Multi-start local search procedure for the maximum fire risk insured capital problem

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Abstract. A recently European Commission regulation requires insurance companies to determine the maximum value of insured fire risk policies of all buildings that are partly or fully located within circle of a radius of 200 meters. In this work, we present the multi-start local search meta-heuristics that has been developed to solve the real case of an insurance company having more than 400 thousand insured buildings in mainland Portugal. A random sample of the data set was used and the solutions of the meta-heuristic were compared with the optimal solution of a MILP model based on the Maximal Covering Location Problem. The results show the proposed approach to be very efficient and effective in solving the problem.

Keywords: Meta-heuristics, Local Search, Solvency II, Continuous Location Problem

1 Introduction

Recently the European Union (EU) has published a new legistative programme - Solvency II - aiming at the harmonization of insurance industry across the European market and defining a policyholders protection framework that is risk-sensitive [1]. Among other aspects Solvency II comprises risk-based capital requirements that need to be allocate in order to ensure the financial stability of insurance companies with assets and liabilities valued on a market consistent basis. More precisely the Solvency Capital Requirement (SRC) should reflect a level of eligible own funds that enables the insurance undertakings to absorb significant losses without compromising the fulfilling of its obligations. As a risk-sensitive and prudential regime, Solvency II wants to take into account all possible outcomes, including events of major magnitude. Therefore, the capital requirement for catastrophe risk should assess all possible catastrophes, as natural catastrophes and man-made catastrophes, and establish how these risks should be quantified to integrate the whole.

In this work we will focus on the **man-made catastrophe risk** which comprises extreme events directly accountable to men (as motor vehicle liability risk; marine risk; aviation risk; fire risk; liability risk; credit and suretyship risk). Specifically we will address the capital requirement for **fire risk** (as fire, explosion and acts of terrorism) that should "(...) be equal to the loss in basic own funds of insurance and reinsurance undertakings that would result from an instantaneous loss of an amount that (...) is equal to the sum insured by the insurance or reinsurance undertaking with respect to the largest fire risk concentration". The fire risk assumes 100% damage on the total sum of the capital insured for each building located partly or fully within a 200 meters radius [2; 3]. Until now, and to best of our knowledge, the choice of 200 meters as the radius for the concentration was based on statistics and expert judgment.

This problem can be stated as: "find the centre coordinates of a circle with a fixed radius that maximizes the coverage of total fire risk insured". This problem can be viewed as a particular instance of the Maximal Covering Location Problem (MCLP) with fixed radius [4]. Church and Revelle, in 1974, were the first authors to address the MCLP under the assumption that both demand and possible site locations are discrete points [5]. Mehrez extended this seminal work by proposing a zero-one integer linear formulation considering as possible site locations the entire plan [6]. A widely used approach to the continuous space optimization has been to discretize the demand region, transforming the problem into a discrete location model [7]. Under the assumption that demand point is either covered or not by the facility, it has been proven that a discrete and finite set contains an optimal coverage solution [5].

This work has been motivated by the real case of an insurance company that, having more than 400 thousand buildings in Portugal covered by a fire insurance policy, needs to determine the maximum accumulated risk within a circle with 200 meters radius. Each building can be viewed as a "demand point" of the MCLP. Such a number of points leads to "enormous" MILP model which only "super" computers might be able to cope with. Although few meta-heuristic approaches already have been proposed to solve the MCLP, to the best of our knowledge, non of them fits our problem. For instance, Bruno et al. have developed a agent-based framework that could suit this problem. However no computational experiments were performed to access the scalability of the proposed framework to large problem instances [8]. Maximo et al. developed a meta-heuristic, named by the authors Intelligent-Guided Adaptive Search, to deal with large-scale instances where both demand points and possible site locations are discrete points [9]. The assumption of having discrete possible site locations may lead to non-optimal solutions, in our problem context.

An algorithm had to be designed so that the insurance company could use it at least once a year. Therefore, we have developed a meta-heuristic - the **Fire Risk Meta-heuristic** - inspired by the pattern search method proposed by Custódio and Vicente [10] that can be run in an ordinary desktop computer.

This paper will develop as follows. In the next section the meta-heuristic will be described in detail. Then test results will be reported to assess the quality of the proposed approach. Lastly, some conclusions and future work are given.

2 Fire Risk Meta-heuristic

The **Fire Risk Meta-heuristic** is a multi-start local search procedure where intensification and exploration strategies have been defined. In a nutshell, this procedure can be stated as: given an initial coordinate point (randomly selected) for the circle centre, determine the total fire risk within a k meters radius; generate and evaluate four neighbourhood points by increasing/decreasing each coordinate by a Δ value; make the best neighbourhood point as the new center.

Initial solution: (x_0, y_0) is randomly selected from the search space where the maximal risk is to be determined.

Objective function: For a given solution s, the objective function value $f(s)$ is the fire risk covered by the circle with centre in s and radius k.

Neighbourhood structure: At iteration i and considering a given Δ_i value, compute four new centre coordinates as shown in figure 1.

The step size Δ_i varies according to the quality of the neighbour solutions.

Stopping rules: Two stopping rules have been defined: one for the local search procedure and one for the multi-start algorithm. For the local search procedure one assumes as stopping criteria a small value for Δ_i, Δ_{min}. The meta-heuristic stopping criteria has two components: minimum number of iterations, i_{min}, and maximum fire risk of a single building, $Best$. The i_{min} is set empirically so that an adequate exploration of the search space is performed. The second component assures that the optimal circle must have a total fire risk (the objective function value) greater than the largest fire risk associated to a single building.

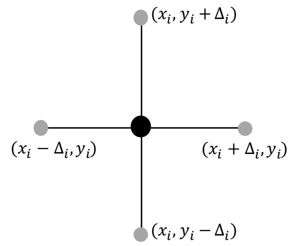

Fig. 1. Neighbourhood structure of a given point (x_i, y_i) [11]

Algorithm description: Algorithm 1 presents the pseudo-code of the Fire Risk Meta-heuristic. Parameters initialization is performed in lines 1 to 5. The local search procedure is presented between in lines 11 to 26. Given an initial solution s_i and the step size Δ_i, a set of four neighbourhood solutions are generated (line 12). The best of the four (in term of the objective function value) is compared with the value of s_i. If the best neighbourhood has a better value (a success), the current solution moves to neighbouring solution (lines 14 and 15). When the algorithm produces two consecutive successes, the step size is increased; the new step size doubles the previous one (lines 16 and 17). With a larger step size, we aim at broadening the search procedure in "promising" areas of the search space. If none of the four neighbours presents better values (line 21) the current solution does not change and the step size decreases - the idea is to intensify the search near the current solution, since it might be a local optimum (lines 22 and 23). The local search procedure stops when the step size is small enough (line 26) and the most promising solution found at the moment is update (or not) - line 27. This local search algorithm is embedded in the multi-start procedure. In the restart step, the random initial solution of iteration i (s_i) is generated and the step size is reset to Δ_0 (lines 7 to 10). The multi-start algorithm ends when both stopping criteria are met.

3 Results

In order to assess the Fire Risk Meta-heuristic solution, the MILP formulation for the maximal covering location problem (MCLP) was adapted from Farahani et al. [12], formulated in GAMS, and solved by CPLEX to optimality [13]. The formulation is given in Appendix. Being a discrete approach, it provides a lower bound to our problem, since the circle centre as to match one of the buildings.

The Fire Risk Meta-heuristic was programmed in R software, a requisite of the insurance company. The `ggplot2` and `ggmap` were the packages chosen to plot all the maps using Google Maps information. All the results were obtained by a PC with Intel® Xeon 10 cores and 32 GB of RAM.

Data

Given the volume of data for a national study and being this work a first step towards the development of an optimization approach, we confined the study to the Lisbon area. The Insurance Company provided a data set which encompasses the chosen geographical area and has 46 843 buildings (points). Each points is defined by the two geographical coordinates (longitude and latitude) and the fire capital insured.

The MILP model is unable to solve real instance with 46 thousand points since it leads to out of memory issues. Our approach was then to select two random samples of tractable size (1000 points): samples A and B. Figure 2 shows the distribution of the insured capitals over the samples areas. Notice that no extreme point exists on sample A (by extreme we mean a building with an insured capital so large that it will obviously belong the optimal circle), while in sample B there are a few of such points. Figure 3 depicts the geographical location of the sample A.

Algorithm 1 Fire Risk Meta-heuristic

Best *Find maximum risk of a single building*
Δ_0 *Set step size*
s_0 *Generate initial solution*
$s^* = s_0$
$i = 0$ *Iteration counter*
While $i < i_{min} \vee f(s^*) < Best$ **do**
  If $i > 0$ **then**
    s_i *Generate iteration i initial solution*
    $\Delta_i = \Delta_0$ *Set step size*
  EndIf
  Do
    $\mathcal{N}(s_i)$ *Generation of candidate neighbours*
    s' *Set the best neighbour* $s' \in \mathcal{N}(s_i)$
    If $f(s') > f(s_i)$ is a better neighbour (success) **then**
      $s_{i+1} = s'$
      If *this is the second consecutive success* **then**
        $\Delta_{i+1} = 2 \cdot \Delta_i$
      else
        $\Delta_{i+1} = \Delta_i$
      EndIf
    else *(unsuccess)*
      $s_{i+1} = s_i$
      $\Delta_{i+1} = \frac{\Delta_i}{2}$
    EndIf
    $i = i + 1$
  Until $\Delta_{i-1} < \Delta_{min}$
If $f(s_i) > f(s^*)$ **then** $s^* = s_i$
EndWhile
Return $s^*, f(s^*)$

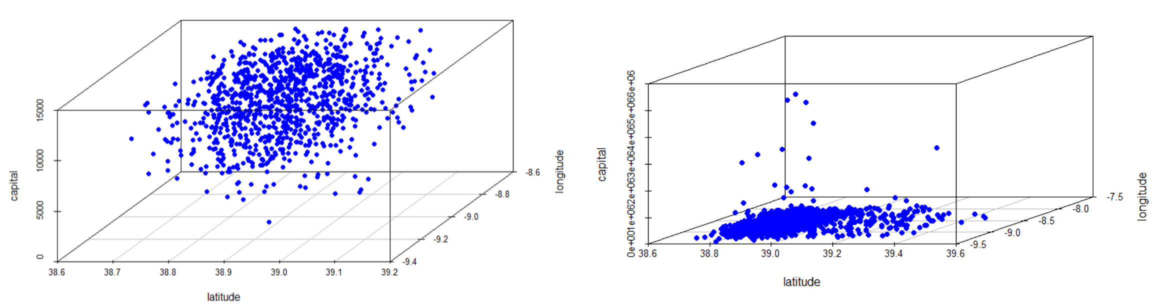

Fig. 2. Distribution of the 1000 insured capitals for samples A (*left*) and B (*right*)

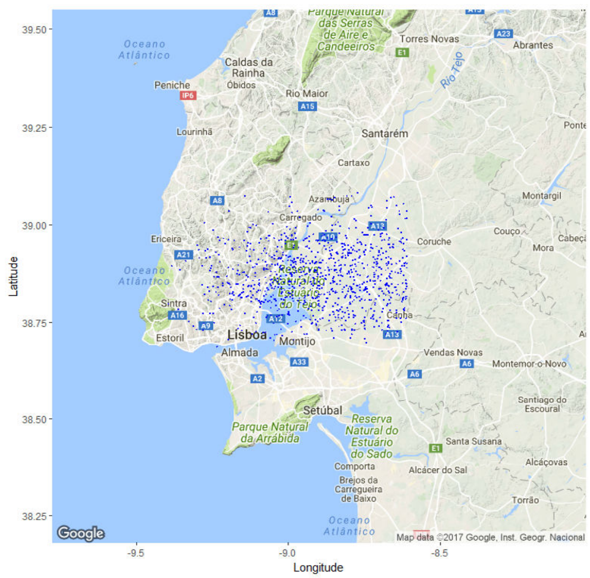

Fig. 3. Geographical location of the 1000 points

In terms of descriptive statistics both samples also present differences. Table 1 shows the maximum and minimum fire insured capital, the amplitude, the average and standard deviation within each sample. With these two sets of points, we aimed at assessing the capability of the algorithm for overcoming the trap of local optimality.

Table 1. Samples descriptive statistics

Sample	Maximum	Minimum	Amplitude	Average	Stand. dev.
A	14490,6	239,2	14251,4	10342,7	2785,9
B	5112903,0	5833,9	5107069,1	123052,0	335729,9

Meta-heuristic validation

The Risk Fire Meta-heuristic parameters were set to $\Delta_0 = 200 \cdot 2^7 = 25\,600$ m, $\Delta_{min} = 50$ m and $i_{min} = 50\,000$ iterations. For each sample 20 runs were performed. All starting point coordinates were randomly chosen while assuring an adequate covering the search space. The results are presented in table 2 together with the optimal values found by GAMS/CPLEX. The columns "no. of buildings" show how many builds lie within the circle corresponding to the "Optimal value" (or "Best value"). The last column presents the average number of iterations needed to find the "Best value".

Table 2. Results

	GAMS/CPLEX		Fire-risk Meta-heuristic		
Sample	Optimal value	No. of buildings	Best value	No. of buildings	Average no. of iterations
A	28857,46	2	33233,56	3	2785,9
B	5112903,0	1	5112903,0	1	30431

For sample A, the optimal fire risk found by GMAS/CPLEX is of €28 857.46 and two buildings are covered. The algorithm was able to find a better value for the covered fire risk (33 323, 56 *vs.* 28 857, 46). In fact, the meta-heuristic only found two different values (not shown): the best value was found in 19 out of the 20 runs, and in one single run the same value as the one proposed by GAMS/CPLEX was found. As mentioned above, the MILP model assumes that the circle centre as being one of the building, while the meta-heuristic allows it to be placed in any point of the plane. The highest fire risk value improves in 15% the optimal value found by the MILP model. Figure 4 shows the two buildings covered of the MILP optimal solution (on the left). On the right side of figure 4, one depicts the three building covered by the 19 runs that provided the best risk value. Lastly, we should refer that the meta-heuristic took on average 54 secs per run and GMAS/CPLEX needed about 0.03 secs to reach the optimal solution.

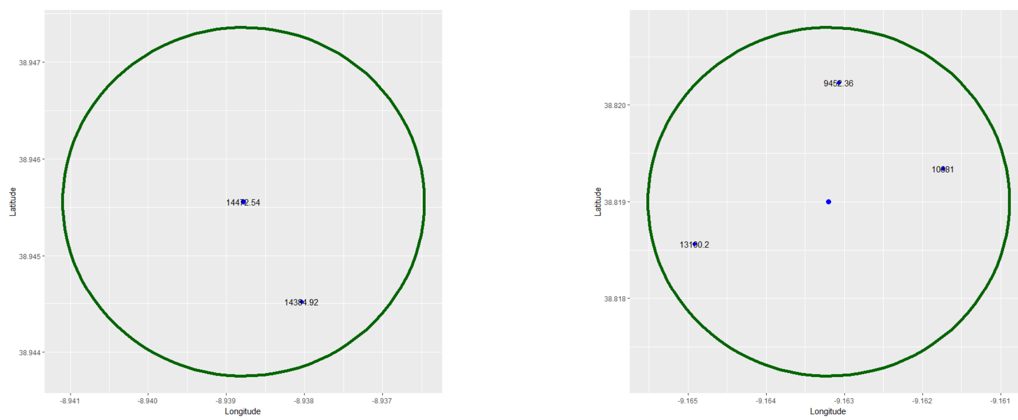

Fig. 4. Sample A: optimal solution from the MILP model (*left*) and meta-heuristic best solution (*right*)

Regarding sample B, one should emphasise that the optimal value was found by the meta-heuristic in all the 20 runs performed. These results suggest the proposed algorithm is able to escape from local optima. Additional tests are nonetheless needed to support this suggestion.

4 Final remarks and future work

In this work we proposed a new meta-heuristic approach to determine the coordinates of a k radius circle that covers the maximum fire risk according to the EU legislative programme Solvency II. Until last year, insurance companies reported only the largest capital that covers one single insured building. The new legislative programme demands companies to solve a much harder problem, which can only be done using computational tools.

In the two tested samples, the Risk Fire Meta-heuristic found optimal value or even a better value than the best one computed by a MILP model. Notice, this MILP model provides a lower-bound for the problem since it is a discrete based model and, therefore, not allowing the circle centre coordinates to be defined outside the existing set of points.

The meta-heuristic has been applied to a data set of more than 46 thousand points [14] and it is currently being used by the insurance company.

As future work we will pursue three main directions. First, we will extended the computational experiments and assess the meta-heuristic solution quality with regard to the optimal solution provided by 01 integer formulation proposed by Mehrez, in 1983 [6]. Second, additional computational experiments will be performed so that statistical tests can be used to validate the usefulness of this method. Third, the computational performance of the algorithm will also be investigated to overcome the long times it is currently taking.

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Appendix

Let (a_i, b_i) be the longitude and latitude coordinates of building i, $i = 1, ..., n$, R_i be the risk associated with building i, and D_{ij} the Euclidean distance between buildings i and j, and consider the set

$$\Delta_{ij} = \{(i, j) : D_{ij} \leq 2k + \epsilon\},$$

with $\epsilon > 0$ and k the circle radius.

Let two binary variables be defined as: $x_{ij} = 1$ if building i is covered by the circle centred at j, 0 otherwise; and $y_j = 1$ if j is the centre of the circle.

$$\begin{aligned}
\max \quad & \sum_{ij \in \Delta_{ij}} R_{ij} x_{ij} \\
\text{s.t.} \quad & D_{ij} x_{ij} \leq k y_i \quad (i, j) \in \Delta_{ij} \\
& \sum_{i:=1}^{n} y_i = 1 \\
& x_{ii} \leq y_i \quad i \in \{1, ..., n\} \\
& x_{ij}, y_i \in \{0, 1\}
\end{aligned}$$

The objective function is defined by the sum of the fire risks insured of the buildings covered by the circle. The first constraint assures that only the buildings distancing less that k from the circle centre y_j will be considered. The second constraint assures that only one circle is determined. The last constraint is needed since one has a maximization model. Notice that $D_{ii} = 0$, $i = 1, ..., n$. Therefore, all $x_{ii} = 1$ will verify the first constraint, whatever the value of y_i.