Slacktivism

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Abstract

Many countries have introduced e-government petitioning systems, in which a petition that gathers enough signatures triggers some political outcome. This paper models citizens who choose whether to sign a petition. Citizens are imperfectly informed about the petition's chance of bringing change. The number of citizens approaches infinity, while the cost of signing is positive but low, falling within certain bounds. In the limit, participation is increasing in the required quota of signatures. Social welfare is decreasing in the quota. Information aggregation may fail if individual signals are sufficiently uninformative.

Keywords: online petitions, collective action, voting, political participation

JEL codes: D72, H41

1 Introduction

Online petitions have become a feature of the political process in many democracies. While some petitions serve simply as a means of signalling opinion, in a number of countries governing institutions have committed to act on a petition if it attracts sufficiently many signatures. In these e-government petitioning systems, a petition signed by a certain number of citizens will be

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debated in the legislature or will trigger some other official response[1]. Although the legislature can still vote to reject the petition, a citizen who signs it is not merely sending a signal, but is directly affecting political outcomes, in a manner similar to voting in a referendum.

Two features of this form of political participation stand out. First, the number of potential participants is very large, essentially including the entire electorate. Second, while there is a cost of participation – the time and effort required to sign an online petition – that cost is usually negligible. Because of the very low cost, political action through online petitions is sometimes referred to, derisively, as slacktivism.

This paper develops a model of participation in such low-cost political action. In the model, there is a large number of citizens, who choose whether to sign a petition, at some cost. If the petition collects a certain quota of signatures, it will be considered by the legislature. The decision of the legislature is represented by a state of the world: in state 0 it will reject the petition, and in the complementary state 1 it will approve it. All citizens receive an imperfect continuous signal about the state. A rejected petition does not change the citizens' payoffs relative to the status quo, while a petition that is approved gives each citizen a positive payoff. Thus, citizens have imperfect information about the petition's chance of success, but are fully informed about its value if it succeeds. Because of this, swing voter's curse does not emerge in the model, and citizens' choice of whether to sign it is driven by the tradeoff between cost of signing the petition and the benefit of increasing its chance of success.

If signing the petition was costless, all citizens would weakly prefer to sign it. On the other hand, if the cost was a positive constant, then a well-known paradox of not voting would emerge: in a sufficiently large electorate, the probability that an individual citizen is pivotal would be low enough that no

[1]For instance, a petition on the UK parliament website signed by at least 100 thousand citizens will be considered for debate in the parliament (UK Parliament, 2018). A similar system used in Germany requires a petition to collect 50 thousand signatures within four weeks to receive a public hearing in the national parliament (Deutsche Welle, 2017). Similar systems are used by the White House in the US; by parliaments in Canada and Latvia; by regional parliaments in Scotland and Queensland, Australia; as well as by a number of municipalities (see Lindner and Riehm, 2009; Mickoleit, 2014; Grover, 2016). These systems are fairly widely used: for example, there are over 12 thousand active petitions on the UK parliament website, several hundred past petitions have received a response from the government, and several dozen have been debated in the parliament.

citizens would sign the petition[2]. Neither outcome seems realistic. The key feature of this model is that the cost is positive but low. Specifically, the paper analyses sequences of equilibria when the number of citizens goes to infinity, while the cost of signing is within certain bounds that are related to the size of the electorate. In the limit, both the upper and the lower bound converge to zero, but at every point along the sequence there is an interval of small but positive costs that satisfy the bounds.

As the size of the electorate approaches infinity, in the limit a randomly selected citizen signs the petition with a probability that is distinct from zero and from one. Hence, positive (although not universal) participation occurs in a large electorate. In the limit, a citizen who signs the petition is almost surely not pivotal – along the sequence, however, the probability that a citizen is pivotal remains positive.

The paper then analyses the equilibria that emerge in large electorates. First, it examines the effect of the required quota of signatures on participation. If the cost was zero, or if it was substantially larger than the upper bound, then participation would not depend on the quota – it would, respectively, be universal, or converge to zero in a large electorate. However, when the cost stays within the bounds of the model, increasing the quota increases the fraction of citizens that sign the petition. Hence, a more conservative petitioning procedure induces greater participation.

The paper also discusses the effect of the minimum quota of signatures on social welfare. In the limit, equilibrium participation depends on the quota in such a way that all quotas produce the same political outcomes. A lower quota, however, reduces the total cost incurred by citizens who sign the petition. For this reason, a lower quota is socially preferable.

Finally, I analyse informational efficiency of the petitioning procedure. A social planner may prefer the petition to be forwarded to the legislature in state 1 but not in state 0. I show that in a large electorate, the petitioning procedure replicates these outcomes (and hence achieves informational efficiency) if and only if individual signals are sufficiently precise.

[2]A similar result would emerge if the cost followed a continuous distribution across voters, unless one assumes that sufficiently many citizens have a negative cost of signing (i.e. that they receive a payoff from the act of signing). Without uncertainty, one could construct an asymmetric equilibrium in which the number of citizens that sign a petition exactly equals the threshold. However, such an equilibrium would be eliminated if uncertainty is introduced (see Palfrey and Rosenthal, 1985, for a similar result in a two-candidate voting setup).

The rest of this section discusses the related literature. Section 2 presents the model. Section 3 derives asymptotic equilibria. Using this equilibrium characterisation, Section 4 analyses the effect of the quota of signatures on participation and social welfare, and studies informational efficiency. It also discusses equilibrium stability, pivotality, and the role of the assumption on the cost of signing the petition. Section 5 concludes. All proofs are in the Appendix.

Related literature. This paper contributes to the literature on petitions and other forms of collective political action. Lohmann (1993), Banerjee and Somanathan (2001), Battaglini and Benabou (2003), and Battaglini (2017) look at citizens who are imperfectly informed about a state of the world. Citizens can engage in political action to signal their information to the decision-maker[3] and induce her to select a policy that is appropriate for the state. This paper differs from that literature in two ways. First, it analyses a limit case in which the number of voters is infinite and the cost of political action is positive but low. In this setting, the paper generates equilibria with positive participation[4]. Second, this model abstracts from the signalling role of petitions. Instead, it looks at e-government petitions, which directly affect political outcomes in a similar way to referenda: political change can happen if the number of citizens who sign the petition exceeds an exogenous threshold[5]. In particular, the fact that the threshold is exogenous makes it possible to analyse the effect of its size on participation and welfare.

The latter aspect links the paper to the literature on discrete public goods – that is, public goods that are provided if and only if the total amount of contributions exceeds a certain threshold (see Palfrey and Rosenthal, 1984, for a classic reference). My model is similar to a common-value discrete public good game in which the set of possible contributions is binary (signing and not signing the petition). Two features, however, distinguish this model from

[3] Or to other voters, as in Lohmann (1994).

[4] In Lohmann (1994), Banerjee and Somanathan (2001), and Battaglini and Benabou (2003), political action is costly, but agents engage in it because their number is finite. If the group becomes very large, participation shrinks to zero. In Battaglini (2017), participation is generally costless; in the variant of the model in which it is costly, participation occurs because a subset of citizens have negative costs of participation.

[5] In contrast, in Lohmann (1993) and Battaglini (2017) the policy-maker changes his decision if the number of voters who engage in collective action exceeds a cutoff that is endogenously determined at the equilibrium.

the rest of the literature. First, rather than looking at public good provision in a group of fixed size, my paper analyses a sequence of discrete public good games in which the group size approaches infinity, and the cost of contributing stays within certain bounds[6]. Second, in my paper there is uncertainty about the threshold amount of contributions required for the common-value public good to be provided (the threshold equals some fraction of the group size in state 1, and infinity in state 0), and individuals receive imperfect private signals about the threshold[7]. This, in particular, underlies the analysis of information aggregation properties of the petition.

The fact that signing an e-government petition is similar to casting a vote in a referendum connects the paper to the extensive literature on voting by imperfectly informed agents. In particular, Battaglini et al. (2007), Krishna and Morgan (2012) and Ambrus et al. (2017) have looked at finite groups of voters who receive private signals about a payoff-relevant state of the world, and face a cost of voting[8]. My paper differs from that literature in two ways. First, I focus on a limit case in which the electorate is large, while the cost belongs to a specific range, which ensures that participation remains positive (but not universal) for any size of the electorate. This allows me to study information aggregation and the effect of quota on participation in a large election with costly voting[9]. Second, in this paper the cost of voting has a very particular form: while signing the petition is costly, voting for the opposite alternative – that is, not signing the petition – is costless. Hence, only voters who prefer to sign the petition face a participation dilemma. This underlies the monotone positive effect of the voting rule on participation.

Finally, the paper is also related to the literature on referenda with approval quorums, in which a proposal is adopted only if a certain number

[6]The role of these bounds is discussed at the end of Section 4.

[7]Nitzan and Romano (1990), McBride (2006), Barbieri and Malueg (2010), and Krasteva and Yildirim (2013) examine discrete public good games in which there is uncertainty about the threshold, but players do not receive private signals about it. More broadly, a number of papers (e.g. Menezes et al., 2001; Laussel and Palfrey, 2003; Martimort and Moreira, 2010) model private-value discrete public good games in which individuals have perfect private information about their valuations of the public good (but not about the technology of producing it).

[8]Costly voting has also been studied in settings that do not involve information aggregation (see e.g. Borgers, 2004; Levine and Palfrey, 2007; Myatt, 2015).

[9]Typically, because of the well-known "paradox of not voting" that emerges when the electorate is large, information aggregation in large elections has been studied under an assumption that voting is cotless (see e.g. Feddersen and Pesendorfer, 1997).

of voters support it. Aguiar-Conraria and Magalhães (2010) analyse such referenda in a model without voting costs. Maniquet and Morelli (2015) examine approval quora in an electorate of finite size. In Herrera and Mattozzi (2010), voting is costly, but voters are not concerned with being pivotal – instead, each voter receives a payoff from voting for her preferred alternative that is depends on the parties' mobilisation efforts. This paper, in contrast, develops a pivotal-voter model with a large electorate and positive, though low, cost.

2 Model

There are $n_r + 1$ citizens, who choose simultaneously whether to sign a petition. Signing the petition carries a cost c_r. The pair (c_r, n_r), will be referred to as the *voting environment* indexed by r. I will focus on equilibrium behaviour when the number of citizens is large. Formally, the paper will focus on sequences of voting environments such that $\lim_{r\to\infty} n_r = \infty$.

The petition will be considered by the legislature if it gathers at least qn_r signatures, where $q \in (0, 1)$.[10] To simplify notation, throughout the paper I will assume that qn_r is an integer for all r. Citizens are uncertain about the legislature's eventual decision[11]. Formally, there is a state of the world $\theta \in \{0, 1\}$. In state 0 the legislature rejects the petition, while in state 1 it approves the petition. If the petition is approved by the legislature, all citizens (both those who signed the petition, and those who did not) receive a payoff of 1, while a petition that is rejected gives a payoff of 0 to all citizens[12]. Ex ante, the probability that the state equals 1 is π.

[10] One could also specify the model by saying that the petition will be considered by the legislature if it gathers at least $q(n_r + 1)$ signatures. Both specifications produce the same results because as $r \to \infty$, qn_r and $q(n_r + 1)$ converge to each other.

[11] For example, citizens might not know which members of the legislature will be present at the session in which the petition will be considered, or what their positions are on the proposal contained in the petition.

[12] In reality, of course, some citizens may oppose the petition. One could easily extend the model to consider such cases. Suppose that a fraction $\alpha \in (0, 1)$ of citizens receive a positive payoff from the petition if it succeeds, while the rest receive a negative payoff. The latter citizens will never sign the petition and can be omitted from the analysis. Then all subsequent results can be derived by replacing n_r with $\tilde{n}_r \equiv \alpha n_r$ and labelling by q the number such that the petition will be considered by the legislature if it gathers at least $q\tilde{n}_r$ signatures.

At the beginning of the game, each citizen i receives a private signal $s_i \in [\underline{s}, \overline{s}]$. In each state θ, signals of all citizens are drawn independently from cdf F_θ with density f_θ. Let s denote a generic realisation of the signal. Given a signal s, let $h(s) = \frac{\pi f_1(s)}{\pi f_1(s) + (1-\pi) f_0(s)}$ be the posterior probability that the true state is 1. I will make the following assumptions about signal distributions:

Assumption 1. $\frac{f_0(s)}{f_1(s)}$ *is strictly increasing in* s.

Assumption 2. $f_1(s)$ *is strictly positive for all* $s \in [\underline{s}, \overline{s}]$.

Assumption 1 imposes a standard monotone likelihood property on signal distributions. In particular, it implies that $h(s)$ is decreasing in s. Assumption 2 ensures that $h(s)$ is positive for any signal realisation.

After citizens observe their signals, they simultaneously choose whether to sign the petition at a cost c_r. In any voting environment (that is, for any size of the electorate) I will assume that the cost is strictly positive, but not too large. Specifically, I will assume the following about the cost:

Assumption 3. *There exists some* $\lambda \in (0,1)$ *and some* $\overline{r}$ *such that* $\lambda^{n_r} \leq c_r \leq \frac{e}{2\pi} \frac{h(\overline{s})}{\sqrt{q(1-q)n_r}}$ *for all* $r \geq \overline{r}$

For any voting environment r, Assumption 3 defines an upper and a lower bound on the cost of signing the petition. The significance of these bounds is discussed in Section 4.

Let $\sigma_i(s)$ be citizen i's strategy, that is, her probability of signing the petition after receiving signal s. The paper will focus on symmetric equilibria, in which $\sigma_i(s) = \sigma(s)$, $\forall i$.

3 Equilibria

First, note that a strategy profile in which every citizen signs the petition irrespective of her signal cannot be an equilibrium, because in this case the number of signatures will be at least qn_r with probability 1, and hence every citizen would gain by deviating. Consider instead a strategy profile in which no citizen signs the petition, irrespective of the signal. If $qn_r > 1$, no citizen is pivotal under this strategy profile, and hence no citizen gains by deviating. Thus, abstention by all citizens is an equilibrium. For the rest of the analysis, I will focus on equilibria in which each citizen signs the petition with a positive probability. I will refer to these as *positive participation* equilibria.

Consider a voting environment r. Take a citizen i who has received signal s_i. If the petition gathers at least qn_r signatures, citizen i will receive a payoff of 1 if $\theta = 0$, and a payoff of 0 if $\theta = 0$. Let $p_r(k)$ denote the probability that at least k citizens sign the petition conditional on the true state being 1. Then citizen i's expected payoff will equal $h(s_i)\, p_r(qn_r - 1) - c_r$ if she signs the petition, and $h(s_i)\, p_r(qn_r)$ if she does not. She then signs the petition if and only if $h(s_i)\,[p_r(qn_r - 1) - p_r(qn_r)] \geq c_n$, where $[p_r(qn_r - 1) - p_r(qn_r)]$ is the usual probability that citizen i is pivotal.

Let y_r be the equilibrium probability that a randomly selected citizen signs the petition when the true state is 1. Then citizen i will sign the petition if and only if

$$h(s_i)\binom{n_r}{qn_r} y_r^{qn_r}(1 - y_r)^{(1-q)n_r} \geq c_r \tag{1}$$

Assumption 1 implies that $h(s_i)$ is strictly decreasing. Hence, an equilibrium in a given voting environment r is characterised by a cutoff s^r such that a citizen signs a petition if and only if her signal is below s^r. The equilibrium in which no citizen signs the petition is given by $s^r = \underline{s}$. A positive participation equilibrium exists whenever there is some $s^r > \underline{s}$ that solves

$$h(s^r)\binom{n_r}{qn_r} F_1(s^r)^{qn_r}\left[1 - F_1(s^r)\right]^{(1-q)n_r} = c_r \tag{2}$$

We can now simplify $\binom{qn_r}{n_r}$ by applying the bounds on factorial derived in Robbins (1955). This yields the following technical result:

Lemma 1. $\frac{\sqrt{2\pi}}{e^2} \leq \binom{n_r}{qn_r} q^{qn_r}(1-q)^{(1-q)n_r}\sqrt{q(1-q)\,n_r} \leq \frac{e}{2\pi}$ *for all* r.

This is sufficient to show that when r is sufficiently large, a positive participation equilibrium exists:

Lemma 2. *There exists* $\overline{r}$ *such that for all* $r \geq \overline{r}$*, there exists at least one* $s^r > \underline{s}$ *at which (2) holds.*

On its own, Lemma 2 does not mean that participation remains strictly positive as $r \to \infty$ (i.e. as the electorate becomes arbitrarily large), because s^r may be converging to $\underline{s}$. If that is the case, participation would converge to zero, and the standard paradox of not voting would emerge. Subsequent analysis, however, will show that this is not the case.

Note that $s^r = \overline{s}$ cannot be an equilibrium by the reasoning given at the beginning of this section. Thus, $s^r \in (\underline{s}, \overline{s})$ in any positive participation equilibrium. By Assumption 2, F_1 has full support on $[\underline{s}, \overline{s}]$, and hence $0 < F_1(s^r) < 1$. Using this, we can express $\binom{n_r}{qn_r}$ from (2), and substitute it into the double inequality in Lemma 1. This yields the following condition that must hold for any positive participation equilibrium cutoff s^r:

$$\left(\frac{\sqrt{2\pi}}{e^2}\right)^{\frac{1}{n_r}} \leq \left[\frac{c_r}{h(s^r)}\sqrt{q(1-q)\,n_r}\right]^{\frac{1}{n_r}} \left[\frac{q}{F_1(s^r)}\right]^q \left[\frac{1-q}{1-F_1(s^r)}\right]^{1-q} \leq \left(\frac{e}{2\pi}\right)^{\frac{1}{n_r}} \tag{3}$$

We are interested in the asymptotic behaviour of s^r as $r \to \infty$. Formally, consider any convergent sequence of positive participation equilibrium cutoffs $\{s^r\}_{r=0}^{+\infty}$. As $\lim\limits_{r\to\infty}\left(\frac{\sqrt{2\pi}}{e^2}\right)^{\frac{1}{n_r}} = \lim\limits_{r\to\infty}\left(\frac{e}{2\pi}\right)^{\frac{1}{n_r}} = 1$, (3) implies that

$$\lim_{r\to\infty}\left(c_r^{\frac{1}{n_r}}\left[\frac{\sqrt{q(1-q)\,n_r}}{h(s^r)}\right]^{\frac{1}{n_r}} \left[\frac{q}{F_1(s^r)}\right]^q \left[\frac{1-q}{1-F_1(s^r)}\right]^{1-q}\right) = 1 \tag{4}$$

This characterises the limit of any sequence of positive participation equilibria, as long as the limit in (4) exists. We can show its existence by showing that $c_r^{\frac{1}{n_r}}$ converges:

Lemma 3. *There exists a subsequence of voting environments for which* $c_r^{\frac{1}{n_r}}$ *converges to some limit* $L \in [\lambda, 1]$.

For these subsequences, s^r converges. Focusing on these subsequences, and letting $\hat{s} \equiv \lim\limits_{r\to\infty} s^r$, we can simplify (4), as the following result states:

Lemma 4. *The limit* $\hat{s}$ *of any convergent sequence of positive participation equilibrium cutoffs* $\{s^r\}_{r=0}^{+\infty}$ *is given by*

$$\left[\frac{q}{F_1(\hat{s})}\right]^q \left[\frac{1-q}{1-F_1(\hat{s})}\right]^{1-q} = \frac{1}{L} \tag{5}$$

For $s \in (\underline{s}, \overline{s})$, let $A(s) \equiv [F_1(s)]^q [1 - F_1(s)]^{1-q}$. For the subsequent analysis, the following simple result will be useful:

Lemma 5. *$A(s)$ is strictly increasing in s for $F_1(s) < q$, and strictly decreasing in s for $F_1(s) > q$.*

We can now derive the first result of the paper – a characterisation of equilibria in large electorates. These equilibria are represented by the values of $\hat{s}$ at which (5) holds. They can be characterised as follows:

Proposition 1. *Take any sequence $\{s^r\}_{r=0}^{+\infty}$ of positive participation equilibrium cutoffs converging to a limit $\hat{s}$. If $L = 1$, then $F_1(\hat{s}) = q$. If $L < 1$, then $\hat{s}$ can take one of two values $\hat{s}_1, \hat{s}_2$, such that $0 < F_1(\hat{s}_1) < q < F_1(\hat{s}_2) < 1$.*

In words, if the electorate becomes arbitrarily large and the cost stays in the interval described by Assumption 3, in the limit there exist equilibria in which the level of participation is substantial. Specifically, there exists a high participation equilibrium at which the fraction of citizens who sign the petition converges to $F_1(\hat{s}_2) > q$, and a low participation equilibrium at which that fraction converges to $F_1(\hat{s}_1) < q$. In both of these equilibria that fraction is distinct from zero and from one. In the special case when $L = 1$, the two positive participation asymptotic equilibria merge into a single equilibrium, at which the fraction of citizens that sign the petition converges to q. In addition, there also exists a zero participation equilibrium, at which no citizen signs the petition (i.e. $\hat{s} = \underline{s}$).

This equilibrium characterisation undelies the results discussed in the next section.

4 Comparative Statics and Discussion

Effect of quota on participation. We can show that all positive participation equilibria are monotone in the quota of signatures:

Proposition 2. *The limit $\hat{s}$ of any convergent sequence $\{s^r\}_{r=0}^{+\infty}$ of positive participation equilibrium cutoffs is strictly increasing with q.*

Hence, in a large electorate, the share of citizens who sign the petition at a positive participation equilibrium increases when the petitioning procedure becomes more conservative.

Equilibrium stability. Consider the general case when $L < 1$. How do the two positive participation equilibrium cutoffs respond to a small perturbation in strategies? Take a sequence of equilibrium cutoffs $\{s^r\}_{r=0}^{+\infty}$ converging to the high participation asymptotic cutoff $\hat{s}_2$. Now suppose citizens deviate to a sequence $\{\widetilde{s}^r\}_{r=0}^{+\infty}$ that converges to an asymptotic cutoff $\hat{s}_2 + \varepsilon$, with $\varepsilon > 0$ (that is, they sign the petition with a higher ex ante probability). When r is sufficiently large (so that $\widetilde{s}^r$ is sufficiently close to $\hat{s}_2 + \varepsilon$), we have $F_1(\widetilde{s}^r) > F_1(s^r) > q$, and so by Lemma 5, $A(\widetilde{s}^r) < A(s^r)$. Thus, as a result of the deviation, $y_r^{qn_r}(1 - y_r)^{(1-q)n_r} = A(\cdot)^{n_r}$ decreases, and so each citizen's best response, given by (1), is to sign the petition at lower values at s_i. Thus, the equilibrium cutoff falls, counteracting the initial deviation. Similarly, if citizens deviate to a sequence converging to a cutoff $\hat{s}_2 - \varepsilon$ (where ε is small enough that $F_1(\hat{s}_2 - \varepsilon) > q$), then $A(\cdot)^{n_r}$ increases, so the best response is to sign the petition at higher values of s_i, again counteracting the deviation. Hence, the high participation equilibrium is stable.

On the other hand, consider a sequence of equilibrium cutoffs $\{s^r\}_{r=0}^{+\infty}$ converging to the low participation asymptotic cutoff $\hat{s}_1$. Suppose citizens deviate to a sequence $\left\{\underset{\sim}{s}^r\right\}_{r=0}^{+\infty}$ that converges to an asymptotic cutoff $\hat{s}_1+\varepsilon$ for some $\varepsilon > 0$ that is small enough that $F_1(\hat{s}_1 + \varepsilon) < q$. When r is sufficiently large, we have $F_1(s^r) < F_1\left(\underset{\sim}{s}^r\right) < q$, and by Lemma 5, $A\left(\underset{\sim}{s}^r\right) > A(s^r)$. Then $y_r^{qn_r}(1 - y_r)^{(1-q)n_r} = A(\cdot)^{n_r}$ increases, and so each citizen's best response is to sign the petition at higher values at s_i, reinforcing the initial perturbation of s^r. Similarly, if citizens deviate to a sequence converging to a cutoff $\hat{s}_1 - \varepsilon$, then $A(\cdot)^{n_r}$ decreases, and the best response is to sign the petition at lower values of s_i, reinforcing the deviation. Thus, the low participation equilibrium is unstable.

To summarise, for $L < 1$, there are two stable asymptotic equilibria (the zero participation equilibrium and the high participation equilibrium), as well as one unstable intermediate equilibrium.

Pivotality. Suppose $L = 1$. When $\theta = 1$, the probability of a given citizen signing the petition approaches q as $r \to \infty$. Hence, the outcome in state 1 is almost surely very close when the electorate is large. This is similar to Theorem 2 in Feddersen and Pesendorfer (1997), which shows a similar result in a large election with costless participation but with uncertainty about the correct alternative.

However, in the more general case when $L < 1$, the probability of a given citizen signing the petition in state 1 converges either to $F_1(\hat{s}_1) < q$, or (at a stable asymptotic equilibrium) to $F_1(\hat{s}_2) > q$. Thus, at a stable asymptotic equilibrium, when $\theta = 1$, the petition almost surely succeeds with a strictly positive margin. In other words, as $r \to \infty$, in the limit citizens who sign the petition know that they are almost surely not pivotal. Along the sequence, however, at every r the probability of being pivotal is sufficiently large that enough citizens strictly prefer to sign the petition.

Social welfare. Which value of q is socially optimal? Proposition 1 implies that in state 1, as $r \to \infty$, the petition almost surely succeeds at the stable high participation equilibrium, and almost surely fails at the other two equilibria. In state 0, of course, the petition will always be rejected. These outcomes do not depend on the value of q. Hence, out of two values of q, we would prefer the one that achieves the outcome at a lower cost to citizens.

At each state $\theta \in \{0, 1\}$, in a given voting environment r, the total cost equals $c_r n_r F_\theta(s^r)$. In the limit, as $r \to \infty$, this converges to $F_\theta(\hat{s}) \lim_{r\to\infty} c_r n_r$, as long as $\lim_{r\to\infty} c_r n_r$ exists. Assumption 3 implies that for all $r \geq \overline{r}$ we have

$$n_r \lambda^{n_r} \leq c_r n_r \leq \frac{e}{2\pi}\sqrt{\frac{n_r}{q(1-q)}}\, h(\overline{s})$$

As $r \to \infty$, the lower bound on $c_r n_r$ converges to zero, while the upper bound becomes infinite. Hence, $\lim_{r\to\infty} c_r n_r$, if it exists, can take any weakly positive value. If $\lim_{r\to\infty} c_r n_r = 0$, then the total cost converges to zero, and hence all petitioning procedures are welfare-equivalent. However, if $c_r n_r$ converges to a finite positive limit, then in the limit the total cost is proportional to $F_\theta(\hat{s})$. Together with Proposition 2, this implies the following result:

Proposition 3. *Suppose $L < 1$. Let $r \to \infty$, and take any sequence of voting environments such that $c_r n_r$ converges to a limit. If $\lim_{r\to\infty} c_r n_r = 0$, then all values of q are welfare-equivalent. If $\lim_{r\to\infty} c_r n_r > 0$, then the efficiency of the petitioning procedure is strictly decreasing with q.*

Hence, a lower quota of signatures is preferable to a higher quota.

Information aggregation. When $\theta = 1$, the petition, if it gathers sufficiently many signatures, gives every citizen a payoff of 1. Since the cost of signing is below 1 when r is sufficiently large, it is socially optimal for the petition to succeed in state 1. Suppose that in addition, a social planner prefers the petition not to be forwarded to the legislature in state 0. This can be because, for example, the time that legislators spend on discussing the petition has some (finite) cost (which is not internalised by individual citizens when they sign the petition), and the social planner would prefer to avoid this cost.

Then a social planner would prefer the petition to gather at least qn_r signatures in state 1, and strictly less than qn_r signatures in state 0. Recall that citizens receive imperfect private signals about the state. We can say that the petition aggregates these private signals if the outcomes replicate the preferred decision of the social planner.

Consider the case when $L < 1$. At the low participation asymptotic equilibrium, the share of citizens who sign the petition in state 1 converges to $F_1(\hat{s}_1) < q$, while at the zero participation equilibrium it converges to $0 < q$. Thus, the petition fails in state 1, and hence these equilibria are not informationally efficient.

Now consider the high participation equilibrium. In the limit, as $r \to \infty$, in state 1 the fraction of citizens who sign the petition converges to $F_1(\hat{s}_2) > q$, so the petition almost surely succeeds. In state 0, that fraction converges to $F_0(\hat{s}_2)$. By Assumption 1, this is smaller than $F_1(\hat{s}_2)$, as monotone likelihood ratio property implies first-order stochastic dominance. This produces the following result:

Proposition 4. *Suppose $L < 1$. As $r \to \infty$, at the unique stable positive participation asymptotic equilibrium the petitioning procedure aggregates private signals if $F_0(\hat{s}_2) < q < F_1(\hat{s}_2)$, and does not aggregate private signals if $q < F_0(\hat{s}_2) < F_1(\hat{s}_2)$.*

Hence, at the stable equilibrium, the petitioning procedure aggregates private signals if and only if $F_0(s)$ is sufficiently far from $F_1(s)$ at $s = \hat{s}_2$. Intuitively, this happens when the distance between F_0 and F_1 is sufficiently large – that is, when individual signals are sufficiently informative. On the other hand, when signals are uninformative (that is, when F_0 and F_1 are similar), informational efficiency is not achieved.

The role of Assumption 3. Assumption 3 defines an interval of admissible values of c_r for any size of the electorate. In the limit, as $r \to \infty$ (and hence $n_r \to \infty$), both the upper and the lower bound of the interval converge to zero, so the interval disappears. Along the sequence, however, there exists a range of c_r for which Assumption 3 holds. This can be seen from the fact that λ^{n_r} is declining faster than $\frac{e}{2\pi}\frac{h(\bar{s})}{\sqrt{q(1-q)n_r}}$ as n_r increases, and hence for a sufficiently large r, λ^{n_r} is strictly smaller than $\frac{e}{2\pi}\frac{h(\bar{s})}{\sqrt{q(1-q)n_r}}$.

The first inequality in Assumption 3 ensures that participation levels are distinct from zero and one. Suppose instead that $c_r = 0$ in some voting environment r. Then the model becomes a collective action problem with zero cost and no uncertainty about the right course of action (in other words, a public good game with no cost of contributing). Then (4) implies that $F_1(\hat{s}) = 0$ or $F_1(\hat{s}) = 1$, so the only symmetric equilibria are zero participation and universal participation. In these trivial (and, arguably, unrealistic) equilibria, participation rate is not sensitive to q or to signal precision, and hence the results of the paper do not hold.

The second inequality places an upper bound on the cost of signing the petition. That upper bound is inversely proportional to the square root of the number of citizens[13]. It is used in the proof of Lemma 2 to show that there exist equilibria at which a citizen is pivotal with probability that is larger than c_r, so signing the petition is a dominant strategy for some signal realisations. This ensures that (2) holds for some value of s^r, and hence a positive participation equilibrium exists.

5 Conclusions

This paper looked at e-government petitions, an instrument of political participation that is becoming increasingly important in a number of democracies.

[13] This can be compared to the result in a two-candidate election with N voters: as N grows, the probability of a voter being pivotal declines at a rate of $\frac{1}{\sqrt{N}}$ if other voters vote for each candidate with probability $\frac{1}{2}$. If other voters behave differently, the probability of being pivotal declines at a much faster rate (Chamberlain and Rothschild, 1981). Note that this result is based on an assumption that strategies of other voters are exogenously fixed. This paper, in contrast, focuses on equilibrium behaviour – thus, a given citizen knows that other citizens' strategies change depending on the size of the group. Consequently, in the comparable case when $q = \frac{1}{2}$, the probability of being pivotal still declines at a rate that is inversely proportional to the square root of the number of citizens – even though in the limit other citizens sign the petition with a probability that is (except in the special case when $L = 1$) distinct from $\frac{1}{2}$, as Proposition 1 shows.

Two salient features of online petitions are a large number of potential signatories, and a low but positive cost of signing. To analyse this form of political participation, the paper developed a model of collective action in which the number of potential participants approaches infinity, while the cost of participation is bounded from above and from below. In the limit, the model generates participation levels that are realistic, that is, distinct from zero and from universal participation.

The paper showed that in the limit, there is a simple monotone relationship between the required quota of signatures and participation level. Because of this, low quotas may be socially preferable. At the same time, the paper showed that a petition can fail to aggregate individual signals unless they are sufficiently informative.

One assumption of the model is that citizens choose whether to sign the petition simultaneously, or at least that they do not condition their decision on the number of citizens that had signed the petition before them. A different model of online petitions could allow citizens to condition their choice on the number of previous signatories. Future research can consider this alternative setup.

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6 Appendix

Proof of Lemma 1. Robbins (1955) shows that $\sqrt{2\pi}k^{k+\frac{1}{2}}e^{-k}e^{\frac{1}{12k+1}} < k! < \sqrt{2\pi}k^{k+\frac{1}{2}}e^{-k}e^{\frac{1}{12k}}$ for all positive integers k. This implies the following weaker result for all positive integers k:

$$\sqrt{2\pi}k^{k+\frac{1}{2}}e^{-k} \leq k! \leq ek^{k+\frac{1}{2}}e^{-k} \tag{6}$$

where the first inequality comes follows the fact that $e^{\frac{1}{12k+1}} > 1$; while the second inequality holds trivially for $k = 1$, and for $k \geq 2$ it follows from the fact that $\sqrt{2\pi}e^{\frac{1}{12k}} \leq \sqrt{2\pi}e^{\frac{1}{24}} < e$. Applying (6) to $\binom{n_r}{qn_r} = \frac{n_r!}{(qn_r)!([1-q]n_r)!}$ yields

$$\frac{\sqrt{2\pi}n_r^{n_r+\frac{1}{2}}e^{-n_r}}{\left[e\left(qn_r\right)^{qn_r+\frac{1}{2}}e^{-qn_r}\right]\left[e\left(\left[1-q\right]n_r\right)^{(1-q)n_r+\frac{1}{2}}e^{-(1-q)n_r}\right]} \leq \binom{n_r}{qn_r}$$
$$\leq \frac{en_r^{n_r+\frac{1}{2}}e^{-n_r}}{\left[\sqrt{2\pi}\left(qn_r\right)^{qn_r+\frac{1}{2}}e^{-qn_r}\right]\left[\sqrt{2\pi}\left(\left[1-q\right]n_r\right)^{(1-q)n_r+\frac{1}{2}}e^{-(1-q)n_r}\right]}$$

This can be simplified as

$$\frac{\sqrt{2\pi}}{e^2}\frac{1}{q^{qn_r+\frac{1}{2}}\left(1-q\right)^{(1-q)n_r+\frac{1}{2}}n_r^{\frac{1}{2}}} \leq \binom{n_r}{qn_r} \leq \frac{e}{2\pi}\frac{1}{q^{qn_r+\frac{1}{2}}\left(\left[1-q\right]\right)^{(1-q)n_r+\frac{1}{2}}n_r^{\frac{1}{2}}}$$

which is equivalent to the statement of the lemma. □

Proof of Lemma 2. Since the left-hand side of (2) is continuous in s^r, it is sufficient to show that for all $r \geq \overline{r}$, there exist values of $s^r > \underline{s}$ for which the left-hand side of (2) is smaller than c_r; as well values of $s^r > \underline{s}$ for which it is larger than c_r. For the former, pick any s^r such that $0 < F_1(s^r) < \left[\frac{c_r}{h(\underline{s})\binom{n_r}{qn_r}}\right]^{\frac{1}{qn_r}}$. Then $h(s^r)\binom{n_r}{qn_r}[F_1(s^r)]^{qn_r}[1-F_1(s^r)]^{(1-q)n_r} < h(s^r)\binom{n_r}{qn_r}[F_1(s^r)]^{qn_r} < \frac{h(s^r)}{h(\underline{s})}c_r < c_r$, where the last inequality follows from Assumption 1. For the latter, let $s^r = F_1^{-1}(q)$. Then

$$\begin{aligned}
& h(s^r)\binom{n_r}{qn_r}[F_1(s^r)]^{qn_r}[1-F_1(s^r)]^{(1-q)n_r} \\
=& h(s^r)\binom{n_r}{qn_r}q^{qn_r}(1-q)^{(1-q)n_r} \geq \frac{\sqrt{2\pi}}{e^2}\frac{h(s^r)}{\sqrt{q(1-q)n_r}} \\
>& \frac{\sqrt{2\pi}}{e^2}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}} \geq c_r
\end{aligned}$$

where the first inequality follows from Lemma 1, the second – from Assumption 1, and the third – from Assumption 3. □

Proof of Lemma 3. By Assumption 3, if $r \geq \overline{r}$, then $\lambda \leq c_r^{\frac{1}{n_r}} \leq \left(\frac{e}{2\pi}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}}\right)^{\frac{1}{n_r}}$. Furthermore, when r is sufficiently large, $\frac{e}{2\pi}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}} \leq 1$, so $\left(\frac{e}{2\pi}\frac{h(\overline{s})}{\sqrt{q(1-q)n_r}}\right)^{\frac{1}{n_r}} \leq 1$. Hence, $\lambda \leq c_r^{\frac{1}{n_r}} \leq 1$ when r is sufficiently large. The statement of the lemma then follows from Bolzano–Weierstrass theorem. □

Proof of Lemma 4. We have $\lim_{r\to\infty}\left(\frac{\sqrt{q(1-q)}}{h(s^r)}\right)^{\frac{1}{n_r}} = 1$, and $\lim_{r\to\infty}\left(\sqrt{n_r}\right)^{\frac{1}{n_r}} = \lim_{n\to\infty} e^{\frac{1}{2n}\ln n} = 1$, where the last equality comes from the fact that $\lim_{n\to\infty}\frac{\ln n}{2n} = \lim_{n\to\infty}\frac{1}{2n} = 0$ by L'Hôpital's rule. Furthermore, $\lim_{r\to\infty}[F_1(s^r)]^q = [F_1(\hat{s})]^q$, and $\lim_{r\to\infty}[1-F_1(s^r)]^{1-q} = [1-F_1(\hat{s})]^{1-q}$. Finally, $\lim_{r\to\infty} c_r^{\frac{1}{n_r}} = L$. Substituting these equalities into (4) yields the result. □

Proof of Lemma 5. We have $\ln A(s) = q\ln[F_1(s)] + (1-q)\ln[1-F_1(s)]$. Thus, $\frac{d\ln A(s)}{dF_1(s)} = \frac{q}{F_1(s)} - \frac{1-q}{1-F_1(s)}$, which is strictly positive for $F_1(s) < q$ and strictly negative for $F_1(s) > q$. Hence, $\ln A(s)$ is increasing in s for $F_1(s) < q$ and decreasing in s for $F_1(s) > q$, which is equivalent to the statement of the lemma. □

Proof of Proposition 1. Let $B(\hat{s}) \equiv \left[\frac{q}{F_1(\hat{s})}\right]^q \left[\frac{1-q}{1-F_1(\hat{s})}\right]^{1-q} = \frac{q^q(1-q)^{1-q}}{A(\hat{s})}$. Note that it is continuous. By Lemma 5, $B(\hat{s})$ has a unique local minimum at $\hat{s} = F_1^{-1}(q)$, at which its value is 1. If $L = 1$, this is the unique value at which (5) holds. Suppose that $L < 1$. By Lemma 5, for $\hat{s}$ such that $F_1(\hat{s}) \in [0, q)$, $B(\hat{s})$ is monotone and takes values between $+\infty$ and 1. Hence, on that interval it equals $\frac{1}{L}$ at exactly one value of $\hat{s}$. The same is true for $\hat{s}$ such that $F_1(\hat{s}) \in (q, 1]$, and hence on that interval too $B(\hat{s}) = \frac{1}{L}$ at exactly one value of $\hat{s}$. □

Proof of Proposition 2. When $L = 1$, $\hat{s} = F_1^{-1}(q)$ and the result follows immediately. Suppose $L < 1$. Then (5) must hold at the two equilibria. Taking the logs of (5) and differentiating it with respect to q yields $\ln q + 1 - \ln[F_1(\hat{s})] - \frac{qf_1(\hat{s})}{F_1(\hat{s})}\frac{\partial\hat{s}}{\partial q} - \ln(1-q) - 1 + \ln[1 - F_1(\hat{s})] + \frac{(1-q)f_1(\hat{s})}{1-F_1(\hat{s})}\frac{\partial\hat{s}}{\partial q} = 0$. Hence,

$$\frac{\partial\hat{s}}{\partial q} = \frac{\ln\left[\frac{q}{F_1(\hat{s})}\right] - \ln\left[\frac{1-q}{1-F_1(\hat{s})}\right]}{\frac{q}{F_1(\hat{s})} - \frac{1-q}{1-F_1(\hat{s})}} \frac{1}{f_1(\hat{s})} \tag{7}$$

Note that when $L < 1$, $F_1(\hat{s}) \neq q$, and hence $\frac{q}{F_1(\hat{s})} \neq \frac{1-q}{1-F_1(\hat{s})}$. Since $\frac{q}{F_1(\hat{s})} > \frac{1-q}{1-F_1(\hat{s})}$ if and only if $\ln\left[\frac{q}{F_1(\hat{s})}\right] > \ln\left[\frac{1-q}{1-F_1(\hat{s})}\right]$, the numerator and the denominator of (7) have the same sign, so $\frac{\partial\hat{s}}{\partial q} > 0$ at any equilibrium. □

Proof of Proposition 3. If $\lim_{r\to\infty} c_r n_r = 0$, then the total cost converges to zero for any q. If $\lim_{r\to\infty} c_r n_r > 0$, then the total cost converges to $F_\theta(\hat{s}) \lim_{r\to\infty} c_r n_r$, which is strictly increasing in $F_\theta(\hat{s})$. By Proposition 2, $F_\theta(\hat{s})$, and hence the total cost, is strictly increasing in q at any positive participation asymptotic equilibrium. □

Proof of Proposition 4. At the asymptotic equilibrium cutoff $\hat{s}_2$, when $\theta = 1$, the share of citizens who sign the petition converges to $F_1(\hat{s}_2) > q$, which corresponds to the planner's preferred choice. When $\theta = 0$, the share of signatures converges to $F_0(\hat{s}_2) < F_1(\hat{s}_2)$. This outcome corresponds to the planner's choice if $F_0(\hat{s}_2) < q$ but not if $F_0(\hat{s}_2) > q$. □