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1-1-2018

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The load carrying capacity of ductile materials degrades as a function of porosity, stress state and strain-rate. The effect of these variables on porosity kinetics is captured by the Cocks–Ashby model; however, the Cocks–Ashby model does not account for material degradation directly. This work uses a yield criteria to form a degradation function that is consistent with Cocks–Ashby porosity kinetics and is a function of porosity, stress state and strain-rate dependence. Approximations of this degradation function for pure hydrostatic stress states are also explored.

Degradation function; Porosity; Damage; Ductile fracture; Strain-rate dependence

Introduction

The load carrying capacity of ductile materials degrades as a function of porosity (Vaz and Andrade Pires [14]). However, this degradation is also a function of stress state (Benzerga et al. [3]) and strain-rate (Rodriguez et al. [12]). Gurson's model (Gurson [7]) for ductile materials accounts for this degradation through a yield criteria that is an explicit function of stress state and porosity. Gurson's model, however, accounts for strain-rate

dependence only implicitly, as Gurson's yield criteria is function of flow stress, and flow stress may be strain-rate dependent.

For strain-rate dependent problems, the Cocks–Ashby model (Cocks and Ashby [5]) provides a porosity evolution (kinetics) expression that is a function of porosity, stress state, and strain-rate exponent. However, their model does not give a corresponding yield criteria to evaluate material degradation. Inversely, since porosity evolution is related to a yield behavior, Cocks–Ashby's porosity evolution expression does not necessary correspond with any general yield criteria. This work proposes a degradation function that is consistent with Cocks–Ashby's porosity kinetics and gives thoughts on its implementation and uses.

Background

Gurson's model gives a yield criteria $\phi(\sigma_h/\bar{\sigma}, f, m) = 0$, where σ_h is hydrostatic stress, $\bar{\sigma}$ is flow stress, f is porosity, and m is strain-rate exponent. Strain-rate exponent characterizes the power-law relation between stress and strain-rate $(\varepsilon) = \varepsilon_o(\sigma_{eq}/\sigma_o)^1/m$ where $\dot{\varepsilon}$ is the rate of small strain evolution, σ_{eq} is equivalent stress, and ε_o, σ_o are material parameters.

Combining mass conservation with the flow rule for plastic strain gives a relation between porosity evolution and Gurson's (or any) yield criteria,

$$f = (1 - f)\, \lambda\, \text{tr}(\partial\phi/\partial\sigma\text{ij}), (1)$$

where $\dot{\lambda}$ is a strain rate multiplier and σ is the Cauchy stress. Cocks–Ashby's porosity evolution expression is given by:

$$\dot{f} = c_1\sinh\left\{c_2\,\frac{2-m}{2+m}\left(\frac{\sigma_h}{\sigma_{eq}}\right)\right\}\left(\frac{1}{(1-f)^{1/m}} - (1-f)\right)\dot{\bar{\varepsilon}}p, (2)$$

where $\dot{\bar{\varepsilon}}p$ is the effective plastic strain rate and Cocks–Ashby's original form has been modified to include two calibration parameters c_1 and c_2. Equation (2) does not adhere to Eq. (1) and (possibly as a result) is used in crystal scale calculations where a yield criteria is not generally known in a functional form. In these cases, phenomenological degradation is used. Specifically, Kweon ([8]) used the degradation function $w = 1 - c_3f$ and Barton et al. ([2]) used the degradation function $w = 1 - \tanh(c_3f)$ where c_3 was a degradation parameter independent of Eq. (2). In both cases $\dot{\bar{\varepsilon}}p$ in Eq. (2) was replaced with the sum of shearing rates in a crystal.

Several yield criteria for strain-rate dependent materials have been proposed. Marin and McDowell ([10]) expressed these criteria in the form

$$\phi = \sqrt{3h1J2 + \Omega} - h_3\bar{\sigma} = 0, (3)$$

where J_2 is the second invariant of deviatoric stress (related to equivalent stress by $\sigma_{eq} = \sqrt{3J_2}$), Ω is a function to be discussed later, and h_1 and h_3 are functions of f and m corresponding to the various yield criteria of Cocks ([4]), Michel and Suquet ([11]) and Duva and Hutchinson ([6]), or Sofronis and McMeeking ([13]).

Theory

A general yield criteria is given as (Vaz and Andrade Pires [14]):

$$\phi = \sigma_{eq} - w\bar{\sigma} = 0. (4)$$

Marin and McDowell ([10]) express porosity evolution in terms of the yield criteria in Eq. (3) as

$$\dot{f} = \frac{1-f}{\sqrt{h_1 2 J_2}} \frac{\partial \Omega}{\partial I1} \dot{\bar{\varepsilon}} p, \ (5)$$

where I_1 is the first invariant of stress.

Marin and McDowell ([10]) equate this to Cocks–Ashby's porosity evolution expression by assuming that $\sinh(x) \approx x$ and integrating with respect to I1 to yield[1] [1]:

$$\Omega = h_1 \left(\frac{2}{9}\right) \left(\frac{2-m}{2+m}\right) I_1^2 \left(\frac{1}{(1-f)^{\frac{1}{m}+1}} - 1\right). \ (6)$$

The yield criteria in Eq. (3) can be written in terms of a degradation function (as in Eq. 4) if Ω can be written as $\Omega = 3 J_2 \Omega^*$. Thus, Eq. (6) could be multiplied by $(3 J_2 / 3 J_2)$ to achieve this form. However, a more natural and general solution is to integrate Eq. (5) exactly without the small sinh assumption to give:

$$\Omega = (3 J_2) h_1 \frac{(2+m)}{(2-m)} \cdot \cosh\left[\frac{2}{3} \frac{(2-m)}{(2+m)} \frac{I_1}{\sqrt{3 J_2}}\right] \left(\frac{1}{(1-f)^{\frac{1}{m}+1}} - 1\right), \ (7)$$

using $\Omega = 3 J_2 \Omega^*$, $\sigma_{eq} = \sqrt{3 J_2}$, $\sigma_h = I_1 / 3$ and including the two calibration parameters c_1, c_2 gives:

$$\Omega^* = \frac{c_1}{c_2} 2 h_1 \frac{(2+m)}{(2-m)} \cdot \cosh\left[c_2 \frac{(2-m)}{(2+m)} \frac{\sigma_h}{\sigma_{eq}}\right] \left(\frac{1}{(1-f)^{\frac{1}{m}+1}} - 1\right). \ (8)$$

With this modification, Eq. (3) can be written as:

$$\sigma_{eq} - \frac{h_3}{\sqrt{h_1 + \Omega^*}} \bar{\sigma} = 0, \ (9)$$

which fits in the form $\phi = \sigma_{eq} - w\bar{\sigma}$ and gives a degradation function consistent with Cocks–Ashby porosity kinetics:

$$w = \frac{h_3(f,m)}{\sqrt{h_1(f) + \Omega^*(f,m,\sigma_h)}}. \ (10)$$

Equation (10) is plotted for several strain-rate exponents in Fig. 1 highlighting the explicit dependence of degradation on m absent from Gurson's model.
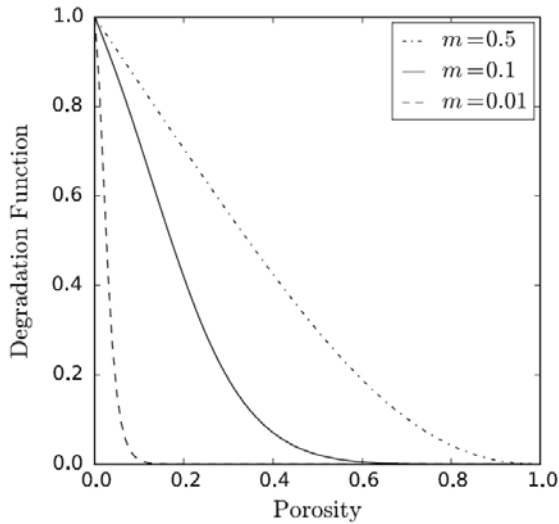
Figure 1. The degradation function's porosity dependence for several strain-rate exponents and a triaxiality of 0.3

This is simply a reinterpretation of the general yield criteria in Marin and McDowell ([10]) (neglecting only the small sinh assumption), but it gives physical insight into the degradation of the material, and gives a functional form for degradation that both captures strain-rate dependence explicitly and is consistent with Cocks–Ashby porosity kinetics.

Implementation

This section implements Eqs. (2) and (4) using the new degradation function in Eqs. (8) and (10), and gives some thoughts on a useful approximation for Eq. (8). These expressions are implemented in the ale3d finite element code developed at Lawrence Livermore National Laboratory (Anderson et al. [1]). Simulations are executed on a single 3D finite element loaded with a constant uniaxial velocity and no lateral constraints. Cocks ([4]) yield criteria is used giving $h_1 = 1 + (2/3)f$ and $h_3 = (1 - f)^{1/(1+m)}$. Kirchhoff's stress τ_{ij} is used in evaluation of the yield criteria, and is converted to Cauchy stress via $\tau_{ij} = J\sigma_{ij}$, where $J = 1/(1 - f)$, for calculation of internal forces. A Swift-type flow stress expression $\bar{\sigma} = \bar{\sigma}o(1 + \bar{\varepsilon}^p/e_o)^{nS}$ is used with parameters set to $\bar{\sigma}_o = 260$ MPa, $e_o = 0.004, n_S = 0.1$. The Young's modulus and Poisson's ratio are 73.3 GPa and 0.33 respectively; The calibration parameters are $c_1 = 0.1, c_2 = 0.5$; The strain-rate exponent is $m = 0.1$, where strain-rate is considered constant; and Initial porosity is 0.07 for the simulation without nucleation.

It is of consequence for implementation that Eq. (2) is singular for pure hydrostatic stress (i.e., $\sigma_{eq} = 0$). The appendix of Cocks and Ashby ([5]) accounts for this by providing a separate $\dot{f}$ expression for pure hydrostatic tension conditions; however, this expression requires both an ad-hoc barrier between high triaxialities (that approach pure hydrostatic stress) and low triaxialities as well as implementation of a mechanism for switching between cases.
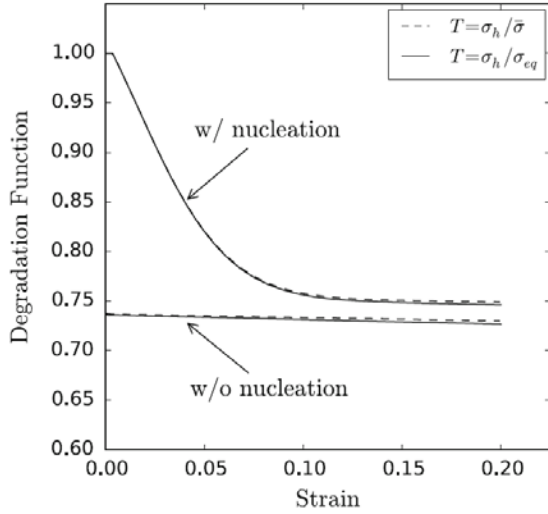
Figure 2. The degradation function with different approximation of triaxiality (T). When nucleation is present no initial porosity is prescribed

To protect against singularity for pure hydrostatic stress states, this work approximates the triaxiality $(T = \sigma_h/\sigma_{eq})$ in Eqs. (2) and (8) by replacing σeq with $\bar{\sigma}$. The same approximation was made by Lee and Dawson ([9]) and was motivated by their observation that σeq is "very close" to $\bar{\sigma}$ for rate-independent inelastic materials. The effect of this approximation on degradation (under a uniaxial load) with and without porosity nucleation are shown in Fig. 2. The scenario with nucleation has no initial porosity and porosity nucleation is defined by:

$$\dot{f}_N = \frac{f_N^o}{s_N\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\bar{\varepsilon}-\varepsilon_N}{s_N}\right)^2\right] \dot{\bar{\varepsilon}}p. \quad (11)$$

In Eq. (11), the porosity available for nucleation is $f_N^o = 0.003$, the reference strain is $\varepsilon_N = 0.01$, and the standard deviation of strain-based nucleation equivalent plastic strain is $s_N = 0.001$.

Figure 2 shows that, while Eq. (10) allows for increasing degradation (with finite initial porosity) before the onset of plastic strain, this elastic degradation has minimal effect (and is barely distinguishable in Fig. 2). Figure 2 also shows that approximating triaxiality using $\bar{\sigma}$ results in negligible error for the loading and parameters studied.

Conclusions

This work shows that the general yield criteria outlined by Marin and McDowell ([10]) can be expressed as a degradation function. This degradation function has the following advantages over its predecessors: It.

- gives physical insight into the degradation of the material which may be opaque from a yield criteria;
- is both consistent with Cocks–Ashby porosity kinetics and an explicit function of strain-rate exponent and stress state;
- is useful for crystal scale (i.e., crystal plasticity) problems where a yield function is not used directly.

Substituting $\bar{\sigma}$ for σ_{eq} in the degradation function protected against a singularity at pure hydrostatic stress and resulted in negligible error for the loading and parameters studied

Acknowledgements

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Footnotes

[1] *a typographic error in Marin and McDowell (1996) gives the leading term as (2 / 8).*