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The Upper Bound Theorem in forging processes: Model of Triangular Rigid Zones on parts with horizontal symmetry

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Abstract

The analysis to determine the necessary forces with which to achieve a plastic deformation in metallic materials, in particular, in forging processes and under conditions of plain strain, has been raised over the years through a double approach; on the one hand, by analytical methods that involve a great complexity in their developments but that allow a direct understanding of the parameters that direct these processes. On the other hand, numerical methods, in which, thanks to the enormous development of computer technology, they provide solutions with a high approximation but, in most cases, do not allow to interpret independently the effect of each one of the parameters that come into play. The development of computers relegated analytical methods to the background. An alternative of great interest to apply these methods comes from the study of the Upper Bound Theorem by means of the Triangular Rigid Zones (TRZ) Model. One of the main limitations in the application of this model come from the fact that it is necessary to define a kinematically admissible velocity field and for complex geometric configurations of parts, this field becomes increasingly complicated. A new approach has delimited, from a theoretical perspective, a modular configuration based on a General Module formed by three TRZ that adapts to any geometry of flat surfaces of the part. Another limitation of the Upper Bound Method is the consideration of the plain strain represented by a flat section with double symmetry. Obviously, this imposition only allows to study a limited number of part configurations, which restricts its application in forging processes since the great majority of forged parts do not present geometrically this double symmetry. The present work releases one of these boundary conditions allowing to expand the possibilities of application of this method.

Keywords: Upper Bound; plain strain; forging; Triangular Rigid Zones

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1. Introduction

The high complexity of the theory of plasticity has conditioned its analytical development. One of the major limitations in the deduction of mathematical relationships that allows us to know the initial conditions of the plastic deformation of a material comes from the irreversible and non-linear nature of this type of deformation. With this objective, several families of methods have been developed for the study of the Processes of Conformation by Plastic Deformation (PCPD) in Metal Alloys [1,2] and, in particular, in forging, both of numerical and analytical nature.

Analytical methods increase the ability of the engineer to evaluate and predict the influence of certain variables on important aspects of the process, for example, the energy invested to achieve it. The initial methods among the proposed ones are based on simple theoretical foundations, where only the geometrical aspects of the part are considered, as well as the distribution of the tension of the plastically deformed area. These methods are the Homogeneous Deformation Method and Local Stress Analysis. Both methods have the advantage of a relative simplicity of application compared to other methods used in PCPD analysis. Unlike the two previous methods, the Sliding Lines Field (SLD) presents a methodological alternative whose application complexity is directly related to the degree of accuracy required in the solution to the problem, since it is based on the definition of a field of flow difficult to define. The complexity of the mathematical approach of the equations limits it only to the study of processes in conditions of plain strain, as well as of geometries of reduced complexity. However, the Bound Analysis Methods, and more specifically, the Upper Bound Theorem [3-7], is a solid alternative to the application of the SLD. This method, a particular case of the SLD, is easier to implement and, nevertheless, provides quite acceptable solutions. At present, the high computational capacity of computers has led to a high development of numerical methods, among which is that of the Finite Elements, to the detriment of these analytical methods. However, the conditions of simplification of the bound analysis methods place these methods in a plane of effectiveness similar to the numerical ones already mentioned.

In the present article, the Upper Bound Method is approached through a Triangular Rigid Zones Model. The exact solutions for plastic deformation problems are difficult to obtain. According to the Limit Theorems, an approximation to them is to confine the solution for the necessary energy of deformation between the lower and upper limits. The power must overcome the resistance of the material to the deformation, as well as the resistance to displacement, the latter due to the friction that appears between the material and the tool. The actual load will be delimited between these upper and lower limits, although the first will be more interesting, since it is the one that ensures that the deformation can be carried out by the calculated load. This method has great advantages for the determination of particular solutions.

The criterion of the Upper Bound applies the principle of maximum work but, from the point of view of deformation, that is, an element is deformed in such a way as to offer the minimum resistance. When deducting a stress system from a hypothetical deformation that is in accordance with the kinematically conditions, the load value obtained will be greater or equal to the one that actually operates. When establishing the appropriate deformation, it will be necessary to define a kinematically admissible velocity field, independently of the tensional conditions, which is usually represented thorough its hodograph. For this it is useful to divide the deformed part into several zones, with a rigid behavior and which are called Triangular Rigid Zones (TRZ) [8-14], in each of which the velocity field and its derivatives must be continuous. The application of this method is done by straight lines, considering that only along them there are velocity discontinuities. The rest of the points that make up each block move at the same speed and with the same cutting direction.

In a particularly unique way, and with reduced technological changes, the forging processes are fully adapted to the application of this method. Modifications aimed at achieving a flow behavior of material very similar to that of flat deformation, which is achieved by designing pieces of straight generatrix in the plane perpendicular to the one under study.

These TRZ allow the incorporation of different variables present in the plastic forming of metal alloys. In this way, and given that the forge constitutes a stationary process, it is possible to determine different natures of the friction (Tresca or Coulomb) acting on the different flat surfaces of contact between part and tool, and even, assigning different values to each surface. Other parameters, such as the temperature of the process, also have the possibility of being analyzed. One of the main limitations in the classic application of the Upper Bound Theorem comes from the imposition of the double symmetry that prevents us from considering a large number of the geometric configurations present in the industry. In the present work, one of these symmetries has been eliminated, thus significantly increasing the range of application of the method.

2. Methodology

The geometric model established is based on the elimination of the horizontal symmetry (with respect to the vertical plane) and therefore of the two fourth parts of the piece located to the right and left of the vertical plane, sufficient for the calculation of the necessary pressure to reach the plastic deformation of the part since the vertical symmetry has not been eliminated in the present study. The part will be arranged between matrices formed by parallel or inclined flat plates adapting to the exterior geometrical configuration.

The Basic Module on which the appropriate combination that represents the initial profile of the section of the part will be established is formed by three TRZ and is considered under the Modular approach (Fig.1). The Basic Module will respond in the evolution of the fluence of the material contained in it with the determination of an output velocity of the Module from the input velocity and geometric characteristics. This evolution can be observed in its corresponding hodograph (Fig.2).

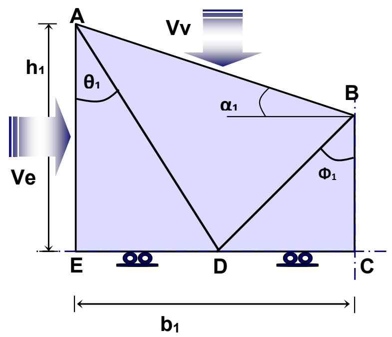

Fig. 1. Configuration of the Basic Module.

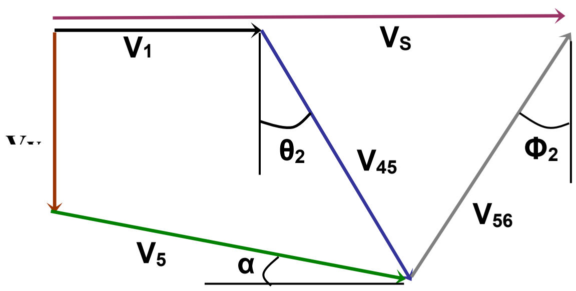

Fig. 2. Basic Module Hodograph.

The output velocity of each Module (Vs) in the hodograph will be (Eq.1):

$$V_s = V_1\left(\frac{tg\theta_1 + tg\phi_1}{1 - tg\alpha\, tg\theta_1}\right) + \frac{V_1 + \dfrac{V_1\left(tg\theta_2 + \dfrac{tg\theta_1 + tg\phi_1}{1 - tg\alpha\, tg\theta_1}\right)\cdot tg\alpha_2}{\cos\alpha_2 - sen\alpha_2\, tg\theta_2}}{\cos\theta_2}\left(sen\theta_2 + \cos\theta_2\, tg\phi_2\right) \quad (1)$$

Where $V_1 = Ve$ is the input velocity to each Module.

For this reason, it will be combinations of this Basic Model that, with the appropriate dimensions, form the section to be studied. Initially the profiles of each of the two quarters will be considered considering double symmetry and obtaining for the two pieces that respond to this configuration the values obtained for the dimensionless relation *p/2k* that shows the increase (or decrease) on the shear stress of the material of the piece. Once these two solutions are presented, the position of the horizontal center of mass will be analyzed and it will be this position that delimits a vertical plane through which the configurations of the part will be studied to the right and left. From this vertical plane and in opposite directions the fluence of the material in the plastic state will occur. The new geometries will be recalculated, which in our case becomes two configurations of three and four Modules respectively.

The Modular approach has been implemented with the possibility of incorporating friction both by adhesion or Tresca (m) (Eq.2) and sliding or Coulomb (μ) (Eq.3) [15], responding in each case to a different expression.

$$\frac{P}{2k}=\frac{1}{2b_2}\cdot\left[\left(\frac{x_2^2+h_2^2}{h_2}+\frac{(b_2-x_2)^2+h_3^2}{h_3}\right)\left(1+\frac{\left(tg\theta_2+\left(\frac{tg\theta_1+tg\phi_1}{1-tg\alpha tg\theta_1}\right)\right)}{\cos\alpha_2-sen\alpha_2\, tg\theta_2}sen\alpha_2\right)+\frac{m\cdot b_2}{\cos\alpha_2}\cdot\frac{\left(tg\theta_2+\left(\frac{tg\theta_1+tg\phi_1}{1-tg\alpha tg\theta_1}\right)\right)}{\cos\alpha_2-sen\alpha_2\, tg\theta_2}\right] \quad (2)$$

$$\frac{P}{2k}=\frac{\left(\frac{x_2^2+h_2^2}{h_2}+\frac{(b_2-x_2)^2+h_3^2}{h_3}\right)\cdot\left(1+\frac{\left(tg\theta_2+\frac{tg\theta_1+tg\phi_1}{1-tg\alpha\, tg\theta_1}\right)\cdot sen\alpha_2}{\cos\alpha_2-sen\alpha_2\, tg\theta_2}\right)}{2b_2\cdot\left(1-\frac{\mu}{\cos\alpha_2}\,\frac{\left(tg\theta_2+\frac{tg\theta_1+tg\phi_1}{1-tg\alpha\, tg\theta_1}\right)}{\cos\alpha_2-sen\alpha_2\, tg\theta_2}\right)} \quad (3)$$

3. Results

We have initially considered a configuration *A* of three Basic Modules with widths of 4, 3 and 5 value respectively computed from left to right and initial height of value 6 (Fig. 3) (the units are not determinant since the calculation is of a dimensionless relation).

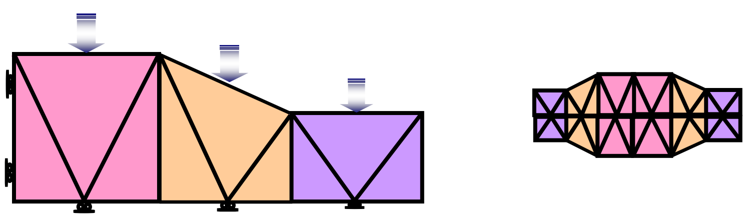

Fig. 3. Configuration *A*.

The values of the ratio *p/2k* for Configuration A are shown in Figure 5 a) and Table 1 a).

Configuration *B* is formed by three Basic Modules with widths of 4, 4 and 3 value respectively computed from left to right and initial height of value 6 (Fig. 4 and 5 b)) (Table 1 b)).

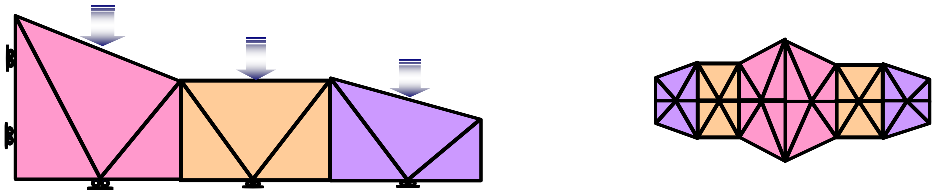

Fig. 4. Configuration *B*.

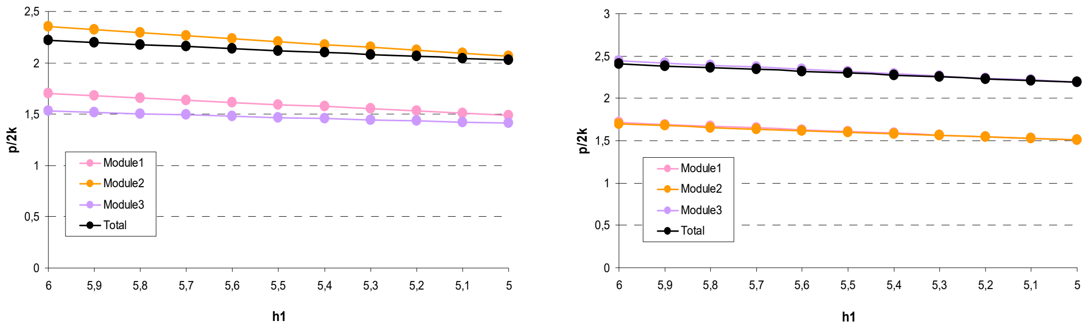

Fig. 5. a) *p/2k* evolution Configuration *A* b) *p/2k* evolution Configuration *B*.

Table 1. a) *p/2k* Modules and Total Values Configuration *A* b) *p/2k* Modules and Total Values Configuration *B*.

h_1	Module 1	Module 2	Module 3	Total
6	1.700	2.349	1.531	2.217
5.9	1.678	2.320	1.518	2.197
5.8	1.657	2.291	1.505	2.177
5.7	1.635	2.262	1.492	2.158
5.6	1.614	2.234	1.480	2.138
5.5	1.593	2.205	1.467	2.119
5.4	1.572	2.177	1.456	2.100
5.3	1.551	2.149	1.444	2.081
.2	1.531	2.121	1.433	2.062
5.1	1.510	2.094	1.422	2.043
5	1.490	2.066	1.411	2.025

h_1	Module 1	Module 2	Module 3	Total
6	1.711	1.696	2.446	2.407
5.9	1.690	1.676	2.420	2.385
5.8	1.669	1.656	2.393	2.362
5.7	1.648	1.636	2.367	2.341
5.6	1.628	1.617	2.341	2.318
5.5	1.607	1.598	2.315	2.296
5.4	1.586	1.579	2.290	2.275
5.3	1.566	1.560	2.265	2.253
5.2	1.546	1.542	2.240	2.232
5.1	1.526	1.524	2.216	2.211
5	1.507	1.507	2.192	2.190

Next we proceed to establish the horizontal value of the center of mass that will mark the vertical plane where the real configuration (*A*+*B*) must be split (Fig. 7) and recalculate the ratio *p/2k* from this plane (Fig. 8, 9) and (Table 2, 3).

Fig. 7. Split Configuration *A* + *B*.

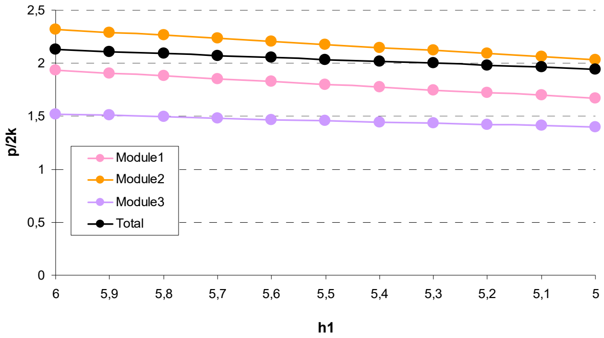

Fig. 8. *p/2k* evolution new split Configuration *A*.

Table 2. *p/2k* Modules and Total Values new split Configuration *A*.

h_1	Module 1	Module 2	Module 3	Total
6	1.930	2.321	1.521	2.130
5.9	1.903	2.292	1.507	2.111
5.8	1.877	2.262	1.494	2.092
5.7	1.851	2.233	1.481	2.073
5.6	1.825	2.204	1.469	2.054
5.5	1.799	2.176	1.456	2.035
5.4	1.773	2.147	1.444	2.017
5.3	1.747	2.119	1.432	1.998
5.2	1.722	2.091	1.421	1.980
5.1	1.696	2.063	1.410	1.962
5	1.671	2.035	1.399	1.944

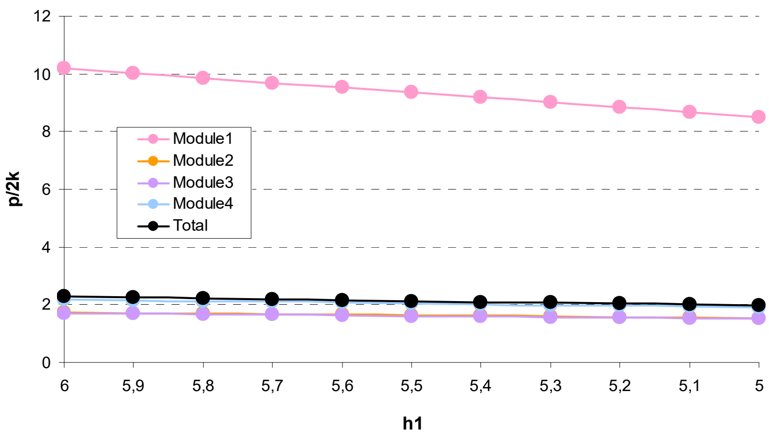

Fig. 9. *p/2k* evolution new split Configuration *B*.

Table 3. *p/2k* Modules and Total Values new split Configuration *B*.

h_1	Module 1	Module 2	Module 3	Module 4	Total
6	10.1991	1.736	1.706	2.184	2.273
5.9	10.030	1.715	1.686	2.156	2.243
5.8	9.861	1.694	1.667	2.129	2.213
5.7	9.692	1.673	1.648	2.101	2.183
5.6	9.523	1.653	1.628	2.074	2.154
5.5	9.354	1.633	1.610	2.047	2.125
5.4	9.185	1.613	1.591	2.020	2.096
5.3	9.016	1.593	1.573	1.993	2.067
5.2	8.847	1.573	1.555	1.967	2.039
5.1	8.679	1.553	1.537	1.941	2.011
5	8.510	1.534	1.519	1.916	1.983

Based on the results obtained, a sensitivity study was conducted to establish the influence of the variation of the width of the additional Module added in Configuration *B*. This Module has a very small width, so the Module is distorted and could affect the result of the total *p/2k* ratio, increasing it significantly. However, this influence is slightly compensated by the reduced value of the area of the aforementioned Module and therefore its influence is reduced (Fig. 10, Table 4).

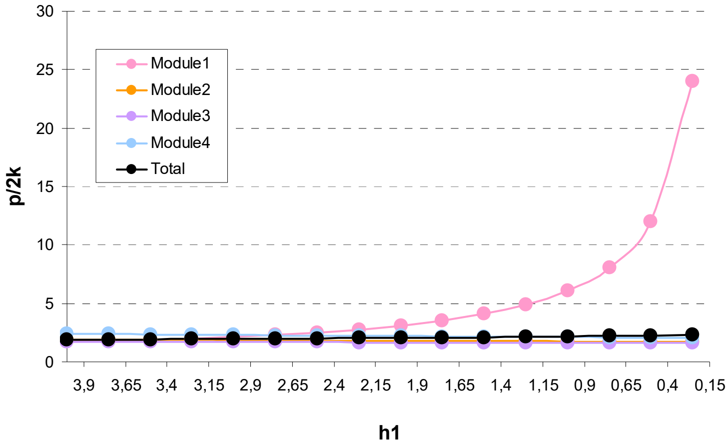

Fig. 10. Sensitivity study of Additional Module.

4. Conclusions

The present work has extended the possibilities of application of the Upper Bound Theorem under the TRZ Method increasing the potentiality of the Modular approach. This potential is reflected in the fact that it can be adapted to practically any geometrical configuration, and with the options of presence of different types of friction, incorporation of the effect of temperature and hardening of the material by means of a low computational cost application, of practically response immediate and, therefore, competitive against numerical methods.

Table 4. Sensitivity studio values of Additional Module.

b_1	Module 1	Module 2	Module 3	Module 4	Total
4.00	1.700	1.879	1.719	2.424	1.898
3.75	1.787	1.868	1.711	2.390	1.911
3.50	1.889	1.858	1.702	2.357	1.926
3.25	2.009	1.847	1.694	2.326	1.942
3.00	2.150	1.837	1.687	2.296	1.959
2.75	2.319	1.826	1.680	2.267	1.978
2.50	2.525	1.816	1.673	2.240	1.999
2.25	2.779	1.805	1.666	2.214	2.021
2.00	3.100	1.795	1.660	2.190	2.045
1.75	3.516	1.784	1.655	2.168	2.072
1.50	4.075	1.774	1.649	2.146	2.100
1.25	4.862	1.764	1.644	2.127	2.130
1.00	6.050	1.753	1.640	2.108	2.162
0.75	8.037	1.743	1.635	2.092	2.197
0.50	12.025	1.732	1.631	2.076	2.234
0.25	24.0125	1.722	1.628	2.062	2.275

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