Frahm and Mikeska Reply: In the preceding Comment by Feingold *et al.*[1] the correspondence of the kicked rotator to Anderson localization of an electron on a one-dimensional lattice with random potentials is used to obtain a more physical interpretation of the level statistics of the kicked rotator presented in our Letter.[2] Discussion of the transition from Wigner to Poisson statistics with increasing $\hbar$, shown in Fig. 2 of Ref. 2, in terms of a one-dimensional Anderson model where the ratio of correlation length ξ to system size N is varied certainly presents an alternative way of looking at our data. This point of view nicely shows that the relation between $\hbar$ and ξ given by Shepelyansky[3] allows one to use the intuitive (and partly quantitative) understanding for the level statistics of the one-dimensional Anderson model to demonstrate that an increasing strength of quantum effects leads to a crossover in the level statistics for a quantum system with a *finite* number N of states. To show this directly for the kicked rotator and to demonstrate agreement with analogous, although less conclusive, results obtained previously for different systems[4] was the aim of our Letter. Compared with existing information on this point, our work, supplemented by new data given below, gives results for much larger values of N than considered so far. Moreover, in the work of Israilev[5] the value $k \cong 20\,000$ is too large to see the ordering influence of quantum effects and in the work of Feingold and co-workers[6,7] the fixed value $\hbar \cong 0.3$ does not allow conclusions about the classical limit.

More interestingly the Comment[1] directs interest to a discussion of the limit of large N: Whereas the analogy to Anderson localization suggests that in the limit $N \to \infty$ level statistics are always Poissonian, implying that the limits $\hbar \to 0$ and $N \to \infty$ do not commute, straightforward comparison of the general quantum system (arbitrary N) and its irregular classical counterpart leads one to expect a transition in $\hbar$ even for $N \to \infty$. This question seems to us undecided in view of the limitations of the mapping as discussed by Feingold and Fishman.[7] While we have addressed this point only by a qualitative remark in our Letter, we now want to report the following new results of further investigations:

(i) We have calculated one more entry in Table I of Ref. 2, diagonalizing a 3043×3043 matrix with the help of a Lanczos algorithm adapted to the symmetry of the problem.[8] For $\hbar = \frac{55}{6084}$, corresponding to the continued fraction [110,1,1,1,1,1,1,1,1,1] and to (following Ref. 3) $\xi/N \simeq 0.6$, we obtain $q = 0.971 \pm 0.044$. This result shows that Wigner statistics persists well into the regime $\xi < N$.

(ii) We have investigated two sets of parameters with constant ratio ξ/N to see whether a dependence on $\hbar$ as additional variable can be established. As shown in Table I this is not the case since q is constant within the numerical errors.

(iii) Extending the arguments of the Comment we have investigated the consequences of the assumption that the level statistics for, say, $\xi/N = 0.1$ corresponds to ten independent systems with level repulsion. A simulation of such a level distribution leads to $q \lesssim 0.3$, much lower than $q = 0.8$ as read off from Fig. 2 in Ref. 2.

TABLE I. Level statistics parameter q for (a) $k = 10.0$, $\xi/N \cong 0.074$ and (b) $k = 5.0$, $\xi/N \cong 0.096$, but different sizes N of the system.

	$\hbar$	q	Δq
(a)	91/1944	0.805	±0.101
	161/6084	0.766	±0.129
(b)	21/128	0.546	±0.600
	29/256	0.611	±0.335
	41/512	0.653	±0.243
	57/1024	0.676	±0.154
	79/1944	0.642	±0.128

We conclude from these results that the crossover from Wigner to Poisson statistics occurs at most at a value of ξ/N considerably lower than 1 and that the issue of a dependence of q on $\hbar$ in the regime $\xi \ll N$ remains undecided with present computational possibilities.

Finally we want to point out that the possibly singular character of the limit $N \to \infty$ loses its importance if the truly δ-function-like kicks of the ideal model are replaced by a more realistic model: Any smearing of the kicks will have the effect of restricting the chaotic dynamics to a finite number of classical resonances. The size of the corresponding Anderson model then diverges as $\hbar^{-1}$ and the crossover in the level statistics of the full system is correctly obtained at finite N.

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