.	422.	400	400.	0.	0.	0.	0.	0.
1217.	1000	817.	668.	552.	469.	419.	400	400.	0.	0.	0.	0.	0.
1217.	1000	817.	667.	550.	467.	417.	400	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	668.	552.	469.	419.	400.	400.	0.	0.	0.	0.	0.

Figure 13. Stress Function, ϕ, for Plate C

1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.98	1.00	1.00	1.00	1.00	1.00	.99	.99	.99	.99	.99
1.02	1.02	1.02	1.01	1.01	1.00	.99	.99	.98	.98	.97
1.01	1.03	1.04	1.03	1.02	1.01	.99	.98	.96	.96	.95
1.02	1.06	1.06	1.05	1.03	1.01	.98	.96	.94	.93	.92
1.08	1.09	1.09	1.07	1.05	1.01	.98	.94	.91	.89	.88
1.12	1.14	1.13	1.10	1.06	1.02	.96	.91	.86	.83	.82
1.17	1.20	1.18	1.14	1.09	1.02	.94	.87	.81	.76	.75
1.29	1.27	1.24	1.18	1.11	1.02	.92	.82	.73	.68	.65
1.38	1.35	1.30	1.23	1.15	1.03	.90	.76	.65	.57	.54
1.53	1.44	1.37	1.29	1.19	1.06	.88	.69	.54	.45	.41
1.62	1.52	1.44	1.36	1.25	1.10	.88	.61	.41	.31	.27
1.69	1.60	1.51	1.43	1.33	1.18	.92	.49	.27	.17	.14
1.79	1.65	1.56	1.48	1.40	1.28	1.04	.31	.11	.05	.04
1.81	1.68	1.59	1.51	1.44	1.37	1.32	0.00	0.00	0.00	0.00
1.79	1.70	1.61	1.52	1.42	1.33	1.35	0.00	0.00	0.00	0.00
1.81	1.70	1.62	1.54	1.45	1.35	1.30	0.00	0.00	0.00	0.00
1.76	1.70	1.64	1.58	1.49	1.37	1.23	0.00	0.00	0.00	0.00
1.75	1.69	1.66	1.62	1.55	1.41	1.13	0.00	0.00	0.00	0.00
1.71	1.67	1.66	1.67	1.62	1.50	.99	0.00	0.00	0.00	0.00
1.70	1.65	1.65	1.70	1.65	1.65	.85	0.00	0.00	0.00	0.00

Figure 14. Stress σ_x for Plate C

0.00	-.01	-.01	-.01	0.00	.01	.03	.04	.06	.07	.07
0.00	0.00	0.00	.01	.02	.04	.06	.08	.09	.10	.10
0.00	0.00	.01	.03	.05	.07	.09	.12	.13	.14	.15
0.00	.01	.02	.05	.07	.10	.13	.16	.18	.19	.20
0.00	.01	.03	.06	.10	.14	.17	.21	.23	.25	.25
0.00	.01	.04	.08	.12	.16	.21	.25	.28	.30	.31
0.00	.01	.05	.09	.13	.18	.23	.28	.32	.34	.35
0.00	.01	.04	.08	.13	.18	.24	.29	.33	.36	.37
0.00	.01	.04	.07	.11	.16	.21	.27	.31	.34	.35
0.00	0.00	.02	.03	.06	.10	.15	.20	.25	.28	.28
0.00	0.00	0.00	-.01	-.01	0.00	.04	.09	.13	.15	.16
0.00	-.01	-.04	-.07	-.10	-.12	-.11	-.06	-.04	-.04	-.05
0.00	-.02	-.07	-.13	-.20	-.28	-.32	-.28	-.30	-.34	-.36
0.00	-.03	-.08	-.16	-.26	-.40	-.56	-.56	-.69	-.78	-.81
0.00	-.02	-.07	-.14	-.22	-.36	-.62	-1.12	-1.32	-1.39	-1.41
0.00	-.02	-.05	-.08	-.09	-.07	0.00	0.00	0.00	0.00	0.00
0.00	-.01	-.03	-.04	-.04	-.02	0.00	0.00	0.00	0.00	0.00
0.00	0.00	-.01	-.03	-.03	-.02	0.00	0.00	0.00	0.00	0.00
0.00	.01	0.00	0.00	-.02	-.03	0.00	0.00	0.00	0.00	0.00
0.00	.02	.05	.05	.04	0.00	0.00	0.00	0.00	0.00	0.00
0.00	.02	.11	.22	.27	.28	0.00	0.00	0.00	0.00	0.00

Figure 15. Stress σ_y for Plate C

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-.01	0.00	0.00	.01	.02	.02	.02	.02	.01	.01	0.00
-.01	0.00	.01	.02	.03	.04	.04	.04	.03	.01	0.00
0.00	0.00	.03	.04	.06	.07	.07	.06	.04	.02	0.00
0.00	.02	.05	.07	.09	.10	.10	.09	.06	.03	0.00
0.00	.04	.08	.11	.13	.14	.14	.12	.09	.04	0.00
.02	.07	.11	.15	.18	.19	.18	.16	.11	.06	0.00
.01	.09	.15	.20	.23	.24	.23	.20	.15	.08	0.00
.02	.11	.18	.24	.28	.29	.29	.25	.18	.09	0.00
.03	.13	.20	.27	.31	.34	.34	.30	.22	.11	0.00
.03	.13	.21	.28	.33	.38	.39	.35	.25	.13	0.00
.04	.12	.19	.26	.33	.39	.43	.39	.27	.13	0.00
.01	.09	.15	.21	.28	.36	.44	.41	.26	.12	0.00
0.00	.05	.09	.13	.18	.26	.40	.38	.19	.07	0.00
0.00	.01	.03	.05	.07	.08	.17	.17	.06	.02	0.00
-.02	-.02	0.00	0.00	.01	.01	0.00	0.00	0.00	0.00	0.00
-.03	-.04	-.03	0.00	.02	.05	.03	0.00	0.00	0.00	0.00
-.03	-.05	-.04	-.01	.02	.07	.04	0.00	0.00	0.00	0.00
-.03	-.05	-.05	-.03	.02	.08	.05	0.00	0.00	0.00	0.00
-.01	-.04	-.05	-.04	0.00	.08	.07	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 16. Shearing Stress τ_{xy} for Plate C

1210.	1000.	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000.	810.	640.	490.	360.	250.	160.	90.	40.	10.	0.	10.	40.
1210.	1000.	809.	639.	489.	360.	250.	161.	92.	43.	13.	3	13.	43.
1211.	1000.	809.	639.	490.	361.	254.	166.	98.	50.	22.	12.	22.	50.
1211.	1000.	809.	640.	492.	365.	260.	175.	109.	63.	36.	27.	36.	63.
1211.	1000.	810.	642.	496.	373.	270.	189.	126.	83.	57.	48.	57.	83.
1212.	1000.	811.	645.	503.	383.	285.	208.	150.	110.	86.	78.	86.	110.
1212.	1000.	812.	650.	511.	397.	304.	233.	180.	144.	123.	117.	123.	144.
1212.	1000.	814.	655.	522.	413.	327.	263.	216.	186.	168.	163.	168.	186.
1213.	1000.	816.	661.	533.	431.	353.	296.	257.	232.	219.	215.	219.	232.
1213.	1000.	818.	667.	544.	449.	378.	330.	300.	282.	273.	270.	273.	282.
1214.	1000.	820.	672.	554.	464.	401.	362.	341.	330.	325.	324.	325.	330.
1214.	1000.	821.	675.	560.	475.	418.	386.	376.	372.	371.	370.	371.	372.
1214.	1000.	821.	677.	564.	481.	426.	400.	400.	400.	400.	400.	400.	400.
1215.	1000.	821.	677.	564.	481.	427.	400.	400.	400.	400.	400.	400.	400.
1215.	1000.	821.	676.	563.	481.	427.	400.	400.	0.	0.	0.	0.	0.
1215.	1000.	820.	675.	562.	479.	426.	400.	400.	0.	0.	0.	0.	0.
1216.	1000.	819.	673.	560.	478.	425.	400.	400.	0.	0.	0.	0.	0.
1216.	1000.	818.	671.	557.	475.	424.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	818.	669.	555.	473.	422.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	667.	552.	469.	419.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	667.	550.	467.	417.	400.	400.	0.	0.	0.	0.	0.
1217.	1000.	817.	667.	552.	469.	419.	400.	400.	0.	0.	0.	0.	0.

Figure 17. Stress Function, ϕ, for Plate D

1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.98	1.01	1.01	1.01	1.01	1.00	.99	.99	.98	.98	.98
1.02	1.03	1.03	1.03	1.02	1.01	.99	.98	.96	.95	.95
1.02	1.06	1.07	1.06	1.04	1.02	.99	.96	.93	.91	.90
1.05	1.10	1.11	1.09	1.06	1.02	.97	.93	.89	.86	.85
1.15	1.16	1.16	1.13	1.08	1.02	.96	.89	.83	.79	.77
1.23	1.23	1.21	1.17	1.11	1.03	.93	.84	.75	.70	.67
1.32	1.32	1.28	1.23	1.15	1.04	.91	.78	.66	.59	.56
1.47	1.41	1.35	1.29	1.19	1.06	.89	.70	.55	.46	.42
1.57	1.50	1.43	1.35	1.25	1.11	.89	.62	.42	.32	.28
1.71	1.58	1.50	1.42	1.33	1.18	.92	.50	.27	.17	.14
1.76	1.64	1.55	1.47	1.40	1.29	1.05	.31	.11	.05	.04
1.78	1.67	1.58	1.50	1.43	1.38	1.33	0.00	0.00	0.00	0.00
1.83	1.69	1.59	1.50	1.41	1.33	1.38	0.00	0.00	0.00	0.00
1.81	1.70	1.60	1.50	1.42	1.35	1.35	0.00	0.00	0.00	0.00
1.78	1.70	1.61	1.53	1.44	1.36	1.31	0.00	0.00	0.00	0.00
1.79	1.70	1.63	1.55	1.47	1.37	1.26	0.00	0.00	0.00	0.00
1.74	1.70	1.65	1.59	1.51	1.38	1.20	0.00	0.00	0.00	0.00
1.75	1.69	1.66	1.63	1.56	1.41	1.11	0.00	0.00	0.00	0.00
1.71	1.67	1.66	1.67	1.62	1.50	.99	0.00	0.00	0.00	0.00
1.70	1.65	1.65	1.70	1.65	1.65	.85	0.00	0.00	0.00	0.00

Figure 18. Stress σ_x for Plate D

0.00	-.01	-.02	-.01	.01	.04	.08	.12	.16	.18	.19
0.00	0.00	.01	.04	.07	.10	.14	.18	.21	.23	.23
0.00	.01	.04	.07	.11	.16	.20	.24	.27	.29	.30
0.00	.02	.05	.10	.15	.20	.25	.29	.32	.35	.35
0.00	.02	.06	.11	.16	.22	.27	.32	.36	.39	.40
0.00	.02	.06	.10	.16	.22	.27	.33	.37	.40	.41
0.00	.01	.04	.08	.13	.18	.24	.30	.34	.37	.38
0.00	0.00	.02	.04	.07	.11	.16	.22	.26	.29	.30
0.00	0.00	0.00	-.01	-.01	0.00	.04	.09	.13	.15	.16
0.00	-.01	-.04	-.07	-.11	-.13	-.12	-.07	-.05	-.05	-.06
0.00	-.02	-.07	-.13	-.21	-.29	-.34	-.29	-.33	-.37	-.38
0.00	-.03	-.08	-.16	-.27	-.41	-.58	-.59	-.72	-.82	-.85
0.00	-.02	-.07	-.13	-.22	-.37	-.65	-1.16	-1.36	-1.44	-1.46
0.00	-.01	-.04	-.07	-.08	-.07	0.00	0.00	0.00	0.00	0.00
0.00	0.00	-.02	-.03	-.02	-.01	0.00	0.00	0.00	0.00	0.00
0.00	0.00	-.01	-.01	-.01	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	-.01	-.02	-.02	-.01	0.00	0.00	0.00	0.00	0.00
0.00	0.00	-.01	-.02	-.03	-.02	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	-.01	-.02	-.04	0.00	0.00	0.00	0.00	0.00
0.00	.02	.04	.05	.03	-.01	0.00	0.00	0.00	0.00	0.00
0.00	.02	.09	.20	.26	.28	0.00	0.00	0.00	0.00	0.00

Figure 19. Stress σ_y for Plate D

0.00	0.00	0.00	0.00	.01	.01	.02	.01	.01	0.00	0.00
-.01	0.00	.01	.03	.04	.05	.05	.05	.04	.02	0.00
-.01	0.00	.03	.06	.08	.09	.09	.08	.06	.03	0.00
0.00	.03	.07	.10	.12	.14	.14	.12	.09	.04	0.00
0.00	.06	.11	.15	.18	.19	.19	.16	.12	.06	0.00
.01	.09	.15	.20	.23	.25	.24	.21	.15	.08	0.00
.04	.12	.19	.25	.29	.30	.30	.26	.19	.10	0.00
.03	.14	.22	.28	.33	.36	.35	.31	.23	.12	0.00
.03	.14	.22	.29	.35	.39	.40	.36	.26	.13	0.00
.03	.13	.21	.27	.34	.40	.44	.40	.28	.14	0.00
.02	.10	.16	.22	.29	.37	.46	.42	.26	.12	0.00
.01	.06	.10	.14	.19	.26	.41	.39	.19	.08	0.00
0.00	.02	.04	.06	.06	.08	.17	.17	.06	.02	0.00
-.01	0.00	0.00	.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-.01	-.02	-.01	0.00	.01	.02	.01	0.00	0.00	0.00	0.00
-.03	-.03	-.02	0.00	.02	.03	.02	0.00	0.00	0.00	0.00
-.03	-.04	-.03	0.00	.02	.04	.02	0.00	0.00	0.00	0.00
-.03	-.04	-.04	-.01	.02	.06	.03	0.00	0.00	0.00	0.00
-.03	-.04	-.05	-.02	.01	.07	.05	0.00	0.00	0.00	0.00
-.01	-.03	-.05	-.03	0.00	.07	.06	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 20. Shearing Stress τ_{xy} for Plate D

B. Results of the Compression Problem

Appendix B gives the critical buckling coefficients for the various plates in terms of the eigenvalues "λ" as obtained from computer outputs. Figure 21 shows the critical buckling coefficients versus the notch aspect ratio. This gives a clear indication as to the effect of the notch size on the critical buckling load for various notches. The graph shows that with the increase of the aspect ratio of the notch, the critical buckling load will decrease. This is not surprising since a larger notch will give a reduced buckling strength. The graph also shows that the critical buckling strength does not vary linearly with the aspect ratio of the notch. Four points on the curve were obtained from the computer analysis. The fifth point for the solid plate (zero aspect ratio)[24] was obtained from a solution in a closed form. All points follow a curvilinear smooth relationship which indicates good agreement between the theoretically obtained point and the values obtained by the finite difference approach.

The relationship between the computed eigenvalue λ and Timoshenko's buckling coefficient value "K" is as follows:

$$K = \pi^2 \lambda$$

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[24]S. Timoshenko and J. H. Gere, *op. cit.*, p. 362.

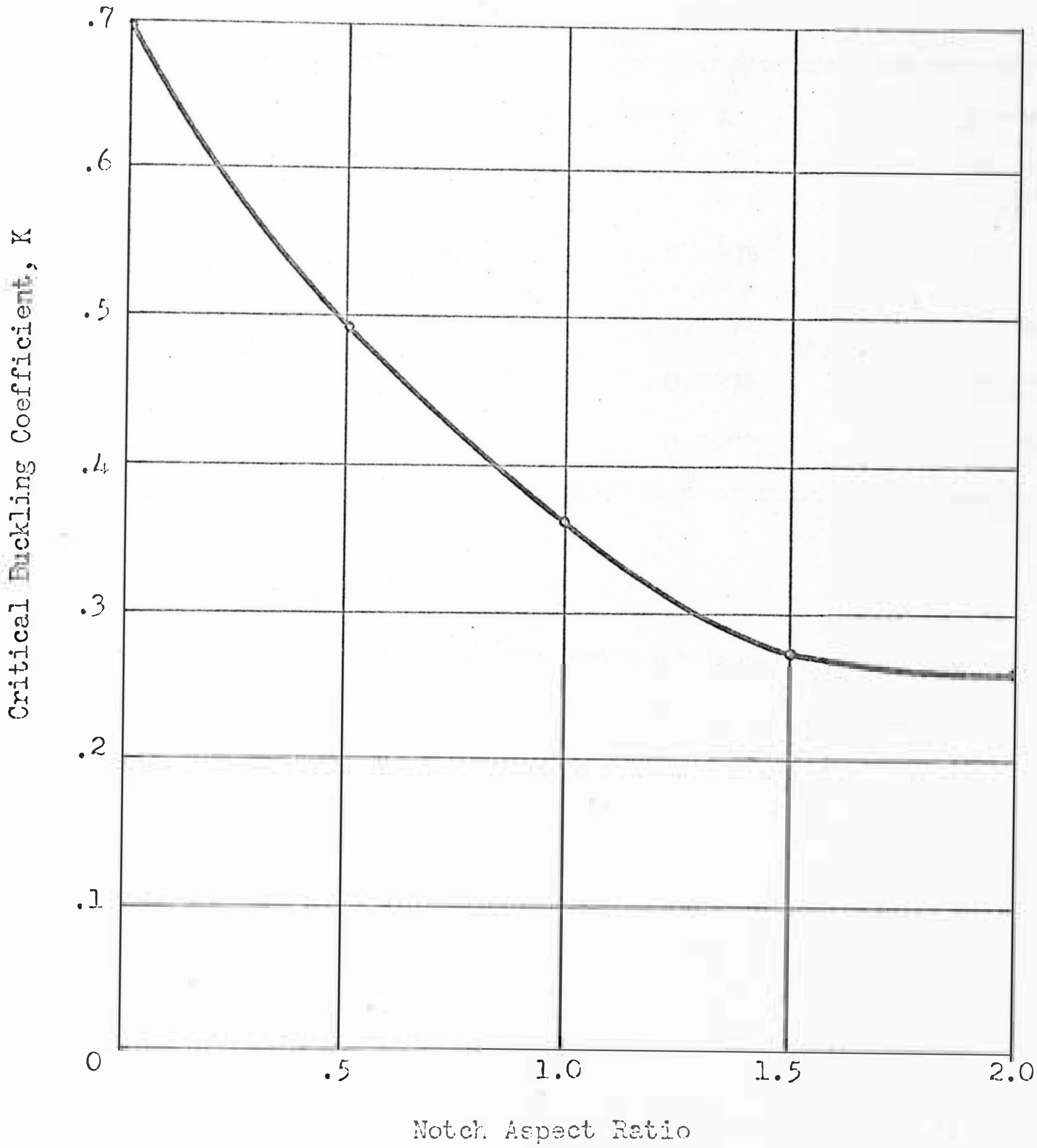

Figure 21. Effect of Notch Size on Critical Buckling

Table 1 summarises the eigenvalue results and includes the solid plate's buckling coefficient.

TABLE 1. Eigenvalue Results

Aspect Ratio	Plate	λ	$K = \pi^2 \lambda$
0	-	-	0.698
1/2	A	0.0505	0.498
1	B	0.0372	0.368
3/2	C	0.0276	0.273
2	D	0.0262	0.258

C. Numerical Example

It is required to determine the critical buckling load for a plate 36 x 72 inches with a square hole of width 14.4 inches. Assuming a steel plate with

$$\nu = 0.3$$

$$t = 1/4 \text{ in.}$$

and

$$E = 29 \times 10^6 \text{ psi},$$

then

$$10h = 36/2 \text{ in.}$$

Therefore,

$$h = 1.8 \text{ in.}$$

Now,

$$\lambda = .0372, \text{ from Table 1,}$$

$$N_{x_{cr}} = \sigma_{x_{cr}} t,$$

and

$$\sigma_{x_{cr}} = \lambda \frac{D}{th^2} .$$

Therefore,

$$N_{x_{cr}} = \lambda \frac{D}{h^2} .$$

But,

$$D = \frac{Et^3}{12(1 - \nu^2)}$$

$$= \frac{29 \times 10^6 \times (1/4)^3}{12(1 - 0.09)}$$

$$= 4.15 \times 10^4 \text{ lb. in.}$$

Therefore,

$$\sigma_{x_{cr}} = \frac{.0372 \times 4.15 \times 10^4}{(1/4) \times (1.8)^2}$$

$$= 1910 \text{ psi}$$

and

$$N_{x_{cr}} = 1910\ (1/4)$$

$$= 477.5 \text{ lb./in.}$$

CHAPTER V

SUMMARY AND CONCLUSIONS

A. General Summary

One of the purposes of this work was to determine the critical buckling load in a rectangularly notched plate under uniformly axial compression. In the analytical approach followed, a determination of the elastic stress distribution in the plate was a necessary first step. The stress value was determined at each of a set of discrete points within a domain which was consistent with the boundary values. With the stresses known, it was possible to incorporate their values into an eigenvalue problem, and the critical buckling coefficient was obtained. The process was followed to determine the critical buckling load for four plates with different notch sizes in order to investigate the effect of the notch size on the load capacity of the plate.

It had been assumed that the plate buckling configuration was symmetrical about both axes of the plate. Due to this fact and for the sake of increasing accuracy within the storage capacity of the computer, the quarter plate was considered.

The numerical approach followed in this work employed the finite differences technique. The entire process was calculated on a digital computer.

The following conclusions may be drawn:

1. This work should be considered as a basis for future investigation. Many similar problems can be solved

by some modification of the basic methods, equations, techniques, and computer programs.

2. The stress contours indicate that the notch corners are points of high stress consideration.

3. The results obtained for the critical buckling coefficients for the four notched plates are reasonably accepted. This is due to the fact that the four values obtained electronically go in harmony with the value for a solid plate obtained in a closed form. This is shown in Figure 21.

4. Certain assumptions have been introduced because of the limited capacity of the computer used. This might affect the accuracy of the results. However, with a wider computer capacity such assumptions can be eliminated, and as a consequence the accuracy can be increased.

5. The computer time required for iteration process, matrix inversion, matrix multiplication, and eigenvalue iteration was relatively long. This time can be reduced by either considering a lesser degree of accuracy or using a faster computer.

6. The finite differences approach, being an approximate numerical method of analysis, introduced small error in the stress distribution values. The finite differences buckling equation caused additional error. Part of this was caused by the inherent properties of finite differences approximation.

The error in finite differences solutions always give buckling loads on the low side. Moreover, with errors of opposite sign, it is evident that a relatively accurate solution would result.

B. Future Areas of Study and Research

The use of numerical techniques in determining critical buckling loads in notched plates is relatively new. There are many problems that can be worked out with the same basic method followed here. Some of the future areas of study and research are listed below.

1. The effect of different types of loading needs to be investigated.
2. The effect of different types of support needs to be investigated.
3. The height to width aspect ratio for the plate considered was constant. Different plates with different overall aspect ratios need to be investigated. This will lead to the determination of the relation between the critical buckling coefficient and the aspect ratios of both the notch and the plate.
4. The top width of the notch was kept constant in this work and the ratio of the width of the notch to the width of the plate was also constant. The effect of variation of this ratio on load capacity needs to be investigated.

5. The notch in the plate considered was rectangular. Effect of other types of notches on loading capacity needs to be investigated.

NOTATION

The following notation is used in this thesis:

a = coefficient of half width of plate at throat section

b = coefficient of half length of notch

c = constant of integration

D = flexural rigidity = $Et^3/12(1-\nu^2)$

d = width of plate

E = modulus of elasticity

h = the spacing dimension for the square network used

K_1 = constant of integration

k = largest eigenvalue

l = length of plate

N_x, N_y = normal forces per unit edge length acting in the x and y directions, respectively

N_{xy} = shear force per unit length acting in the xy-plane

R = half width of plate at the throat

S = half width of plate at the solid section

t = thickness of plate

w = deflection perpendicular to the middle surface of the web plate

x = variable distance along the x-axis

y = variable distance along the y-axis

λ = smallest eigenvalue

ν = Poisson's ratio

σ_n = axial compressive stress applied at top of plate

σ_p = stress at top of plate = σ_n

σ_r = axial resulting compressive stress at throat of plate

σ_x = stress in the x-direction

σ_y = stress in the y-direction

τ_{xy} = shearing stress at any point

ϕ = the stress function

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APPENDIX A

COMPUTER PROGRAMS

PROGRAM I

```
C     A FORTRAN II PROGRAM FOR SATISFYING THE
C     BIHARMONIC MOLECULE AND COMPUTING SIGX, SIGY
C     AND TXY AS APPLIED TO A RECTANGULAR PLATE
C     WITH A RECTANGULAR CENTRAL HOLE .

C     NOTCH 12 X 8
C     LEFT QUARTER CONSIDERED .
C     AXIAL UNIFORMLY DISTRIBUTED COMPRESSION LOAD APPLIED

    3 FORMAT(11F7.0/3F7.0)
    4 FORMAT(F10.4,F10.4)
    5 FORMAT(11F7.0/3F7.0,I3)
C     CLEAR THE ARRAYS
      DIMENSION F(23,14)
    6 DO 7 I=1,23
      DO 7 J=1,14
    7 F(I,J)=0.0
C     F EQUALS STRESS FUNCTION AT NODAL POINTS
C     READ LIMITS ON BIHARMONIC AND STRESS SWEEPS
    8 READ 3,((F(I,J),J=1,14),I=1,23)
      ICARD=-1
C     START AN ITERATION -SWEEP THE BIHARMONIC MOLECULE
C     TEST EQUALS LARGEST ERROR VALUE
C     DCT EQUALS LARGEST ERROR VALUE DECIMAL
   12 TEST = 0.0
C     ELEM EQUALS COMPUTED F VALUES AT NODAL POINTS
      DCT=0.0
   13 DO 29 I=3,15
      DO 29 J=3,12
   15 ELEM=(8.*(F(I+1,J)+F(I,J+1)+F(I-1,J)+F(I,J-1))
     C-2.*(F(I+1,J+1)+F(I+1,J-1)+F(I-1,J+1)+F(I-1,J-1))
     C-1.*(F(I,J+2)+F(I+2,J)+F(I,J-2)+F(I-2,J)))/20.
C     CHECK ERROR. ECK MEANS ERROR CHECK
      ECK=ABSF(F(I,J)-ELEM)
      IF(ECK-TEST)25,23,23
   23 TEST=ECK
C     DECK MEANS DECIMAL CHECK
   25 DECK=ABSF(ECK/ELEM)
      IF(DECK-DCT)29,27,27
   27 DCT=DECK
   29 F(I,J)=ELEM
  113 DO 39 I=16,21
      DO 39 J=3,7
   35 ELEM=(8.*(F(I+1,J)+F(I,J+1)+F(I-1,J)+F(I,J-1))
     C-2.*(F(I+1,J+1)+F(I+1,J-1)+F(I-1,J+1)+F(I-1,J-1))
     C-1.*(F(I,J+2)+F(I+2,J)+F(I,J-2)+F(I-2,J)))/20.
      ECK=ABSF(F(I,J)-ELEM)
      IF(ECK-TEST)45,33,33
```

\- CONTINUED

```
   33 TEST=ECK
   45 DECK=ABSF(ECK/ELEM)
      IF(DECK-DCT)39,37,37
   37 DCT=DECK
   39 F(I,J)=ELEM
 1000 DO 1001 I=3,15
 1001 F(I,13)=F(I,11)
 1002 DO 1003 I=3,15
 1003 F(I,14)=F(I,10)
 1004 DO 1005 J=3,7
 1005 F(23,J)=F(21,J)
      PRINT 4,TEST,DCT
      IF (SENSE SWITCH 1)73,12
   73 PUNCH 5,((F(I,J),J=1,14),ICARD,I=1,23)

      IF (SENSE SWITCH 2)74,12
    1 FORMAT(11F7.2)
      DIMENSION SIGX(22,12),SIGY(22,12),TXY(22,12)
   74 DO 72 I=2,22
      DO 72 J=2,12
      SIGY(I,J)=0.0
      TXY(I,J)=0.0
   72 SIGX(I,J)=0.0
      DO 200 I=2,16
      DO 200 J=2,12
  200 SIGX(I,J)=(F(I,J+1)+F(I,J-1)-2.*F(I,J))/20.
   75 DO 201 I=17,22
      DO 201 J=2,8
  201 SIGX(I,J)=(F(I,J+1)+F(I,J-1)-2.*F(I,J))/20.
      PUNCH 1,((SIGX(I,J),J=2,12),I=2,22)
   76 DO 300 I=2,16
      DO 300 J=2,12
  300 SIGY(I,J)=(F(I-1,J)+F(I+1,J)-2.*F(I,J))/20.
   77 DO 301 I=17,22
      DO 301 J=2,8
  301 SIGY(I,J)=(F(I-1,J)+F(I+1,J)-2.*F(I,J))/20.
      PUNCH 1,((SIGY(I,J),J=2,12),I=2,22)
   78 DO 400 I=2,16
      DO 400 J=2,12
  400 TXY(I,J)=(F(I-1,J-1)+F(I+1,J+1)-F(I+1,J-1)
     C F(I-1,J+1))/80.
   79 DO 401 I=17,22
      DO 401 J=2,8
  401 TXY(I,J)=(F(I-1,J-1)+F(I+1,J+1)-F(I+1,J-1)
     C F(I-1,J+1))/80.
      PUNCH 1,((TXY(I,J),J=2,12),I=2,22)
      STOP
      END
```

PROGRAM II

PROGRAM FOR INVERTING N X N MATRIX

```
  1 FORMAT (I3)
  2 FORMAT (16F5.0)
  4 FORMAT (8F10.4)
    DIMENSION A(47,47),AI1(46)
    READ 1,N
    DO 3 I=1,N
  3 READ 2,(A(I,J),J=1,N)
    PUNCH 13
 13 FORMAT (40H THE FOLLOWING ARE ELEMENTS OF A INVERSE)
100 DO 12 K=1,N
    N1=N+1
    A(1,N1)=1.0
    DO 7 I=2,N1
  7 A(I,N1)=0.0
    DO 8 J=2,N1
  8 A(N1,J-1)=A(1,J)/A(1,1)
    DO 11 I=1,N
    AI1(I)=A(I,1)
    DO 11 J=2,N1
 11 A(I,J-1)=A(I,J)-AI1(I)*A(N1,J-1)
    TYPE 1,K
    DO 12 I=1,N
    DO 12 J=1,N
 12 A(I,J)=A(I+1,J)
    DO 14 I=1,N
 14 PUNCH 4,(A(I,J),J=1,N)
    STOP
    END
```

PROGRAM III

PROGRAM FOR MULTIPLYING N X N ARRAY BY N X N ARRAY

```
1 FORMAT (I3)
2 FORMAT (8F10.4)
3 FORMAT (8F10.5)
  DIMENSION B(46,46),A(46),G(46)
  READ 1,N
  DO 4 I=1,N
4 READ 2,(B(I,J),J=1,N)
  PUNCH 5
5 FORMAT (43H THE FOLLOWING ARE ELEMENTS OF THE G MATRIX)
6 DO 9 M=1,N
  READ 2,(A(J),J=1,N)
  DO 8 K=1,N
  SUM=0.0
  DO 7 L=1,N
7 SUM=SUM+A(L)*B(L,K)
8 G(K)=SUM
9 PUNCH 3,(G(K),K=1,N)
  STOP
  END
```

PROGRAM IV

PROGRAM FOR COMPUTING THE EIGENVALUE

```
  2 FORMAT (8F10.5)
  3 FORMAT (I3)
  7 FORMAT (E14.6)
    DIMENSION G(46,46),X(46),Y(46)
    READ 3,N
    DO 1 I=1,N
  1 READ 2,(G(I,J),J=1,N)
    ITERA=0
    X(1)=1.
    DO 10 I=2,N
 10 X(I)=0.
    PUNCH 100
100 FORMAT (24X,25HITERATION      EIGENVALUE//)
    DO 9 I=1,N
    DO 9 J=1,N
  9 G(I,J)=-1.*G(I,J)
 23 DO 20 I=1,N
    Y(I)=0.
    DO 20 J=1,N
 20 Y(I)=Y(I)+G(I,J)*X(J)
    DO 30 I=2,N
 30 X(I)=Y(I)/Y(1)
    EIGEN=1./Y(1)
    ITERA=ITERA+1
    TYPE 7,EIGEN
    PUNCH 101,ITERA,EIGEN
101 FORMAT (I30,8X,E14.6)
    IF(SENSE SWITCH 1)24,23
 24 PUNCH 8
  8 FORMAT (28H ELEMENTS OF THE EIGENVECTOR/)
200 PUNCH 7,(X(I),I=1,N)
    STOP
    END
```

APPENDIX B

EIGENVALUE COMPUTER OUTPUTS

PLATE A

ITERATION	EIGENVALUE
1	23.255813E+01
2	-62.669528E-02
3	55.435571E-03
4	48.403261E-03
5	50.574847E-03
6	50.503149E-03
7	50.504847E-03
8	50.505159E-03
9	50.505287E-03
10	50.505351E-03
11	50.505403E-03
12	50.505417E-03
13	50.505421E-03
14	50.505424E-03
15	50.505424E-03

PLATE B

ITERATION	EIGENVALUE
1	13.106159E+01
2	54.946739E-02
3	37.476701E-03
4	37.203403E-03
5	37.199505E-03
6	37.199438E-03
7	37.199435E-03

PLATE C

ITERATION	EIGENVALUE
1	20.283975E+01
2	-14.394623E-02
3	29.449732E-03
4	27.169145E-03
5	27.708849E-03
6	27.575969E-03
7	27.608415E-03
8	27.600479E-03
9	27.602421E-03
10	27.601947E-03
11	27.602063E-03
12	27.602034E-03
13	27.602041E-03
14	27.602040E-03
15	27.602041E-03
16	27.602039E-03
17	27.602039E-03
18	27.602040E-03
19	27.602040E-03
20	27.602038E-03
21	27.602039E-03
22	27.602041E-03
23	27.602040E-03
24	27.602040E-03
25	27.602040E-03
26	27.602039E-03
27	27.602040E-03
28	27.602040E-03
29	27.602040E-03
30	27.602039E-03
31	27.602040E-03
32	27.602040E-03
33	27.602039E-03
34	27.602040E-03
35	27.602039E-03
36	27.602040E-03
37	27.602040E-03
38	27.602040E-03
39	27.602040E-03

PLATE D

ITERATION	EIGENVALUE
1	13.404825E+01
2	-10.791506E-02
3	33.907101E-03
4	22.313809E-03
5	28.667006E-03
6	24.794134E-03
7	27.018742E-03
8	25.694491E-03
9	26.466680E-03
10	26.010860E-03
11	26.278009E-03
12	26.120777E-03
13	26.213087E-03
14	26.158813E-03
15	26.190701E-03
16	26.171955E-03
17	26.182970E-03
18	26.176498E-03
19	26.180298E-03
20	26.178067E-03
21	26.179378E-03
22	26.178608E-03
23	26.179059E-03
24	26.178795E-03
25	26.178948E-03
26	26.178860E-03

APPENDIX C

FREE BOUNDARY EQUATIONS

FREE BOUNDARY EQUATIONS

Figure 22 represents a cross section of a plate with a free edge. The free edge is extended a distance 2h beyond the boundary, where "h" represents the mesh size. In this figure, w_a, w_b, w_1, and w_2 represent the deflections of the plate at points a, b, 1 and 2 respectively. Points a and b are on the plate while points 1 and 2 are beyond the plate. If the slope is assumed to be constant between points b and 2, then

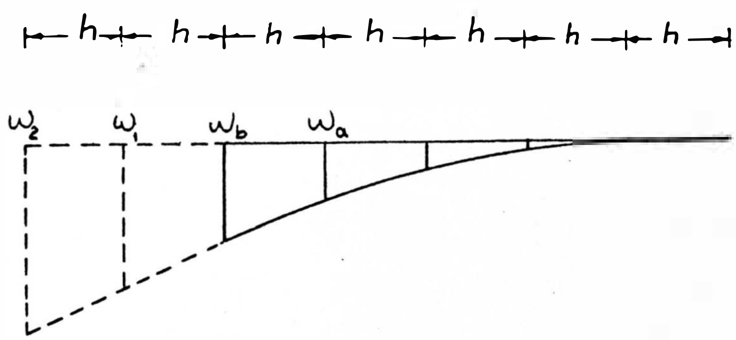

Figure 22. Approximation Curve for Deflections of a Free Boundary

$$w_1 = w_b + \frac{w_b - w_a}{h} h$$

$$= 2w_b - w_a \quad \text{(a)}$$

and

$$w_2 = w_1 + \frac{w_1 - w_b}{h} h$$

$$w_2 = 2w_1 - w_b \quad \text{(b)}$$

Substituting for w_1 in Equation (b), then

$$w_2 = 3w_b - 2w_a \qquad \text{(c)}$$

Equations (a) and (c) express w_1 and w_2 in terms of w_a and w_b.

APPENDIX D

EXAMPLE ON ITERATION TECHNIQUE FOR DETERMINATION OF EIGENVALUE

EXAMPLE ON ITERATION TECHNIQUE FOR DETERMINATION OF EIGENVALUE

Consider the three linear equations:

$$10x_1 + 2x_2 + x_3 = \lambda x_1$$
$$2x_1 + 10x_2 + x_3 = \lambda x_2$$
$$2x_1 + x_2 + 10x_3 = \lambda x_3$$

$$D(x) = \begin{vmatrix} 10-\lambda & 2 & 1 \\ 2 & 10-\lambda & 1 \\ 2 & 1 & 10-\lambda \end{vmatrix} = 0$$

$$(10-\lambda)^3 - 7(10-\lambda) + 6 = 0$$

Solve,

$$10-\lambda = \begin{bmatrix} -3 \\ +1 \\ +2 \end{bmatrix}$$

Therefore

$$\lambda_1 = 13$$
$$\lambda_2 = 9$$
$$\lambda_3 = 8$$

The eigenvalue is the largest and is equal to 13.

The same problem is solved now by the iteration method. The procedure is explained in Chapter III.

$$\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 3 \end{bmatrix} = 12 \begin{bmatrix} 1 \\ 1 \\ .25 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ .25 \end{bmatrix} = \begin{bmatrix} 12.25 \\ 12.25 \\ 5.50 \end{bmatrix} = 12.25 \begin{bmatrix} 1 \\ 1 \\ .45 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ .45 \end{bmatrix} = \begin{bmatrix} 12.45 \\ 12.45 \\ 7.50 \end{bmatrix} = 12.45 \begin{bmatrix} 1 \\ 1 \\ .60 \end{bmatrix}$$

The procedure continues until the factored coefficient is repeated over and over. Repeating the row-into-column multiplication with $x_1^{(i)} = 1$, the successive approximations become

$$12.6 \begin{bmatrix} 1 \\ 1 \\ .7 \end{bmatrix} ; \quad 12.7 \begin{bmatrix} 1 \\ 1 \\ .8 \end{bmatrix} ; \quad 12.8 \begin{bmatrix} 1 \\ 1 \\ .9 \end{bmatrix} ;$$

$$12.9 \begin{bmatrix} 1 \\ 1 \\ .93 \end{bmatrix} ; \quad 12.93 \begin{bmatrix} 1 \\ 1 \\ .95 \end{bmatrix} ; \quad 13 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ;$$

$$13 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

or

$$\lambda_1 = 13, \text{ the eigenvalue.}$$

The same problem has been worked out on the computer and the results are shown on the next page.

EXAMPLE PROBLEM

ITERATION	EIGENVALUE
1	-10.000000E+00
2	-10.600000E+00
3	-11.188678E+00
4	-11.704889E+00
5	-12.114824E+00
6	-12.415475E+00
7	-12.623357E+00
8	-12.761305E+00
9	-12.850363E+00
10	-12.906842E+00
11	-12.942259E+00
12	-12.964307E+00
13	-12.977975E+00
14	-12.986422E+00
15	-12.991636E+00
16	-12.994849E+00
17	-12.996829E+00
18	-12.998047E+00
19	-12.998798E+00
20	-12.999260E+00
21	-12.999544E+00
22	-12.999719E+00
23	-12.999827E+00
24	-12.999893E+00
25	-12.999934E+00
26	-12.999959E+00
27	-12.999974E+00
28	-12.999984E+00
29	-12.999989E+00
30	-12.999994E+00
31	-12.999995E+00
32	-12.999997E+00
33	-12.999998E+00
34	-12.999998E+00
35	-12.999999E+00
36	-12.999999E+00
37	-13.000000E+00
38	-13.000000E+00
39	-13.000000E+00
40	-13.000000E+00
41	-13.000000E+00
42	-13.000000E+00
43	-13.000000E+00

APPENDIX E

DETERMINATION OF STRESS FUNCTION VALUES FOR THE SELECTED PLATE WITH FOUR DIFFERENT NOTCH SIZES

DETERMINATION OF STRESS FUNCTION VALUES FOR THE SELECTED PLATE WITH FOUR DIFFERENT NOTCH SIZES

According to equation (12a) for the side A-B of Figure 3,

$$\phi = \sigma_n \frac{(10h)^2}{2}$$

Let the value of the mesh spacing be considered as unity and the applied stress σ_n be equal to 20 for convenience; then, for each mesh point along the side A-B, as shown in Figure 3, the stress will be as shown in Table 2.

TABLE 2. Stress Function Values Along Boundary A-B

x	ϕ
1	10
2	40
3	90
4	160
5	250
6	360
7	490
8	640
9	810
10	1000
11	1210

Hence at point B,

$$\phi = 1000 \ .$$

Then, at point B from equation (11b)

$$\frac{\partial \phi}{\partial x} = 200 \ .$$

From equation (14a), the stress function along the side B-C is constant and its value is

$$\phi = 1000$$

Also, from equations (15) and (16) respectively

$$\frac{\partial \phi}{\partial y} = 0$$

and

$$\frac{\partial \phi}{\partial x} = 200 \ .$$

Along the boundary C-D, from equation (9)

$$\sigma_r = \frac{10}{a} \sigma_n$$

Then from equation (17),

$$\frac{\partial \phi}{\partial x} = (\frac{10}{a})(20)x + c_5$$

Therefore,

$$c_5 = \frac{\partial \phi}{\partial x} - \frac{200}{a} x \ .$$

At point B:

$$\frac{\partial \phi}{\partial x} = 200, \text{ and}$$

$$x = 10.$$

Therefore,

$$c_5 = 200 - \frac{200}{a}(10)$$

$$= 200\left(1 - \frac{10}{a}\right)$$

From equation (18), the stress function value along the boundary C-D becomes

$$\phi = \left(\frac{10}{a}\right)(\sigma_n)\left(\frac{x^2}{2}\right) + 200\left(1 - \frac{10}{a}\right) x + c_6$$

But at point x = 10,

$$\phi = 1000.$$

Then,

$$1000 = \frac{10}{a}(20)\left(\frac{100}{2}\right) + 200\left(1 - \frac{10}{a}\right)10 + c_6$$

Therefore,

$$c_6 = 1000\left(\frac{10}{a} - 1\right).$$

Substituting the values of c_5, c_6, and σ_n in equation (17) and (18) respectively, then

$$\frac{\partial \phi}{\partial x} = \frac{200}{a} x + 200\left(1 - \frac{10}{a}\right)$$

and

$$\phi = \frac{100}{a} x^2 + 200\left(1 - \frac{10}{a}\right) x - 1000\left(1 - \frac{10}{a}\right).$$

Upon substitution of values of x in this equation, the stress function values at nodal points along the x-axis for the boundary C-D are obtained. These values are tabulated in Table 3.

TABLE 3. Stress Function Values Along Boundary C-D

x	point	ϕ
11	c'	$1200 + \frac{100}{a}$
10	c	$1000 + 0$
9		$800 + \frac{100}{a}$
8		$600 + \frac{400}{a}$
7		$400 + \frac{900}{a}$
6		$200 + \frac{1600}{a}$
5		$0 + \frac{2500}{a}$
4		$-200 + \frac{3600}{a}$
3		$-400 + \frac{4900}{a}$
2		$-600 + \frac{6400}{a}$
1		$-800 + \frac{8100}{a}$
0		$-1000 + \frac{10,000}{a}$

Along the side D-E, the stress function value is constant.

$$x = 10 - a$$

Therefore,

$$\phi = \frac{100}{a}(10 - a)^2 - \frac{200}{a}(10 - a)^2 + \frac{1000}{a}(10 - a)$$

or

$$\phi = \left(\frac{10 - a}{a}\right)\left[1000(10-a) - 200(10 - a) + 1000\right].$$

Along the side E-L

$$\frac{\partial \phi}{\partial x} = c_8$$

At point D, along the side C-D,

$$\frac{\partial \phi}{\partial x} = \sigma_r x + c_5$$

and

$$x = 10 - a.$$

Therefore, equation (17) becomes

$$c_8 = \sigma_r x + c_5.$$

Substituting for the values of σ_r, x and c_5,

$$c_8 = \left(\frac{200}{a}\right)(10 - a) - \frac{200(10 - a)}{a},$$

or,

$$c_8 = 0 \cdot$$

Therefore, along the side E-L, from equation (21),

$$\phi = c_8 x + c_9 ,$$

which can be reduced to

$$\phi = c_9 ,$$

= constant, same value at point E from the side D-E.

Therefore, for the side E-L,

$$\phi = \left(\frac{10 - a}{a}\right) \left[100(10 - a) - 200(10 - a) + 1000\right]$$

The above completes the boundary stress function calculations for the quarter plate.

With reference to Figure 3 the stress function values immediately beyond the boundaries A-B, D-E, and E-F are determined by extrapolation. Their values are equal to those on the respective boundaries. The values immediately beyond the boundary B-C are linearly interpolated between the extrapolated values at points B' and C'.

APPENDIX F

CONTOURS FOR ϕ, σ_x, σ_y, τ_{xy} FOR PLATE A

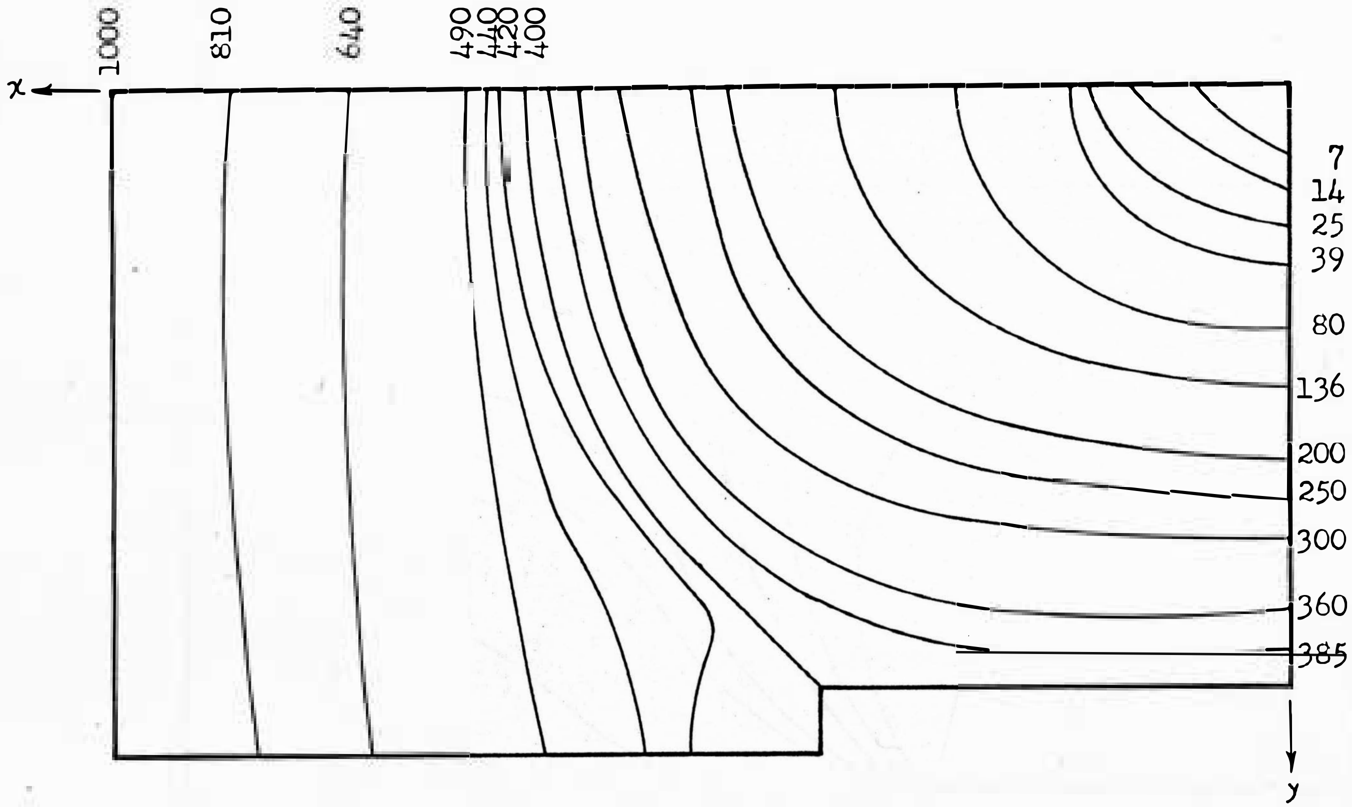

Figure 23. Stress Function Contours for Plate A

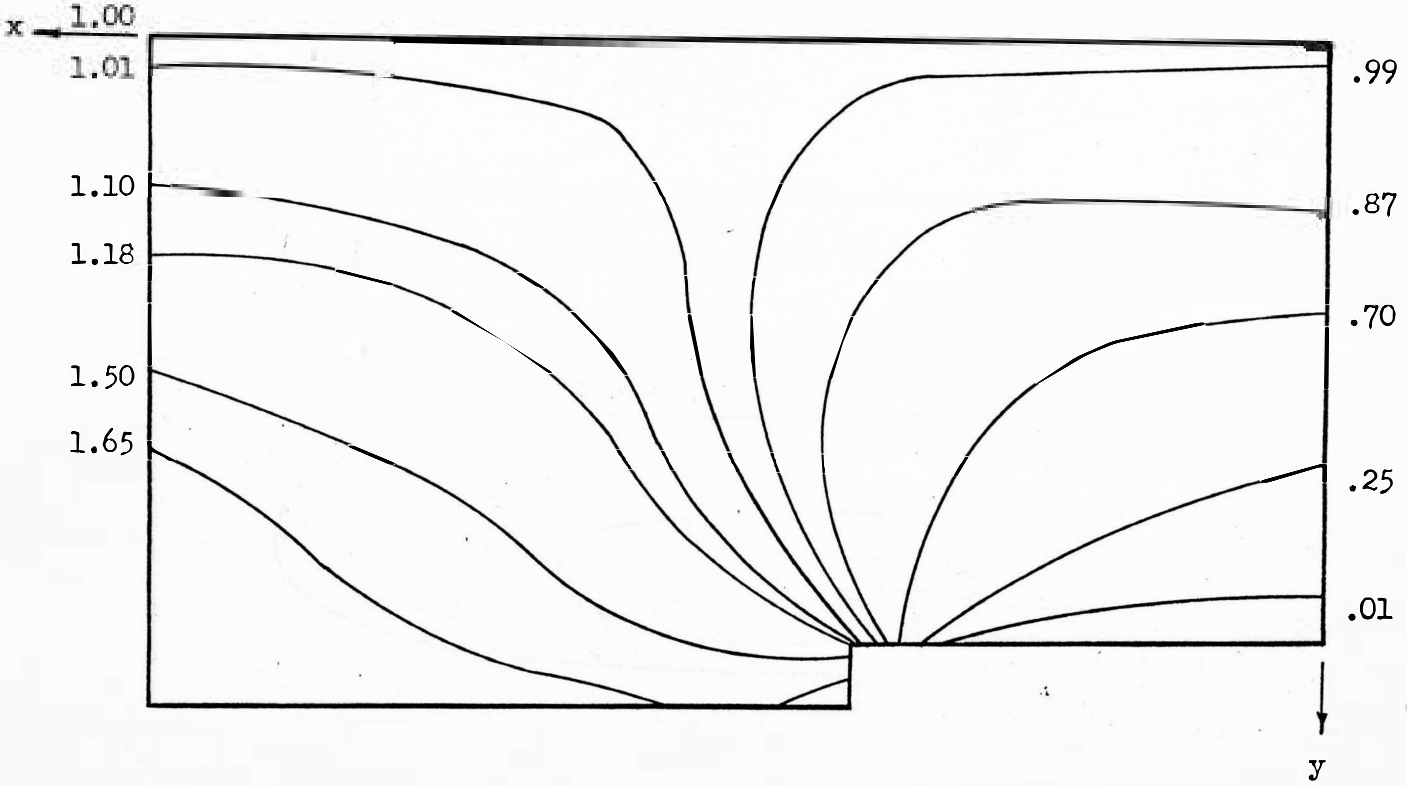

Figure 24. Stress σ_x Contours for Plate A

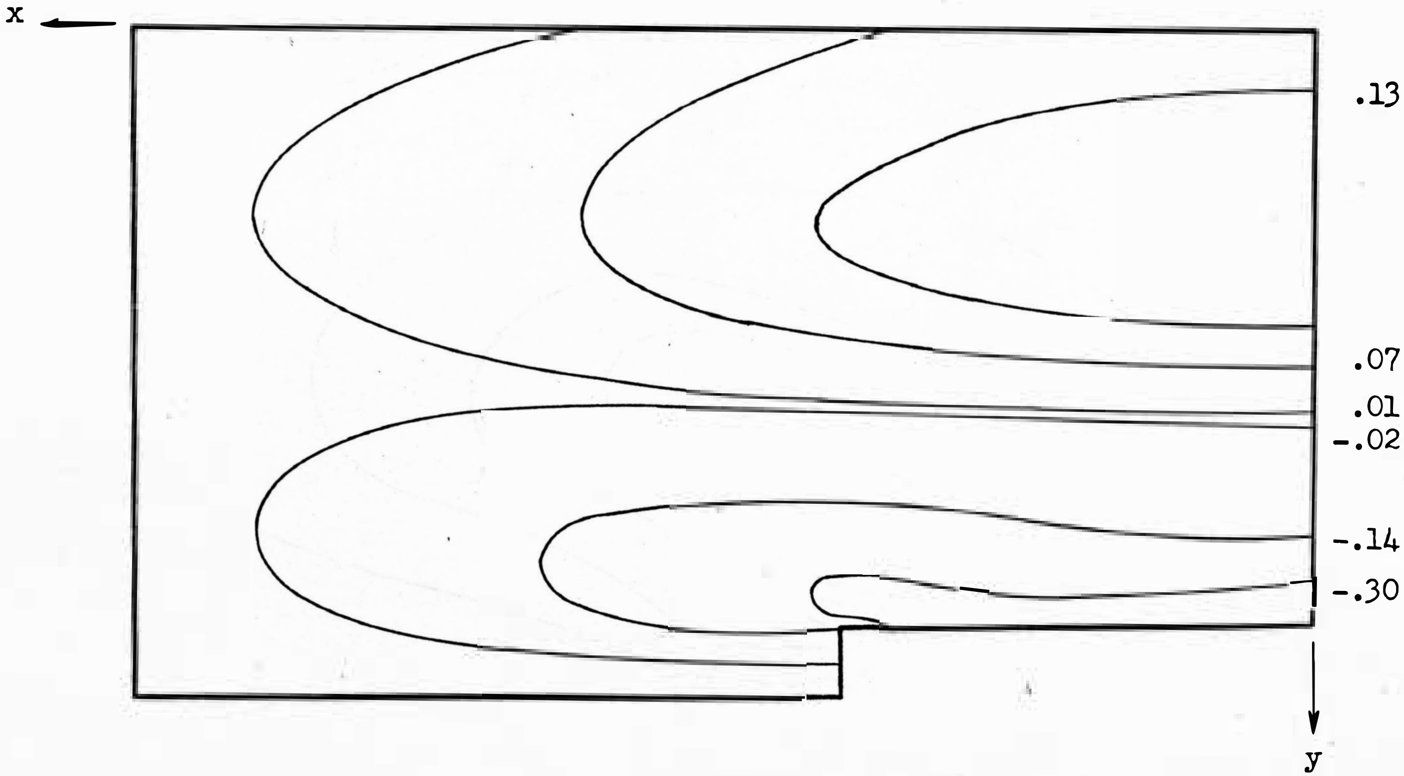

Figure 25. Stress σ_y Contours for Plate A

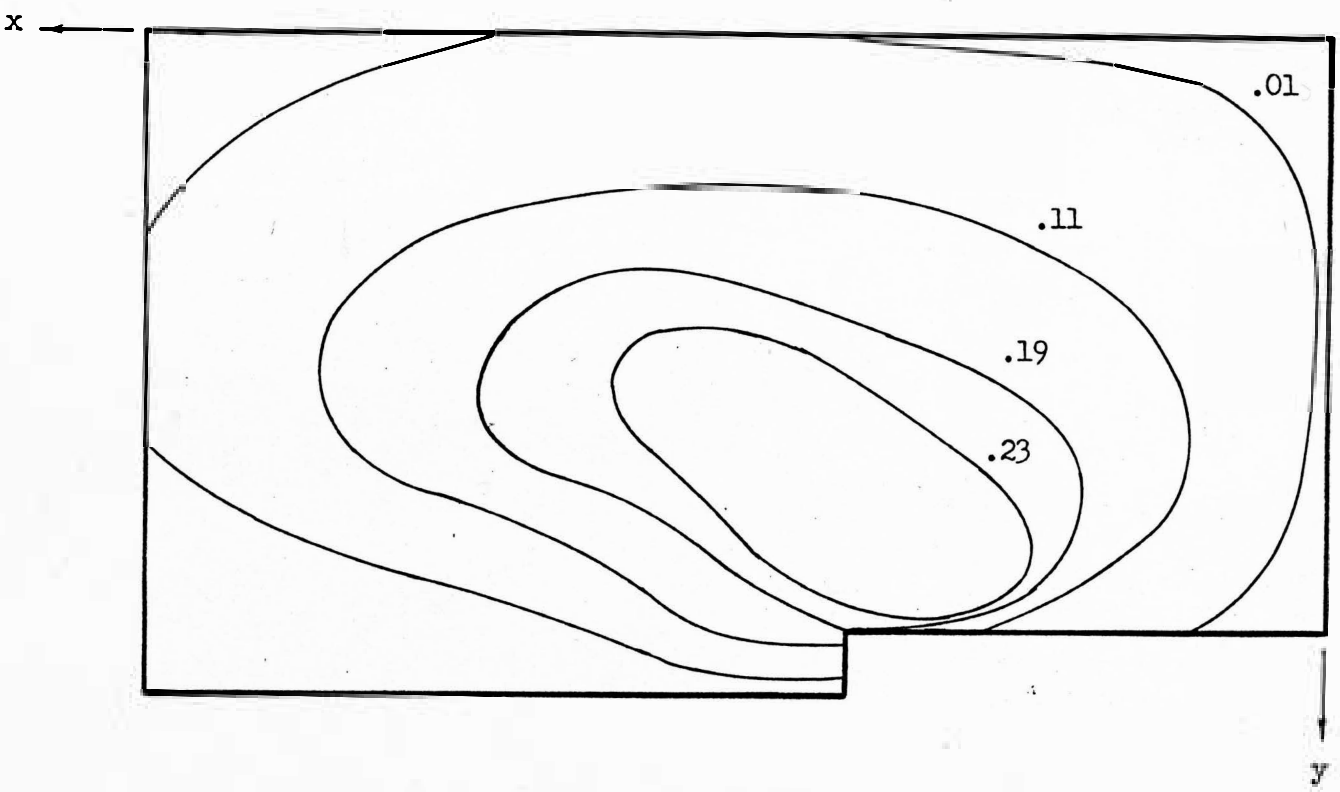

Figure 26. Shearing Stress τ_{xy} Contours for Plate A