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Exploring the relationship between mathematics content knowledge and pedagogical content knowledge among pre-service teachers

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EXPLORING THE RELATIONSHIP BETWEEN MATHEMATICS CONTENT
KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE AMONG PRE-
SERVICE TEACHERS

A Dissertation

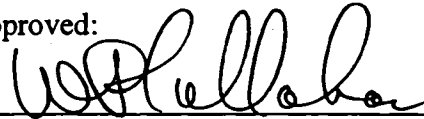
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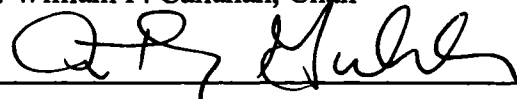
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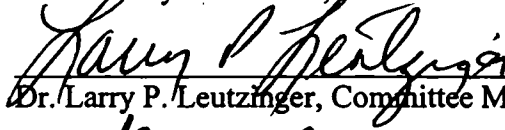
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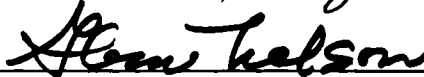
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An Abstract of a Dissertation

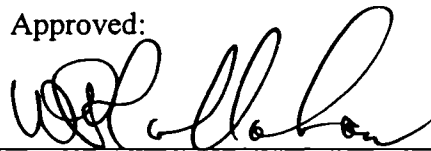
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ABSTRACT

This study replicates and extends a previous study (Nathan & Petrosino, 2003) that explored the relationship between the content knowledge and pedagogical content knowledge of pre-service mathematics teachers.

In that study, Nathan and Petrosino (2003) examined and reported evidence supporting the counterintuitive hypothesis that, in some situations, having a high degree of content knowledge may be associated with the “expert blind spot” (symbol-precedence) in pedagogical content knowledge. Pre-service teachers with various levels of expertise in mathematics subject matter were given a series of mathematics problems and asked to rank order their difficulty. Nathan and Petrosino (2003) reported that, on the average, pre-service teachers with more advanced mathematics education courses and fewer pedagogical content knowledge courses rank ordered the problems in ways that were inconsistent with actual patterns of student performance. This suggested that they had less insight into how students think about and solve these problems. In contrast, students who had taken fewer advanced mathematics courses but more pedagogical content knowledge courses rank ordered the problems in a way that was more consistent with actual student performance.

The present study builds upon this work. Forty pre-service teachers majoring or minoring in mathematics education were surveyed to assess their knowledge of algebra and aspects of their knowledge regarding how to teach algebra. They were asked to rank order Nathan and Petrosino’s (2003) problems in the Difficulty Factor Analysis task.

Based on their assessment scores, teacher candidates in three categories were selected for follow-up interviews. The categories were: high content knowledge (CK) and symbol-precedence pedagogical content knowledge (PCK), low CK and verbal-precedence PCK, and high CK and verbal-precedence PCK. The importance of both elements (CK and PCK) for pre-service education majors and professional development programs was also investigated, although no causal relationship was implied.

Quantitative results replicated previous findings--students with higher CK showed they used symbol-precedence to teach algebra significantly more than students with lower CK. Follow-up interview data suggest a more complicated relationship between content and pedagogical content knowledge. Those findings revealed that (a) the high CK pre-service teacher with symbol-precedence was knowledge-centered in her teaching perspectives; (b) the low CK pre-service teacher with verbal-precedence was problem-centered in her teaching perspectives; and (c) the high CK pre-service teacher with verbal-precedence was response-based in his teaching perspectives.

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CHAPTER 1

INTRODUCTION TO THE STUDY

Purpose of the Study

Research on “expert blind spot” (e.g., demonstrate symbol-precedence pedagogical content knowledge) was supporting the counterintuitive hypothesis that, in some situations, having a high degree of content knowledge might in fact be associated with the “expert blind spot” in pedagogical content knowledge (Koedinger, Alibali & Nathan, 1998; Nathan & Koedinger, 2000a; Nathan & Koedinger 2000b; Nathan & Petrosion, 2003). The purpose of this study was to explore the components of the “expert blind spot” (Nathan & Petrosino, 2003) produced in the relationship between content knowledge and pedagogical content knowledge.

Precisely, the study attempted to accomplish the following:

1. Explore the relationship between algebraic content knowledge and pedagogical content knowledge of pre-service teachers majoring or minoring in mathematics education.
2. Replicate Nathan and Petrosino’s (2003) study on the “expert blind spot” (hereafter, EBS) to assess if it is replicated in pre-service teachers majoring or minoring in mathematics education.
3. Extend Nathan and Petrosino’s (2003) study of the EBS to the pre-service mathematics teachers’ content knowledge and pedagogical content knowledge of algebra.

The study of teacher knowledge by elementary and secondary education majors was based on their different levels of understanding of algebraic content knowledge.

Prior research focusing on teacher knowledge (Fennema & Franke, 1992) and teachers' perspectives of students' performances (Nathan & Koedinger, 2000b) were used to demonstrate how educators recognize the nature of teacher knowledge as an important component in preparation, instruction, assessment, and reflection on teaching processes and students' learning.

Background of the Study

Mathematics education research has focused on the relationship between teachers' content knowledge and their pedagogical content knowledge in the last decade (Ball, 1988a, 1991; Even, 1990; Grossman, 1990; Shulman, 1986, 1987). Members of the Conference Board of Mathematical Sciences (CBMS; 2001) stated that reform efforts in mathematics education are calling for a change in teaching for understanding and meaning rather than developing isolated procedures and skills. To approach the goal of teaching for understanding, the National Council of Teachers of Mathematics (NCTM) suggested teacher preparation programs in mathematics tended to help pre-service teachers develop solid knowledge of content (NCTM, 2000). This requirement also included teaching pre-service teachers not only to understand mathematics content knowledge but also to build their perspectives of pedagogy.

Leinhardt, Zaslavsky, and Stein (1990) pointed out that the study of algebra is one of the earliest points in mathematics where students use a symbolic system to represent their arithmetic concepts. To help students represent their arithmetic concepts by a symbolic system, NCTM (1989) suggested that, instead of just manipulating symbols and developing isolated skills, students should be able to (a) model real world phenomena

with a variety of algebraic symbols, (b) visualize and analyze algebra in various representational forms, (c) translate between representational forms; and (d) develop an understanding of operations of algebra and the general behavior of classes of algebra.

Many of the reform efforts in mathematics education were based on an image of mathematics teaching that emphasized student thinking as the springboard for mathematical discourse and learning (Ball, 1991; Lampert, 1989; NCTM, 1991). Such an approach required teachers to have highly developed reasoning skills (Barnett, 1991; Fennema & Franke, 1992). Researchers (Barnes, 1989; McDiarmid, Ball & Anderson, 1989; Shulman, 1987) reported that teaching subject matter (such as arithmetic or algebra) required knowledge of the concepts, ideas, and principles that make up the content of the discipline. When teaching subject matter, teachers should understand how the discourse within a discipline relates to the teaching of the subject and how fundamental ideas could be transformed into appropriate representations that made this knowledge comprehensible to learners (Shulman, 1987). Teachers needed to know the way students comprehended the subject matter so that teachers could use different representations of content knowledge to benefit students (Barnett, 1991).

The comparisons between expert and novice teachers in mathematics education fields (e.g. Brown & Borko, 1992; Carpenter, 1992; Leinhardt & Smith, 1985) have indicated that teachers' mathematics content knowledge and pedagogical content knowledge must be well connected each other. Expert teachers displayed more effective representation skills from content knowledge to pedagogical content knowledge (Leinhardt & Smith, 1985). When teaching algebra, expert teachers also were "more

elaborate, interconnected, and accessible in their conceptual systems and cognitive schemata where storing content knowledge” (Brown & Borko, 1992, p. 213).

Alternatively, several studies suggested that pre-service teachers lacked a deep and connected understanding of the various algebraic concepts and tended to rely on a limited definition of algebra (Even, 1993; Kuchemann, 1981; NCTM, 1989).

Leinhardt and Smith (1985) and Nathan and Petrosino’s (2003) studies, examining the relationship between mathematics content knowledge and pedagogical content knowledge, focused on pre-service teachers’ core courses in their teacher preparation program. Normally, pre-service teachers enhanced their knowledge of teaching through interactions with mathematical content knowledge courses and method courses based on building professional teaching experiences (NCTM, 2000). However, there was only a small base of information on the extent of teachers’ mathematics content knowledge and on the relationship between the content knowledge and the development of pedagogical content knowledge. The challenge to enhance teacher knowledge through this interaction with content courses and methods courses was that pre-service teachers did not understand how teaching experience could contribute to content knowledge (Fennema & Franke, 1992). Since knowledge was open to change and development with experience, it was important to conduct research on knowledge of teaching with pre-service teachers in an environment where they had an opportunity to learn knowledge-based teaching (Shulman, 1986).

Kieran (1992) and Nathan and Koedinger’s (2000a) studies found that pre-service teachers with different content knowledge had different beliefs about the knowledge of

teaching. Because of these findings, it was beneficial to examine the level of pedagogical content knowledge between pre-service teachers with high content knowledge and pre-service teachers with low content knowledge. Specifically, the conceptions of the pre-service teachers' algebraic knowledge and the perceptions of teaching algebra also needed to be studied in this investigation. The purpose of this study was to explore whether pre-service teachers have the EBS hypothesis (Nathan & Petrosino, 2003) produced in the relationship between content knowledge and pedagogical content knowledge. This exploration further sought to discover the nature of the EBS (Nathan & Petrosino, 2003) in both pre-service teachers with high content knowledge and pre-service teachers with low content knowledge.

Significance of the Study

In mathematics education, it was significant to explore pre-service teachers' knowledge of content and how their content knowledge impacted their mathematics teaching. Furthermore, examining the relationship between pre-service mathematics teachers' content knowledge and pedagogical content knowledge could be used to support, improve, and even reform mathematics teacher education programs.

Mathematics education should help pre-service teachers gain a better appreciation of mathematics and help them integrate the content and pedagogy necessary for proper teaching (Cooney, 1999). Shulman (1987) stated that knowledge for effective teaching could be categorized into seven components. Pedagogical content knowledge, which focused on the knowledge for teaching (Shulman, 1986, 1987), was used to combine the conceptual and the methodological framework of teaching. To improve the quality of

teaching in pre-service teacher education, the best pre-service teacher preparation would provide pre-service teachers with the conceptual and methodological framework so they could anticipate, recognize, articulate, and incorporate facets of teaching that improved teaching in the classroom (Grossman, 1990; Hiebert & Lefevre, 1986; Hiebert & Carpenter, 1992).

Normally, a pre-service teachers' education program was designed to help pre-service teachers understand the components of pedagogical content knowledge at the completion of the pre-service teachers' preparation program. Yet, pre-service teachers' education programs could not possibly achieve the desired level of application because the understanding of pedagogical content knowledge was based on experience. Pedagogical content knowledge could not be totally taught but must be experienced through actual practice, reflection on the practice, and reconstruction of the practice over an extended time.

A possession of mathematical knowledge was positively correlated with being an effective teacher. According to Shulman (1987), an effective teacher had to understand what was to be taught and how it was to be taught. Pre-service teachers were in the best position to explore new concepts in instruction (Grossman, 1990; Shulman, 1987). Shulman (1987) mentioned that to be effective at teaching, teachers should first comprehend the subject matter content knowledge so they could flexibly transform that content through pedagogical consideration. Pre-service teachers developed their ability to teach for understanding as they grew from being students, to expert learners, to pre-service teachers, to novice teachers, and finally, to effective teachers (Shulman, 1987).

Going through this process from expert learners to expert teachers, it was significant to discover how pre-service teachers transformed content knowledge to pedagogical content knowledge.

In this study, the content knowledge of algebra, as a specific domain, was used to investigate pre-service teachers' perceptions of their pedagogical content knowledge in teaching algebra. Different learning backgrounds and preparatory courses for pre-service teachers' algebra content knowledge were used to categorize their ability to teach algebra with high content knowledge or low content knowledge. Wu (2000) advocated that pre-service teachers with higher-level content knowledge could more successfully develop their pedagogical content knowledge in the future. However, Nathan and Petrosino's (2003) article suggested that pre-service teachers with higher-level content knowledge might be associated with the "expert blind spot" (symbol-precedence) in their pedagogical content knowledge. It was important in this study to discover if pre-service teachers with high content knowledge were potentially able to develop an appropriate pedagogical content knowledge (both symbol-precedence and verbal-precedence).

It was necessary for teachers to understand the content knowledge they teach because the more connected and broad a teacher's content knowledge was, the richer the teacher's learning environment for students would be. However, teachers also needed to understand how to present the content knowledge of a specific domain (e.g., algebra) so that students could learn and understand it (Kilpatrick, Swafford & Findell, 2001). While teachers' knowledge was dynamic and dialectical in terms of content knowledge, knowledge of pedagogy, knowledge of students' cognition, and teachers' beliefs, it was

also situated in practice (Thompson, 1992). Expert teachers had a better-connected schema of content and pedagogy because of their richer mental plans and representation skills and, therefore, could respond to students' questions more effectively (Leinhardt & Smith, 1985). In this way, the teachers' content knowledge affected the teachers' pedagogical content knowledge and, thus, impacted students' opportunity to learn. However, Koedinger, Alibali and Nathan (1999) and Nathan and Koedinger (2000a, 2000b) reported that teachers with unreflective content knowledge created a barrier between their content knowledge and their pedagogical content knowledge. Even pre-service teachers with high content knowledge had an "expert blind spot" when they predicted students' difficulties in learning algebra (Nathan & Petrosino, 2003). Based on this information, it was important to replicate and then to determine whether the "expert blind spot" occurred in pre-service teachers with different levels of content knowledge. This present study replicated the "expert blind spot" hypothesis (Nathan and Petrosino, 2003), and then explored the relationship between pre-service teachers' perspectives of pedagogical content knowledge in different levels of content knowledge.

Limitation of the Study

First, the researcher's educational experience was a limitation of the study. The researcher's mathematics background in teaching and learning was based on Asian mathematics instruction, which was similar to the traditional British teaching style. The researcher's experience in teaching and learning mathematics in the United States might be a limitation. It was possible that some limitation in understanding the United States pedagogical content knowledge might occur. However, the researcher's experiences and

observations in the United States K-12 mathematics classes might help in an understanding of mathematics education in the United States.

Second, this present study was limited to exploring the relationship between mathematics content knowledge and pedagogical content knowledge without considering the variables in cultural differences in mathematics education and social values discrepancy between Asian countries and the United States.

Third, mathematics content knowledge in this study was limited to 7th grade algebraic content knowledge and pedagogical content knowledge in the United States. This study was limited to early algebraic concepts, representing the relationship between result-unknown (arithmetic) and start-unknown (algebra).

Fourth, the select samples of the study were limited to a Midwest region comprehensive university, University of Northern Iowa. The education preparation program at the University of Northern Iowa followed the Iowa State Department of Education requirements.

Delimitation of the Study

First, the EBS hypothesis in Nathan and Petrosino's (2003) article was tested on pre-service teachers who took Calculus I or higher-level classes and those who took only classes below Calculus I. The present study involved pre-service secondary mathematics education majors and pre-service elementary education majors with a mathematics minor. According to the different majors, the participants were divided into two groups by an Algebra Content Knowledge test. Participants with above-median scores on the Algebra Content Knowledge test were categorized in the higher content knowledge group.

Participants with below median scores on the Algebra Content Knowledge test were categorized in the lower content knowledge group.

Second, in this study, the specific domain in algebra focused on the concepts of variables represented as symbol equations, word problems and story problems. The specific domain in algebra was also designed to be either result-unknown (arithmetic) or start-unknown (algebra) problems.

Third, as in Nathan and Petrosino's (2003) study, the goal of this study was to collect data from pre-service teachers with high content knowledge and pre-service teachers with low content knowledge.

Fourth, the study attempted to explore the differences in pedagogical content knowledge in both the high and the low content knowledge groups.

Fifth, the study also investigated whether the pedagogical content knowledge was different between pre-service elementary education teachers with a mathematics minor and pre-service secondary education mathematics majors.

Sixth, if the pre-service teachers' perceptions in this study matched Nathan and Petrosino's (2003) EBS hypotheses, the study then focused on the components of the "expert blind spot" and why pre-service teachers, who had almost finished their teaching preparation program, already had an "expert blind spot" in their beliefs system.

Seventh, SPSS 11.5 version has used to analyze the algebra content knowledge test and ranking task (Nathan & Petrosino, 2003).

Research Problem

The purpose of the study was to explore whether the EBS hypothesis (Nathan & Petrosino, 2003) was extended to pre-service teachers with high content knowledge or low content knowledge. Nathan and Petrosino's (2003) EBS hypothesis was that "teachers with greater mathematics knowledge tend to expect students to follow a normative process of development that mirrors the structure of the domain of mathematics" (p. 910). The EBS hypothesis (Nathan & Petrosino, 2003) was investigated and then the study explored the components of the "expert blind spot" in pre-service teachers' perceptions.

A mathematics teacher was the teacher who had greater content knowledge (Third International Mathematics and Science Study [TIMSS], 1995, 1999). Teachers with a higher-level understanding of content knowledge of algebra enhanced their effective pedagogy in teaching algebra (Wu, 2000). On the other side, previous research (Ball, 1988a; Usiskin, 2003) have reported pre-service teachers' understanding of content knowledge has limited benefit when applied as teaching knowledge. Nathan and Koedinger (2000a, 2000b), and Nathan and Petrosino (2003) have shown there is a mathematics educators' "expert blind spot" between teachers' content knowledge and pedagogical content knowledge. The "expert blind spot" decreased a teacher's knowledge of students' understanding. When pre-service teachers were nearly finished with their preparation program, they were expected to have reduced the "expert blind spot" in their pedagogical perceptions. Curiously, Nathan and Petrosino's (2003) research showed that

pre-service teachers with high mathematics content knowledge often produced the “expert blind spot” when they predict students’ needs in learning algebra.

In this study, an Algebra Content Knowledge test was administered in order to divide participants into a high content knowledge group and a low content knowledge group. To extend the research problem, the same “Difficulty Factors Analysis” ranking task (Nathan & Petrosino, 2003) was conducted in high and low algebra content knowledge groups, composed of pre-service teachers of two different majors to assess if the study results replicated Nathan and Petrosino’s (2003) results. After conducting Nathan and Petrosino’s (2003) ranking task, interviews with eight participants were used to explore the nature of the “expert blind spot” and pedagogical content knowledge. The eight participants were selected from the high content knowledge group and the low content knowledge group.

The research questions were:

1. What is the level of pre-service teachers’ algebra content knowledge?
2. How do the high content knowledge participants and low content knowledge participants perform on the same Difficulty Factors Analysis ranking task (Nathan & Petrosino, 2003)?
3. What are the pre-service teachers’ perspectives in teaching algebra?

The first and second research questions were used to examine pre-service teachers’ algebra knowledge and their pedagogical content knowledge by conducting quantitative research design. The third research question was based on the findings of first and second research questions to explore pre-service teachers’ perspectives in

teaching algebra. The “expert blind spot” hypothesis was discussed in the pre-service teachers with high content knowledge and symbol-precedence groups. Pedagogical knowledge was also discussed in the pre-service teachers with verbal-precedence groups.

Definition of Terms

Pre-service mathematics teachers included pre-service elementary education teachers with a mathematics minor and pre-service secondary mathematics education teachers. These pre-service teachers were students, who had enrolled at a Midwest university in the spring of 2005 and nearly completed teacher education courses. These pre-service teachers were either ready for student teaching or nearing completion of all core courses in a mathematics education major or minor.

Mathematics content knowledge referred to a teachers’ personal mathematics understanding in algebra. In this study, the algebraic content knowledge included algebraic conceptual knowledge, algebraic procedure knowledge, and the ability to solve algebraic equation problems and story problems.

Mathematics pedagogical content knowledge referred to knowledge about the representation of content knowledge in a form that makes it comprehensible to students within the classroom (Shulman, 1986). In this study, the pedagogical content knowledge referred to the pre-service teachers’ understanding of what makes learning specific aspects of a concept easy or difficult. Teaching algebra in this study was used as the specific domain to explore pre-service teachers’ pedagogical content knowledge. For teaching algebra, the pedagogical content knowledge included useful forms of representation for promoting understanding of key characteristics of algebra or problem

solving. It also included knowledge of powerful analogies, illustrations, examples, explanations, metaphors, and demonstrations that can promote student understanding. Pedagogical content knowledge in teaching algebra (Nathan & Petrosino, 2003) might also include the understanding of the preconceptions that students of different mathematical achievement and backgrounds brought with them when learning algebra.

Teachers' "expert blind spot" referred to Nathan and Petrosino's (2003) EBS hypothesis that, in some situations, having a high degree of content knowledge might in fact be associated with the "symbol-precedence" in pedagogical content knowledge. In general, the "expert blind spot" in this study referred to the pre-service teachers who used symbol-precedence in teaching algebraic knowledge.

Summary of Chapter 1

This chapter established the foundation for the study. Purpose, background, significance, limitations, delimitations, research questions and the definition of terms were discussed in this chapter. The following chapter is the review of the related literature. The literature review presents teachers' knowledge, a model of teachers' content knowledge and classroom teaching, review of the literature on school algebra learning and teaching and Nathan and Petrosino's Difficulty Factors Analysis ranking task. Chapter 3 explicates the design of the study, including methodology, introduction of methodology, procedure and data collection and data analysis. Chapter 4 presents the results of the study and discussion. Chapter 5 summarizes the study with further discussions of implications and limitations.

CHAPTER 2

REVIEW OF LITERATURE

Introduction

The following literature review is guided and shaped by the work of Gall, Borg, and Gall as illustrated in their textbook, *Educational Research: An Introduction* (1996). This review relates the method, theory, and prior research to the present descriptive study. It presents a selection of available literature that reports the knowledge of pre-service teachers. The review is intended to present evidence to justify and motivate the present study. First, teacher knowledge is discussed. The knowledge needed for teaching includes knowledge of mathematics, knowledge of the connections within the subject as well as with other subjects, and knowledge of students' understandings and misunderstandings. Second, a model of teachers' knowledge and classroom instruction is presented. A structure of how content knowledge and pedagogical content knowledge can combine in the context of the classroom is presented. Third, a review of the literature on school algebra learning and teaching is presented. The research topics reviewed are categorized as school algebra, difficulty in learning and teaching algebra, and pre-service teachers' concept about teaching mathematics. In the final section of this chapter, Nathan and Petrosino's Difficulty Factors Analysis ranking task is presented. A structural development of the six questions in Nathan and Petrosino's (2003) ranking task from 1999 to 2003 is presented.

Teacher Knowledge

Teacher knowledge that includes synthesized teaching processes is the instrument of change in students' learning (Shulman 1986, 1987). The available research on teacher's knowledge led Fennema and Franke (1992) to propose a five-component framework of teaching knowledge. The framework includes content knowledge, pedagogical knowledge, beliefs, knowledge of students' cognition, and knowledge of the context of classrooms. This study explored two of the five domains of teachers' knowledge – that is, content knowledge and pedagogical knowledge. In addition, pedagogical knowledge can be specified as general pedagogical knowledge and pedagogical content knowledge (Shulman, 1986).

According to Shulman (1987), the seven categories of teacher knowledge are: (1) content knowledge; (2) general pedagogical knowledge, with special reference to the broad principles and strategies of classroom management and organization that appear to transcend subject matter; (3) curricula knowledge, with a particular group of the materials and programs that serve as “tools of the trade” for teachers; (4) pedagogical content knowledge, each teachers' own special form of professional understanding; (5) knowledge of learners and their characteristics; (6) knowledge of educational contexts, ranging from the workings of the groups or classroom, the governance and financing of school districts, to the character of communities and cultures; and (7) knowledge of educational purposes, values, and their philosophical and historical grounds.

Shulman (1987) argued that teachers' understanding of content is critical and paramount, irrespective of the pedagogy that is employed by the pedagogues. However,

among these categories, pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching (Shulman, 1987). It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to diverse interests and abilities of learners and how they are presented for instruction (Shulman, 1987).

Moreover, “Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue” (Shulman, 1987, p. 8). In essence, it is the teacher who plans the “learning trajectories” (Shulman, 1987, p. 8) of the students. Thus, the teacher is central and inextricable from the learning episodes that occur in the classroom. Furthermore, Shulman (1987) mentioned, “the manner in which that understanding is communicated conveys to students what is essential about a subject and what is peripheral” (p. 9). Pre-service teachers in teacher preparation programs need to know how to communicate to diverse students (Shulman, 1987). In this approach, “teachers must have a flexible comprehension in their content knowledge so that they can use alternative explanations of the same concepts of principles” (Shulman, 1987, p.9).

Grossman (1990) analyzed Shulman’s concept of pedagogical content knowledge of six secondary English teachers and presented a model of teaching specific subject matter that includes: “(1) what it means to teach a particular subject, (2) knowledge of curriculum in particular field, (3) knowledge of students’ understanding and potential misunderstandings of a subject area, and (4) knowledge of instructional strategies and representations for teaching particular topics” (p. 25).

Leinhardt and Smith's (1985) research focused on experienced and novice teachers' cognitive structures, and developed a model, composed of three concepts – agenda, scripts and routines. The model makes claims about the forms of teacher knowledge as an agenda in which teachers impose on the mathematical content to facilitate pedagogy. Scripts are specific plans for dealing with specific topics that allow teachers to interpret the mathematical content for pedagogy. Routines are “scripted sets of behaviors that allow teachers to carry out some activities in a relatively automated manner and with minimum cognitive load” (Sherin, Sherin, & Madanes, 1999, p. 361). Leinhardt's cognitive structure model explains teachers' behaviors based on data gathered from videotaping teachers in the classrooms, viewing the videotapes, and inferring from the videotapes teachers' cognitions and then using the inferred cognitive structures of teacher knowledge to explain the scripts and routines of the teachers (Sherin, Shein, & Madanes, 1999). The model is the most appropriate for analyzing teachers' pre-active, active, and post-active phases of instruction; always remaining focused on actual classroom practice.

Schoenfeld and the Teacher Model Group at Berkeley University have attempted to “explain why a teacher does what he or she does during the moment of instruction...to be able to account for different teaching styles and different types of lessons” (Sherin, et al., 1999, pp. 362-363). The model shows how teachers can teach at all levels, from the specific (grain size) to the general. The goals of the Teacher Model Group are to construct a model of teaching that “(1) accommodates all teaching in its architecture; (2) works at all levels of grain size, from planning curricula to planning lessons to

utterance-by-utterance interactions; and (3) provides a fine-grained explanation of how and why any teacher does what he or she does, in the midst of learning interactions” (Schoenfeld, 1999, p. 244).

The focus of the present study is not on the practices of teachers. Rather, it categorizes and catalogues the level of understanding algebraic content knowledge in pre-service elementary education teaching majors with a mathematics minor and in pre-service secondary education mathematics teaching majors. Understanding pre-service teachers’ knowledge of mathematics and pedagogy with respect to particular mathematical strands is useful knowledge for planning the effective education of the future mathematics teachers. The model from the Teacher Model Group does not quite address the interests of the present study because as a model, it is primarily concerned with teacher problem solving while teachers are in the act of teaching.

Shulman’s model was chosen for the present research over other models of teachers’ knowledge (Sherin, et al., 1999) because (a) it explains the phenomena of teacher knowledge; (b) Shulman’s model methodologically supports the use of interviews as a means to elaborate on observations and inquiry; (c) Shulman’s model does not attempt to explain behavior; rather, it is a model for analyzing the content of teachers’ knowledge. By Shulman’s (1987) concepts, it is reasonable to direct the exploration of pedagogical content knowledge in a specific subject matter. To analyze teacher knowledge in specific subject matter, Grossman’s (1990) model further developed pedagogical content knowledge.

Teacher knowledge can be considered in a different cultural context. Fennema and Franke's (1992) five-component framework of teachers' knowledge, investigations, and a comparison of different cultural contexts also shows some valuable factors in teacher knowledge, especially in the relationship between content knowledge and pedagogical content knowledge. Ma (1999) stressed the importance of the culture of teacher education on the efficacy of teachers. For example, she assumed that the differences between Chinese elementary teachers' and United States elementary teachers' understanding of elementary mathematics are partly explained by different educational experiences. Ma (1999) stated that "teachers' subject matter knowledge develops in a cyclic process of schooling, teacher preparation, and teaching" (p. 144). However,

...in China, the cycle spirals upward, when teachers are still students, they attain mathematical competence. During teacher education programs, their mathematical competence starts to be connected to a primary concern about teaching and learning school mathematics. Finally, during their teaching careers, they develop a teacher's subject matter knowledge, which I call in its highest form PUFM [Profound Understanding of Fundamental Mathematics]. Unfortunately, this is not the case in the United States. It seems that low-quality school mathematics and low-quality teacher knowledge of school mathematics reinforce each other. (Ma, 1999, p. 145)

Theoretically, Ma's (1999) categorization of teachers' knowledge along the dimensions of connectedness, multiple perspectives, basic ideas, and longitudinal coherence presents an appropriate way to categorize pre-service teachers' understanding of mathematics, its place in school mathematics, and its pedagogical implications.

To be effective at teaching, the teacher should first comprehend the subject matter with different degrees of flexibility and adaptability in order to transform the subject matter into powerful pedagogical content knowledge (Shulman, 1987). This present study

focuses on attempting to understand whether or not pre-service teachers with different levels of algebraic content knowledge have the necessary pedagogy that would foster meaningful student learning. It also investigates how these pre-service teachers intend to use their knowledge structures to facilitate students' learning.

A Model of Teachers' Content Knowledge and Classroom Teaching

Previous research (Ball, 1988b; Fennema & Franke, 1992; Leinhardt & Smith, 1985; Shulman, 1987) has recognized content knowledge and pedagogical content knowledge as being important cognitive aspects of teacher knowledge. Fennema and Franke (1992) put forth a model of teachers' knowledge as it occurs in the context of the classroom. Carpenter and Fennema (1991) proposed a model that illustrated how teachers' knowledge and beliefs influence classroom instruction and students' learning. These models provided a representation of how teachers' content knowledge and pedagogical content knowledge are used in classroom situations.

In building their model, Fennema and Franke (1992) examined Shulman's (1987) and Grossman's (1990) framework as well as the work of many others, including an examination of teacher knowledge as situated knowledge. The importance of their work on situated knowledge comes from examining the knowledge of pre-service teachers in their teachers' preparation program versus the difficulty of transferring their knowledge to an actual teaching situation. Knowledge acquired is not independent of the situation in which it is learned and used, and this component remains a referent by which knowledge is retrieved, interpreted, and used. It is necessary for knowledge to be set in a context that enables a learner and a teacher to connect knowledge to his or her broader culture.

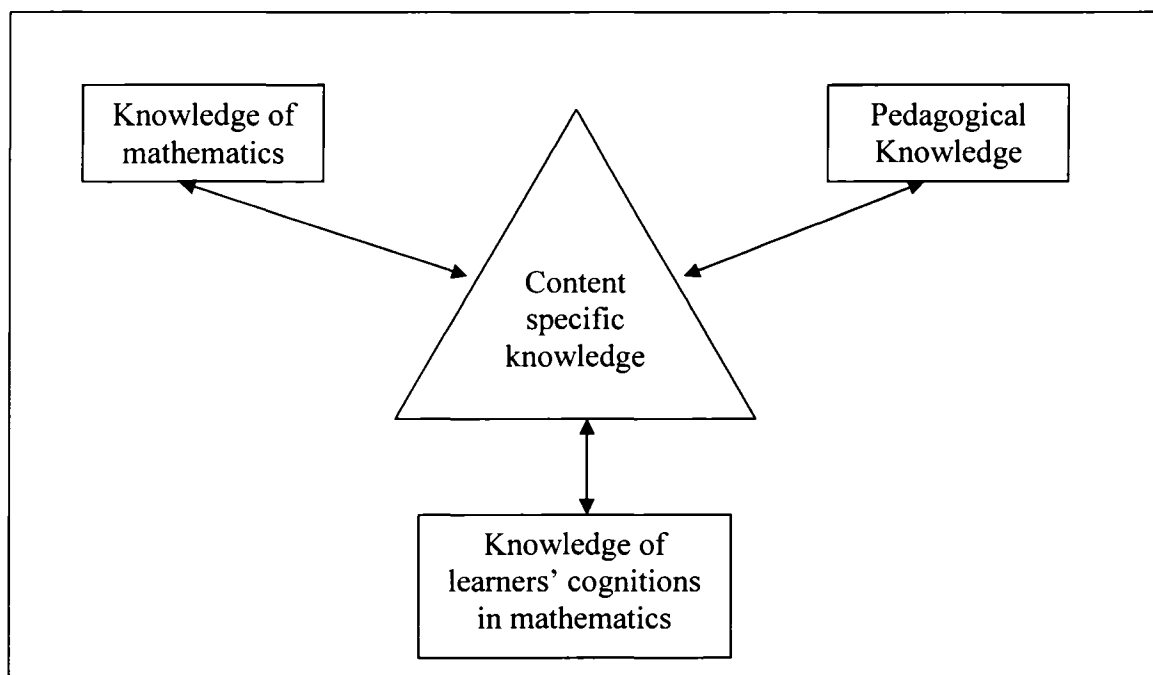


Figure 1. Teachers' knowledge in the classroom

Note: From "Teachers' Knowledge and Its Impact" by E. Fennema and M. Franke, 1992, *Handbook of Research on Mathematics Teaching and Learning*, p, 162.

In synthesizing previous work, Fennema and Frank (1992) used a model that centers on teacher knowledge as it occurs in the context of the classroom (see Figure 1). This model showed the interactive and dynamic nature of teacher knowledge. The larger rectangle (the frame of the figure) represented the context. The components in three smaller rectangles (knowledge of mathematics, pedagogical knowledge, knowledge of learners' cognitions in mathematics) are based on previous work and are comparable to Shulman's (1987) conceptualization. Shulman (1987) stated:

...the key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to *transform* the content

knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students. (p. 15)

Fennema and Franke (1992) considered the key word here to be “transform.” This transformation is not simple but must change as the students who are being taught change. The center triangle represents the coming together of knowledge in a given context. These authors stated,

The context is the structure that defines the components of knowledge and beliefs that come into play...to create a unique set of knowledge that drives classroom behavior...If the context in which the teacher is a part was to change..., the knowledge drawn upon by the teacher will also change. (p. 162)

Fennema and Franke (1992) felt that the future lies in understanding the dynamic interaction between these components as they relate within a context and how this transformation of knowledge leads to new knowledge formed from adaptive behavior.

The model proposed by Carpenter and Fennema (1991) integrated the perspectives of cognitive and instructional science to study teachers’ content knowledge, pedagogical content knowledge, teachers’ beliefs, and how teachers’ knowledge and beliefs influence classroom instruction and students’ learning (see Figure 2). Programs based on this model embody principles of instruction consistent with what is known about students’ learning, thinking, and problem solving within a domain and what is known about teachers as active, thoughtful professionals.

The framework provides a basis to study teachers’ knowledge about different content, the relative difficulty of aspects of the content, and teachers’ knowledge of the different strategies that students use to solve problems within the content. It is then possible to establish links between teachers’ knowledge and how that knowledge is used

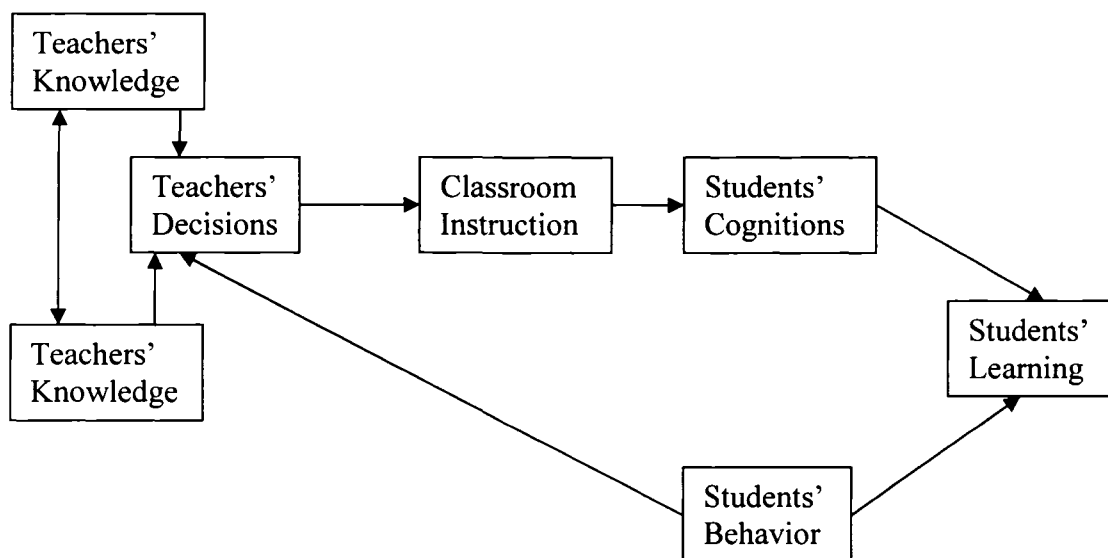


Figure 2. General model for research and curriculum development integrating cognitive and instructional science

Note: From “Research and cognitively guided instruction” by T. P., Carpenter and E. Fennema, 1991, *Integrating Research on Teaching and Learning Mathematics*, p. 16.

in specific classroom activities or to investigate how particular instructional activities are reflected in students’ learning within a domain. This implies an important role between teachers’ content and pedagogical content knowledge and teachers’ decision-making.

Due to the importance of the role of mathematical understanding, which plays in the educational and practical lives of individuals, an examination of the role of subject matter knowledge is warranted in this dynamic and complex process representing teacher knowledge. The importance of subject matter knowledge has not always been the case, though the common sense argument has appeared time and again. Ball (1991) followed

the research on teacher subject matter knowledge through three phases. The earliest research compiled characteristics of teachers whom others perceived as effective. In Begles' (1979) survey of research in mathematics education, he found that there was no rational defense for the number of mathematics courses we ask prospective teachers to take and no grounds for asking for more. He found that the more the teacher knows, as measured in past studies by the number of college courses taken or performances on standardized exams, does not demonstrate a strong positive correlation with the amount of students' knowledge. The counter-intuitiveness of this data leads one to examine the assumptions on which these studies are based. The questions of the assumptions would likely be: Are the number of college courses taken a good measure of teacher subject matter knowledge, and are the measures used for what students know reflective of educational goals?

Research then turned from teachers' characteristics as predictors to teachers' behaviors because it is what a teacher does that affects students' learning (Grouws & Koehler, 1992). This perspective has subject matter as part of the context, but not as a focus for observation because it wasn't considered an integral part of teachers' behaviors. As researchers expand the concept of behavior to include teacher decision-making, the role of teacher subject matter knowledge begins to reappear as a potentially significant variable (Ball, 1991).

So, we are at a point in time when subject matter knowledge is worthy of investigation in a meaningful manner. Although Fennema and Franke (1992) lend equal weight to the three knowledge components in their model, the recent research that

highlights the serious lack of meaningful subject matter knowledge in perspectives and certified teachers (Ball, 1988a, 1988b, 1990; Ball & Feiman-Nemser, 1988; Borko et al., 1992; Leinhardt & Smith, 1985). This indicates the necessity for a concerted effort to investigate the role this knowledge has in conjunction with other components of the model and of its effects on classroom practices and the decision-making process.

For example, Ball and Feiman-Nemser's (1988) work on beginning teachers' use of textbooks and teachers' guides highlighted the effect of teacher subject matter knowledge on decisions concerning the use of curricula materials. Many of the beginning teachers lacked a strong or connected knowledge of the material in the textbooks. Unsure of how to adapt textbook material appropriately, they would make modifications that distorted the point of the lesson, or they would omit information that they realized several lessons later was important for the development of a coherent system of knowledge. These teachers also lacked the knowledge about which mathematics knowledge is essential for evaluating the appropriateness of the material for their classes.

If we accept the premise that the goal of a teacher education program is to help teachers implement programs of instruction that develop deep understanding of mathematics, it seems reasonable to expect that teachers have and are continuing to develop a well-connected and extensive knowledge base to bring to mathematics teaching. This knowledge base has been shown to effect teachers' instructional decisions and classroom practices. It is important to have a clear understanding of what role content knowledge and pedagogical content knowledge of mathematics, and in particular of algebra, play in conjunction with other factors in classroom teaching.

Review of the Literature on School Algebra Learning and Teaching

Some aspects of algebraic concepts have not been well understood by students (Kieran, 1992). To address this problem, it is important to understand what pre-service teachers “know” or “don’t know” about algebraic concepts and about the students’ understanding of the concepts. The following studies reviewed in this section are categorized under school algebra as difficulty in learning algebra, teaching algebra, and pre-service teachers’ concept in teaching mathematics. These studies will examine pre-service teachers’ understanding of algebraic concepts and their developing pedagogical content knowledge of the concepts.

School Algebra

Usiskin (1988) conducted an investigation on conceptions of school algebra. The algebra taught in school has quite a different cast from the algebra taught to mathematics majors (Usiskin, 1988). He defined school algebra as having to do with the understanding of “letters” (today we usually call them variables) and their operations. He offered five different algebra problem forms produced by two numbers: (1) $A = LW$ (a formula); (2) $40 = 5x$ (an equation to solve); (3) $\sin x = \cos x - \tan x$ (an identity); (4) $1 = n * (1 / n)$ (a property); and (5) $y = kx$ (an equation of a function of direct variation). Five different equation forms will produce five different results. According to Usiskin’s (1988) article, algebra can be reduced to the concepts of (a) algebra as generalized arithmetic; (b) algebra as a study of procedures for solving certain kinds of problems; (c) algebra as the study of relationships among quantities; and (d) algebra as the study of structures.

Chazan (2000) illustrated the claim that there are conflicting conceptualizations of relationships between arenas of mathematical exploration (sub-disciplines like Linear Algebra and Abstract Algebra) and school algebra. Debates about the future of school algebra can be conceptualized as conflicts over how to present students with the nature of the x 's and y 's of school algebra. Chazan (2000) offered the following different perspectives in school algebra. One way to define the notion of algebra, according to George Peacock (1791-1858), is "algebra as a generalization of arithmetic", a point of view that Peacock called arithmetical algebra. People who are looking to the theory of equations for guidance will see polynomial equations (e.g., linear equations, quadratics, cubics, quartics, quintics, etc.) as the fundamental object of study in school algebra. Applied mathematicians might suggest that school algebra is fundamentally about mathematical models for non-mathematical situations. Those grounded in the field of abstract algebra might suggest that the true objects of study in school algebra are not the calculation procedures built up out of addition and multiplication operations on number systems. Instead, they believe algebra is about binary operations in general. Commonly used addition and multiplication are just two examples, which operate on sets—number systems being one type of set. This point of view is behind many of the innovations of the "New Math" (Thom, 1986 as cited in Chazan, 2000). If approaches to school algebra were based on Abstract Algebra, students might be asked to move beyond the number system. While students might work with equations with coefficients, for example: $x(a - bx)^2 = 0$, the literal symbols might stand for other mathematical objects, such as matrices, polynomials, or sets, rather than numbers.

To sum up, it is difficult to re-conceptualize high school algebra courses (Chazan, 2000). While mathematics educators debate the future direction of the algebra curriculum, they have not arrived at a consensus, and it is unclear whether many teachers are informed about this debate (Chazan, 2000).

Difficulty in Learning Algebra

Booth (1988) conducted a research project with the algebra strand of Strategies and Errors in Secondary Mathematics (SESM) in the United Kingdom. The students involved in this research were in grades 8-10. Fifty students were interviewed in-depth. Four trends of errors were observed in the responses. First, in arithmetic, the focus of activity was in finding particular numerical answers. In algebra, however, the focus was on the derivation of procedures and relationships and the expression of these in general, simplified form. Second, students' attempts to simplify expressions such as $2a + 5b$ would produce $7ab$ as an answer. This was evidence showing that children who have never studied algebra before have a strong tendency to "simplify" an expression such as $a + b$ to ab (Booth, 1984). Third, the letters "m" and "c" for instance, may be used in arithmetic to represent "meters" and "centers," rather than representing the number of meters or the number of centers, as in algebra. Confusion over this change in usage may result in a "lack of numerical referent" problem in students' interpretation of the meaning of letters in algebra. Fourth, algebra is not separate from arithmetic. Indeed, it is in many respects "generalized arithmetic." To understand the generalization of arithmetical relationships and procedures requirements should be to apprehend the arithmetical context.

To assess a teachers' pedagogical content knowledge, Booth (1988) asked teachers to respond to students' misconceptions or errors. After interviews, Booth concluded, "Children's difficulties in learning algebra are by no means exhaustive." This information may, however, serve to shed some light on the kinds of difficulties children are likely to experience when they begin to study algebra.

Kieran's (1992) article investigated several questions: (1) What compels many students to memorize the rules of algebra? (2) What makes school algebra difficult to learn? (3) Is the content of algebra the source of the problem or is it the way it is taught that causes students to misunderstand the subject? Kieran distinguished the way in which the terms "procedure" and "structural" are being used to explore the three research questions. Procedure refers to arithmetic operations carried out on numbers to yield numbers. Structural, on the other hand, refers to a different set of operations that are carried out, not on numbers, but on algebraic expressions.

Kieran (1992) reported most students never reach the structural part of the procedure-structural cycle. Students tend to memorize a pseudo-structural content. Students can develop structural conceptions of certain aspects of algebra if they are provided with experiences involving the field properties in both arithmetic and algebraic settings.

Teaching Algebra

Chalouh and Herscovics (1988) carried out a study of a teaching experience involving six children (12 – 13 years of age). The investigators sought, first, to determine the feasibility of a geometric approach to constructing meaning for algebraic expression

and, second, to uncover the cognitive obstacles associated with such an approach. Chalouh and Herscovics designed an instructional sequence involving algebra representations for arrays of dots, line segments, and areas of rectangles. It was found that the lessons helped the children develop meaning for expressions such as $2a + 5a$, but that most of the children were unable to interpret this expression as $7a$. These results suggest that constructing meaning for algebraic expressions does not necessarily lead to spontaneous development of meaning for the simplification of algebraic expression. This study also showed that beginning algebraic concepts are different from those concepts used in arithmetic.

Kieran (1988) assessed two different approaches to teaching algebraic concepts. Arithmetic and algebra approach groups were designed in a three-month teaching experiment for six 7th grade participants. The arithmetic approach focused on the given operations. The algebra approach focused on the inverses of the given operations. Examples are $5 + a = 12$ and $2c + 15 = 29$. Arithmetic approach groups treat the letter as a number. Algebra approach groups answers referred to the inverse operations necessary to find the value of the letter. The way the students' solutions evolved throughout the project showed three sublevels may be involved in the learning process. The first is reconstructing one's view of the letter in an equation to encompass the notion of letter as a number within the given sequence of operations. The second involves translating this conceptualization of a letter into the related equation-solving process of trial-and-error substitution. The third requires replacing the process of substitution by the process of performing the same operation on both sides. In conclusion, those students who belonged

to the algebra group preferred to use the transposing solutions. Those who belonged to the arithmetic group were able to make sense of the taught procedures of performing the same operation on both sides and used these procedures by the end of the project. These findings suggest that the construction of meaning for this solving procedure of the learning processes may take time. Instruction that provides for both of these precedences will probably be more successful than instruction that is geared to just one.

Pre-service Teachers' Concepts in Teaching Mathematics

Ball's (1990) research reported on the subject matter knowledge of 252 pre-service elementary and secondary mathematics teachers. These prospective teachers were examined on their understanding of mathematics by questionnaires. Based on this data, the article explored three common assumptions about learning to teach elementary or secondary mathematics: (1) that traditional school mathematics content is not difficult, (2) that pre-college education provides teachers with much of what they need to know about mathematics, and (3) that majoring in mathematics ensures subject matter knowledge.

Ball's (1990) article reported that the elementary pre-service teachers and the secondary pre-service teachers (who were majoring in mathematics) had significant difficulty "unpacking" or understanding the meaning of division with fractions. These results fit with evidence from interviews that the teacher education students' substantive understandings of mathematics were both rule bound and compartmentalized. To answer the first assumption of this article, traditional teacher education implied a message: "If you can 'do' these topics, then you can teach them" (p. 462). Throughout the interviews

in the article (1990), many college students had difficulty working beneath the surface procedural level of so-called “simple” mathematics. To answer the second assumption in this article, the article showed that prospective teachers study very little school mathematics content as part of their formal teacher preparation. Even mathematics majors, who had taken over seven college mathematics courses, did not demonstrate meaningful understanding or even the knowledge of mathematical concepts, procedures, or terms. To answer the third assumption, the article showed that even though secondary teacher candidates, who were mathematics majors, had taken more mathematics, this did not afford them a substantial advantage in articulating and connecting underlying concepts, principles, and meanings. Moreover, studying calculus does not usually afford students the opportunity to revisit or extend their understanding of arithmetic, algebra, or geometry, the subjects they will teach. The data in Ball’s (1990) article also suggests the biggest difference between the two groups was that most of the secondary candidates were confident that they knew mathematics and were less tentative in their responses. If they could not figure something out, they assumed they were “rusty” (p. 464). The elementary candidates were more anxious and more convinced that they did not know mathematics.

Ebert (1993) examined prospective secondary teachers’ subject matter and pedagogical content knowledge on functions and their graphs. Ten pre-service teachers were given a written assessment of subject matter knowledge and were asked to respond to vignettes concerning students solving problems about functions. The vignettes included problems on the motion of projectiles, on piece-wise defined functions, on

composition of functions, and on inverses. These were selected to both evaluate the subjects' understanding of difficult concepts as well as to assess their response to common student problems and misconceptions. Three of the ten participants had misconceptions, an inability to define functions, and poor use of notation. The majority had a good knowledge base, although four still exhibited the picture-as-graph misconception. They also had some difficulty expressing the distinctions between constant and variable functions, especially in the context of the composition of functions.

With regard to pedagogy, a majority of the prospective teachers addressed misconceptions as points where relearning was necessary, often resolving issues by simply telling students the correct procedure or answer. Three of the ten, those who also had good subject matter knowledge, exhibited markedly different beliefs about approaching students' misconceptions. They felt students could and should examine various representations of functions to test conjectures and that making connections between representations was an important part of doing mathematics. They also emphasized the role technology could hold in testing these conjectures.

Nathan and Koedinger (2000a) used a Difficulty Factors Analysis ranking task to investigate elementary, middle, and high school mathematics teachers. One hundred five subjects answered the Difficulty Factors Analysis ranking task, which is based on problem-solving difficulty. Analyses (2000a) suggest that teachers hold a symbol-precedence view of students' mathematical development. High school teachers were most likely to hold the symbol-precedence view and made the poorest predictions of students' performances.

Nathan and Koedinger (2000b) used the Difficulty Factors Analysis ranking task on mathematics teachers and educational researchers. The predictions deviated from algebra students' performances but closely matched a view implicit in textbooks. The Symbol Precedence Model of development of algebraic reasoning was contrasted with the Verbal Precedence Model, which provided a better quantitative fit of students' performance data. The meaning of symbol precedence model was that "through reliance on and repeated exposure to textbooks, teachers internalize the symbol precedence view as a basis for their predictions of problems difficulty for students" (Nathan & Koedinger, 2000a). The verbal precedence model, based on students' problem-solving-process data, suggested "verbal competence and the associated reliance on guess-and-test and unwinding strategies are hypothesized to precede symbol-manipulation skill for both arithmetics (result-unknown) and algebraic (start-unknown) problems" (Nathan & Koedinger, 2000a). The two models, symbol-precedence and verbal precedence were used in this study to explore pre-service teachers' pedagogical content knowledge.

Nathan and Petrosino (2003) examined the relationship between pre-service secondary education teachers' subject-matter expertise in mathematics and their judgments of students' algebra problem-solving difficulties. The hypothesis of the "expert blind spot" (hereafter, EBS) of this study is that well-developed knowledge of subject matter can lead people to assume that learning should follow the structure of the subject-matter domain rather than the learning needs and developmental profiles of novices. Forty-eight participants completed a ranking task. Three trends of EBS were observed in the response. First, based on the research and Nathan's previous research

(Nathan & Koedinger, 2000a), it appears that educators with greater subject-matter knowledge tend to view students' development through a domain-centric lens and, consequently, tend to make judgments about students' problem-solving performance and mathematical development that differ from actual performance patterns in predictable ways. This study showed educators with high mathematics knowledge tend to follow a symbol-precedence. Based on Nathan and Petrosino's (2003) article, it is evident among pre-service teachers, regardless of their affiliation with secondary mathematics, that their teacher knowledge was influenced by the choice of curriculum. Second, if participants who were strong in mathematical problem solving were simply drawing on their own experiences, the symbolic equations and story problems composed of the same quantitative relations would be ranked as similar in difficulty level. Third, pre-service teachers possessed both subject matter knowledge and pedagogical content knowledge on the study, but their knowledge may be inadequate or even include conflicting elements. This was because educators with a symbol-precedence view make inaccurate predictions about students' problem solving performance. Fourth, the study showed that educators who exhibit EBS may have the requisite subject matter knowledge and pedagogical content knowledge for the general topic at hand, but as they apply that knowledge to a specific area of mathematics, such as algebra instruction, the bodies of knowledge come into conflict. The conflicting ideas may lead to a view of student development and performance that was influenced by the view derived from the prevailing knowledge of the profession.

Nathan and Petrosino (2003) reported that educators with requisite subject matter knowledge and pedagogical content knowledge could make inaccurate predictions about problem difficulty. The major issue in this study is to explore what it means to have the proper pedagogical content knowledge to make knowledge-based decisions. This exploration would be in theory and in practice about the development and application of educators' pedagogical content knowledge.

Nathan and Petrosino's Difficulty Factors Analysis Ranking Task

The Difficulty Factors Analysis ranking task is reported by Nathan's several articles (2000a, 2000b, 2003). Nathan and Koedinger (2000a) described the hidden structure of the six problem models in the ranking task (see Appendix E). There are two categories in the hidden structure. First, problems 1, 2, and 3 are in arithmetic format and problems 4, 5, and 6 are in algebra format. Second, problems 1 and 4 are symbolic equations presented by symbolic models. Problems 2, 3, 5, and 6 are verbal presentations. Problems 2 and 5 are word equations. Problems 3 and 6 are story problems. The ranking task was used to predict that arithmetic problems are easiest within each level of representational format no matter what the symbolic and verbal presentation is, and that the ability to solve symbolic forms strictly precedes the ability to solve story and word problems, called symbol-precedence (Nathan & Koedinger, 2000a).

According to Nathan and Koedinger (2000a), using the ranking task showed the predictions of high school mathematics teachers and mathematics education researchers deviated from algebra students' performances but closely matched a view implicit in textbooks. Students in the original study ($n = 76$) exhibited much lower performance

levels on algebra problems (50% of students correctly solved algebra problems) than arithmetic problems (64% of students correctly solved arithmetic problems). The next year, students ($n = 171$) replicated the six problems and showed a similar pattern of the results (46% of students correctly solved algebra problems and 70% of students correctly solved arithmetic problems).

Nathan and Koedinger (2000b) used the ranking task with elementary, middle, and high school mathematics teachers ($n = 105$). High school teachers in Nathan and Koedinger's (2000b) study were most likely to hold the algebra view and made the poorest predictions of students' performances. Middle school teachers' predictions in Nathan and Koedinger's (2000b) study were most accurate. According to the results, Nathan and Koedinger (2000b) suggested that high school teachers with their extensive content training might be particularly susceptible to an EBS. They overestimated the accessibility of algebra representations and procedures for students' learning of introductory algebra.

Nathan and Petrosino (2003), using the ranking task on pre-service teachers ($n = 48$), examined the relationship between pre-service secondary teachers' content knowledge and their judgments of students' algebra problem-solving difficulty. The study in Nathan and Petrosino (2003) reported that participants with more advanced mathematics education, regardless of their program affiliation or teaching plans, were more likely to view symbolic reasoning and mastery of equations as a necessary prerequisite. This view is in contrast with students' actual performance patterns.

Summary of Chapter 2

The review of the literature revealed what mathematics teachers understand about teaching algebra and how students have difficulty in learning algebra. Mathematics teachers' knowledge of teaching algebra was inferred from studies on the connections from arithmetic to algebra. Students tended to answer algebra problems by using arithmetic concepts. However, teachers liked to represent algebra problems to students by their own expert algebraic concepts. Based on the literature review, this study tried to explore the gap between pre-service teachers' mathematics content knowledge and their perspectives in teaching knowledge. The two models (Nathan & Koedinger, 2000a), symbol-precedence and verbal precedence, were used to explore pre-service teachers' pedagogical content knowledge. The literature reviews also supported three coding points: (a) teaching algebra meaningfully; (b) understanding students' needs; and (c) connecting algebra to students' understanding. The three coding points could help to understanding the relationship between pre-service teachers' mathematics content knowledge and pedagogical content knowledge.

CHAPTER 3

DESIGN OF THE STUDY

Methodology

Introduction

This study explored the relationship between mathematics content knowledge and pedagogical content knowledge of pre-service teachers in algebra. The study was divided into two sections. In the first section, each participant completed an Algebra Content Knowledge test and then replicated a Difficulty Factors Analysis ranking task from Nathan and Petrosino's (2003) articles. The second section of this study involved in-depth interviews with participants who have different performances in their algebra content knowledge test and Nathan and Petrosino's (2003) ranking task (e.g., high content knowledge/symbol-precedence; low content knowledge/verbal-precedence, etc.). The interviews investigated the participants' mathematics content knowledge and their perceptions of pedagogical content knowledge in teaching algebra. The interview categories included: (a) what it means to teach algebra, (b) knowledge of curriculum in algebra, (c) knowledge of students' understanding and potential misunderstandings of algebra, and (d) knowledge of instructional strategies and representations for teaching algebra topics (Grossman, 1990). Furthermore, the interviews provided the researcher with data for an in-depth analysis of the pre-service teachers' understanding of the mathematics knowledge for teaching algebra.

This chapter discusses the methodology, description of methodologies, procedures and data collection, data analysis, and research questions.

Methodological Foundations

This study was built on previous research by Booth (1988); Kieran (1992); Koedinger and Nathan (1999); Nathan and Koedinger (2000a, 2000b); and Nathan and Petrosino (2003). The first section of the study used descriptive statistics to report participants' content knowledge level in algebra by an Algebra Content Knowledge (ACK) test and then used Pearson's correlation (r), t test, Fisher's transformation z and χ^2 to analyze the Difficulty Factors Analysis (DFA) ranking task (Nathan & Petrosino, 2003). In the second section, an interview with participants who have different results between their performances in the ACK test and the DFA ranking task was conducted. Kahan, Cooper, and Bethea (2003) and Stake (1995, 2000) supported the specific categories in a case study to analyze the interview data. The choice of participants for case studies in the second section was based on the first section. In this study, the participants were selected for case studies when the participants had (a) high performances in the ACK test and low performances in the DFA ranking task; (b) low performances in the ACK test and high performances in the DFA ranking task; and (c) high performances in ACK test and high performances in the DFA ranking task. Ideally, the three case studies were used to discover the participants' pedagogical content knowledge under Grossman's (1990) structure. To support the analyses of pre-service teachers' pedagogical content knowledge, Kahan, Cooper, and Bethea's (2003) study was used to technically analyze the data. Interviews presented the best method for learning the reasons and motivations of the three participants' actions and choices of explanations (Stake, 2000). For the present study, the interviews were conducted to identify how pre-

service mathematics teachers organized and categorized the domain of algebra and how they applied their pedagogical content knowledge of algebra.

Description of Methodology

Participants

The targeted population for this study was pre-service teachers who were nearing the completion of a mathematics teaching certification program. Pre-service teachers who had enrolled in the classes Introduction of Algebraic Thinking for Elementary Teachers and Problem Solving for Teaching of Secondary Mathematics at a Midwest comprehensive university in the 2005 fall semester were invited to participate in the study. Normally, when students are near the completion of their educational courses, they take these final classes. Most students in these classes are either elementary education majors with a mathematics minor or secondary mathematics education students. None of the participants had a teacher's certificate.

Participants in a pilot study were selected from pre-service teachers who are elementary education majors with a mathematics minor ($n = 17$) and current secondary education mathematics teachers ($n = 13$). Thirty participants completed the algebra content knowledge test (Appendix C). The 30 participants included pre-service teachers and current elementary, middle school and secondary education mathematics teachers. Of the 13 current secondary education mathematics participants, three experienced middle school teachers and one secondary mathematics teacher were invited to participate in the pilot interview. They answered all of the interview questions (Appendix E) and provided comments used to revise the interview questionnaire.

Instrumentation

There were two sections in the instrumentation of the study. Before processing the study, a consent form was used to solicit participation in the study (Appendix A). It included a brief description of the purpose of the study and the research procedures. The processing of the study was conducted after participants returned the form.

Algebra Content Knowledge test (ACK test). An algebra test was developed to assess conceptual knowledge and meaningful understanding of algebra. The test was designed to investigate pre-service mathematics teachers' understanding of the complexities of algebraic computation ideas. The test of algebraic knowledge combined symbolically conceptual and procedural knowledge of algebra because the purpose of the research was to investigate the participants' cognition of symbolic concepts based on algebraic consideration. The test focused on the level of participants' mathematics content knowledge of algebra. The test addressed the following aspects of algebra knowledge: (1) transformation from arithmetic to algebra; (2) the concepts of variables; (3) the concepts of patterns and relationships; (4) graphing; (5) algebraic expression; (6) symbolic computations; and (7) the use of algebra in solving and modeling mathematics problems.

The algebra content knowledge test was developed after reviewing experts' recommendations on what pre-service teachers ought to know about the content knowledge of algebra (Conference Board of Mathematical Sciences, 2001; Usiskin, 1988), and reviewing instruments used in a mathematics department scholarship test. In particular, the algebra content knowledge test measured fundamental notions of variables,

patterns, relationships, and graphing. Professors in mathematics education, mathematics, and educational measurement evaluated the test and made suggestions for improving it. A revised test was used in the pilot study. The test was refined again to be the final algebra content knowledge test.

There were ten questions in the final Algebra Content Knowledge test. All questions were symbolic questions related to algebraic computation and graphing. The figure 3 contains a holistic rubric for scoring the ten items in the Algebra Content Knowledge test.

Forty points were possible in the test of Algebra Content Knowledge. Descriptive statistics of mean, range, and standard deviation were used to analyze the scores of the participants.

Difficulty factors analysis ranking task (DFA task). The first section of this study also included the Difficulty Factors Analysis ranking task, previously conducted by Nathan and Petrosino (2003). Six kinds of mathematics problems were ranked from the easiest to the hardest in the survey. The Nathan and Petrosino's (2003) DFA ranking task was used to predict whether pre-service teachers with high content knowledge tended to have the "expert blind spot" (hereafter, EBS) which might indicate symbol-precedence in their pedagogical content knowledge.

The null hypothesis in the DFA ranking task was that participants' rank ordered scores (Pearson's r) was equal to 1. It indicated that participants were likely to favor symbol-precedence (Nathan & Koedinger, 2000a) in their pedagogical content knowledge (in Nathan and Petrosino's study [2003] called "expert blind spot"). In the pilot study for

the present study, participants with high content knowledge highly matched the “expert blind spot” hypothesis.

4 points	All procedures and solutions are accurate, complete and appropriate.
3 points	Have minor errors in at most one part of the solution process, even though the conceptual knowledge of solving problems is correct.
2 points	Have serious and major errors in procedural knowledge and conceptual knowledge.
1 point	Do not understand the problem and try to use an irrelevant concept or procedure to solve the questions.
0 point	Left the answer space blank

Figure 3. The holistic rubric for scoring the Algebra Content Knowledge test

Interview. An in-depth interview was conducted with three kinds of participants. Participants with high content knowledge (e.g., high performance in ACK test) and symbol-precedence in pedagogical content knowledge (e.g., low performance in DFA ranking task), participants with low content knowledge and verbal-precedence in pedagogical content knowledge, and participants with high content knowledge and verbal-precedence in pedagogical content knowledge were interviewed. These participants were chosen after an analysis of the data from the ACK test and DFA ranking task (Nathan & Petrosino, 2003).

The interview components of pedagogical content knowledge were directed by the concepts of Grossman's (1990) analysis of pedagogical content knowledge. Grossman's (1990) ideas about pedagogical content knowledge, used as basic inquiries, were extended to the specific subject domain, teaching algebra. Interview questions to explore pedagogical content knowledge of algebra were developed after gathering information on pre-service teachers' content knowledge of algebra, their perceptions of teaching algebra, and how they represented their algebra content knowledge to students. The interviews provided the interviewees with opportunities to respond how they would deal with common student misconceptions about algebra in the classroom. Further, the interviews provided additional situations for analyzing student understanding of the content of algebra and their pedagogical content knowledge by using metaphorical thinking.

Procedure and Data Collection

Participants in the study were selected during the fall semester, 2005. The researcher visited classes of pre-service teachers to recruit participants for the study. Participants in the study were encouraged to help further the mathematics education field's understanding of pre-service teachers' knowledge of school mathematics. Moreover, the researcher emphasized pre-service teachers' responsibility to safeguard participant anonymity by not conferring with their peers about their participation status. In addition, interviewees were told that the audiotape of the interview was kept secure and confidential.

All instruments were piloted in June 2004. Four summer school students were enlisted for the pilot study. The four students offered comments to refine the instruments, the interview questions, and the understanding of mathematics education. The pilot study revealed that some participants whose mathematical knowledge was superior to pre-service undergraduate students had difficulty defining algebra and representing the meaning of variable. According to the results of the pilot study, a 10-item Algebra Content Knowledge test was created for use in the main study (see Appendix F). This 10-item Algebra Content Knowledge test was selected from an algebra test designed by mathematics education professors at the University of Northern Iowa. It was submitted to a mathematics educator, Dr. Leutzinger. He revised the items based on his expertise of mathematics education, to divide all participants as high content knowledge group (above-median group) and low content knowledge group (below-median group).

First Section

The study was organized into two sections. The first section involved quantitative investigations of algebraic content knowledge and exploration of pedagogical content knowledge. Pre-service teachers enrolled in the classes *Introduction of Algebraic Thinking for Elementary Teachers*, or *Problem Solving for Elementary Teachers* or *Teaching of Secondary Mathematics* were invited to participate in the study.

All participants were invited to answer the ACK test. The median score of the test results was used to divide participants into the high content knowledge group (above median score) and the low content knowledge group (below median score). Before the test, there was a demographic survey that asked participants' academic major and course

history (Appendix F). After finishing the ACK test, all participants replicated the DFA ranking task (Nathan & Petrosino, 2003). The instruments were administered in the following order: the ACK test with demographic survey and the DFA ranking task (Nathan & Petrosino, 2003).

The ACK test was utilized to explore the participants' conceptual and computational knowledge in algebraic fields. The DFA ranking task (Nathan & Petrosino, 2003) was used to evaluate participants' prediction of learners' needs in order to ascertain their knowledge of students' understanding of algebra. The two instruments were used to explore pre-service teachers' understanding of algebra content knowledge and their cognition of pedagogical content knowledge of algebra. The two instruments were completed individually and independently. The instruments in this section of the study were completed during a single class period of 50 minutes.

Second Section

After an initial analysis of the data from the first section of the study, three kinds of participants were invited to conduct one-on-one case studies. The subjects for the participations were selected from the participants in the first section. The selection of interviewees is based on participants' performance in the ACK test and the ranking task. In this study, the ACK test was used to test pre-service teachers' content knowledge level while the Pearson's r of Nathan and Petrosino's (2003) ranking task was used to test pre-service teachers' pedagogical content knowledge level. Based on the findings in the ACK test and ranking task, three kinds of pre-service teachers were selected for interview. Pre-service teachers with high content knowledge and symbol-precedence pedagogical

content knowledge (Pearson's r less than 0.25), those with low content knowledge and verbal-precedence pedagogical content knowledge (Pearson's r more than 0.75) and those with high content knowledge and verbal-precedence pedagogical content knowledge were selected for an interview concerning their perspectives in pedagogical content knowledge. The participants chosen for the case studies were four high Algebra Content Knowledge test score pre-service teachers with symbol-precedence pedagogical content knowledge, one high algebra content knowledge test score pre-service teacher with verbal-precedence pedagogical content knowledge, and three low algebra content knowledge test score pre-service teachers with verbal-precedence pedagogical content knowledge. The interviews attempted to uncover the complexities of the relationships between the content knowledge of algebra and knowledge of pedagogy specific to the teaching of algebra ideas.

The interview questions were designed to analyze participants' perspectives in teaching algebra. Grossman's (1990) four elements about pedagogical content knowledge were used to collect data. Kahan, Cooper, and Bethea (2003) specified Grossman's ideas as a two-dimensional array that illustrated the interaction of three elements of teaching algebra with four teaching processes. In the first dimension, three elements of teaching algebra were measured: (1) selections of tasks and representations; (2) motivation of content; and (3) development: connectivity and sequencing. In the second dimension, four teaching processes were measured: preparation, instruction, assessment, and reflection (see Table 1). In Table 1, each cell, as an interview category, represented an

intersection of teaching processes and elements using the data that the researcher used to demonstrate how content knowledge played out in the teaching behavior.

Table 1.

Category for interview on pedagogical content knowledge

Elements of Teaching	Process of Teaching			
	A Preparation	B Instruction	C Assessment	D Reflection
1. Selection of tasks and representations	1A	1B	1C	1D
2. Motivation of content	2A	2B	2C	2D
3. Development: connectivity and sequencing	3A	3B	3C	3D

The primary evidence of task and representation selection was in class preparation (cell 1A), but the framework lead one to also consider task selection during instruction (cell 1B), as when selecting a task to catch an unplanned teachable moment; assessment (cell 1C); or reflection on the lesson (cell 1D) when the teacher considered modifying or replacing tasks and representations (Kahan, et al., 2003).

In element 2, motivation of content indicated the teacher's ability to address or preempt student questions such as "Why is this important to know?" and "When are we ever going to use this?" In teaching variables, teachers needed to be able to plan a lesson (cell 2A), share the lesson (cell 2B) of the concept effectively with students, and motivate students to become interested in learning the concept of variables (Kahan, et al., 2003). Motivation of content (cell 2C) contributed relevance with the question "How does the assessment engage students in relevant, purposeful work on worthwhile mathematical activities?" (NCTM, 1995, p. 14). Finally, the pre-service teachers reflected on whether

the examples they thought would motivate students' interest (cell 2D) were successful (Kahan, et al., 2003). In this teaching element, content knowledge was used to enhance a teacher's sense of the structure of algebra.

The mathematical development of a lesson or unit was important for effective teaching. The content should not appear to be a collection of disjointed, isolated topics. It should be sequenced so that topics are studied in a sensible order with prerequisite content being taught or reinforced as needed. The evidence of the role of algebraic knowledge in connectivity and sequencing was in a teacher's preparation (cell 3A) and instruction (cell 3B; Kahan, et al., 2003). It was valuable to explore whether a teacher's assessments asked a student to reflect on how the ideas in a unit connect (cell 3C) or how pre-service teachers reflect on ways the ideas in a unit might have been reorganized to have maximum effect (cell 3D; Kahan, et al., 2003).

Each interviewee could select either to write an answer on the paper for each interview question or answer questions verbally. When asking questions about Nathan and Petrosino's (2003) six questions in the survey, the researcher prepared six cards for the six questions and wrote one question on each card. The interviewee could use the cards and rank order the questions from the easiest to the hardest. The cards could also be used to categorize (a) the algebra questions and arithmetic questions, (b) symbol equation, (c) the word equation or story problems.

Data Analysis

The first section described how to analyze the ACK test and the DFA ranking task (Nathan & Petrosino, 2003). The second section described how to analyze the interview.

The First Section

Algebra Content Knowledge test (ACK test). The first analysis was the description and categorization of the items on the Algebra Content Knowledge test. Each item in this test was categorized alone by level of difficulty, representational modality, knowledge structure, process, importance and nature of importance, and projected success rate. Each item was assigned a low, medium, or high level of difficulty depending on the computational knowledge and pre-requisite knowledge required for accurate resolution of that item. Each item was assigned one of the following representational modalities: variables, symbols, patterns and relations, graphing, and notions of function depending on the primary mode of the expected response. Each item was assigned one of the following processes: factual, recall, or problem solving. Finally, each item was ascribed a success rate depending on the level of difficulty ascertained from the pre-requisite knowledge, representational systems, conceptual depth of knowledge, and the process (recall or problem solving) associated with the item. The previous descriptions and categorizations served as an interpretive framework for the discussion of the results.

The ACK test was analyzed quantitatively. The quantitative aspects of the analysis involved scoring the 10 items for correctness. Each item had a possible score, ranging from 0 to 4 points. The total score possible for all correct responses in the test of algebra was 40. Descriptive statistics (mean, median, standard deviation) were used to analyze the participants' scores. A high content knowledge group included participants whose test score was above the median score. A low content knowledge group included

participants whose test score was below the median score. To examine the relationship between participants' content knowledge and pedagogical content knowledge, the study focused on participants with high performance in the ACK test and participants with low performance in the ACK test. If participants' ACK test scores were higher than the third quarter of the ACK test scores or lower than the first quarter of the ACK test, these participants were selected and their results of DFA ranking task were examined.

Difficulty Factors Analysis ranking task (DFA ranking task). The replicated ranking task in this study followed Nathan and Petrosino's (2003) quantitative analysis. Descriptive statistics including mean and standard deviation were represented. A DFA ranking hypothesis from the easiest to the most difficult problem in this task was the same as six problems ordering in the DFA ranking task (Nathan & Petrosino, 2003) 1, 2, 3, 4, 5, to 6. The average Pearson's correlation value (r) was used to compare the hidden ranking in each group. A t-test and χ^2 analysis was applied to compare the means of the six questions in the ranking task and 95% confidence interval was used to measure the t value for symbol-precedence ranking in the two groups.

In this study, participants were considered to have symbol-precedence pedagogical content knowledge ("expect blind spot") when their ranking result (Pearson's r) was higher than 0.75. Participants were considered to have verbal-precedence pedagogical content knowledge when their ranking result (Pearson's r) was lower than 0.25.

The Second Section

Interview. The goal in the second section was to explore the discrepancy of pedagogical content knowledge between the high content knowledge group and low content knowledge group.

The data, gathered from interviewing pre-service teachers in the case studies, were supported by Grossman's (1990) components of pedagogical content knowledge. Several specific pedagogical content knowledge considerations (Kahan, et al., 2003) were used to analyze in-depth interviews in high content knowledge and symbol-precedence pedagogical content knowledge groups, in low content knowledge and verbal-precedence pedagogical content knowledge groups and in high content knowledge and verbal-precedence pedagogical content knowledge groups.

Research Questions

The main research question in this study explored the relationship between mathematics content knowledge and pedagogical content knowledge among pre-service teachers. To conduct this exploration, the research questions discussed were related to: (a) participants' algebraic content knowledge, (b) participants' DFA ranking task (Nathan & Petrosino, 2003), and (c) participant's perspectives in teaching algebra.

1. What is the level of pre-service teachers' algebra content knowledge?

The ACK test was used to help answer this question. Scores from the ACK test were used to analyze the pre-service teachers' understanding of (a) algebra expressions; (b) ratio computations; (c) linear equations; (d) variables computations; (e) logarithm computations; and (f) square roots with variables. To compare the high and low content

knowledge participants' performance in the algebra content knowledge test, the specific questions were as follows:

- Is there significant difference in ACK test between secondary mathematics majors and elementary education teaching majors with a mathematics minor?

The null hypothesis for this research question is that the mean of elementary education teaching majors' ACK test is equal to the mean of secondary mathematics education majors.

- Is there a significant difference in ACK test between the high content knowledge group (above median score) and the low content knowledge group (below median score)?

The null hypothesis for this research question is that the mean of high content knowledge group is equal to the mean of low content knowledge group.

2. How do the high content knowledge participants and low content knowledge participants perform on the same DFA ranking task (Nathan & Petrosino, 2003)?

Nathan and Petrosino's (2003) ranking task was used in pre-service teachers with high mathematics knowledge (had completed calculus or above) and low mathematics knowledge (had not completed pre-calculus) in their current work. In this study, the ranking task was used in pre-service teachers whose ACK test scores were higher than the median score, and whose ACK test scores were lower than the median score. To compare this study and Nathan and Petrosino's (2003) study, the specific questions were as follows:

- Is there a significant difference between this study and Nathan and Petrosino's (2003) study?

The null hypothesis for the research question is that there is no difference between this study and Nathan and Petrosino's (2003) study.

- Is there significant difference of the DFA ranking task between high content knowledge pre-service teachers and low content knowledge pre-service teachers?

According to Nathan and Petrosino's (2003) "expert blind spot" hypothesis, they predicted that high content knowledge would correlate better with the symbol-precedence pedagogical content knowledge. Therefore high content knowledge in DFA ranking task is exhibited by the problem ranking (using the problem numbers shown in the survey): 1 2 3 4 5 6. This ranking predicts that the ability to solve symbolic forms precedes the ability to solve story problems. The null hypothesis is that the problem ranking in high content knowledge group is the same as in low content knowledge group.

3. What are the pre-service teachers' perspectives in teaching algebra?

Three kinds of participants were selected to be interviewees. Participants who had (a) high performance in the ACK test and symbol-precedence in ranking task (Nathan & Petrosino, 2003), (b) low performance in the ACK test and verbal-precedence in ranking task, and (c) high performance in the ACK test and verbal-precedence in ranking task (Nathan and Petrosino, 2003), were invited to be interviewees. The interviews were analyzed by coding themes to address the following questions:

- What are the pre-service teachers' perspectives of teaching algebra meaningfully?

- How do the pre-service teachers understand students' needs when representing the knowledge of algebra?
- How do pre-service teachers connect their algebraic knowledge to students' understanding?

Summary of the Chapter 3

The design of this study connected the previous studies and developed a new exploration. Eighteen pre-service teachers with secondary mathematics education majors and 22 elementary education majors joined the present study. In the first section, all participants took an ACK test and a DFA ranking task from Nathan and Petrosino's (2003) study. The ACK test in this study was quantitatively used to group participants as high content knowledge group and low content knowledge group. The replicated DFA ranking task (Nathan & Petrosino, 2003) in this study was used to show if the result of this proposed study was similar to Nathan and Petrosino's study (2003). Based on the first section result, participants with high/low ACK test score and symbol/verbal-precedence ranking task were invited to join the second section. In the second section, an interview activity was conducted with eight selected participants. The interview was used to explore differences of pedagogical content knowledge among the participants and to analyze the components of "expert blind spot" of participants who had high/low content knowledge and symbol/verbal-precedence in pedagogical content knowledge.

CHAPTER 4

FINDINGS AND INTERPRETATIONS OF RESULTS

Chapter 4 data on study findings are presented in two sections. In the first section, quantitative analyses are reported. This section examines the relationship between pre-service teachers' algebra content knowledge and their pedagogical content knowledge as indexed by their rankings on the algebra problem difficulty task. In the second section, qualitative analyses of follow-up interviews with selected participants are reported to further explore the relationship between algebra content knowledge of pre-service teachers and their pedagogical content knowledge of algebra. Specifically, three case examples of pre-service teachers who represent different knowledge profiles are described.

Relationship between Content Knowledge and Pedagogical Content Knowledge:

Quantitative Analysis

This section reports the results of quantitative analyses in the Algebra Content Knowledge test (ACK test) and the Difficulty Factors Analysis (DFA) ranking task (Nathan & Petrosino, 2003).

ACK Test Analyses

Scores on the ACK test were used to answer the research question: Is there a significant difference in the ACK test scores between elementary education teaching majors with a mathematics minor and secondary mathematics education majors? A comparison was conducted on the ACK scores of the elementary education majors and secondary mathematics education majors. Table 2 summarizes the performance of

participants on the ACK test. It was assumed that the samples of the two majors represented the population distribution. Based on that assumption, a one-way ANOVA was used to compare the two majors' scores. As shown in Table 2, the results, $F(1, 38) = 16.09$, $p < 0.01$ (two tailed), indicated that secondary mathematics majors, on average, outperformed elementary majors on the ACK Test. These participants were rank ordered on the ACK test and a median split was performed. As shown in Table 2, only 7 of 22 elementary education majors (31%) were above the median score, and 13 of 18 secondary mathematics education majors (72%) were above the median score. The results showed elementary education majors' performance in ACK test was lower than secondary mathematics education majors.

Table 2.

Performance on ACK Test by Type of Pre-service Education Major

Pre-service Education Major	Performance On Algebra Content Knowledge Test				
	n	Mean (whole points = 40)	% correct	SD	# students scoring above median score (29.0)
Elementary Education	22	25.68	64%	4.92	7 out of 22
Secondary Mathematics Education	18	31.33	78%	3.74	13 out of 18

To examine the relationship between content knowledge and pedagogical content knowledge, the researcher conducted a median split of all participants' performance on the ACK test to get a high content knowledge group and a low content knowledge group.

Based on the median split, the researcher examined each group's pedagogical content knowledge performance by the results of the DFA ranking task.

To conduct the median split, the researcher rank ordered participants on the ACK test (Table 3). The highest score on the test was 38 points (95% correct in ACK test). The lowest score on the test was 17 points (30% correct in ACK test). The median score was 29. Following the design of the study, the 40 participants were divided into two groups (high content knowledge group and low content knowledge group). Participants whose ACK test results were higher than the median score (29) were placed in the high content knowledge group, and participants whose ACK test results were lower than the median score were placed in the low content knowledge group.

Table 3.

<i>Performance on ACK Test by median split group</i>				
Pre-service teachers' median split group	Performance On ACK Test			
	n	Raw score mean (whole points = 40)	% correct	SD
High content knowledge group	20	32.30	80.8%	3.08
Low content knowledge group	20	24.65	61.8%	3.41

To compare the ACK test performance of the two groups, a *t* test was used to show whether or not the two groups had a significant difference. The *t* test showed that the ACK test scores in the high content knowledge group ($M = 32.30$, $SD = 3.08$) were significantly higher than the scores in the low content knowledge group ($M = 24.65$, $SD = 3.41$), $t(38) = 7.204$, $p < .05$ (two-tailed).

DFA Ranking Task Analyses

The findings in the DFA ranking task were used to answer the question: Is there a significant difference in the ranking task scores between high content knowledge pre-service teachers and low content knowledge pre-service teachers?

Forty participants took the DFA ranking task after they took the ACK test. Based on the ACK task, 40 participants were separated by median split to be a high content knowledge group and a low content knowledge group. Each participant's ranking correlation was estimated by Pearson's r . To predict whether participants had perfect correlation, a Fisher transformation (z) was applied to transform each participant's Pearson's r . If a participant's Pearson's r was 0.76 or above, then the participant's Fisher's z was 1 or greater than 1. Also Fisher's z could approximately transform Pearson's r to be a normal distribution. According to these two conditions, Fisher's transformation (z) was also applied to count each participant's 95% confidence interval. If a participant's 95% confidence interval of Fisher's z included 1.0 (for instance, $.45 \leq z \leq 1.14$), then it statistically indicated that the participant might have perfect correlation with symbol-precedence (match "expert blind spot" hypothesis). If a participant's 95% confidence interval of Fisher's z did not include 1.0 (for instance, $.11 \leq z \leq .45$), then it indicated that the participant statistically did not have a perfect correlation with the symbol-precedence (did not match "expert blind spot" hypothesis).

As shown in Table 4, 18 of 20 participants in the high content knowledge group matched the "expert blind spot" hypothesis, and only 2 of 20 participants in the high content knowledge did not match the "expert blind spot" hypothesis. In the low content

knowledge group, 11 of 20 participants matched the “expert blind spot” hypothesis, while 9 of 20 participants did not match the “expert blind spot” hypothesis. Chi-square, $\chi^2(1, N = 40) = 6.14, p = .01$, indicated that the participants in the high content group more often matched the “expert blind spot” hypothesis. A t test was used to compare the Fisher’s z in the high content knowledge group with the low content knowledge group. The t test showed that the Fisher’s z in the high content knowledge group ($M = 1.28, SD = 0.50$) were significantly higher than the scores in the low content knowledge group ($M = 0.45, SD = 0.45$), $t(38) = 3.25, p < .05$ (two-tailed).

Table 4.

Pre-service teachers’ median split group	95% Conference Interval from Fisher’s z		
	n	# include 1	# less than 1
High Content knowledge group	20	18	2
Low Content knowledge group	20	11	9

The findings in the DFA ranking task were also used to answer the question: Is there a significant difference between this study and Nathan and Petrosino’s (2003) study?

There is a limitation in understanding how Nathan and Petrosino analyzed their data. In the DFA ranking task (Nathan & Petrosino, 2003), the first three questions were designed in an arithmetic question format, result-unknown (arithmetic symbol equation,

arithmetic word equation, and arithmetic story problem). The second three questions were designed in an algebraic question format, start-unknown (algebraic symbolic question, algebraic word equation, and algebraic story problem).

The results of the DFA ranking task (Table 5) in this study showed that the average ranking across the high content knowledge group (e.g., above-median group, $n = 20$) moved from the easiest (arithmetic symbol equation and arithmetic word equation), to more difficult (arithmetic story problem), to hard (algebra symbol equation), and finally, to the most difficult (algebra word equation and algebra story problem). This ranking was indistinguishable from that predicated by the symbol-precedence view (Nathan & Keodinger, 2000a), $r = .971$, $p < .001$. Analyses of individual rankings of each participant showed an average correlation with symbol-precedence view of $r = .86$, $SE = .50$. The rankings of the high content knowledge group paralleled Nathan and Petrosino's (2003) hypothetical ranking predicated from the symbol-precedence view.

To compare this study with Nathan and Petrosino's (2003) study, the first step was to predict whether or not the high content knowledge group had a high probability of having a high value of Pearson's r . To do this in this study, two outliers in high content knowledge group were removed. Again, Fisher's transformation (z) was used to transfer the high content knowledge group's ($n = 18$) Pearson's r to a normal distribution and then measure this group's 95% confidence interval. As shown in Table 5, the high content knowledge group's transformed scores had a 95% confidence interval ($0.78 \leq z \leq 1.78$). These results indicated that pre-service teachers with high content knowledge statistically had the symbol-precedence ranking.

Table 5.

Performance on DFA Ranking Task by Algebra Content knowledge

Pre-service teachers' median split group	Performance on Nathan's et al., DFA Ranking Task					
	Results of the study			Results of Nathan et al., 2003 study		
	n	Pearson's r	Fisher's z	n	Pearson's r	Fisher's z
High Content knowledge group	18	0.86	$0.78 \leq z \leq 1.78$	16	.72	$0.64 \leq z \leq 1.19$
Low Content knowledge group	20	0.42	$0 \leq z \leq 0.90$	13	.48	$0.35 \leq z \leq 0.71$

This result also replicated Nathan and Petrosino's (2003) study in the high content knowledge group (MathSci, $n = 16$) group. To compare this study and Nathan and Petrosino's (2003) study in the high content knowledge group, Fisher's z values in these two studies were used. The comparison of two Fisher's z value, $F(18, 16) = 1.03$, $p > 0.05$, showed the results of the study paralleled Nathan and Petrosino's (2003) study. Also, the Pearson's r in this study showed the high content knowledge group had higher correlation to symbol-precedence (Pearson's $r = 0.86$) than Nathan and Petrosino's (2003) study in their high content knowledge group (Pearson's $r = 0.72$).

The low content knowledge group exhibited an average ranking on the DFA results from easiest (arithmetic symbol equation and arithmetic word equation), to medium difficulty, to hardest (algebra symbol equation). As shown in Table 5, this

ranking was distinguishable from that predicated by the symbol-precedence view, $r = .86$, $p < .05$. The average Pearson's correlation was $r = .42$, $SE = .45$. The rankings of the low content knowledge group were distinguishable from the symbol-precedence view. Table 5 also showed that Fisher-transformed ranking correlations for the low content knowledge group produced a 95 % confidence interval ($0 \leq z \leq .90$). The results indicated that the low content knowledge group statistically had verbal-precedence (did not match the "expert blind spot" hypotheses). These results also replicated Nathan and Petrosino's (2003) study.

Justification of Follow-up Interviewees Chosen

Three groups of pre-service teachers (high content and symbol-precedence group, low content and verbal-precedence group and high content and verbal-precedence group) were involved in the second stage of this study. Based upon their performance in the first stage (ACK test and DFA ranking task), some were interviewed and assigned to one of three groups. High ACK test scores referred to high algebra content knowledge. If Pearson's r was high, it meant that the predictions for the pre-service teachers were highly correlated to the "expert blind spot" hypotheses (symbol-precedence pedagogical content knowledge).

Twenty pre-service teachers in the high content knowledge group (ACK test score $>$ median score = 29) and 20 pre-service teachers in the low content knowledge group (ACK test score $<$ median score = 29) were considered to be selected for further follow-up interviews. These pre-service teachers were selected based on the following: (a) their ACK test scores were in the upper quarter (third quarter) of the test and Pearson's r in the

ranking task was above 0.76 (high ACK test/symbol-precedence pre-service teachers), (b) their ACK test scores were in the lower quarter (first quarter) of the test and Pearson's r in the ranking task was below 0.36 (low ACK test/verbal-precedence pre-service teachers), and (c) their ACK test scores were in the third quarter of the test and Pearson's r in the ranking task was below 0.358 (high ACK test/verbal-precedence pre-service teachers).

As shown in Appendix H, 11 participants were identified with higher content knowledge (the ACK test scores > 34 , the third quarter = 32) from the high content knowledge group. Among these 11 participants, only one person (named Trevor in the study) had verbal-precedence (Pearson's r is below 0.36) in his pedagogical content knowledge. Therefore, as a high ACK test/verbal-precedence pre-service teacher, Trevor was selected for a follow-up interview. In the ACK test score, participants with the top four scores (named Jessica, Cheryl, Megan, and Andy in this study) all had symbol-precedence (Pearson's $r > 0.76$) in their pedagogical content knowledge. These 4 pre-service teachers were selected, as high ACK test/symbol-precedence pre-service teachers, to be interviewed.

Also shown in Appendix H, in the low content knowledge group, 12 participants' scores were in the first quarter of the test. However, only 3 participants (named Ryan, Jennifer and McKenzie in this study) had verbal-precedence (Pearson's r behind 0.36) in their pedagogical content knowledge. Therefore, they were categorized in the low ACK test/verbal-precedence pre-service teachers group.

Table 6 shows the performance of the eight participants selected for follow-up interviews.

Table 6.

Interviewees' Performance in the First Stage

	High ACK test/symbol-precedence group				Low ACK test/verbal-precedence group			High ACK/verbal-precedence group
	Jessica	Cheryl	Andy	Megan	Ryan	Jennifer	McKenzie	Trevor
ACK test (% of correct)	95%	95%	95%	85%	63%	60%	53%	90%
DFA ranking task (Pearson's r)	.714	.796	.729	.919	.230	-.257	-.169	.200

Cheryl's case: High ACK test/symbol-precedence pre-service teacher. Four high ACK test/symbol-precedence pre-service teachers were invited to join the follow-up interview. They had similar mathematics education backgrounds (see Appendix I). They had taken 15 academic hours of common core courses (including *Calculus I, II, and III*, and *Linear Algebra for Application*), 21 academic hours in mathematics teaching courses, and 9 academic hours in mathematics elective courses. The interview data on Jessica, Andy and Megan showed they had stronger geometry than algebra skills and that they liked to teach geometry. Cheryl's favorite subject matter was algebra. In her interview, she also showed an interest in teaching algebra. Therefore, Cheryl's interview data was used as the example of high ACK test/symbol-precedence pre-service teachers.

Jennifer's case: Low ACK test/verbal-precedence pre-service teachers. Three low ACK test/verbal-precedence pre-service teachers were invited to join the follow-up

interview. They all were elementary education majors with a mathematics minor. Ryan and McKenzie had taken 24 academic credit hours for their mathematics minor. Jennifer had not taken all the 24 academic credits for her mathematics minor (see Appendix I). In her interview data, she had more unique perspectives regarding the relationship between content knowledge and pedagogical content knowledge. She thought that knowing how to teach mathematics was more important than knowing what should be taught. She believed that mathematics teachers did not need higher mathematics content knowledge than their students because mathematics teachers can learn content knowledge from mathematics textbooks. She said the difference between mathematics teachers and their students is that mathematics teachers know how to teach the content knowledge.

Trevor's case: High ACK test/verbal-precedence pre-service teacher. Only one high ACK test/verbal-precedence pre-service teacher was invited to join the follow-up interview. Like all secondary mathematic education majors, Trevor had taken 15 academic hours of common core courses, 21 academic hours in mathematics teaching courses, and 9 academic hours of mathematics elective courses. He wanted to teach both algebra II and trigonometry in a small high school. He felt confident in his mathematics content knowledge and had strong confidence in his ability to teach algebra concepts to 7th grade students.

Relationship between Content Knowledge and Pedagogical Content Knowledge:

Qualitative Analyses

The findings in the second section of the study were used to answer this research question: What are the pre-service teachers' perspectives in teaching algebra? Three sub-

questions, constructed by coding themes in literature reviews, were designed to address views of the third research question. These three sub-questions are:

- a. What are pre-service teachers' perspectives of teaching algebra meaningfully?
- b. How do pre-service teachers understand students' needs when representing the knowledge of algebra?
- c. How do pre-service teachers connect their algebraic knowledge to students' level of understanding?

According to the justification in choosing the interviewees' backgrounds and interview data, Cheryl could be an example of high ACK test/symbol-precedence pre-service teachers, Jennifer could be an example of low ACK test/verbal-precedence pre-service teachers, and Trevor could be an example of high ACK test/verbal-precedence pre-service teachers. The following are the analyses of the three cases through the three coding themes.

Cheryl: An Example of High ACK Test/Symbol-Precedence Pre-service Teachers

This section shows Cheryl's perspectives in (a) teaching algebra meaningfully, (b) understanding students' needs, and (c) connecting algebra to students' level of understanding.

Teaching algebra meaningfully. Four main interview questions were used to explore Cheryl's perspectives of teaching algebra meaningfully. They were:

- a. What is your definition of algebra?

- b. Suppose you are going to teach 7th grade students how to solve this question (Solve for D : $D \times 4 + 25 = 68.36$). What is your strategy for teaching this question? What is your strategy for teaching 7th grade algebra?
- c. In this question (Solve for D : $D \times 4 + 25 = 68.35$), how would you determine that a student can solve similar problems?
- d. Once you are certain that your students can solve the problem (Solve for D : $D \times 4 + 25 = 68.36$), what is the next problem you will offer to your students? Why?

Overall, high ACK test/symbol-precedence pre-service teachers had a learning hierarchy in mind so that they predicted students' needs based on their own learning hierarchy. Cheryl's interview data reported a good case to show such a learning hierarchy. She defined algebra as being the knowledge of patterns and relationships and then she followed her definition to approach the meaning of teaching algebra to 7th grade students. She clearly stated her learning hierarchy and then used the learning hierarchy to offer a mathematics context connected to her algebraic concepts. Following are the specific findings and interpretations.

Define algebra as concepts of patterns and relationships. Cheryl had a strong tendency to define the meaning of algebra based on her mathematics content knowledge. The meaning of algebra for her was showing the concepts of patterns and relationships. She could show the concepts of patterns and relationships by different mathematics contexts (using number sequence, geometric picture or story problems). She did not define algebra as using letters to refer to variables. She tried to show the role of algebra in

the whole mathematics knowledge system. Knowledge of algebra for her was “finding unknowns in the format of variables, which can be shown in geometry or an equation system.”

The interview data showed that Cheryl’s learning hierarchy in her mind was composed of symbolic reasoning and symbolic computation skills. This phenomenon was also seen in the other three high ACK test/symbol-precedence pre-service teachers’ (Andy, Megan, and Jessica) interview data. Cheryl used her algebraic perceptions to explain the meaning of solving for an algebraic problem. She said the meaning of solving an algebraic problem is, “solving a problem by discovering the connections between the parts.” The same perspectives were seen in Megan’s definition of algebra “looking for the relationships and patterns”, Jessica’s definition “the study of pattern”, and Andy’s definition “a representation of patterns in a generalized form.”

Guide students to understand patterns and relationships. In Cheryl’s learning hierarchy, she emphasized symbolic formulas (for example the distributive property, $a(b + c) = a \times b + a \times c$). She felt that those symbolic formulas in the knowledge of algebra could give students the easiest way to show the pattern and relationship. Those symbolic formulas, in her learning hierarchy, could help her connect the easiest to the hardest algebraic concepts so that she had great confidence to teach 7th grade algebra.

Patterns and relationships, in Cheryl’s mind, were the core concepts in learning algebra. Therefore, she believed teaching students the concepts of patterns and relationships were the main points in teaching algebra. Instead of using mathematics textbooks to facilitate her teaching procedure, she developed her own way of representing

the concepts of patterns and relationships for her students. She was confident about transiting her content knowledge to her teaching knowledge. For example, she mentioned that 7th grade mathematics was easier than what she was learning in the mathematics department. However, when asked how she connected her level of mathematics content knowledge to her students' levels of mathematics content knowledge, she skillfully used different strategies to demonstrate her mathematics content knowledge. She showed her mathematics content knowledge without understanding her students' levels of mathematics content knowledge first. Her main consideration was how to effectively represent her algebra content knowledge to 7th grade students rather than to facilitate 7th graders understanding of algebra. Her interview showed that she could use different teaching strategies to effectively transit her content knowledge to 7th grade students instead of different strategies to understand her students' needs. Her perspectives on teaching algebra meaningfully were based on her learning hierarchy to judge students' algebraic content knowledge. Therefore, her pedagogical content knowledge, as a bridge, connected her mathematics content knowledge to her learning hierarchy.

Emphasize transition from verbal to symbol. Using a problem (Solve for D : $D \times 4 + 25 = 68.36$) in the ranking task (Nathan & Petrosino, 2003) as an example, this researcher tried to understand how Cheryl figured out the questions and represented them to her 7th grade students in her algebra classes. Overall, she focused on how to clearly show the computational procedure rather than on how to develop the conceptual knowledge. When looking at the six problems in the ranking task (Nathan & Petrosino, 2003), she easily figured out that the algebra symbol equation (problem number 4) was

implied in a story problem (problem number 6). In her teaching perspectives, she would teach students how to transit the algebra story problem to be a symbol equation because she thought most students would have difficulty in the transition from the verbal problem to a symbol equation. Cheryl's perspectives of teaching showed that verbal-precedence was an application of symbol-precedence (Nathan & Koedinger, 2000). Therefore, she would make sure students could compute the symbol equation, and then she would use story problems to show how the symbolized concepts could be used in the real world.

Computational knowledge as major consideration. Once students could solve the problem (Solve for D : $D \times 4 + 25 = 68.36$), the researcher wanted to know what Cheryl's next teaching strategy would be for her 7th grade students. Her answers showed that she would offer more difficult symbolic problems to challenge students' mathematics content knowledge. She thought the difficult symbolic problems could be used to make sure that students understood how to compute the problem (Solve for D : $D \times 4 + 25 = 68.36$). She also wanted to make sure the students understood computation knowledge first because that skill was the major consideration in her teaching. When she was certain that her students could solve more difficult problems, then she would consider presenting story problems which were equivalent to the advanced symbol equations. The story problems would give Cheryl a chance to teach students how to apply their computational knowledge to a real situation. Therefore, she would use symbol precedence first and then verbal-precedence to connect the application to "real life."

Understanding students' needs. The following interview questions were used to discover teaching algebra meaningfully by high ACK test/symbol-precedence pre-service teachers and to explore their perspectives in understanding students' needs:

- a. What are you going to do so that your students will be interested in learning the content knowledge?
- b. Which kind of problem (symbol equation, word equation, or story problem) would you like to teach 7th grade students first and why?
- c. Why do you think problem number 6 (depending on interviewee's ranking task) is the hardest for students to understand and problem number 1 (also depending on interviewee's ranking task) is the easiest for students?

To create students' interest in learning algebra and solving algebraic symbol equations (e.g. solve for D : $D \times 4 + 25 = 68.36$, in Nathan and Petrosino, 2003), Cheryl considered using students' experience to introduce algebraic concepts. For example, she thought counting money would be a good way to let students gain interest in learning algebraic thinking. To do that, Cheryl had to convert story problems to algebraic problems. However, no matter what kind of precedence Cheryl used (symbol-precedence or verbal-precedence), her teaching was focused on the transition between symbol equations and story problems.

Facilitate students' symbolic concepts. Based on teaching for understanding (NCTM, 2000), the problem (Solve for D : $D \times 4 + 25 = 68.36$) was used to explore how Cheryl got students interested in solving the problem. She was concerned about the meaning underlying the symbolic equation. She would use students' common interests to

introduce a symbolic equation. As an example, she used putting an amount of money in a bank and then asked students to count the interest per month or per year. She thought this example could facilitate students' symbol concepts and also could train students to set up a correct symbolic equation. Therefore, in her teaching process, story problems were used to facilitate students' understanding of using symbol concepts in real life.

Focus on teaching symbolic equation. In the ranking task (Nathan & Petrosino, 2003) one question was, "Which one is easiest and which one is the hardest for your students?" Cheryl's interview data showed that she thought symbolic equations were easier than story problems for her students. According to her belief, she would show students algebraic concepts by using symbolic equations rather than story problems. In her teaching style, she would use symbol-precedence rather than verbal-precedence. She thought symbol-precedence would make it easier for students to gain clear algebraic concepts. She felt that if she used story problems to show algebraic concepts, some students would struggle to understand the meaning of the language in the story problem. Therefore, she would use symbol-precedence and then use verbal-precedence to support her students.

Connecting algebra to students' levels of understanding This section presents the way high ACK test/symbol-precedence pre-service teachers correct students' misunderstandings. The following is an exploration of teaching algebra meaningfully by high ACK test/symbol-precedence pre-service teachers. To explore their perspectives in connecting students' level of understanding, they were asked three interview questions:

- a. How will you teach $x \cdot y$ and $x + y$ for 7th grade students?

- b. If students do not understand that $2a + 5b$ is not $7ab$, how can you explain this to 7th grade students?
- c. Can you explain to 7th grade students why negative 2 times negative 4 is positive 8?

In general, Cheryl took basic mathematics concepts for granted so she was not skilled with connecting the basic mathematics concepts to the “real life” situation. She moved from understanding a formula to remembering the formula. However, she could show algebraic concepts by using geometry. She could also show algebra by using arithmetic rules, but she was not motivated to consider the best way to show her students.

Few strategies to teach basic mathematics. Cheryl taught mathematics based on her own level of understanding of the concepts. The higher levels of mathematics gave her difficulty when explaining her strategy (how will you teach $x \cdot y$ and $x + y$ for 7th grade students). She definitely understood the difference between $x \cdot y$ and $x + y$, but she did not think of an effective way to connect her understanding of the difference to her students’ level of understanding. Even though she finally offered strategies to show the differences between $x \cdot y$ and $x + y$, she did not think she offered good strategies. She certainly understood basic mathematical concepts, but she did not have enough training in how to teach them. For example, she agreed that $-2 \times -4 = 8$, but she could not explain why negative 2 times negative 4 is positive 8. She had memorized the rule that a negative number multiplied by a negative number would be a positive number, but she did not understand why. Her answer was, “...what I would do is tell them that that’s a fact, in other words, they have to accept it to be able to do it this way.”

Connect algebra to geometry. In answering the question “How will you teach $x \cdot y$ and $x + y$ for 7th grade students”, Cheryl showed she could easily connect different mathematics content knowledge to interpret the difference between $x \cdot y$ and $x + y$. She used the area of rectangle to show $x \cdot y$ and half of perimeter of rectangle to show $x + y$. She drew a rectangle with a base that equaled y and a height that equaled x . She would tell her students that $x + y$ was half the perimeter of the rectangle, and x times y was the area of the rectangle. Cheryl’s teaching knowledge has used different mathematics content knowledge to help students understand the difference by visualization.

Also, Cheryl thought about a way to substitute numbers for x and y so that students could see the difference between $x \cdot y$ and $x + y$. She said, “Put in actual numbers and show the resulting difference.” She used the same concept to interpret students’ misunderstanding of $2a + 5b = 7ab$. For example, Cheryl went back to earlier arithmetic concepts to show students that “ $2a$ ” means “ $a + a$ ” and “ $5b$ ” means “ $b + b + b + b + b$.” Cheryl thought the arithmetic procedural knowledge would help students understand that $2a + 5b$ would not equal $7ab$. However, she did not think the computational procedure was a good way to show the difference between $x \cdot y$ and $x + y$. She preferred to use geometry to show the difference between $x \cdot y$ and $x + y$.

Jennifer: An Example of Low ACK Test/Verbal-Precedence Pre-Service Teachers

This section showed the perspectives of a pre-service teacher with low content knowledge but with verbal-precedence pedagogical content knowledge in (a) teaching algebra meaningfully, (b) understanding students’ needs, and (c) connecting algebra to students’ level of understanding. Jennifer’s case study could be an example of low ACK

test/verbal-precedence group. Overall, pre-service teachers in this group had a general problem-solving philosophy in mind that made it easy to understand their student's thoughts and strategies. The following analyses report Jennifer's case study by the three coding themes.

Teaching algebra meaningfully. Following is an exploration of teaching algebra meaningfully by low ACK test/verbal-precedence pre-service teachers, using findings from Jennifer's interview data.

Generally, Jennifer defined knowledge of algebra as the knowledge of dealing with unknowns. She thought that teaching algebra to 7th grade students was teaching them the meaning of variables and how to use the variables. Jennifer had a high motivation to teach how to use variables in real life. She did not have strong connections among her mathematics content knowledge, so she did not have higher symbolic reasoning to support her problem-solving ability. However, she had a general problem-solving philosophy as common strategies in her mind to solve for algebraic problem. As to her content knowledge, she preferred to use mathematics textbooks as her content knowledge resources to teach 7th grade students. This gave her more time to design more effective hands-on mathematics activities for students. Teaching algebra was meaningful for Jennifer because she reviewed earlier arithmetic skills students needed to help them make more sense of new concepts. Her evaluations focused on whether students could do mathematics rather than whether they understood mathematics. The specific analyses of Jennifer's understanding of teaching algebra meaningfully are described in the following paragraphs.

Define algebra as knowledge of dealing with unknowns. For Jennifer, algebra was one of basic mathematics concepts, resulting in “variables” or “representing mathematics by letters.” In the ranking task (Nathan & Petrosino, 2003), she thought both the first problem: $(68.36 - 25)/4 = P$ and the fourth problem (Solve for D : $D \times 4 + 25 = 68.36$), were algebra problems for 7th grade students because both used a letter to refer to an unknown. Jennifer thought that both result-unknown questions and start-unknown questions were algebraic questions. Unlike Cheryl who defined algebra by conceptual knowledge (algebra as finding the patterns and relationships), Jennifer used computational knowledge, defining algebra by using a problem-solving situation. Therefore, the meaning of algebra for Jennifer was solving for unknowns and representing unknowns with letters.

Use textbooks to develop the concept of unknowns. Jennifer did not have a strong sense of how she would teach 7th grade algebra. However, she thought the concept of unknowns and how to use letters to refer to an unknown in mathematics were a main teaching point. She believed she could teach 7th grade mathematics without having high mathematics content knowledge to support her teaching. Textbooks for Jennifer were used as a resource for mathematics content knowledge. She could get help from the mathematics textbook, and consequently, have more time to understand students’ needs and design teaching activities. She emphasized teaching activities more than understanding mathematics content knowledge. She said, “The key point is to know how to teach. The content knowledge will be shown in the textbooks. You can follow the textbook to teach them [students].”

She believed that basic algebraic thinking was enough to teach 7th grade algebra. Therefore, she felt she did not need to solve all 7th grade algebra problems. If she had content knowledge problems, she could learn along with her students. She was not worried about how limited her content knowledge was. She emphasized how she could interpret the concepts of algebra well. Therefore, Jennifer needed to use mathematics textbooks to have a good orientation for a teaching unit.

Emphasize doing mathematics (verbal-precedence). An example (Solve for D : $D \times 4 + 25 = 68.36$) from the ranking task (Nathan & Petrosino, 2003) was used to determine how pre-service teachers would figure out the question and then teach it to students. Jennifer would teach algebra by doing mathematics and then interpret algebraic concept by verbal-precedence. To teach the question (D : $D \times 4 + 25 = 68.36$), she would prepare manipulatives in her class for some hand-on mathematics. She would lead students in a simple game. First, Jennifer would replace the decimal number with a natural number. It could be D times 25 is equal to 50. Jennifer said “When students played the game, they were learning the equation with one variable.” Therefore, students might not remember the process of computation without understanding it. After playing the game, students could understand, as Jennifer said, “You subtract here and now you’re adding it here.”

Use arithmetic concepts to help algebraic concepts. Jennifer was concerned about the transitional knowledge from arithmetic to algebra. When using a letter to refer to a variable, she worried that students might not understand the meaning of the letter. Therefore, after she taught students how to solve the algebra symbol problem (Solve for

$D: D \times 4 + 25 = 68.36$), in the next question, she would offer the same symbol equation but change the letter D to another letter. She worried that students would misunderstand the meaning of letters in a mathematics statement.

Understanding students' needs. The following section explores how low ACK test/verbal-precedence pre-service teachers understand students' needs. The same interview questions were used with all interviewees.

Searching for a good way to conduct verbal-precedence. Jennifer's interview data showed that she would examine students' interests, and then she would connect her students' interests to some hands-on mathematics activities. She had a high ability to connect students' interests with her teaching subject and she emphasized this in her teaching perspective. For instance, to teach the problem (Solve for $D: D \times 4 + 25 = 68.36$), she would invite students to play some designed activities to show algebraic thinking rather than showing students the computational knowledge of how to isolate D.

Learn mathematics in context. Six problems in the ranking task (Nathan & Petrosino, 2003) were also used to explore her knowledge of students' needs. Jennifer predicted that students would feel story problems were easier than symbolic equations. However, she also understood that some students struggled with a story-context problem and felt symbolic equations were easier. According to Jennifer, she did not like teaching the algebra by symbol-precedence because she believed story-context problems could effectively help students understand mathematics. She preferred using algebra story problems to show algebraic thinking because she thought verbal-precedence would be easier to understand than symbol-precedence. If students struggled with story problems,

she would use the strategy of “learning from discussing the problem in the class.” She believed that “helping students’ conceptual knowledge was more important than teaching them how to solve for D.” She also thought that the meaning of teaching algebra was to develop students’ conceptual knowledge. Therefore, in her teaching process, she would not focus on whether students had computational knowledge. Instead, she would emphasize how to teach conceptual knowledge more effectively.

Connecting algebra to students’ levels of understanding. The following is the exploration of connecting algebra to students’ perceptions by low ACK test/verbal-precedence pre-service teachers. To explore their perspectives in understanding students’ needs, the previous interview questions were used.

Use tables to clearly show basic arithmetic concepts. Jennifer showed that she had enough algebra content knowledge to facilitate 7th grade students’ algebra instruction with basic arithmetic concepts. She declared that the difference between $x \cdot y$ and $x + y$ could be orally interpreted to students and that she would also need to draw a table to show how x , y , $x \cdot y$, and $x + y$ were different. She emphasized that visualization for students was more important than interpretation by conversation. Jennifer’s attitude showed that even though teachers could easily use arithmetic knowledge to show the meaning of algebra, students also needed a visual way to relate arithmetic to algebra. She also liked to do some activities using teaching manipulatives so students’ perceptions could be transited from doing mathematics to learning mathematics.

Use natural language. Students’ misunderstanding of $2a + 5b = 7ab$ was used to explore how pre-service teachers connect their content knowledge to students’

perceptions. Jennifer used conceptual contradiction to show the meaning of symbolic difference. She used natural language to show the contradiction. She assigned ‘a’ for “apples” and ‘b’ for “bananas.” She thought the contradiction (apples plus bananas could not be fruit) would help students understand that $2a + 5b$ is not equal $7ab$. She said:

You could say, ok, add two apples plus five bananas. What do you get? Well 7 fruit, yes, but would fruit be a and b? No, fruit is a new word. Then you’d have to say that doesn’t work. What do I have? I have 2 apples and 5 bananas. I would do something like that.

Trevor: An Example of High ACK Test/Verbal-Precedence Pre-service Teachers

According to results in the first section of this study, only one pre-service teacher (Trevor) was in the high ACK test/verbal-precedence group. Overall, Trevor showed that his flexible teaching strategies enabled him to represent content knowledge to his students, based on students’ response. Three coding themes in this study were used to analyze his interview data.

Teaching algebra meaningfully. The following section explores Trevor’s perspective of teaching algebra meaningfully using the same interview questions as before.

Different algebraic meanings for different people. Trevor (CK = 34, $r = .200$) was the only pre-service teacher who qualified to be a high ACK test/verbal-precedence pre-service teacher. His interview data showed he had flexible ways to define the meaning of algebra. When asked the definition of algebra, he said he would use a different level of content knowledge to define algebra for different people. To other mathematics teachers, he defined algebra as symbolic formulas. To middle school students, he defined algebra

as the meaning of variables. To mathematics majors in colleges, he defined algebra as the patterns and relationships in number sense or geometry.

Trevor did not think he could narrow the meaning of algebra down to one sentence because the meaning of algebra was too broad. Trevor refused to show his definition of algebra to students because he expected that students could define the meaning of algebra on their own. Trevor thought that students could broaden their meaning of algebra by solving different kinds of algebraic problems. His job in teaching algebra was to offer algebra questions to students, based on students' mathematics levels; and then saw how his students responded to these algebra questions. He emphasized that some students might understand the concepts of algebra by using formulas, but other students might make more sense of algebra by using the concepts in a real situation.

In general, Trevor showed it was not necessary to tell students a teacher's meaning of algebra because a teacher's definition would effect students' development of algebra skills. An effective teacher, Trevor explained, could offer different kinds of algebra problems and let students develop their own meaning of algebra by solving those kinds of algebra problems.

Guide students by students' progress. In Trevor's algebra class, he would not worry about how many algebra concepts he could teach his students because he thought students' mathematics ability was wide ranging. However, Trevor thought that 7th graders would somehow understand arithmetic. Therefore, when he taught algebra, he would use their arithmetic knowledge to stretch their understanding of algebraic concepts. If

students understood the concept of fractions, Trevor would begin with this previous knowledge and then add in algebraic concepts to the concept of fractions.

Trevor's knowledge for teaching algebra was based on gradually assessing what algebraic concepts he taught to his students. He assessed students' learning through specific mathematics questions and students' responses. For example, Trevor used percents to show his pedagogical content knowledge. If he wanted to know if students understood the meaning of 110%, he would design a simple problem with the answer 110%. Trevor would help students easily get that answer. However, he would ask students to explain why 110% could answer the problem (conceptual knowledge) and how to get 110% (procedural knowledge). He thought if students could easily answer the mathematics question, then he could make sure that students understood not only the procedural knowledge but also the conceptual knowledge. Therefore, Trevor not only wanted to teach the procedural knowledge to students with specific problems but also wanted to connect conceptual knowledge to students' previous knowledge.

Trevor thought an effective teaching process was to teach mathematics by problem-solving. In that way students could show their ability by solving problems, and teachers could assess students' learning results. He said, "If I'd like them to know what a percent is, I wouldn't let my students sit around and ask me why a percent is like that. I'd like them to learn what a percent is first. Then I would ask them to show me what's a 110%."

Emphasize students' levels of understanding. Problem solving in mathematics should involve students' conceptual knowledge and procedure knowledge (Hilbert,

1986). Trevor emphasized that both conceptual knowledge and procedural knowledge controlled students' progress. Trevor thought that effective mathematics teachers could match students' levels of understanding and select either conceptual or procedural knowledge strategies to facilitate students' progress. Trevor also thought that an effective mathematics teacher could easily transform a mathematics problem to a symbolic format or story-context format. He used the ranking task as an example and mentioned that a mathematics question could be shown as a symbolic equation (Solve for D : $D \times 4 + 25 = 68.36$) or as a story problem. He pointed out that the algebra symbolic equation (problem 4 in Nathan and Petrosino's ranking task) was used to emphasize the procedural knowledge so that students would focus on how to solve for D . Trevor also understood that, in the DFA ranking task, the algebra story problem could be transformed to be an algebra symbol equation. He would use the story problem with students who preferred to learn mathematics based on a context and use the symbol equation with students who preferred to learn procedural knowledge first. He said, "If I want them to know how to solve the equation [algebra symbol equation], I would teach them the multiplication rule which is multiplication/division is first and addition/subtraction is second."

Trevor explained that story problems are more effective in helping students "understand what the heck is going on" so that students would find it easy to build on their conceptual knowledge. Therefore, he would offer several algebra story problems to them. Having made sure the algebra story problems gave students enough conceptual knowledge, Trevor would focus on how to solve the symbol equations.

Concern for students' performance. Trevor's teaching knowledge was built on evaluating students' performance with his effective connection to different content knowledge. For example, if students had high performance in solving the symbol equation in the ranking task, then Trevor would offer more difficult symbolic equations. A longer sentence like $2x + 3 = 1 - x$ would be given to see if his students could solve for x . If students preferred to learn algebra in real life situations as a mathematics context, then Trevor would design more story problems to match students' cognitive development. Trevor also understood that there was more critical thinking used in story problems to teach algebra, so he would produce more critical thinking points to improve students' learning results. He said, "If students are being successful with a basic story problem, I'd basically throw more variables as critical thinking points." Trevor thought that when he facilitated students' algebraic thinking, he would connect symbol equations and story problems so that students could easily understand how to transit a story problem to a symbol equation.

Understanding students' needs. The following examines Trevor's perspectives of understanding students' needs using the interview questions previously.

Appropriate ways to answer students' questions. To understand students' needs, Trevor carefully examined students' responses to choose appropriate ways to answer students' questions. Trevor thought that knowledge of mathematics was separate from knowledge of teaching mathematics. He said he liked to use a sequence of problems in his teaching process. He would offer his students easy problems and then more difficult problems so that he could more easily understand his students' learning. Trevor thought

assessment played a role in connecting students' learning to his mathematics content knowledge. As a teacher, Trevor would use specific mathematics questions to logically assess students' learning. He used the six problems in the ranking task (Nathan & Petrosino, 2003) as an example. If he wanted to teach the algebra story problem in the ranking task, he said he preferred to design several algebra story problems from easy to more difficult. He said, "I'd like to use a story problem tied to students' interest. It would hold their interest a little bit more. They have to actually think about it to get started."

Trevor would also use a line of story problems to help assess students. If students did not have high performance on some story problems, Trevor would either use easier story problems or begin by teaching simple symbol equations. He would not let students completely develop their own way to learn mathematics. He would develop a curriculum to fit what students needed to learn in his algebra class.

Build upon students' conceptual knowledge. Trevor understood that students might struggle with the transition from a story problem to a symbol equation. However, he did not directly teach students how to transit mathematics knowledge in a story problem to a symbol equation since "students can do it well if they have a strong enough conceptual knowledge." Trevor focused on showing conceptual knowledge to students. However, he emphasized that students' needs would show in each student's performance, not in a teachers' prediction. He said, "Students know what they need; not me."

Connecting algebra to students' levels of understanding. The six problems in the ranking task (Nathan & Petrosino, 2003) were used to explore Trevor's perspectives in teaching algebra. Trevor agreed that the arithmetic symbol equation (problem 1 one in the

ranking task) was the easiest question; but he also agreed that some students felt the algebra story problem (problem 6 in the ranking task) was the easiest question for them. Different question styles could fit different students' learning styles, so Trevor's conclusion was that his instruction needed to fit the students' learning style. The following section explores his perspectives on using the interview questions previously.

Use both symbol problems and story problems to help students' learning. Trevor used two strategies in his teaching process. He said that if students did not know the meaning of variable computations, he would put $x \cdot y$ and $x + y$ in a story problem context which could effectively let students focus on the variable computations. He said that an easy way to show the difference between $x \cdot y$ and $x + y$ was to substitute natural numbers and then show the result to students. However, he preferred designing a simple story problem to show the meaning of $x \cdot y$ and $x + y$. He said, "I would use numbers or some simple form of word problem and then just work the variables differently." Trevor emphasized differences between arithmetic and algebra, and he wanted to show the differences to his students. He said, "Basically, x times y creates a new term that is a combination of both. If you leave $x + y$ as the variables, you can't actually combine $x + y$ to be $x \cdot y$."

If students knew the meaning of variables, he would design an appropriate story problem for students because he believed story problems could increase students' conceptual knowledge. When he was asked how to make a simple story problem to help students figure out $x \cdot y$ and $x + y$, Trevor did not directly make a simple story problem to show the difference between x times y and x plus y .

Use geometry to learn algebra. To explain the difference between $x \cdot y$ and $x + y$, Trevor said some teachers would use “a” to replace “apples” and “b” to replace “banana.” However, he did not do this because he said that would not clearly explain the meaning of “ab” and students would easily confuse the relationship between mathematics “ab” and natural language “fruits.” He would show students by using the length and width of a rectangle so students could visually see the two things (length and width) added together.

To explain students’ misunderstanding, “Depending upon how deep I think their misunderstanding goes,” Trevor said he would monitor students’ expressions to find the specific problems causing the misunderstanding. Then, he would work through similar problems to correct the students’ misunderstanding. As to the problem using in the interview ($2a + 5b = 7ab$), Trevor could not come up with a good explanation without using an individual student’s problem as an example. He said, “The green are ‘a’ and the red are ‘b’ and those things can’t be combined into a third color, ‘ab’ because ‘ab’ is different from ‘a’ and different from ‘b’.”

Summary of Chapter 4

This chapter discussed data from the study that answered the three research questions:

1. What is the level of pre-service teachers’ algebra content knowledge?
2. How do the high content knowledge participants and low content knowledge participants perform on the same Difficulty Factors Analysis ranking task (Nathan & Petrosino, 2003)?
3. What are the pre-service teachers’ perspectives in teaching algebra?

The findings in the first section answered the first and second research questions. Quantitative method, *t-test*, Pearson's *r*, and Fisher's transformation value (*z*) were used to indicate the comparison between pre-service teachers in the high content knowledge group and low content knowledge groups.

In the first section, the Algebra Content Knowledge test showed the high content knowledge group's performance was significantly higher than that of the low content knowledge group's. That indicated it was meaningful to divide the participants into high and low content knowledge groups. The DFA ranking task, in this study, replicated Nathan and Petrosino's (2003) study. In this study, 95% confidence interval (Fisher's *z*, $0.78 < z < 1.78$) in the high content knowledge group included 1. It indicated that pre-service teachers with high content knowledge have symbol-precedence in their pedagogical content knowledge. A 95% confidence interval (Fisher's *z*, $0 < z < 0.90$) in the low content knowledge group did not include 1. This result statistically indicated that pre-service teachers with low content knowledge might have verbal-precedence pedagogical content knowledge. These quantitative results would be valid when the study added up qualitative interview data.

The findings in the second section, answering the third research question, were based on interviews with selected pre-service teachers who had (a) high ACK test/symbol-precedence pedagogical content knowledge (PCK), (b) low ACK test/verbal-precedence PCK, and (c) high ACK test/verbal-precedence PCK (Table 4). Three coding themes, based on the summary of the literature review, explored the relationship between content knowledge and pedagogical content knowledge. High ACK test/symbol-

precedence pre-service teachers showed their perspectives as knowledge-centered in teaching algebra. Low ACK test/verbal-precedence pre-service teachers presented their perspectives as problem-centered in teaching algebra. The high ACK test/verbal-precedence pre-service teacher showed his perspective as response-based in teaching algebra. Three kinds of characteristics will be discussed in Chapter 5.

Table 7.

The summary of findings in the second stage

Coding Themes (main interview questions)	High ACK test/symbol- precedence group	Low ACK test/verbal- precedence group	High ACK test/verbal- precedence group
Teaching algebra meaningfully (Discuss the six problems in DFA ranking task)	They have a learning hierarchy in mind so that they predict students' needs based on their own learning hierarchy	They have a general problem-solving philosophy in mind that makes it easy to understand their students' thoughts and strategies	Their flexible teaching strategies enable them to represent content knowledge to their students, based on students' response
Understanding students' needs (Based on the DFA ranking task, discuss which problem is easy and difficult; and the reasons)	Students need to know the transition from symbolic problems to story problems	Search for students' needs in learning algebra first; and then design a verbal context match students' needs	Students' understanding is based on specific responses
Connecting algebra to students' levels of understanding (Interpret the different between $x + y$ and $x \times y$)	Explain $x + y =$ a half of perimeter in a rectangle and $x \times y$ is a area of a rectangle	Use tables to clearly show x , y , $x + y$ and $x \times y$ so that students can be easier to see the differences	Create story problems to show the differences between $x + y$ and $x \times y$; and also use number sense to show the differences

CHAPTER 5

DISCUSSIONS AND CONCLUSIONS

Chapter 5 presents a discussion of the relationship between content knowledge and pedagogical content knowledge in pre-service teachers' perspectives. Previous "expert blind spot" research (Nathan & Keodinger 2000; Nathan & Petrosino, 2003) has quantitatively shown that pre-service teachers with high content knowledge are most likely to have symbol-precedence pedagogical content knowledge. In this study, the findings in the first stage of Chapter 4 replicated Nathan and Petrosino's (2003) "expert blind spot" study and showed again that high content knowledge pre-service teachers have symbol-precedence pedagogical content knowledge. The findings in the second stage of Chapter 4 showed the characteristics of pre-service teachers' perspectives in teaching algebra. Based on these findings, the discussion in this chapter presents the components of "expert blind spot" in high ACK test/symbol-precedence pre-service teachers' perspectives in (a) teaching algebra meaningfully, (b) understanding students' needs, and (c) connecting algebra to students' levels of understanding. Moreover, this chapter also presents possible teaching blind spots in low ACK test/verbal-precedence pre-service teachers and high ACK test/verbal-precedence pre-service teachers. The conclusions address pre-service teachers' curricula design, the value of the "expert blind spot" research to schools, the value of the "expert blind spot" research to mathematics education reform and the further research of this study.

Research Questions and Discussions

According to the findings in Chapter 4, the results of both the first and second stages briefly answer the following questions.

1. What is the level of pre-service teachers' algebra content knowledge? Forty pre-service teachers took the ACK test with a maximum score of 40. The test median, 29, was used to divide pre-service teachers into a high content knowledge group (above median) and a low content knowledge group (below median). Pre-service teachers in the high content knowledge group ($M=32.3$, $SD = 3.1$) were significantly different from the pre-service teachers in the low content knowledge group ($M = 24.7$, $SD = 3.4$), $t(36) = 7.20$, $p\text{-value} < 0.05$ (two-tailed). Among the 20 pre-service teachers in the high content knowledge group, 65% were secondary mathematics majors. Among the 20 pre-service teachers in low content knowledge group, 75% were elementary education majors with a mathematics minor. Results showed that the secondary mathematics majors had higher content knowledge than elementary education majors with a mathematics minor.

2. How do the high content knowledge participants and low content knowledge participants perform on the same Difficulty Factors Analysis ranking task (Nathan & Petrosino, 2003)? Pre-service teachers in the high content knowledge group ($n = 20$) had 18 members whose Pearson's r in the DFA ranking task was higher than 0.75. The result indicated that 90% of the pre-service teachers in the high content knowledge group were likely to have symbol-precedence pedagogical content knowledge. Therefore, pre-service teachers in the high content knowledge group may possibly have an "expert blind spot" in their teaching perspectives. Only nine pre-service teachers with low content knowledge

($n = 20$) had a Pearson's r in the DFA ranking task higher than 0.75. This result indicated that only 45% of those with low content knowledge were likely to have symbol-precedence pedagogical content knowledge. It showed that pre-service teachers with low content knowledge had little "expert blind spot" in their teaching perspectives.

3. What are the pre-service teachers' perspectives in teaching algebra? According to the Chapter 2 literature review, three coding themes could be used to explore their perspectives. These are (a) teaching algebra meaningfully, (b) understanding students' needs, and (c) connecting algebra to students' levels of understanding. As an example of high ACK test/symbol-precedence pre-service teachers, Cheryl assumed that her mathematics content knowledge could effectively connect to students' levels of understanding through her teaching knowledge. However, based on her perspectives, she would guide students to learn algebra through symbol-precedence, but she could not interpret her high content knowledge by either symbol-precedence or verbal-precedence. Therefore, when she represented her content knowledge to fit students' levels of understanding, a possible "expert blind spot" would be that she did not have an effective teaching knowledge to connect students' levels of understanding. Based on Cheryl's case study, the researcher could say that high ACK test/symbol-precedence pre-service teachers have a knowledge-centered perspective regarding their pedagogical content knowledge.

Jennifer, as an example of a low ACK test/verbal-precedence pre-service teacher, felt that pedagogical knowledge was more important than content knowledge. She liked to represent content knowledge based on hands-on mathematics so that students could

solve problems by manipulating physical objects. Her teaching processes were developed to verbally guide students' "real life" sense to clarify their mathematics sense. Using hands-on mathematics with verbal guidance described her perspectives of teaching knowledge. In her teaching processes, textbooks offered the mathematics content knowledge. Even though she had enough algebra content knowledge, Jennifer tried to imagine herself as a 7th grade student. She would use a general problem-solving philosophy with her students because she believed it was an appropriate way to understand students' thoughts and problem-solving strategies. However, with her perspective, she preferred to follow the textbook's progression rather than be guided by her algebra content knowledge. To explain this phenomenon, the research would suggest that she needed textbooks to support her mathematics content knowledge so that she could have more time to focus on her teaching. Therefore, a possible blind spot in her perspective might be in her content knowledge. She could effectively connect the basic mathematics content knowledge shown in the textbook to more advanced mathematics content knowledge when doing hands-on mathematics. Based on the results of her case study, the researcher could say that low ACK test/verbal-precedence pre-service teachers use problem-centered perspectives in their pedagogical content knowledge.

In Trevor's case, an example of high ACK test/verbal-precedence pre-service teacher, he would focus on both showing his mathematics content knowledge to students and, at the same time, searching students' levels of understanding. To do that, he would modify his teaching strategies to better match his students' performance in learning, either symbol-precedence or verbal-precedence. Students' responses in his teaching

processes were his primary means to understand the students' learning situation. Ideally, he would develop an appropriate way to connect his content knowledge to students' level of understanding. When asking a real question (for instance, the question: Why $-2 \times -4 = +8$?), however, Trevor also struggled with use of the verbal-precedence to show the question, $-2 \times -4 = +8$, to his students. A possible problem in his teaching knowledge was how to show his understanding of mathematics content knowledge by using both symbol and verbal-precedence. Based on the results of Trevor's case study, we might say that high ACK test/verbal-precedence pre-service teachers recognize the difficulties in connecting the teaching strategies to their mathematics content knowledge. Therefore, they preferred to use response-based perspectives to connect teachers' content knowledge to students' levels of understanding.

Different groups of pre-service teachers have different perspectives in their pedagogical content knowledge based on their different levels of mathematics content knowledge. It is not reasonable to compare all kinds of teaching perspectives (Grossman, 1987). The following discussion individually addressed the three groups' possible teaching blind spots.

Teaching Blind Spots in the High ACK Test/Symbol-Precedence Pre-service Teacher

The first stage of this study showed that in the DFA ranking task (Nathan & Petrosino, 2003), 76% of all pre-service teachers are in the symbol-precedence pedagogical content knowledge group. Moreover, 90% of those with high content knowledge have symbol-precedence pedagogical content knowledge while only 40% of those with low content knowledge have symbol-precedence pedagogical content

knowledge. The discrepancy in pedagogical content knowledge indicated that the “expert blind spot” hypothesis was shown in this study and that pre-service teachers with high content knowledge have symbol-precedence pedagogical content knowledge. This result replicated Nathan and Petrosino’s (2003) study.

To explore the components of the teaching blind spots, a follow-up interview was conducted with four pre-service teachers who had high content knowledge and symbol-precedence pedagogical content knowledge. Cheryl was the pre-service teacher who qualified as an example of the high ACK test/symbol-precedence group.

Blind spots in teaching algebra meaningfully. According to the findings in Cheryl’s case study, this pre-service teacher had a knowledge-centered perspective when teaching mathematics. Her definition of algebra showed her higher levels of mathematics content knowledge. Based on her definition of algebra, she tended to guide her students by her own mathematical skills, and she felt confident to show patterns and relationships in any mathematics subject matter. A possible “expert blind spot” in teaching algebra meaningfully for her was that she believed that her students have to learn computational skills rather than conceptual knowledge. It was easier for her to use symbol-precedence to express her content knowledge, and verbal guidance to show how the content knowledge from symbol-precedence could be applied in a “real life” problem. Therefore, she preferred to use symbol-precedence as her major teaching strategy when teaching algebra.

Blind spots in understanding students’ needs. Understanding students’ needs for Cheryl meant that students needed to know some supplemental content knowledge in

order to learn the current subject matter. Even though Cheryl used students' experience to motivate students' interests in learning algebra, she expected that students could make more sense of computational skills in symbol-precedence. She thought students could more easily understand computational skills than verbal problems. A teaching blind spot might be that she neglected the fact that different students needed different ways to connect mathematics content knowledge to their learning strategies.

Blind spots in connecting algebra to students' levels of understandings. Cheryl assumed that her students know basic mathematics. However, when she was asked how to teach a concept of a computational property (for instance, $-2 \text{ time } -4 \text{ is equal to } +8$), she took a long time to recall the meaning of the computational property. As shown in her interview data, she preferred to emphasize how to use the property in her teaching knowledge of a computational process rather than show students why the computational property can work. Therefore, Cheryl might have the "expert blind spot" from her precedence to connect students' levels of understanding. To sum up, the interview results showed that Cheryl believes her effectiveness as a teacher was affected by her content knowledge of mathematics. The following is the analysis based on the interview findings.

Teaching Blind Spots in Low ACK Test/Verbal-Precedence Pre-service Teachers

Twenty pre-service teachers were in the low content knowledge group. Among these 20 pre-service teachers, 11 pre-service teachers (45% of 20) had verbal-precedence pedagogical content knowledge (their Pearson's r is below 0.25). The statistics showed that pre-service teachers with low content knowledge have a high probability of having verbal-precedence pedagogical content knowledge. A follow-up interview was conducted

with three pre-service teachers with low content knowledge but verbal-precedence pedagogical content knowledge. The interview results in Jennifer's case showed that she had strong problem-centered perspectives in teaching mathematics. Her teaching strategies would be to use hands-on mathematics to connect students' real life experiences to mathematics knowledge. Her problem-centered perspective, however, might cause some teaching blind spots because of her stronger reliance on verbal-precedence. The following is the analysis based on the interview findings from Jennifer.

Blind spots in teaching algebra meaningfully. Jennifer is an expert in teaching students mathematics content knowledge by verbal-precedence because she believed that teaching algebra was 'doing algebra', so she was determined to be a good mathematics activity designer. However, she did not have high content knowledge to build on her mathematics ability, so she needs textbooks to arrange her content knowledge. Jennifer did not work to improve her mathematics content knowledge or any other content knowledge. Instead, she thought that increasing her teaching knowledge was more important than improving her content knowledge. She believed that if she had high teaching knowledge she could teach any kind of subject matter to students. A possible teaching blind spot might be her deficiency in mathematics content knowledge. She might implement her teaching knowledge of the basic concepts very well, but she might not offer higher levels of mathematics content knowledge to students.

Blind spots in understanding students' needs. Jennifer had a high ability to convey mathematics knowledge by understanding students' problem-solving strategies and using visual materials. To teach algebra, she would use a table and lower grade arithmetic to

show the concepts of unknowns. She would develop her teaching lessons based on monitoring students' interests. Therefore, she understood students' needs very well and could design good activities to develop algebraic knowledge. However, she would focus on teaching conceptual knowledge rather than procedural knowledge. She would even skip teaching computational processes if time were limited. She believed that most students were interested in solving story problems by guess-and-check strategies. Jennifer's problem-centered teaching strategies could help her understand students' needs and then benefit students' learning situations. However, a possible teaching "blind spot" for her was that the problem-centered perspective may cause students not to learn enough mathematics content knowledge from Jennifer. Moreover, she might not have a strong enough ability to combine verbal and symbol-precedence even though she could figure out the difference between verbal and symbolic representational questions.

Blind spots in connecting algebra to students' levels of understandings. When students misunderstood algebraic concepts, Jennifer preferred to use verbal examples to correct the misunderstandings. For instance, the question: $2a + 5b = 7ab$, Jennifer would use apples, bananas, and fruit as metaphors to figure out the symbolic procedural knowledge. In the similar symbol question (Why $-2 \times -4 = +8$?), Jennifer did not have an idea how to explain this to students. However, she believed that mathematics textbooks would effectively explain why negative 2 multiplied by negative 4 is equal to positive 8. She would use the textbooks' interpretation as a reference, and then she would develop more interpretation to support the textbooks' interpretation for students.

Different from knowledge-centered perspectives, Jennifer, with her problem-centered perspectives, tried to put herself on the students' levels of understanding. She thought this was the best way to connect with students' needs for understanding. While most pre-service teachers used arithmetic concepts verbally to show the difference between x times y and x plus y , Jennifer would show the difference between x times y and x plus y by using a visualized table. She believed that if she just interpreted the difference verbally without any visualization, then students might easily forget the concepts. Therefore, she emphasized the use of a table and actual numbers to show students the difference visually. She also tried to use verbal-precedence to match students' level of understanding through real life examples. Different from knowledge-centered perspectives, Jennifer focused on students' understanding in conceptual knowledge. However, Jennifer's case also showed that she did not have a strong sense of understanding the meaning of higher level mathematics content knowledge. Therefore, Jennifer, or pre-service teachers with low content knowledge and verbal-precedence pedagogical content knowledge, might have a possible "blind spot" with connecting the basic content knowledge to higher levels of mathematics content knowledge.

Teaching Blind Spots in the High ACK Test/Verbal-Precedence Pre-service Teachers

According to the results in the first stage, only one pre-service teacher, Trevor, qualified to be a high content knowledge and verbal-precedence pedagogical content knowledge. In Trevor's interview data, he had response-based perspectives in teaching mathematics. He emphasized how he used questions to assess students' level of understanding and then increased students' understanding by a sequence of assessments.

He clearly showed his teaching processes were based on students' responses to his teaching process. However, he could not clearly show how an effective response to his students could be used in teaching a real subject matter (e.g., algebra). Therefore, some blind spots might be in the connection between his understanding of students' mathematics problems and students' real performance in mathematics.

Trevor's interview results showed he had good perspectives between mathematics content knowledge and pedagogical content knowledge. However, concerning pedagogical knowledge, he could not effectively connect his mathematics cognition to a real teaching process. Therefore, his "expert blind spot" was that even though he knew the difference in pedagogical knowledge between verbal-precedence and symbolic precedence, he still could not effectively offer a flexible teaching strategy in his pedagogy.

Trevor's teaching strategy was based on students' levels of understanding. Although he was concerned with students' level of understanding, he still potentially expected students to follow his way of understanding mathematics content knowledge. He needed more teaching experience to increase his response-based skills so that he could gain more ideas on how to connect his mathematics content knowledge to the students' learning progression.

Conclusion

This study investigated mathematics pre-service teachers' pedagogical content knowledge and different mathematics content knowledge levels. It was designed to discover the perspectives that arise from their high/low mathematics content knowledge.

More importantly, this study explored three kinds of pre-service teachers' perspectives based upon their level of content knowledge.

First, pre-service teachers with high content knowledge but symbol-precedence pedagogical content knowledge are likely to produce knowledge-centered perspectives in their teaching style. Nathan and Petrosino's (2003) study has shown these pre-service teachers have little pedagogical content knowledge (expert blind spot). They use their content knowledge to think about their teaching knowledge so that teaching algebra is sharing their understanding in algebra. The meaning of increasing students' content knowledge is based on what pre-service teachers feel is easy for them, not on what students feel is easy. They have their own way of connecting the easy content knowledge to the difficult content knowledge. They like to understand students' needs by their own rationale of content knowledge. Therefore, when trying to correct students' misunderstandings, they used their own learning experience to perceive students' misunderstanding, and then they could explain the question/problem to students' by using other means.

Second, pre-service teachers with low content knowledge but verbal-precedence pedagogical content knowledge were likely to produce student-centered perspectives in their teaching knowledge. These pre-service teachers liked to increase their teaching knowledge rather than mathematics content knowledge, because they did not think teachers needed to have higher content knowledge than their students. These pre-service teachers thought that students needed hand-on mathematics. They were familiar with using mathematics manipulatives and also would design some activities for their students.

They believed teaching mathematics by visualization was the best way to meet students' needs. However, they did not have a strong ability to connect different mathematics knowledge. They believed that pedagogical content knowledge was different from content knowledge so they did not use their understanding in content knowledge to support their pedagogical content knowledge. They still separate content and pedagogy.

Third, pre-service teachers with high content knowledge and verbal-precedence pedagogical content knowledge were likely to produce response-based perspectives in their teaching practice. Content knowledge for these pre-service teachers was not a major issue. Their teaching processes combined both mathematics content knowledge and students' levels of understanding of mathematics content knowledge. Their assessments of students' progress gave them a direction for selecting different teaching strategies to maximize students' levels of understanding. The limitation for these pre-service teachers was that they could not offer an effective assessment to show how they might implement their perspectives in a real teaching situation. Real teaching experience would help them gain the knowledge to relate the mathematics content knowledge to students' level of understanding.

The Pre-service Teachers' Curricula Design

In this study, the sample data were from pre-service teachers who used different curricula for different goals. Elementary education majors who wanted to get a mathematics minor for teaching K-6 were in one curriculum (Appendix J) under the college of education. Twenty-four semester hours were required in this curriculum. In these 24 hours, only 9 hours were working on mathematics content knowledge

(*Mathematics in Decision Making, Introduction to Statistical Methods, and Introduction to Mathematical Modeling*). Students in this curriculum were not required to take *Calculus I, II, and III*. The other 15 hours (66% of total semester hours) in this curriculum focused on students' pedagogical content knowledge.

Secondary mathematics majors were in the other curriculum (Appendix J) under the college of natural sciences. They were required to take 45-46 semester credits for the mathematics teaching major program. In this curriculum, 15 semester hours were in common core courses (including *Calculus I, II, III* and *Linear Algebra for Application*). Students in the program also needed to take 21 semester hours for teaching core courses (including *Introduction to Mathematics Modeling, Modern Algebra I, Introduction to Modern Geometry, Probability and Statistics, History of Mathematics, The Teaching of Middle School/Junior High Mathematics, and The Teaching of Secondary Mathematics*). In these 7 courses, the first 5 were used to increase students' thinking in their content knowledge, and the last two courses were focused on the knowledge of teaching. Students in this program were developing their teaching knowledge only in the last two courses (13% of total semester hours).

When comparing the two programs (elementary education major and secondary mathematics teaching major), the researcher found some phenomena in both programs. In the mathematics teaching minor program, for the elementary education major students, some courses focused on both content knowledge and teaching knowledge (for instance, *Algebraic Thinking* and *Problem Solving*). However, these elementary education students used the concepts of hands-on mathematics and neglected to connect the concepts of

hands-on mathematics to their higher levels of mathematics content knowledge. If they could effectively improve their content knowledge, they could gain more mathematical sense for developing their teaching knowledge. Also, they would have more ideas to help develop their hands-on mathematics for their students.

In the secondary mathematics teaching major program, the courses were separated to either focus on content knowledge or teaching knowledge. Not one class was designed to combine content knowledge and teaching knowledge. For instance, in the group of teaching core courses, the first five classes (*Introduction to Mathematics Modeling*, *Modern Algebra I*, *Introduction to Modern Geometry*, *Probability and Statistics*, and *History of Mathematics*) were potentially used to increase students' content knowledge. These classes did not show how this specific content knowledge could effectively connect to students' level of understanding. In the secondary mathematics teaching major, only 6 semester credits were in courses which discussed how to teach. Students in this program received mathematics content knowledge and teaching mathematics separately so they easily developed knowledge-centered perspectives.

The Value of the "Expert Blind Spot" Research to Schools

The "expert blind spot" research offers a scientific method to examine the relationship between mathematics content knowledge and pedagogical content knowledge. This research showed that most high content knowledge mathematics teachers are likely to have blind spots in their teaching knowledge. These results offer valuable insights for K-12 mathematics teachers, principals, and current higher education professors who are teaching mathematics methods courses. Specifically, current

principals in middle schools may have a dilemma when hiring new mathematics teachers. Since there are not a lot of high ACK test/verbal-precedence pre-service teacher candidates, principals will most likely hire either high ACK test/symbol-precedence pre-service teachers or low ACK test/verbal-precedence pre-service teachers. A major issue to consider is which kind of pre-service teacher may more likely reduce his/her blind spots and benefit students' learning. People may think that high ACK test/symbol-precedence pre-service teachers might be the best choice as future teachers, because many studies (Wu, 1997, 2000; Ma, 2000) showed that teachers' high content knowledge could effectively help their teaching knowledge. However, this research has shown that if teachers increase their content knowledge, they might create some teaching blind spots in their teaching progress. They were not aware of the "expert blind spot" within their teaching knowledge (Nathan & Koedinger, 1999, 2000a, 2000b; Nathan & Petrosino, 2003). The research showed that high ACK test/symbol-precedence pre-service teachers did not have high motivation to understand students' needs. So, it is more probable for high ACK test/symbol-precedence pre-service teachers to teach mathematics content knowledge which ignores this "expert blind spot" concerning their low teaching knowledge.

The low ACK test/verbal-precedence pre-service teachers understood their limitation in mathematics content knowledge even though they did not think it was very important to being a good math teacher. These pre-service teachers have been trained to have high skills in understanding students' needs. They more often designed an appropriate activity to fit different students' learning situation. If low ACK test/verbal-

precedence pre-service teachers understand the value of content knowledge, they might expect to increase their content knowledge in order to develop their pedagogical content knowledge, based on mathematics content. Increasing content knowledge for these pre-service teachers would not cause them to change from their problem-centered perspective to a knowledge-centered perspective. Instead, they may increase their content knowledge by using more creative methods of instruction.

The Value of the “Expert Blind Spot” Research to Mathematics Education Reform

Mathematics education in the USA has been discussed in national newspapers and magazines (The International Commission on Mathematical Instruction [ICMI], 1998). The focus of attention has been on the so-called “Math Wars” which concerned the reform in the school mathematics curriculum and teaching. This study was related to a major conflict between the traditional approach and a new approach to teaching for understanding. The major conflict happened in 1987 when the California Department of Education (CDE) published the Mathematics Model Curriculum Guide (California Department of Education, 1987) which included 88 pages devoted to “teaching for understanding” with classroom examples.

University of California, Berkeley, mathematics Professor H. Wu assessed the traditional approach and a new approach to teaching for understanding (Wu, 1997) based on both his mathematical and educational perspective. Because he was worried about students’ academic mathematics ability, his thinking is worthy of examination in terms of academic mathematics preparation. When discussing the relationship between mathematics content knowledge and pedagogical content knowledge, he felt many errors

in the new mathematics curricula needed to be corrected. He thought some important topics were omitted. He emphasized that there was an ambiguous mixture of pedagogical statements with content statements in the new curricula. For example, he mentioned the omission of the division algorithm in the elementary grades and the Fundamental Theorem of Algebra in the higher grades (Wu, 1997).

The result of this research might offer some evidence in explaining relationships between mathematics content knowledge and pedagogical content knowledge. First, mathematics reform must not focus only on the content knowledge. Wu suggested that mathematics teachers should increase their mathematics content knowledge in order to have solid pedagogical content knowledge. Based on this study, the research would suggest that even though content knowledge is very important for mathematics teachers, it does not mean that elementary education teachers should be required to take *Calculus I, II, and III* to become effective mathematics teachers. Elementary mathematics teachers should understand students' learning strategies first so that they could use different teaching strategies to match students' needs in learning mathematics. To do that, the focus of increasing their mathematics content knowledge should have a pedagogical content knowledge focus that connects the mathematics content knowledge to different students' needs in learning mathematics.

Second, as shown in this study, high ACK test/symbol-precedence pre-service teachers have the "expert blind spot" in their pedagogical content knowledge. The results indicated that high content knowledge pre-service teachers extend their understanding of mathematics content knowledge to their teaching knowledge. They do not develop their

teaching knowledge by focusing on students' levels of understanding. High ACK test/symbol-precedence pre-service teachers might create their teaching knowledge based on their experience of learning higher mathematics content knowledge. They might find it difficult to think about ways to interpret the computational knowledge by verbal-precedence. They did not have enough understanding about using either symbol or verbal-precedence to teach the same mathematics concepts. Therefore, they are likely to have an "expert blind spot" in their understandings.

Finally, as shown by the high ACK test/verbal-precedence pre-service teacher, these pre-service teachers combined their high mathematics content knowledge with their high curricula knowledge. They provided a good model for a future mathematics preparation program. Even though the high ACK test/verbal-precedence pre-service teacher interviewee could not offer effective assessments, he was highly aware of an appropriate way to connect his content knowledge to fit students' needs. Normally, we would expect that high school mathematics teachers should have high mathematics content knowledge. If so, the high school mathematics curriculum should not only prepare students who need to pass academic examinations, but should also prepare students who are interested in learning applied mathematics. Therefore, high content knowledge pre-service teachers should learn how to convey their symbolized knowledge in real life situations.

The Further Research of This Study

Future research can focus on the following suggestions.

1. In this study, 40 participants were either mathematics education majors or elementary education majors with a mathematics minors. They had almost finished their major courses and they were ready to be pre-service teachers in the coming semester so that their mathematics content knowledge and pedagogical content knowledge was reliable and measurable. If the future research could increase the size of the participant group, the statistical reliability could be improved. Also, a future study may find more participants who have both high content knowledge and verbal-precedence pedagogical content knowledge so that reliability could be greater.

2. A limitation to understanding pre-service teachers' pedagogical content knowledge was that they did not have full-time teaching experience. Their perspectives of teaching knowledge were based on reflections of their academic learning experience. Follow-up research on the subjects in this study could track those selected pre-service teachers' pedagogical content knowledge and their students' achievements when they are in a full-time teaching position.

3. The research topic of this study, exploring the relationship between mathematics content knowledge and pedagogical content knowledge, used pre-service teachers as participants. A further study can use in-service teachers as participants to compare the results in this same topic.

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Appendix A

APPROVAL IRB FORM AND CONSENT LETTER

This IRB was approved by Dr. Mary Losch
Date to approve: 2004, May

Office Use Only: Protocol # 03-0240

**University of Northern Iowa
Human Participants Review Committee Application**

Note: Before Completing Application, Investigators Must Read **Information for Investigators**
(<http://www.grad.uni.edu/research/policy.asp>)

All items must be completed and the form must be typed or printed electronically.

**Submit 3 hard copies to the Human Participants Review Committee, Graduate
College, 122 Lang Hall, 0135**

Title of proposal: Exploring the relationship between mathematics content knowledge and pedagogical content

Project Faculty/Staff Research Class Project Thesis/Dissertation

Grant/Contract Other, Specify _____

Name of Principal Investigator (PI): Hsueh-I Martin Lo

Status: Faculty Undergraduate Student Graduate Student Staff

PI Department: Curriculum and Instruction Faculty Advisor
Dept(if different): _____

PI Phone: 222-5951 PI Email: martinlo@uni.edu

PI Campus Mailing Address/Mail Code: 301 F St. Cedar Falls, IA50613

Source of Funding: Student funded

Agency's Number (if assigned): _____

Data collection dates: Upon Approval Through May 2005

Project New Renewal Modification Grant-Compet. Renewal Grant-

blind spot") that in some situations, having a high degree of content knowledge may in fact be associated with symbol-precedence pedagogical content knowledge. Pre-service teachers' with various levels of mathematics subject-matter expertise were given a series of algebra problems and asked to rank order their difficulty. Nathan & Petrosino (2003) reported that on average, pre-service teachers with more advanced mathematics education rank ordered the problems in ways that were inconsistent with actual patterns of student performance, suggesting that they had less insight into how students think about and solve these problems (pedagogical content knowledge). In contrast, students who had taken fewer advanced mathematics courses rank-ordered problems in a way that was more consistent with actual student performance. In the proposed study, we build on this work by using a direct measure of content knowledge (an algebra content knowledge test) and explore additional aspects of algebraic pedagogical content knowledge through an interview with select participants.

Significance of the study: The findings of this study may have important implications for policy makers and teacher educators alike. For example, the expert blind-spot hypothesis has implications for various alternative teacher certification proposals that suggest that teacher candidates would be better served if preparation programs emphasized more subject-matter (i.e., mathematics) course-work and less (if any) pedagogical (e.g., education) coursework. This study will help establish the robustness and limits of the "expert blind spot" finding identified by Nathan and Petrosino (2003). If the finding is replicated, it may suggest that both content knowledge and pedagogical content knowledge need to be evaluated for the appropriate level of emphasis necessary for successful teacher preparation in teacher education programs.

- B. RESEARCH PROCEDURES INVOLVED.** 1. Provide a complete description of:
 a. the study design, and b. all study procedures that will be performed (e.g., presentation of stimuli, description of activity required, topic of questionnaire or interview, name of psychological test). Provide this information for each phase of the study (pilot, screening, intervention and follow-up). Attach study flow sheet, if desired.

Attach questionnaires, interview questions/topic areas, scales, and/or examples of stimuli to be presented to participants.

Three different groups of pre-service teachers (secondary math education, elementary education majors with math minors and middle school education majors) who are

enrolled in a first education methods course in the Fall (04) will be invited to participate in this study. Participants will be given an algebra achievement test to assess their content knowledge of algebra. Based on participants' scores on the algebra test they will be identified as "high" (above median score) or "low" (below median score) in content knowledge and randomly assigned to one of two types of test conditions: within the first two weeks of the semester (pre-methods instruction) or within the last three weeks of the end of the semester (post-methods instruction). After assignment to condition, arrangements will be made to have participants complete the algebra problem difficulty ranking task to assess their pedagogical content knowledge. Copies of these two instruments are attached.

Finally, a select sample of participants who display characteristics of interest (e.g., high content knowledge/symbol-precedence pedagogical content knowledge; low content knowledge/verbal-precedence pedagogical content knowledge, etc.) will be invited for follow-up interviews. Interviews will focus on exploring participants' understanding of other aspects of pedagogical content knowledge related to algebra learning.

- C. DECEPTION:** If any deception or withholding of complete information is required for this activity, explain why this is necessary and attach a protocol explaining if, how, when, and by whom participants will be debriefed.

There is no deception in the study.

D. PARTICIPANTS

1. Approximately how many participants will you need to **complete** this study?

Number 90 Age Range(s) 20-50

2. What characteristics (inclusion criteria) must participants have to be in this study? (Answer for each participant group, if different.)

The groups of participants will be preservice teachers with a mathematics teaching major or minor enrolled in a mathematics method course with Dr. Leutzinger or Dr. Miller.

3. Describe how you will recruit your participants and who will be directly involved in the recruitment. (Attach advertisements, flyers, contact letters, telephone contact protocols, scripts, web site template, etc.)

A contact letter (in Appendix) will be distributed to the prospective participants' during their class meeting time with their UNI mathematics instructors, Dr. L. Leutzinger, Dr. Nelson, or Dr. C. Miller. The professors will exit the classroom during the recruitment, testing, and surveying portions of the study.

4. How will you protect participants' privacy during recruitment? (Attach letters of cooperation & agreement from any and all agencies, institutions or others involved in participant recruitment.)

Each potential participant will return the detachable portion back from the initial contact letter indicating interest in continuing in this study or declining participation. Participant's privacy will be respected during the recruitment phases and no public release of names of those who agree to participate will be disclosed. Professors will exit the room during the recruitment, testing, and surveying portions of this study.

5. Explain what steps you will take during the recruitment process to minimize potential coercion or the appearance of coercion.

All potential participants will reply back to the researcher if they are willing or not willing to be considered as subjects for the algebra achievement test, algebra ranking survey and interviews. After the initial explanation of Mr. Lo's study, the professor will be asked to exit the classroom so students may complete the consent forms with less possibility of the feeling of coercion. No coercion or the appearance of coercion is expected to take place. All participants will have the option to discontinue participation at any time during the testing, ranking survey or interview protocols.

6. Will you give participants gifts, payments, services without charge, or course credit?

No Yes If yes, explain:

7. Where will the study procedures be carried out? If any procedures occur off-campus, who is involved in conducting that research? (Attach copies of IRB approvals or letters of cooperation from non-UNI research sites if procedures will be carried out elsewhere.)

On campus Off campus Both on- and off-campus

8. Do offsite research collaborators have human participants protection training?
 No Yes Don't know Not applicable – no offsite collaborators

E. RISKS AND BENEFITS

1. All research carries some social, economic, psychological, or physical risk. Describe the nature and degree of risk of possible injury, stress, discomfort, invasion of privacy, and other side effects from all study procedures, activities, and devices (standard and experimental), interviews and questionnaires. Include psychosocial risks as well as physical risks.

The risk for participants to engage in this study may include some psychological risk since they will be asked to share personal reasons why their pedagogical content knowledge will be successful. All interviewed participants will be given pseudonyms. Details of all of the participants' test and ranking data will be reported in aggregate form.

2. Explain what steps you will take to minimize risks of harm and to protect participants' confidentiality, rights and welfare. (If you will include protected groups of participants which include minors, fetuses, prisoners, pregnant women, or cognitively impaired or economically or educationally disadvantaged participants, please identify the group(s) and answer this question for each group.)

Participants will be given an opportunity to review the data regarding their interviews and correct or eliminate any information they do not want reported. The participants will not be harmed nor will their rights or welfare be impeded. The interviews will be recorded by audio or video tape and transcribed. Only the researcher (and possibly the dissertation chair) will hear or have access to the audio tapes. The audio tapes will be kept in a locked file.

Upon final approval by UNI faculty of the dissertation paper, the tapes will be erased to prevent others from hearing or viewing them.

All tests, ranking documents, and interview transcripts will be kept for a period of one year following the completion of the doctoral degree. No personally identifying information will be kept. All coding of participants will be destroyed so only anonymous coded documents are available.

3. Study procedures often have the potential to lead to the unintended discovery of a participant's personal medical, psychological, and/or psycho-social conditions that could be considered to be a risk for that participant. Examples might include disease, genetic predispositions, suicidal behavior, substance use difficulties, interpersonal problems, legal problems or other private information. How will you handle such discoveries in a sensitive way if they occur?

There is no anticipated risk of a personal type that will emerge from the testing or ranking activities of this study. The reporting of private information during the interview will be at the discretion of the interviewee. In addition, interview participants will be given an opportunity to review the data regarding their personal interviews and correct or eliminate any information they do not want reported.

4. Describe the anticipated benefits of this research for individual participants in each participant group. If none, state "None."

No direct benefits to participants exist. However, this study will be contributing to a better understanding of the roles of content knowledge and pedagogical content knowledge in the preparation of pre-service mathematics teachers.

Participants will be adding to the knowledge base in the teaching field of mathematics.

5. Describe the anticipated benefits of this research for society, and explain how the benefits outweigh the risks.

The findings of this study may have important implications for policy makers and teacher educators alike. For example, the expert blind-spot hypothesis has implications for various alternative teacher certification proposals that suggest that teacher candidates would be better served if preparation programs emphasized more subject-matter (i.e., mathematics) course-work and less (if any) pedagogical (e.g., education) coursework. This study will help establish the robustness and limits of the "expert blind spot" finding identified by Nathan and Petrosino (2003). If the finding is replicated, it may suggest that both content knowledge and pedagogical content knowledge need to be evaluated for the appropriate level of emphasis necessary for successful teacher preparation in teacher education programs.

F. CONFIDENTIALITY OF RESEARCH DATA

1. Will you record any direct participant identifiers (names, Social Security numbers, addresses, telephone numbers, locator information, etc.)

Yes If yes, explain why recording identifiers is necessary and describe the coding system(s) you will use to protect against disclosure.

In order to contact students for the interview portion of this study, pre-service teachers will be assigned a coded number upon receipt of their consent form. This coded number and identifying information will be stored in a locked drawer in the principal investigator's office. It will not be stored in the same location as the tests, ranking surveys, or interview tapes or transcripts.

2. After data collection is complete, will you retain a link between study code numbers and direct identifiers after the data collection is complete?

No Yes If yes, explain why this is necessary and for how long you will keep this link.

3. Describe how you will protect data against disclosure to the public or to other researchers or non-researchers. Other than members of the research team, explain who will have access to data (e.g., sponsors, advisers, government agencies) and how long you intend to keep the data. If data will be collected via web or internet, please include information on security measures, use of passwords, encryption, access to servers, firewalls, etc.

The tests, ranking survey, tapes, transcript data and other results of data recording and analysis will be secured in a locked file. The data will be retained until final approval of the research paper requirement for UNI and then destroyed.

4. Do you anticipate using any data (information, interview data, etc.) from this study for other studies in the future?

No Yes If "Yes," explain and include this information in the consent form.

G. ADDITIONAL INFORMATION

1. Will you need access to participants' medical, academic, or other personal records for screening purposes or during this study?

No Yes. If yes, specify types of records, what information you will take from the records and how you will use them.

2. Will you make sound or video recordings or photographs of study participants?

- No Yes. If yes, explain what type of recordings you will make, how long you will keep them, and if anyone other than the members of the research team will be able to see them.

Audio or video recordings will be made during the interviews with approval from the interviewees. The Principal Investigator will be the only person with access to the recordings. The recordings will be destroyed upon final approval of dissertation paper.

H. CONSENT FORMS/PROCESS Check all that apply.

- Written** (Attach a copy of all consent and assent forms for each participant group.)
- Oral** (Attach a written script of oral consent and assent for each participant group and justification for waiver of documentation of consent)
- Elements of Consent Provided via Letter or Electronic Display** (Attach written justification of waiver of documentation of consent along with text of consent for letter or display)
- Waiver of Consent** (Attach written justification of waiver of consent process. Note that waiver of consent would only be granted if the consent process itself posed a greater risk to participants than did participation in the research)

Consent Letter

Project Title: An Investigation of Pre-service Mathematics Teachers' Content Knowledge
and Pedagogical Content Knowledge of Algebra

Principal Investigator: "Martin" Hsueh-I Lo

Faculty Sponsor: Dr. William Callahan

Dear Pre-service Teacher,

You are invited to participate in a research project about pre-service teachers' pedagogical content knowledge. This study is being conducted by a doctorate student, "Martin" Hsueh-I Lo, for his dissertation project. The following information is provided to help you make an informed decision about whether or not to participate.

Background of the Study

It is essential that mathematics teachers are able to provide content knowledge as well as pedagogical content knowledge when teaching algebra. The primary purpose of this study is to determine what kind of pedagogical content knowledge pre-service mathematics teachers will want to use in their classroom, and what is essential pedagogical content knowledge needed to teach algebra.

Participants in the Study

To participate in this study, you must be enrolled at UNI as a pre-service teacher. You must be planning to finish your undergraduate coursework within one year.

You will be administered an algebra content knowledge test, and a 10-minute ranking task. Eight participants will be selected to be interviewed so the researcher can gather more detailed information about perceptions about teaching algebra.

Methodology

If you agree to join this study as a participant, you will be notified in writing and a testing/survey date will be scheduled during your class. Your identity will be protected. If you are one of the eight participants selected to be interviewed, you will be notified in writing after the testing/survey is completed. The time of interview for each participant is about 15 to 20 minutes. Mr. Lo will record the content of interview. After Mr. Lo completes his dissertation, the tape will be destroyed. Upon completion of the interview, transcripts or interview summaries will be made available to you for correction, elimination, or clarification of responses. Interview responses will be analyzed by the researcher. Quotes and story summaries from the interviews may be used to explain and

clarify the results of the testing/surveys conducted. The results of this study will be reported in a graduate dissertation for the Department of Curriculum Instruction. No other dissemination of this research project is planned. Through your participation in this study, you will be contributing to a better understanding of the pedagogical content knowledge and content knowledge of pre-service teachers. The participants in Mr. Lo's dissertation do not have direct benefit.

Your professor will **NOT** attend the testing, survey, or interview sessions. Your professor will **NOT** have access to any identifying information connecting you with your test, survey, or interview tapes/transcripts. Your professor will not have knowledge of who does or does not participate in the study until after grades have been assigned. You may discontinue this research at anytime. No consequences for withdrawal from this study will occur. No physical psychological, social, legal, and/or economic risk(s) or cost(s) on your part are expected to result from this research other than minimal risks such as inconvenience.

If you have questions about the study or desire information in the future regarding your participation or the study in general, you may contact "Martin" Hsueh-I Lo at 319-222-5951. His email address is: martinlo@uni.edu. You also may contact the project investigator's faculty advisor Dr. William Callahan at 159A Schindler Education Center, University of Northern Iowa, Cedar Falls, IA 50614-0606. You may email him at bill.callahan@uni.edu or phone his at 319-273-2719. You may contact the Office of the Human Participants Coordinator at the University of Northern Iowa at 319-273-2748 for answers to questions about rights of research participants and the participant review process.

If you agree to participate in the study, “An Investigation of Pre-service Mathematics Teachers’ Pedagogical Content Knowledge and Pedagogical Content Knowledge of Algebra”, please sign and complete the TWO copies of the Agreement of Participation form on the next two pages. The first copy is retained by you and the second copy is given to Mr. Lo.

Agreement

I am fully aware of the nature and extent of my participation in this project as stated above and the possible risks arising from it. I hereby agree to participate in this project. I acknowledge that I have received a copy of this consent statement. I am 18 years of age or older.

(Signature of participant)

(Date)

(Printed name of participant)

(Signature of investigator)

(Date)

(Signature of instructor/advisor)

(Date)

Agreement

I am fully aware of the nature and extent of my participation in this project as stated above and the possible risks arising from it. I hereby agree to participate in this project. I acknowledge that I have received a copy of this consent statement. I am 18 years of age or older.

(Signature of participant)

(Date)

(Printed name of participant)

(Signature of investigator)

(Date)

Appendix B

PROFESSOR CONSENT FORM

I grant permission for “Martin” Hsueh-I Lo to test, survey, and interview pre-service teachers from my course, _____, regarding their pedagogical content knowledge. I understand that the testing/surveying/interviews are part of a Mr. Lo’ s dissertation project, “An Investigation of Pre-service Mathematics Teachers’ Content Knowledge and Pedagogical Content Knowledge of Algebra.” No more than 80 pre-service teachers will be tested/surveyed and no more than 8 will be interviewed. The content of the interviews are perceptions of teaching mathematics in middle school, algebra content knowledge, perceptions of math curricula knowledge, and perceptions of pedagogical content knowledge linked to content knowledge. I understand that all participation is voluntary and any participant may withdraw from the research at any time. No other dissemination of information about the study is planned beyond its use for a graduate dissertation of the Department of Curriculum and Instruction. I agree to schedule an appropriate time for Mr. Lo to administer the test and survey during my class meeting time. I agree that I will NOT attend the recruiting, testing, surveying, or interviewing sessions and will leave the room and vicinity prior to the non-participants exiting the classroom. I will NOT have access to any identifying information connecting my students with their tests, surveys, or interview transcripts.

Professor’s Signature

Professor’s Name Printed

Date

Course Name and Number

Dates and Times Course Meets

Appendix C

ALGEBRA CONTENT KNOWLEDGE TEST SAMPLE
(Solving the following question without using calculator)

I. Patterns:

$$x + y, 3x + y, 6x + y, \dots$$

What is the tenth one?

II. Symbol computation:

$$\text{Solve for } D: \frac{2}{3}D - (-1.2) = -7.3$$

III. Graph:

Graph each equation

a. $xy = 24$

b. $y + 5 = -\frac{3}{4}(x - 6)$

IV. Symbolizing Word Problems

Mikes' parents allow him to work 30 hours a week. He would like to use this time to help out in his parents' hardware store, but it pays only \$5 per hour. He could mow lawns for \$7.50 per hour, but there is less than 20 hours of lawn work available. What is the maximum amount of time Mike can work in his parents' store and still make at least \$175 per week?

Appendix D

DIFFICULTY FACTORS ANALYSIS RANKING TASK

Below are six problems that represent a broader set of problems that typically are given to public school students at the end of an Algebra 1 course—usually eighth- or ninth-grade students. My colleagues and I would like you to help us by answering this brief (10 min) survey. We will have an opportunity to discuss these problems later.

What I would like you to do:

Rank these problems, from the ones you think will be easiest **for students to solve** to the ones you think will be hardest for them to solve. You can have ties if you like. For example, if you think the fourth problem was the easiest, the third was the most difficult, and the rest were about the same, you would write:

4 (easiest)

2 1 5 6

3 (hardest)

Problems:

- 1) $(68.36 - 25)/4 = P$
- 2) Starting with 68.36, if I subtract 25 and then divide by 4, I get a number. What is it?
- 3) After buying a basketball with her four daughters, Ms. Jordan took the \$68.36 they all paid and subtracted out the \$25 she contributed. She then divided the remaining amount by 4 to see how each daughter contributed. How much did each daughter pay?
- 4) Solve for D : $D \times 4 + 25 = 68.36$
- 5) Starting with some number, if I multiply it by 4 and then add 25, I get 68.36. What number did I start with?
- 6) After buying a basketball with her daughters, Ms. Jordan multiplied the amount each daughter had paid by 4 (because all four sisters paid the same amount). Then Ms. Jordan added the \$25 she had contributed and found the total cost of the ball to be \$68.36. How much did each daughter pay?

Problems Designed by Dr. Mitchell Nathan

Please rank the 6 problems in the space below:

(Easiest)

(Hardest)

If you like, you may provide an explanation and any assumptions you made in the space below:

Thank you for the help!!

Appendix E

Interview Protocol Sample

The following are key questions to be used when interviewing the 6 pre-service teachers:

Interviewee Code _____

Date _____

Subject Area Experience and Perceptions

1. What are your majors/minors/endorsement areas?
2. Have you ever taught or worked with K-12 children? If yes, how?
3. What grades do you plan to teach?

Cognition of Pedagogical Content Knowledge

1. What is your definition of algebra? (If the participants cannot offer a clear definition, the question will be) Can you add to or revise the following definition of algebra? Algebra encompasses the relationships among quantities, the use of symbols, the modeling of phenomena, and the mathematical study of change.
2. What content knowledge in algebra will be most difficult for you to teach? (For example: moving from arithmetic to algebra, algebraic problem solving, etc.?)
Why do you think that will be difficult for you to teach?
3. How would you teach 7th graders the meaning of x in the equations $4x - 3 = 11$?
4. How would you teach 9th graders the meaning of x in the equation $y = x^2 + x - 3$?

5. If students cannot understand the subject matter of algebra, how will you address their misunderstandings or misconceptions?

Appendix F

ALGEBRA CONTENT KNOWLEDGE TEST

Demography:

Name: _____

Gender: ___ Male; ___ Female

Major(s): ___ Mathematics: Teaching (for 7-12 certification)

___ Mathematics: Applied

___ Mathematics (general and graduate school preparation)

___ Elementary Education

___ Elementary Education/Early childhood

___ Early Childhood

___ Middle Level Education

___ Others _____

Minor(s): ___ Mathematics (k-6), teaching (for elementary education majors)

___ Mathematics, teaching (for 7-12 certifications)

___ Mathematics, general

___ Middle school education endorsement

___ Others _____

In your major, do you need to complete your student teaching?

___ Yes; ___ No

If yes, when will you complete your student teaching?

___ 2005; ___ 2006; ___ 2007; ___ 2008

___ Summer

___ Fall

___ Spring

THANK YOU VERY MUCH!!!

The following are 10 algebra questions. Please answer each question. You may write down your computation procedure on the sheet.

1. If a and b are constants and $(x-1)(x+a) = x^2 + bx - 1$, then b equals

- a) -5
- b) -1
- c) 1
- d) 0
- e) 5

2. If $\frac{a}{b} = \frac{1}{3}$ and $\frac{b}{c} = 4$, what is the ratio of $(a+b)$ to $(b+c)$?

- a) $\frac{3}{4}$
- b) $\frac{5}{6}$
- c) $\frac{8}{9}$
- d) $\frac{11}{12}$
- e) $\frac{16}{15}$

3. The y -coordinate of the vertex of the graph of $y = x^2 - 2x + 3$ is

- a) $y = 3$
- b) $y = 2$
- c) $y = -2$
- d) $y = 1$
- e) None of these

4. If $f(x) = \frac{3x-1}{x+1}$, which of the following is the expression for $f\left(\frac{1}{x}\right)$?

- a) $\frac{3-x}{1+x}$

- b) $\frac{x+1}{3x-1}$
- c) $\frac{x+2}{x-1}$
- d) $\frac{3x+1}{x-1}$
- e) None of the above

5. If $x + \frac{1}{x} = 6$, then $x^2 + \frac{1}{x^2} =$

- a) 36
- b) 216
- c) 34
- d) 38
- e) None of these above

6. If $x \neq 0$, $\frac{x^{n^2-1}}{x^{n+1}}$ equals

- a) x^{n-2}
- b) x^{n^2-n-2}
- c) x^{n^2-n}
- d) x^{n-1}
- e) None of these above

7. Solving for m , $y = mx - 3$ through points $(2,3)$ and $(-1,2)$

8. Solve $\sqrt{x-3} = 4$

9. Solve the equations

$$3x + 2y = 1$$

$$5x - 3y = 8$$

10. Find the equation of the line which is parallel to the line $3x - y = 4$ and which intersects the y -axis at $y = 7$.

Appendix G

INTERVIEW QUESTIONS AND SCRIPTS

General questions:

Introduce myself and show the goal of the interview:

My name is Martin Lo, an international student in Curriculum and Instruction. I am studying the relationship between content knowledge and pedagogical content knowledge of teaching algebra. In doing this dissertation, there are two phases in my study. We have done the first phase. In the second phase, I would like to interview some pre-service teachers about their teaching knowledge. May I ask you to answer some questions about your perceptions of teaching algebra?

OK!

Before the interview, there are several things I need to announce. First of all, I would like to mention that all your answers will be saved in a secure place. No one can read or listen to your answers. Second, if you don't feel comfortable in this interview, you can refuse to answer the questions. You also may stop the interview any time.

If you're ready, may I begin to interview you?

Yes.

General questions:

Great! Thank you very much.

The following questions will help me to know your background and your perceptions about your knowledge of mathematics. The first question is

1. What subject matter are you expecting to teach?

I expect to teach middle school mathematics because it will be fun and relative to my background. Besides, middle school math is not like high school math. There's too much academic (pure) mathematics required.

2. How do you feel about your mathematical content knowledge? Good enough, fair, not good enough.

Can you select one of them?

I think fair.

The next question is

3. How do you feel about teaching 4-8 grade level mathematics? Strongly confident, fairly confident, confidence, little confidence, no confidence.

Can you select one of them and tell me why you feel this way?

I think I will choose fairly confident because my content knowledge is good enough to teach 4-8 grade mathematics content knowledge. I think I can teach that content knowledge very well.

Categories for interview on pedagogical content knowledge (Grossman, 1990; Kahan, Cooper, & Bethea, 2003):

Elements of Teaching	Process of Teaching			
	A Preparation	B Instruction	C Assessment	D Reflection
1 Section of tasks and representations	1A	1B	1C	1D
2 Motivation of content	2A	2B	2C	2D
3 Development: connectivity and sequencing	3A	3B	3C	3D

Pedagogical content knowledge interview questions (21 questions will be asked of interviewees):

The following are several questions about your teaching knowledge. Are you ready to answer those questions?

Yes, I am ready!

The first question is

a. Please give me your definition of algebra?

Algebra is a symbolic system, representing the concepts of patterns and relationships by variables.

The variable in arithmetic system is referred by the term "unknown." In arithmetic, the question

format is result unknown. For example, " $2 + 3 = X$ " is a result known. In algebra, the question format will be " $X + 3 = 5$ ", a start unknown.

- b. After having taken some teacher preparation classes at UNI, can you describe what 7th graders need to know in your algebra class? (1A)

In 7th grade level mathematics, to know the concepts of variables is the key point to learning algebra. So, they need to know how to use variables. They need to know how to solve a story problem by using variables. They need to know $2a + 7b$ can be treated as a number. They need to know $2a + 7b$ cannot be $9ab$.

- c. Describe or tell me what the difference is between these two problems?

$$(68.36 - 25)/4 = P$$

$$\text{Solve for } D: D \times 4 + 25 = 68.36$$

The first question is arithmetic question because the unknown is in the end. The second question is a kind of algebra question because the unknown is in the beginning.

- d. Suppose you are going to teach a 7th grade student how to solve this question

$$\text{(Solve for } D: D \times 4 + 25 = 68.36\text{)}. \text{ What processes are you going to use? (1B)}$$

*I will go back to arithmetic concepts first. I will turn the questions back to $2*3 + 5 = D$. Once I make sure students can answer the question, I will change the question to be "solve for $D: D*3 + 5 = 11$." When students understand the concepts of balancing two sides, then I can make the same format question more difficult as this question "solve for $D: D*4 + 25 = 68.35$.*

- e. In this question (Solve for $D: D \times 4 + 25 = 68.35$), how can you determine that a student can solve other similar problems? (1C)

*The key point is the concept of balance. If a student can solve this problem, $D * 4 + 25 = 68.35$, then I can change to a word problem, which is solved by the symbol question.*

- f. Once you are certain that your students can solve the problem (Solve for D : $D \times 4 + 25 = 68.36$), what is the next problem you will offer to your students? Why? (1D)
- I will give them the questions with variables on both sides. For example, solve for D ; $2D + 3 = 3D - 2$. I also like to use symbol questions with fractions and decimals to make sure they don't have any problem in computation. Then, I will give a word problem and story problem to them.*
- g. (I will show 6 problems in the ranking task and ask): Tell me what the difference is between problems 1, 2, 3, and 4, 5, 6?
- I guess problem 1 and 4 are symbol problems and 3 and 6 are word problems. I am not sure if it is right but I feel this way.*
- h. Some colleagues said problems 1, 2, and 3 are arithmetic problems and 4, 5, and 6 are algebra problems. What do you think about these 6 problems?
- I think problem 1, 2, 3 are arithmetic problems and 4, 5, 6 are algebra problems*
- i. Suppose you are going to teach your students how to solve the question (Solve for D : $D \times 4 + 25 = 68.36$). What mathematical concepts do your students need to know? (2A)
- The first, I think they need to know the meaning of the D . If they don't know how to use a symbol in a statement, they won't understand how to solve the question. So, they need to know the meaning of symbols. The second thing they need to know is the meaning of "equal" signs. In this problem, the equal sign means two sides are equal. So, they might use the arithmetic concept to guess the answer. That's O.K. However, we need to point out the concepts of balance so that they can understand it.*
- j. When you teach your students how to solve the problem (Solve for D : $D \times 4 + 25 = 68.36$), what are you going to do so that your students will be interested in learning the content knowledge? (2B)

I think I will teach the balance concept to them. I will do some activities about balance. For example, I can draw a bucket which refers to the D and draw pencils which refer to numbers. Then, I will change the question like $D + 3 = 7$. Students will know there are 4 pencils in the bucket. I think students will have fun in these kinds of activities.

- k. In the ranking task, the problems 4, 5 and 6 are three kinds of algebraic problems.

The problems can be categorized as symbol equation, word equation, and story problem. Which is which?

I think problem 4 is a symbol problem. Problem 5 is a word equation. Problem 6 is a story problem.

- l. Which kind of problem (symbol equation, word equation or story problem) would you like teach first to your 7th grade students? Explain why. (2B)

First, I would teach symbol problems to my students, because I have to make sure they don't have any problem on the computation. I mean the computation ability includes the symbol with fraction, decimal, ratio and so on. I have to make sure they can count different kind of questions so that I can teach application skills.

- m. How do you assess that your students have the confidence to solve problems 4, 5, and 6? (2C)

I will use a paper/pencil test to make sure they know the computational procedure. Then, I will design some more creative problems so that students can not only compute the problem but also think about the problems.

- n. Why do you think the problem (6) [depending on interviewee's ranking task] is the hardest for students to understand and problem (1) is much easier for students to understand? (2D)

Since the story problem for me is too complex, I think students will not do it well. Students like to answer questions very quickly.

- o. How will you teach $x \cdot y$ and $x + y$? How will you connect your teaching to students' previous content knowledge? (3A)

It is not easy to show the differences between x multiplied by y and x plus y . First of all, I have to teach them a key concept, which is that two variables mean two groups of numbers. X multiplied by y means the two numbers multiple each other. I will tell them that we don't need to know which number is selected in each group but we show the relationship of the two group numbers. It is the same thing in " x plus y ."

- p. A student (a 7th grader) asks, "Would you please explain again the difference between the operation of multiplication $x \cdot y$ and addition $x + y$ on variables?" How would you respond? (3B)

I would illustrate more examples to the student to show the meaning of " x multiple y " and " x plus y " so that the student can make sense that the sentence is used to show the relationship between two group numbers.

- q. As you are walking around the room monitoring students' work, you hear one student comment, "I just don't get it. How can negative 2 times negative 4 be positive 8?" How would you respond to the student? (3B)

I would try to understand of which procedure the student cannot make sense. Then I will make more examples to point out their misunderstandings. After that, I would monitor their work to ensure that they are using the correct procedure.

- r. When most students demonstrate that they have a misunderstanding of

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{5} \text{ or } 2a + 5b = 7ab, \text{ how would you respond to them? (3B)}$$

I would go back to their previous work on concepts of fraction or the concepts of variables. Then I would show them the right way. After that, I would use more examples to discuss with them.

- s. How do you know students understand (not memorize the rule) that negative 2 times negative 4 should be positive 8? (3C)

A computation test will cause them to remember the rule without understanding the reason. So, I will ask them to solve word problems which can demonstrate that negative times negative is equal to positive.

- t. Show a way to teach that negative 2 times negative 4 is equal to positive 8.

Negative means "take off" and positive means "add up." I will draw a picture like (-+, -+, -+, -+) and (-+, -+, -+, -+). Then "negative 2" means "take off twice." "Negative 4" means 4 negative signals. So, "negative 2 times negative 4" will be taking twice 4 negative signals. After doing that, the rest of the picture will be twice 4 positive signals. It means positive 8.

- u. How many ways can you show that negative 2 times negative 4 is equal to positive 8? (3D)

I think there are several ways. I can design the concepts as story problems so that students can make sense why negative times negative is positive.

- v. Show those ways. (3D)

For example, I go to refund 12 pencils at Wal-Mart. I paid 12 dollars for the 12 pencils. Suppose each pencil is one dollar. The "negative 12" means "take 12 items off." The "negative 1" means "each item refers to the signal, -1." The money I can gain from the refund is negative 12 times negative 1. So we get positive 12 dollars back from the refund.

Appendix H

PERFORMANCE IN ACK TEST RANKING TASK RESULTS AND SELECTED
INTERVIEWEES

	ACK Rank	ACK Sum	% of ACK correct	DFA Pearson's <i>r</i>	Fisher's <i>z</i>	95% interval from Fisher's <i>z</i>	<i>z</i> < / > 1	Name
High content knowledge group	1	38	95	.714	0.90	.42 ≤ <i>z</i> ≤ 1.37	1	Jessica
	2	38	95	.796	1.09	.61 ≤ <i>z</i> ≤ 1.56	1	Cheryl
	3	38	95	.729	0.93	.45 ≤ <i>z</i> ≤ 1.40	1	Andy
	4	36	90	.200	0.20	-.27 ≤ <i>z</i> ≤ .68	<1	Trevor
	5	34	85	.919	1.58	1.11 ≤ <i>z</i> ≤ 2.06	>1	Megan
	6	33	82.5	.913	1.55	1.07 ≤ <i>z</i> ≤ 2.02	>1	
	7	33	82.5	.764	1.01	.53 ≤ <i>z</i> ≤ 1.48	1	
	8	33	82.5	.943	1.76	1.29 ≤ <i>z</i> ≤ 2.24	>1	
	9	32	80	.862	1.30	.83 ≤ <i>z</i> ≤ 1.78	1	
	10	32	80	.886	1.40	.93 ≤ <i>z</i> ≤ 1.88	1	
	11	32	80	.941	1.75	1.27 ≤ <i>z</i> ≤ 2.22	>1	
	12	30	75	.943	1.76	1.29 ≤ <i>z</i> ≤ 2.24	>1	
	13	30	75	.886	1.40	.93 ≤ <i>z</i> ≤ 1.88	1	
	14	30	75	.828	1.18	.71 ≤ <i>z</i> ≤ 1.66	1	
	15	30	75	.771	1.02	.55 ≤ <i>z</i> ≤ 1.50	1	
	16	30	75	.729	0.93	.45 ≤ <i>z</i> ≤ 1.40	1	
	17	30	75	.029	0.03	-.45 ≤ <i>z</i> ≤ .50	<1	
	18	29	72.5	.794	1.08	.61 ≤ <i>z</i> ≤ 1.56	1	
	19	29	72.5	.600	0.69	.22 ≤ <i>z</i> ≤ 1.17	1	
	20	29	72.5	.926	1.63	1.15 ≤ <i>z</i> ≤ 2.10	>1	
Low content knowledge group	21	29	72.5	-.143	-0.14	-.62 ≤ <i>z</i> ≤ .33	<1	
	22	28	70	.000	0.00	-.48 ≤ <i>z</i> ≤ .48	<1	
	23	28	70	.086	0.09	-.39 ≤ <i>z</i> ≤ .56	<1	
	24	28	70	-.971	-2.11	-2.58 ≤ <i>z</i> ≤ -1.63	<1	
	25	27	67.5	.143	0.14	-.33 ≤ <i>z</i> ≤ .62	<1	
	26	27	67.5	.555	0.63	.15 ≤ <i>z</i> ≤ 1.10	1	
	27	27	67.5	.914	1.55	1.08 ≤ <i>z</i> ≤ 2.03	>1	
	28	27	67.5	-.086	-0.09	-.56 ≤ <i>z</i> ≤ .39	<1	
	29	26	65	.914	1.55	1.08 ≤ <i>z</i> ≤ 2.03	>1	
	30	26	65	.703	0.87	.40 ≤ <i>z</i> ≤ 1.35	1	
	31	25	62.5	.236	0.24	-.23 ≤ <i>z</i> ≤ .72	<1	Ryan
	32	25	62.5	.886	1.40	.93 ≤ <i>z</i> ≤ 1.88	1	
	33	25	62.5	.714	0.90	.42 ≤ <i>z</i> ≤ 1.37	1	
	34	24	60	-.257	-0.26	-.74 ≤ <i>z</i> ≤ .21	<1	Jennifer

35	23	57.5	.829	1.18	$.71 \leq z \leq 1.66$	1	
36	21	52.5	.771	1.02	$.55 \leq z \leq 1.50$	1	
37	21	52.5	-.169	-0.17	$-.65 \leq z \leq .30$	<1	McKen
38	20	50	.688	0.84	$.37 \leq z \leq 1.32$	1	
39	19	47.5	.478	0.52	$.05 \leq z \leq 1.00$	1	
40	17	30	.721	0.91	$.43 \leq z \leq 1.39$	1	

Appendix I

INTERVIEWEES' BACKGROUND/HOURS

Course works	Trevor	Cheryl	Jennifer
	Secondary mathematics major High CK with verbal-precedence PCK	Secondary mathematics major High CK with symbol-precedence PCK	Elementary major with math minor Low CK with verbal-precedence PCK
Calculus I	X	X	
Calculus II	X	X	
Calculus III	X	X	
Linear Algebra for Applications	X	X	
Introduction to Mathematical Modeling	X	X	
Introduction to Modern Geometries	X	X	
History of Mathematics	X	X	
Modern Algebra I	X	X	
Technology for Secondary Math Teachers	X	X	
Geometric Transformations	X	X	
Combinatorics	X	X	
Probability and Statistics	X	X	
Teaching Middle School/Jr High Mathematics	X	X	
Teaching Secondary Mathematics	X	X	

Mathematics for Elementary Teachers	X
Technology for Elementary School Mathematics Teachers	X
Algebraic Thinking for Elem. Math Teachers	X
Introduction to Geometry and Measurement for Elementary Teachers	X
Topics in Mathematics for Elementary Teachers	X
Problem Solving in Mathematics for Elementary Teachers	X
Teaching Math in the Elementary School	X
Mathematics for Elementary Students with Special Needs	X

Appendix J

CURRICULA OF MATHEMATICS TEACHING MAJOR AND MINOR

MATHEMATICS MINOR (K-6) - TEACHING

One of the following:		Hours	Credit
800:023	Mathematics in Decision Making	--3	hours
800:072	Introduction to Statistical Methods	--3	hours
800:092	Introduction to Mathematical Modeling	--3	hours

Each of the following:

800:030	Mathematics for Elementary Teachers	--3	hours
800:037	Technology for Elementary School Mathematics Teachers	--3	hours
800:111	Introduction to Analysis for Elementary Teachers	--4	hours
800:112	Introduction to Geometry and Measurement for Elementary Teachers	--3	hours
800:113	Topics in Mathematics for Elementary Teachers	--3	hours
800:114	Problem Solving in Mathematics for Elementary Teachers	--4	hours

One of the following:

800:137	Action Research for Elementary School Mathematics Teachers	--1	hour
800:192	Mathematics for Elementary Students with Special Needs	--1	hour

Total: 24 hours

MATHEMATICS MAJOR - TEACHING

Required Courses:		Hours	Credit
Common Core:			
800:060	Calculus I	--4	hours
800:061	Calculus II	--4	hours
800:062	Calculus III	--4	hours
800:076	Linear Algebra for Applications	--3	hours
Teaching Core:			
800:092	Introduction to Mathematical Modelling	--3	hours
800:160	Modern Algebra I	--3	hours
800:165	Introduction to Modern Geometries	--3	hours

800:173	Probability and Statistics	--3 hours
800:180	History of Mathematics: To the Calculus	--3 hours
800:188	The Teaching of Middle School/ Junior High Mathematics	--3 hours
800:190	The Teaching of Secondary Mathematics	--3 hours

Two of the three courses:

800:144	Elementary Number Theory	--3 hours
800:162	Modern Algebra II	--3 hours
800:189	Geometric Transformations	--3 hours

and one of the following:

810:030	BASIC Programming	--3 hours
810:031	FORTRAN Programming	--3 hours
810:032	Pascal Programming	--3 hours
810:034	COBOL	--3 hours
810:035	C Programming	--3 hours
810:051	Introduction to Computing	--4 hours

Total: 45-46 hours