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A RESONANCE METHOD FOR MEASURING THE PHASE DIFFERENCE OF CONDENSERS.

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It is a well known fact that when an alternating electromotive force is impressed upon a condenser the resulting current does not lead the electromotive force by the theoretical ninety degrees. This is particularly noticeable in the case of paper condensers of the telephone type so often used in laboratories for the study of the alternating current circuit. When an attempt is made to interpret such experiments by graphic methods it is sometimes found that the power factor of the condensors is so great that it must be taken into account. The power factor angle or the angle by which the current lags behind the theoretical ninety degree lead is known as the phase difference of the condenser.[1]

This paper is devoted to a description of a method by which the phase difference of paper condensers can be determined from the same data that are obtained in an experimental study of the phenomena of current resonance.

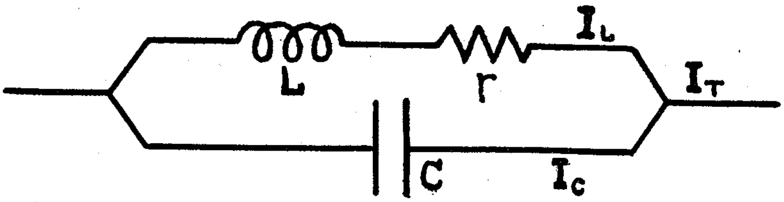

FIG. 16.—Diagram showing circuit for determining phase differences.

Figure 16 represents the circuit which consists of two parallel branches, one containing a coil with inductance L and resistance r, the other a variable condenser C. An alternating electromotive force is impressed upon this circuit and the value of the three currents I_T, I_L, I_C, read as the capacity of the condenser is increased by equal steps. The current in the inductive branch will remain constant. In the condensive branch the current

[1] F. W. Grover, Bureau of Standards, Bull., vol. 7, 495, 1911; vol. 3, 371, 1907. These papers contain a very complete discussion of condensers and their properties, with numerous references to other work upon this subject.

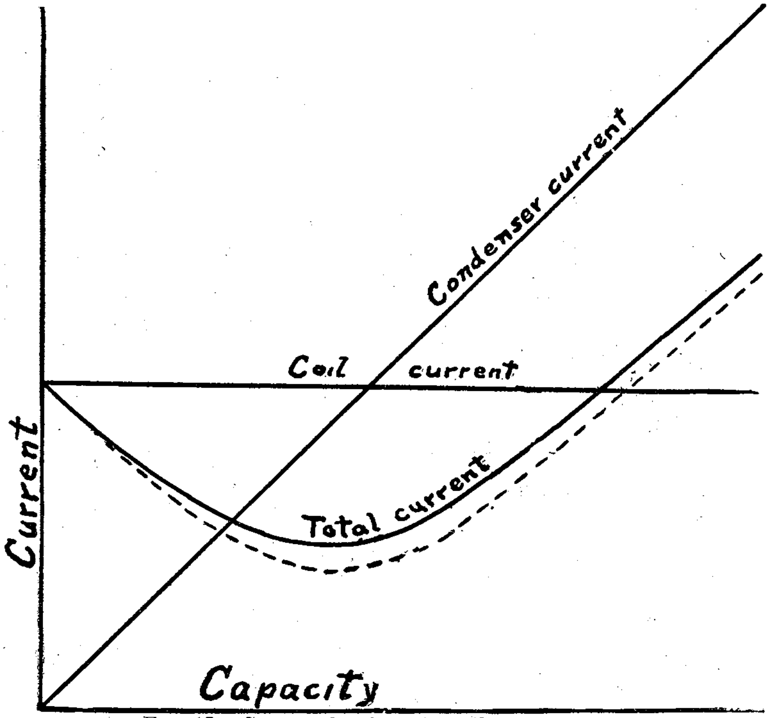

FIG. 17.—Curves showing strength of currents.

will increase uniformly as the capacity is increased. The total current will, however, decrease at first, pass through a minimum value and then steadily increase. This is a standard procedure in the study of current resonance and figure 17 represents the results that would be obtained by plotting such a series of observations.

Figure 18 is a vector diagram of the currents assuming zero phase difference in the condensers. If the impressed electromotive force be taken as the axis of reference the vector OL represents the current in the coil which lags by the angle ϕ given by the relation $Cos\ \phi = \frac{r}{z}$ in which expression r is the resistance of the coil and z its impedance. The vectors OC_1, OC_2, etc., represent different values of the current in the condensers, and the vectors OR_1, OR_2, etc., are the corresponding resultant or total currents as found by the parallelogram method. The values of the total current obtained by this graphic method are represented in figure 17 by the dotted line. It should, of course, fall exactly upon the line of observed values and the discrepancy is to be attributed to the fact that the phase difference of the condensers has not been taken into account.

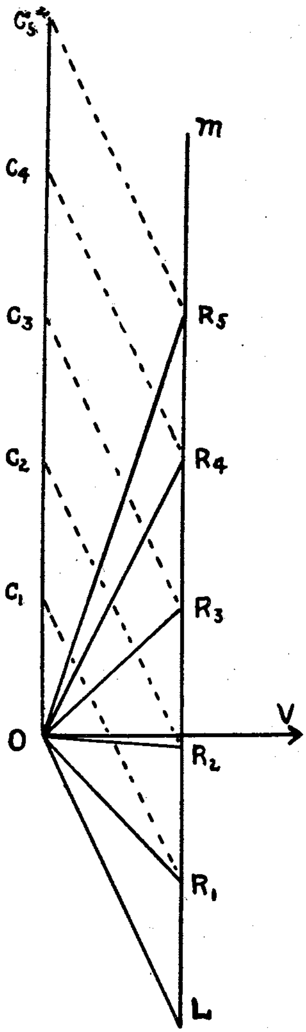

FIG. 18.

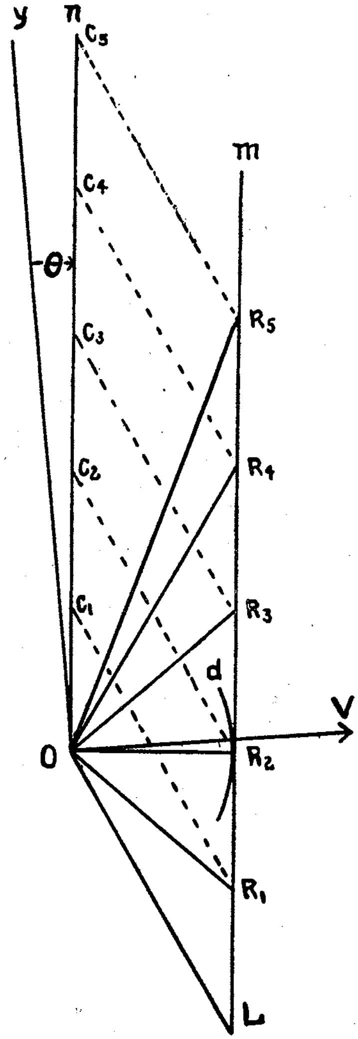

FIG. 19.

FIG. 18.—Vector diagram of currents assuming zero phase difference.
FIG. 19.—Vector diagram assuming known phase difference.

If we assume that the phase difference θ is known and construct the vector diagram, figure 19 will be the result. The angle θ determines the slope of the line *On* along which fall the vectors representing the different values of the condenser current. The line *Lm* upon which end the vectors representing the various values of the total current is, of course, parallel to *On*. If the angle θ has been taken correctly all of these total current vectors as found by the parallelogram method will check exactly with the observed values. If θ is taken too small the resultants will all be too short, if θ is too long the resultants will be larger than the observed values. A cut and try method could be employed but there is one of the total current vectors which has such properties as to make it possible for it to be used to determine the position of the line *L m*. This vector is the one shown in figure 19 as $O\ R_2$. It represents the minimum value of the total current and as the figure shows must be perpendicular to the line *L m*. *O L*, $O\ R_2$, and $L\ R_2$ must then form a right angled triangle of which two sides are known. Thus the slope of *L m* is determined and with it the slope of *O n* and ultimately the value of the angle θ which is the phase difference of the condensers. The method of procedure is as follows:

The required data are secured for a parallel circuit such as that of figure 16 and plotted as in figure 17, smooth curves being drawn which are more accurate than the single observations. The vector *O L* is drawn with its length proportional to the current in the coil. As the impressed electromotive force is taken as the axis of reference, *O L* lags behind *O Y* by an angle determined by the resistance and impedance of the coil. With *O* as a center and with a radius corresponding to the minimum value of the total current the arc *d* is described. From the point *L* a line *L m* is drawn tangent to this arc. From *O* a line *O n* is drawn parallel to *L m*, this line giving the direction of the capacity current vectors. *O y* is drawn ninety degrees ahead of the axis of reference. The angle θ between *O y* and *O n* is therefore the phase difference of the condensers.

The phase difference may also be determined from the following formula

$$\theta = Cos^{-1} \frac{r}{z} - Cos^{-1} \frac{I_L}{I_T}$$

in which r is the resistance of the coil, z its impedance, I_L the coil currents, and I_T the minimum value of the total current.

In the development of the above theory a sine wave electromotive force has been assumed. If there are higher harmonics the effect will be to increase the apparent value of the phase difference. In this case there will be no value of the capacity for which perfect resonance will be secured. All values of the total current will be larger than for sine wave conditions and for a given value of the coil current $O\ L$, the radius of the arc d will be increased resulting in a greater value of the phase angle θ.

In the ordinary laboratory experiment this is not a serious matter unless the harmonics are very prominent. In our own laboratory this difficulty is avoided by the introduction of considerable inductance in series with the parallel circuit.

The above method of measuring the phase difference of condensers is not proposed as a standard or accurate method. It is, however, of some theoretical interest and has proven to be of great practical value in a laboratory where students are studying the alternating current circuit with the use of the commercial paper condenser.

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