de Mecánica Computacional

Mecánica Computacional Vol XXVI, pp.513-522
Sergio A. Elaskar, Elvio A. Pilotta, Germán A. Torres (Eds.)
Córdoba, Argentina, Octubre 2007

STRATEGIC ECONOMIC GROWTH WITH DECREASING RATES OF TIME PREFERENCE IN A TWO-AGENT ECONOMY

Fernando A. Tohmé[a,b] and Juan M.C. Larrosa[a,b]

[a]*CONICET*
[b]*Departamento de Economía, Universidad Nacional del Sur, San Juan y 12 de Ocubre, 8000 Bahía Blanca, Argentina*

Keywords: Decreasing rate of time preference, dynamic system, equilibrium.

Abstract.

This paper presents a two-agent economy, in which each agent has a consumption-dependent time preference. The optimal dynamic paths of accumulation will tend to one of many possible steady states, depending on the location of the initial capital level. The qualitative properties of this economic system have been analyzed elsewhere (Tohmé and Dabús, 2000). It has been shown that the interaction between the two agents may drag the poorest agent towards a higher steady state, getting her out of the potential poverty trap in which she could fall in isolation.

We are interested now in studying specific functional forms of the joint production function, the utility functions and the psychological discount rates. The goal is to characterize both the individual and joint steady states in order to assess the advantages of interaction.

Following the lead of (Dockner and Nishimura, 2004) we will obtain the subgame perfect equilibria of the economy seen as a two-person non-zero sum game. We will show that the non-linear convergence path towards the steady state examined by Tohmé and Dabús also obtains in a closed-loop solution..

1 INTRODUCTION

Since Ramsey's seminal paper the optimal economic growth has been represented by a single agent that maximizes an intertemporal welfare function. His basic construction has remained as the fundamental building block of models of optimal growth. The welfare function consists of the discounted sum of instantaneous utilities. In most models the rates of time preference were assumed to be constant and therefore independent over time. Instead, some researchers considered a variable rate of time preference: (Uzawa, 1968; Beals and Koopmans, 1969; Iwai, 1972; Blanchard and Fischer, 1989) and in the last decade, (Mantel, 1993, 1995, 1997). The key difference among these approaches lies on the particular characteristics of the preference rate. Uzawa, for example, assumes that it increases with the income level, making the optimal growth paths independent of the initial conditions. On the contrary, following a suggestion of (Fisher, 1930), (Mantel, 1997) postulates that the degree of impatience (the rate of time preference) should be a *decreasing* function of consumption and thus, indirectly, of the income level. The lower the income, the higher the sacrifice of postponing present consumption in exchange for future consumption.[1]

A line of work on optimal growth, based on assuming a single agent's decreasing rate of time preference, concludes that these preferences imply a monotone evolution of capital in time, depending on the initial capital levels (Mantel, 1967, 1995, 1997). These results seem more satisfactory than Uzawa's. The assumption of decreasing time preference rates, and the conclusion that the initial capital determines the path of growth, are certainly more realistic. In particular, a model with these features can generate poverty traps when the representative agent has a low initial level of capital. However, this is still far away from a realistic depiction of real world economies, which in general do not exhibit monotone paths of growth, and where the heterogeneity among agents cannot be easily reduced to a representation with a single agent. These features of real economies are not unrelated: the *mutual influence* among agents makes capital paths interdependent and therefore more prone to non-monotonicity.

In this paper we present a variation in the literature of optimal growth with varying time preferences in a differential game setting. To introduce heterogeneity, our model considers two agents, who differ in their initial capital levels, and therefore in their degrees of impatience, that depend on consumption and indirectly on income. This is intended as a highly stylized representation of closely integrated economies, as for instance those belonging to an economic union. In spite of this empirical interpretation of our framework, we will keep our discussion in an abstract setting. Nevertheless, our final goal is to develop an explicit treatment of interacting economies in growth.

For simplicity we assume that the agents share the yields of a single productive unit (like in a private ownership economy *à la* Arrow-Debreu). This leads to cross-over effects between the individual capital paths. A positive interaction means that accumulation (deaccumulation) by one agent leads to accumulation (deaccumulation) by the other. This generates monotone paths for both agents. On the other hand, if the interaction is negative, we can obtain non-monotone paths.

In this sense, one of the main results of our model arises in comparison with single-agent

[1] A quite different way of including variable discount rates has been largely explored in the literature on Behavioral Economics. The presence of time inconsistency in intertemporal choices indicates that the agents have hyperbolic discount rates (Laibson, 1997). On the contrary, in our approach we assume time consistency and focus instead on the existence of multiple steady-states.

models. More precisely, a straightforward implication of the latter is that if two agents choose their optimal plans separately, a kind of poverty trap may arise.[2] That is, two different agents can be such that, if there were no interaction, one would become "rich" and the other "poor" because of their respective accumulation policies. In our model, since they interact, an initially growing gap in capital accumulation can be reduced later. In fact, this is the main contribution of this paper: *while in single-agent models low initial levels of capital lead to paths of deaccumulation, and therefore imply the existence of poverty traps, our model allows, in some cases, to avoid them, thanks to the interaction among the agents.*

The plan of the paper is as follows. Section 2 presents the model. Section 3 is devoted to the characterization of optimal paths. Section 4 characterizes the steady states of the system. Section 5 summarizes the dynamics of this two-agents system. Finally, section 6 discusses our results and compares them with those obtained in an economy with a single agent.

2 THE MODEL

The economy considered here consists of two agents and a single productive unit. We assume, as in a Arrow-Debreu private ownership economy, that both agents get a constant share of the outcome. We want to characterize the equilibrium path of the economy as a function of the preferences, initial levels of capital and outcome shares of the agents.

Each agent i ($i = 1, 2$) has a prospective utility function:

$$\underset{\{c_t^i\}}{\text{Max}} \sum_{t=0}^{\infty} \beta_t u_i \left(c_t^i\right)$$

s.t.

$$c_t^i + k_t^i = \theta_i f \left(k_{t-1}^i + k_{t-1}^j\right)$$

with

$$\theta_i + \theta_j = 1$$

where $u_i\left(c_t^i\right)$ is i's instantaneous utility of consuming c at time t, and the real-valued function $\alpha(c_t)$ is the psychological factor of time preference.

$$\beta_t = \alpha\left(c_t^i\right) \beta_{t-1}$$

In turn, the rate of time preference function is $\rho(\cdot) \equiv \frac{1}{\alpha(.)} - 1$. Since $\alpha(\cdot)$ is increasing in consumption (and therefore in income), $\alpha(\cdot)$ can also be conceived as being increasing in income. ρ is decreasing, and acts as a discount rate.

The prospective utility function has the following properties:

1. $u^i(c)$ and $\alpha(c)$ are continuous on R_+ and twice differentiable for $c > 0$.
2. $u^{i'} > 0 > u^{i''}$, and $\lim_{c\to 0^+} u^{i'}(c) = +\infty$, $\lim_{c\to\infty} u^{i'}(c) = 0$, $u^i(0) \geq 0$.
3. $\alpha' > 0 > \alpha''$, $\alpha(0) > 0$.

[2] A *poverty trap* is defined as any self-reinforcing mechanism which causes poverty to persist (Azariadis, 2006). Surveys of the literature on poverty traps can be found, among others, in (Hoff, 2000; Easterly, 2001; Azariadis, 2004, 2006).

4. $0 < \alpha(c) \leq \bar{\alpha} < 1$ for some constant $\bar{\alpha}$, for all $c \geq 0$.

The technology consists in a simple neoclassical aggregate production function, which satisfies Inada's conditions. Therefore, it can be summarized by a real-valued production function $f(k)$, where k is the positive per capita capital. In turn, $f(k)$ has the following properties:

1. It is continuous, twice continuously differentiable for $k > 0$.
2. $f(0) = 0$, $f' > 0$, $f'' < 0$, $lim_{k \to 0^+} f'(k) = +\infty$
3. There exists a $k_m > 0$ such that $f(k_m) = k_m$.
4. $0 < k_t < k_m$ for all t.

Particularly, it takes a Cobb-Douglas functional form like:

$$f(k_t^i, k_t^j) = \theta_i \left(k_t^i + k_t^j\right)^\gamma$$

There is only one good, used both for consumption and for accumulation. Furthermore, $f(\cdot)$ is net of depreciation and of maintenance costs. To simplify the analysis the labor force is assumed constant, and all relevant variables are expressed in *per capita* terms. A capital path ${}_0k$ is admissible and feasible for an initial capital stock k if $k_0 = k$ and for $0 \leq t$:

$$0 \leq k_{t+1} \leq f(k_t)$$

Now we can postulate the optimization problem faced by both agents. Without loss of generality, we assume that the functional form of their psychological factor of time preference $\alpha(\cdot)$ is the same for both agents. Since ρ is decreasing in income, it will reach different values for different initial levels of capital, being this sufficient to induce heterogeneity between the agents, without resorting to different functions of psychological time preference.

Then, for each agent i the problem is to determine the optimal value of the prospective utility, deciding how much to consume and save at each time period, i.e. to find:

$$v^i(k_0^i) = \text{ Max}_{(c_0^i, c_1^i, \ldots)} \sum_{t=0}^{\infty} \beta_t u^i(c_t^i)$$

s.t.

$$k_{t+1}^i \leq \theta_i f(k_t^i + \bar{k}_t^{-i}) - c_t^i$$

$$\beta_{t+1} \leq \alpha(c_t^i)\beta_t$$

$$k_0^i \text{ and } {}_0\bar{k}^{-i}\text{given; } \beta_0 = 1$$

where θ_i is i's share of the income, which is assumed constant and $\theta_i + \theta_{-i} = 1$ (where variables and parameters subindexed by i correspond to agent i while those with subindex $-i$ correspond to the other agent). Moreover, without loss of generality, we assume that $\theta_i > \theta_{-i}$. It is assumed that these shares are enforceable and not open to renegotiation. Of course, c_t^i and k_t^i are, respectively, i's consumption and savings level at period t. The total amount of capital in the economy at t is $k_t = k_t^i + k_t^{-i}$. Finally, notice that β_t actually is a shorthand for the recursively determined weight of $u^i(c_t^i)$, i.e. $\beta_t = \prod_{s=0}^{t-1} \alpha(c_s)$.

Finally, ${}_0\bar{k}^{-i}$ is the optimal plan of the $-i$ agent. That is, each agent's optimal solution constrains the decisions of the other agent. Therefore, we should consider whether there exist equilibria in the choices of ${}_0k^i$ and ${}_0k^{-i}$. Since the instantaneous utility functions are continuous and the spaces of individual consumptions are compact,[3] this game has subgame-perfect equilibria (Harris, 1985; Hellwig and Leininger, 1987).[4] Those equilibria obtain by (the limit of) a backward induction process.

3 OPTIMAL PATHS

Expressions (5) and (6) in the previous section summarize the properties of optimal paths of capital accumulation. Since the policy functions g^i and g^{-i} determine the amount of capital at $t+1$ in terms of the current amount of capital at t, their derivatives define qualitatively the behavior of k^i and k^{-i}.
First, let us note that the sign of g^i defines the monotonicity (or its absence) of $\{k_t^i\}_{t=0}^{\infty}$:

Proposition 1 *If $g^i : K^i \to K^i$, where $K^i \subseteq \Re^+$ is the space of feasible values of k^i, is continuous and differentiable, its first derivative verifies that $g^{i\prime} > 0$ if and only if the capital path $\{k_t^i\}_{t=0}^{\infty}$ is monotone. That is, either $k_t^i \leq k_{t+1}^i$ for all t or $k_t^i \geq k_{t+1}^i$ for all t.*

4 STEADY STATES

Lemma 1 *k^i and k^{-i} must reach their steady states simultaneously.*

This result is consistent with our analysis of the agents' capital stocks behavior. They move in the same direction (except possibly for the first period) because of their *mutual influence*. So, if one variable reaches a steady state the other has to reach its own too.
Since the steady states must be reached simultaneously, the previous condition has to be fulfilled by both of them:

$$1 = \frac{\theta_i}{\theta_{-i}} \frac{\alpha\left(\theta_i f(k^i + k^{-i}) - k^i\right)}{\alpha\left(\theta_{-i} f(k^i + k^{-i}) - k^{-i}\right)} \frac{\left[1 + \frac{dk^{-i}}{dk^i}\right]}{\left[1 + \frac{dk^i}{dk^{-i}}\right]}$$

$$\theta_i \gamma \left(k_t^i + k_t^j\right)^{\gamma-1} = \frac{\left(\theta_i \left(k_t^i + k_t^j\right)^{\gamma} - k_{t+1}^i\right)^2}{\theta_i \left(k_{t-1}^i + k_{t-1}^j\right)^{\gamma} - k_t^i}$$

Introducing a new function to summarize the information involved in this characterization we have

$$\begin{aligned} \phi(k^i, k^{-i}) \equiv\ & \theta_i \alpha \left(\theta_i f(k^i + k^{-i}) - g^i(k^i)\right) \left[1 + \frac{dk^{-i}}{dk^i}\right] \\ -\ & \theta_{-i} \alpha \left(\theta_{-i} f(k^i + k^{-i}) - g^{-i}(k^{-i})\right) \left[1 + \frac{dk^i}{dk^{-i}}\right] \end{aligned}$$

The condition $\phi(k^i, k^{-i}) = 0$ is a necessary and sufficient[5] condition for (k^i, k^{-i}) to be a steady state. Then, the set of steady states of the system is:

[3] $c_t \in [0, f(k_m)]$ for all $t \geq 0$.

[4] Even if the agents choose their consumptions simultaneously at each period t, this is not an essential deviation from the perfect information setting assumed for this result.

[5] Sufficiency follows from the fact that the envelope condition is defined upon a *solution* to problem (1).

$$\hat{k}^i \times \hat{k}^{-i} \;=\; \left\{(k^i, k^{-i}) \in K^i \times K^{-i} : \phi(k^i, k^{-i}) \;= 0\right\}$$

The characterization of steady states is given by the following:

Theorem 1 $\emptyset \neq \hat{k}^i \times \hat{k}^{-i} \;=\; \left\{(k^i, k^{-i}) : \frac{dk^{-i}}{dk^i} = \frac{\theta_{-i}}{\theta_i}\frac{\alpha\left(\theta_{-i}f(k^i+k^{-i})-k^{-i}\right)}{\alpha(\theta_i f(k^i+k^{-i})-k^i)}\right\}$.

Proof of Theorem 1 *Consider a generic steady state,* $(\hat{k}^i, \hat{k}^{-i})$. *Then:*

$$\begin{aligned}\phi(\hat{k}^i, \hat{k}^{-i}) \;&=\; \theta_i\alpha\left(\theta_i f(\hat{k}^i + \hat{k}^{-i}) - \hat{k}^i\right)\left[1 + \frac{dk^{-i}}{dk^i}\right] \\ -\;\; &\theta_{-i}\alpha\left(\theta_{-i} f(\hat{k}^i + \hat{k}^{-i}) - \hat{k}^{-i}\right)\left[1 + \frac{dk^i}{dk^{-i}}\right] \;=\; 0\end{aligned}$$

It follows trivially that

$$\frac{dk^{-i}}{dk^i} \;=\; \frac{\theta_{-i}}{\theta_i}\frac{\alpha\left(\theta_{-i}f(\hat{k}^i + \hat{k}^{-i}) - \hat{k}^{-i}\right)}{\alpha\left(\theta_i f(\hat{k}^i + \hat{k}^{-i}) - \hat{k}^i\right)}$$

On the other hand, for each $(\hat{k}^i, \hat{k}^{-i})$ *such that* $\frac{dk^{-i}}{dk^i} = \frac{\theta_{-i}}{\theta_i}\frac{\alpha\left(\theta_{-i}f(\hat{k}^i+\hat{k}^{-i})-\hat{k}^{-i}\right)}{\alpha\left(\theta_i f(\hat{k}^i+\hat{k}^{-i})-\hat{k}^i\right)}$ *it is trivial that* $\phi\left(\hat{k}^i, \hat{k}^{-i}\right) = 0$.
Now let us consider this relation between k^i *and* k^{-i} *at* $(k^i, k^{-i}) = (0, 0)$. *At that point it is trivial that* $k^{-i} = \frac{\theta_{-i}}{\theta_i}\frac{\alpha(0)}{\alpha(0)}k^i$ *(since at* $(0, 0)$, $f(0 + 0) = 0$ *and* $g^i(0) = 0 = g^{-i}(0)$*). Therefore* $\frac{dk^{-i}}{dk^i} = \frac{\theta_{-i}}{\theta_i}\frac{\alpha(0)}{\alpha(0)}$, *i.e.* $(0, 0)$ *is a steady state.* □

This means that at least $(0, 0)$ is a steady state and that at each steady state the vector field has its direction defined by the relation between the parameters θ and the psychological factors of time preference computed at that point. On the other hand, Theorem 1 is not enough as a characterization of steady states. A steady state like $(0, 0)$ is not economically meaningful. Therefore we are interested in other (positive) values of the steady state variables. We cannot show that these entities exist in general since it depends clearly on the shape of ϕ. We can, instead, provide sufficient conditions for their existence:

Proposition 2 *If there exist two pairs* (k^i_{or}, k^{-i}_{or}) *(close to* $(0, 0)$*) and* $(k^i_m, k^{-i}_m) \gg (0, 0)$ *(with* $k^i_m + k^{-i}_m = k_m$*) such that* $\phi(k^i_m, k^{-i}_m) < 0$ *while* $\phi(k^i_{or}, k^{-i}_{or}) > 0$, *there exist* $(\hat{k}^i, \hat{k}^{-i}) \gg (0, 0)$ *such that* $(\hat{k}^i, \hat{k}^{-i}) \in \hat{k}^i \times \hat{k}^{-i}$.

Proof of Proposition 1 *By assumption, we have that* $(k^i_{or}, k^{-i}_{or}) \in K^- = \{(k^i, k^{-i}) \in K^i \times K^{-i} : \phi(k^i, k^{-i}) \geq 0\}$ *and* $(k^i_m, k^{-i}_m) \in K_- = \{(k^i, k^{-i}) \in K^i \times K^{-i} : \phi(k^i, k^{-i}) \leq 0\}$. *We have that* $K^i \times K^{-i} = K^- \cup K_-$, *and therefore the convex combination of* (k^i_{or}, k^{-i}_{or}) *and* (k^i_m, k^{-i}_m) *is in* $K^- \cup K_-$. *Then, a straightforward application of the Knaster-Kuratowski-Mazurkiewicz Theorem (see (Border, 1985)) yields that there exist* $(\hat{k}^i, \hat{k}^{-i}) \in K^- \cap K_-$. *Furthermore,* $(\hat{k}^i, \hat{k}^{-i})$ *is an element of the segment that joins* (k^i_{or}, k^{-i}_{or}) *and* (k^i_m, k^{-i}_m). *This means, on one hand, that* $\phi(\hat{k}^i, \hat{k}^{-i}) = 0$, *and on the other that* $(\hat{k}^i, \hat{k}^{-i}) \gg (0, 0)$ *(since only* (k^i_{or}, k^{-i}_{or}) *may have a* 0 *component, but it does not belong to* $K^- \cap K_-$*).* □

This means that, if there are at least two pairs of capital levels, one close to the origin, and the other in the technical efficiency frontier, such that the vector field given by g^i and g^{-i} points towards the interior of $K^i \times K^{-i}$ there must exist an interior steady state.

So far, the cardinality of $\hat{k}^i \times \hat{k}^{-i}$ is not determined. But notice that in a compact set, like $K^i \times K^{-i}$, if the steady states are isolated means that they must be finite in number. The advantages of this feature for the analysis of the global dynamics are clear: a finite number of steady states allows a partition of the phase space into a finite collection of attraction basins. The position of the initial capital levels determines the optimal path of the entire system.

As a previous step for this line of analysis, let us give a definition of *non-degeneracy* of steady states:

5 THE STRUCTURE OF THE PHASE SPACE

Lemma 2 *For each pair of steady states, $\hat{k}^j_{i,-i}$, $\hat{k}^l_{i,-i}$ we have that*

1. *either $\hat{k}^j_{i,-i} < \hat{k}^l_{i,-i}$ or $\hat{k}^j_{i,-i} > \hat{k}^l_{i,-i}$. That is, steady states can be linearly ordered.*
2. *if $\hat{k}^j_{i,-i} < \hat{k}^l_{i,-i}$, and both are stable steady states, there must exist another steady state $\hat{k}^s_{i,-i}$, such that $\hat{k}^j_{i,-i} < \hat{k}^s_{i,-i} < \hat{k}^l_{i,-i}$ and for each $\hat{k}^j_{i,-i} < (k^i_0, k^{-i}_0) < \hat{k}^l_{i,-i}$ either $(k^i_t, k^{-i}_t) \overset{t\to\infty}{\to} \hat{k}^j_{i,-i}$ or $(k^i_t, k^{-i}_t) \overset{t\to\infty}{\to} \hat{k}^l_{i,-i}$, or $(k^i_t, k^{-i}_t) = \hat{k}^s_{i,-i}$ for all t.*

Now, to complete the characterization of the dynamics of this system, notice that the basin of attraction towards an interior steady state $\hat{k}^j_{i,-i}$ $(j \neq 0)$ is determined by its two neighboring unstable steady states:

$$\hat{k}^j_{i,-i} \;=\; \{(k^i, k^{-i}) \in K^i \times K^{-i} : \hat{k}^{j-1}_{i,-i} < (k^i, k^{-i}) < \hat{k}^{j+1}_{i,-i}\}$$

while at the boundary, if the steady state is stable we have:

$$\hat{k}^0_{i,-i} \;=\; \{(k^i, k^{-i}) \in K^i \times K^{-i} : \hat{k}^0_{i,-i} < (k^i, k^{-i}) < \hat{k}^1_{i,-i}\}$$

while if it is unstable:

$$\hat{k}^0_{i,-i} \;=\; \{(0,0)\}$$

In any case, it must be clear that even if

$$\bigcup_{j\in\{0,\ldots,n\}} \hat{k}^j_{i,-i} \;\neq\; K^i \times K^{-i}$$

any dynamical path beginning at any element of $K^i \times K^{-i}$ will get, at $t = 1$, to $\bigcup_{j\in\{0,\ldots,n\}} \hat{k}^j_{i,-i}$.

6 A GOLDEN RULE

We have shown, until now, that the optimal solutions to the problem represented in (1) can exhibit a multiplicity of steady states. This is because the psychological discount factors, and then the rate of time preference depend on the income level. If they were assumed constant, say α^i, α^{-i}, from the envelope condition (10) we would have that at a steady state $(\hat{k}^i, \hat{k}^{-i})$ (using that at a steady state $\frac{dk^{-i}}{dk^i} = \frac{\theta_{-i}}{\theta_i}\frac{\alpha^{-i}}{\alpha^i})$:

$$f'(\hat{k}^i + \hat{k}^{-i}) \;=\; \frac{1}{\theta_i\alpha^i + \theta_{-i}\alpha^{-i}}.$$

Since f is concave, there is a unique value of $\hat{k}^i + \hat{k}^{-i}$ that verifies this relation. That is, $\hat{k}^i \times \hat{k}^{-i} \subset \left\{(\hat{k}^i, \hat{k}^{-i}) : f'(\hat{k}^i + \hat{k}^{-i}) = \frac{1}{\theta_i \alpha^i + \theta_{-i}\alpha^{-i}}\right\} \cup \{(0,0)\}$. But, according to Lemma 3 steady states are linearly ordered. Therefore there can exist only one steady state in $\left\{(\hat{k}^i, \hat{k}^{-i}) : f'(\hat{k}^i + \hat{k}^{-i}) = \frac{1}{\theta_i \alpha^i + \theta_{-i}\alpha^{-i}}\right\}$.

If, instead, the rate of time preference is decreasing in income, we can obtain several steady states. This allows us to compare our results with those in (Mantel, 1997). There the dynamical path of accumulation of the single agent in the economy is determined both by the initial capital level and the preferences over time. Besides, it follows also that only one steady state exists with a constant or increasing rate of time preference, and several otherwise.

From the results in the previous sections follows that at steady state the marginal productivity of (total) capital equals an expression based on the rates of time preference. This relation is actually a version of the *modified Golden Rule* of Economic Growth theory. More precisely:

Proposition 3 *If* $f'(k^i + k^{-i}) > (<, =) \frac{1}{\theta_i \alpha\left(\theta_i f(\hat{k}^i + \hat{k}^{-i}) - \hat{k}^i\right) + \theta_{-i}\alpha\left(\theta_{-i} f(\hat{k}^i + \hat{k}^{-i}) - \hat{k}^{-i}\right)}$, *in a neighborhood of a stable steady state* $(\hat{k}^i, \hat{k}^{-i})$, (k_t^i, k_t^{-i}) *will increase (decrease, remains constant) towards* $(\hat{k}^i, \hat{k}^{-i})$.

7 DISCUSSION

An important difference between Mantel's and our approach is the implication of presence of more than one agent; in particular, the fact that the dynamics depends on their interaction. While if agents are left on their own one may fall while the other grows sustained, the interaction makes them move in the same direction after $t = 1$. Therefore, while in a single-agent economy with low initial capital the agent fall in a poverty trap, in our two-agent economy the most wealthy may reverse the motion of the poorest and pull him toward a higher income steady state (see Figure 2). In turn, it is also may happen (almost theoretically) that the poorest agent pushes the richest into a lower steady state. In any case both will move monotonically in the same direction after $t = 1$.

How can be our results interpreted in terms of economic development? This question is natural given the original aims of Mantel, who intended his model to provide an explanation of increasing discrepancies between poor and rich countries. In this sense, he provided a formal argument for the origin of poverty traps. In our model, instead, any initial discrepancy can be reduced, or even eliminated, leading to possibility of avoiding poverty traps. As shown in Proposition 2, when the accumulation of one agent surpasses the deaccumulation of the other, the latter reverts his path.

The mechanism that yields this reversion has an interpretation in Development Theory: two countries that constitute a common market, with one investing more than any possible disinvestment of the other, can jointly grow through their interaction. The latter country would then begin to invest and then avoid the poverty trap to which it would fall if it were on its own.

Finally, a natural extension of this work is to incorporate specific functional forms and solve the model explicitly, in order to find out the conditions under which poverty traps may be avoided.

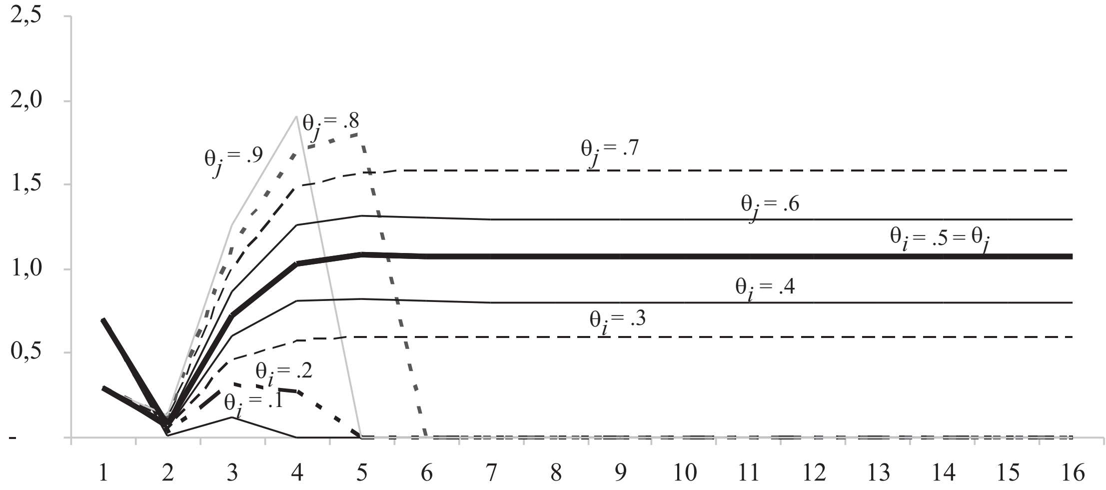

Figure 1: Simulated Paths for 16 periods

8 FIGURES

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