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Detecting the process' 1.5 sigma shift: A quantitative study

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DETECTING THE PROCESS' 1.5 SIGMA SHIFT

A QUANTITATIVE STUDY

A Dissertation

Submitted

in Partial Fulfillment

of the Requirements for the Degree

Doctor of Industrial Technology

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DEDICATION

I want to dedicate this work to my family who never doubted my ability to complete this work. My wife Nancy is also my soul mate and my best friend. Without her support I would not have been able to continue this work. To John and Emily, I want you to always pursue your dreams and achieve whatever your hearts desire. Your pursuit will not be easy; however, the satisfaction derived from your achievement will be indescribable.

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Gratitude is an inadequate word to express the feelings I have for the people and organizations that enabled me to complete this study. I had unwavering support from my wife, Nancy who permitted me to follow this and other life's dreams. My colleague, Scott Vandenbos who spent may hours of discussion with me on this topic and was always willing to give me honest and critical feedback. To my advisor, Dr. Ali Kashef and my committee members for encouraging me to continue through this effort and complete it on time. Especially Dr. Kirmani who found time to guide me through independent studies in mathematics. They advised me, challenged me and made this result much better than I believed it could become. I am indebted to the professors at the University of Northern Iowa who gave me all the benefit of their teaching experience and insights into research. Although I had this small army of support, I alone am responsible for errors or omissions and the quality of this result. If the reader should ever begin a journey toward obtaining a doctorate I hope she has the support and encouragement I received. This became a labor of love. I will never be worthy of this support, but I appreciate it more than words can express.

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ABSTRACT

Process behavior can change with time. In this study an attempt was made to discover whether the Six Sigma™ claim of changes in the process mean stayed within +/- 1.5 sigma units. Several process groups were examined for a particular firm that made metal castings, machined parts, tested major components and assembled these into a vehicle that was a product sold to the customer. As the assembly progressed, deficiencies were identified and recorded. Analyses employed cumulative sum (CUSUM) sequence charts, Autoregressive Integrated Moving Average (ARIMA) time series analyses, minimum mean square error (MMSE) exponentially weighted moving average (EWMA), Shewhart control charts and Analysis of Variance (ANOVA) to identify the shift in the process mean, M/s_w, the duration of the shift, λ_B, and the proper choice of EWMA smoothing coefficient, λ_{EWMA}. Kruskal-Wallis analysis of the relationship of these measures to process group (assembly, foundry, heat treatment, machining, shaving, test machine, grinding, turning, warranty and yield) was also performed. The method used was generally applicable for all these processes. The process group and the ARIMA type also influenced the measurement of M/s_w, λ_B, and λ_{EWMA}.

CHAPTER I

INTRODUCTION

In this introductory chapter historical background, statements of the problem and the purpose of this investigation, as well as the need for this study and the justification for it are introduced. As in most scientific work, the hypotheses, research questions, assumptions, delimitations, limitations and definition of terms are addressed in order to prepare the reader for ensuing chapters. Internal validity of this study and a description of statistical methods and other analyses employed close the chapter.

Background

In the summer of 1980 an NBC broadcast, "If Japan Can... Why Can't We?" awakened America to the fact that Japan was taking over U.S. markets with products that had superior quality. Citing the works of Americans like Dr. W. Edwards Deming and Dr. Joseph M. Juran the broadcast proposed that the Japanese were taking market share from American companies using American ideas (Box, 2006; Reuven, 1980). Leaders of corporate America today are aware that Japan continues to compete successfully using the methods identified 29 years earlier. At the root of the quality issue was variation. The Japanese products were seen as superior in quality level and consistent from one item to another.

In 1987 the United States Department of Commerce announced an American award for quality, the Malcolm Baldrige Award. An early recipient of this award was Motorola, Inc. who demonstrated superior product quality through a program it called Six Sigma™. Soon corporate giants like Honeywell (now Allied Signal), General Electric, Raytheon and others adopted Six Sigma™ (iSixsigma, 2004). DuPont (2002) the oldest American company touted the benefits of Six Sigma™ in their annual report. Hahn, Doganaksoy, and Hoerl (2000) and Folaron (2003) trace the evolution of Six Sigma™ and conclude that Six Sigma™, with a focus on continuous and unending improvement, will endure.

Motorola established a training institute, Motorola University, to handle the demand from other American businesses to share the method and teach them how to develop skill levels within individuals who would lead their improvement efforts. At first this was quality focused, but Van Tiem (2004) and Barney (2002) report that the second generation of Motorola Six Sigma™ is now a business improvement strategy. Six Sigma™ is also used to design new products and processes (Treichler, Carmichael, Kusmanoff, Lewis, & Berthiez, 2002; Watson, 2005).

The American educational system has not done a good job of meeting business needs for trained people, even in the basic skills required to use standard control charts (Alwan & Radson, 1995). While many companies send their managers to Motorola to become trained as Six Sigma Black Belts, those who manage projects using the Six Sigma™ methods, educators and quality professionals offer guidance to those who feel they can educate themselves (Abraham & Mackay, 2001; Bailey, 2001; Breyfogle, Enck & Meadows, 2001; Hill, 2001; Hoerl, 2001a, 2001b; Montgomery, Lawson, Molnau & Elias, 2001; Pyzdek, 2001; Snee, 2000, 2001, 2006). Consultants have made lucrative incomes posing as masters of Black Belt methods. A search on the World Wide Web site, Monster.com for Black Belt returns 154 postings for the time period June 2008 through July 2008. Many positions offer salaries in excess of $150,000 per year.

According to Praveen Gupta (2006), Motorola engineer Bill Smith is responsible for the Six Sigma™ concept. He also incorporated the 1.5 sigma shift in the model, but Gupta is not certain how the value was derived. He reports that it was associated with control charts and argues that a subgroup of 4 will detect a mean shift or drift in the process at 3*sigma/sqrt(4) which is equal to 1.5 sigma. The ability of a process to satisfy specifications was advanced by Kane in 1986 when he introduced the Cp and Cpk calculations. Cp was taken as the tolerance range divided by the process spread of 6 sigma. Sigma was calculated from the sample of 30 units gathered from a process that was in statistical control. A process was able to produce within specification if the Cpk were at least one. Many businesses, following the ideas of Juran, Gryna and Bingham (1974) opted for a minimum Cpk of 1.33 to allow for shifts and drifts over time. Six Sigma™ moved that value to 1.5 (Harry & Stewart, 1988).

Deming (1942) identified the purpose of data as information for action. He found it useful to distinguish between two types of problems that confront the statistician in his job of making predictions. Type A problems in which action is based on a prediction regarding future measurements of a product already in existence. Type B problems in which action is based on a prediction regarding future measurements of a product not yet subject to measurement. Six Sigma™ methods today are tailored for Type A problems with DMAIC (Define, Measure, Analyze, Improve and Control) methods and tools. Type B problems are treated with DMADV (Define, Measure, Analyze, Design and Verify) phases and gates to create new products or services. Each is focused on identification of variation and its minimization.

Tadikamella (1994), Lucas (2002), and Gnibus (2000) illustrate the prediction calculations applied to Six Sigma™. The calculation includes a 1.5 sigma allowance for drift in the mean. This allowance, first described by Harry and Stewart (1988) at Motorola, Inc. has been a point of confusion and controversy since its introduction. No process studies are cited to justify the 1.5 sigma in the formula.

Burns (2006, 2007) challenges the whole premise of Six Sigma™. He believes that processes should be distinguished from customer needs and sigma is a measure of process variation. He is especially wary of the calculations of sigma and the justification for six of them. He argues that processes vary within plus and minus three sigma limits if normally distributed. His communication with Michael Harry, principal founder of Motorola University and now head of Six Sigma Academy, identifies the other three sigmas from the drift in the process mean over a long period of time. The mean can drift plus or minus 1.5 sigma from the original position. Harry and Stewart (1988) have never produced any scientific studies for this value, but most practitioners agree that some drift happens and 1.5 sigma is reasonable. In 1998 Harry wrote that "In fact, research has shown that a typical process is likely to deviate from its natural centering condition," but cites no studies. As early as 1970, Juran and Gryna had proposed capability analysis of processes and recommended a capability ratio, Cr, of 0.67 for new machines and 0.75 for older ones. Today the common practice is to use the reciprocal of this value, Cp, which is 1.5.

Statement of the Problem

The problem of this study is to determine whether the process mean shifts within a 1.5 sigma distance about a target.

Statement of Purpose

The purpose of this study was to identify the magnitude of the drift in the process mean relative to the inherent variation in the process. This study determined how processes collectively behave relative to the shifts and drifts in the mean over time. The important conclusion was whether the value of the shifts and drifts were less than 1.5 times the common cause variation in the process. Four methods were employed depending on the nature of the process as depicted in its data: 1. Shewhart control charts for processes with data that show a stationary mean with random variation and constant sigma, 2. Analysis of Variance for stationary processes with shifts in the mean with random variation and constant sigma, 3. CUSUM and EWMA control charts to analyze stationary processes with slowly shifting and drifting independent means with large random variation and constant sigma and 4. Autoregressive Integrated Moving Average on non-stationary processes with auto correlated means with random variation and constant sigma.

Statement of Need/Justification

The need / justification for the study are based on the following factors:

1. American businesses in the last quarter of the 20th century were losing market share to Japanese companies (Reuven, 1980) due to competition through quality.

2. The United States government responded to this challenge by instituting the Malcolm Baldrige National Quality award to encourage businesses to improve quality.

3. Motorola, Inc. created the Six Sigma™ program to improve their quality and capture the Baldrige Quality Award in 1987. A key premise of this method is allowance of 1.5 sigma for the shift and drift in the process mean.

4. Methods to analyze process variation and shifts and drifts in the mean were created in the early part of the 20th century. Control charts, Analysis of Variance, and regression analysis were widely known by the 1950s. In the 1950s the Cumulative sum and exponentially weighted moving average methods were developed. The most recent development in analysis tools was the time series analysis using Autoregressive Integrated Moving Average to separate common cause from total process variation.

5. By 1994 Box and Luceño (1994, 1995) wrote about statistical methods of process monitoring and engineering methods of process control and explained how ARIMA modeling could monitor and PID could control processes.

6. In 1995, Jack Welch, president and CEO of General Electric, stated that the use of Six Sigma tools accounted for the dramatic increase in sales and profits at GE (Welch, 2000).

7. By 1997, Box and Luceño published their work describing how to use statistical control methods of ARIMA to monitor and feedback control information. They related this to six sigma in 2000 (Box & Luceño, 2000)

8. Beginning in June 2001, the American Society for Quality offered a certification for Six Sigma Black Belts as those knowledgeable in the use of Six Sigma tools were called.

9. In 2007 quality leaders recognized that time series analysis of processes was necessary in order to understand process behavior and control its parameters with traditional EWMA and Shewhart control charts (Hunter, 2007a; 2007b).

This study characterized process mean location over time using the statistical methods of control charts, ANOVA, CUSUM, EWMA and ARIMA. Conventional wisdom would predict that the process mean drifts within a band of 1.5 sigma units above and below the process target.

Hypotheses/Research Questions

The hypotheses employed in this study were:

1. The process mean shifts less than 1.5 sigma units from its target during normal operation. The alternative hypothesis is that the mean drifts more than 1.5 sigma units in at least one process.

2. No process measurements are related to others over time. The alternative hypothesis is that at least one process parameter measurement is related to itself over time.
3. The autoregressive coefficients are zero for all processes. The alternative hypothesis is that the autoregressive coefficients are not zero for at least one process.
4. Likewise, the moving average coefficients are zero for all processes. The alternative hypothesis is that the moving average coefficients are not zero for at least one process.
5. The autoregressive and moving average coefficients are simultaneously zero for all processes. The alternative hypothesis is that these coefficients are not equal to zero for at least one process.
6. ARIMA time series analysis separates the drift in the process average from the common cause variation inherent in the process. The size of the drift would be less than or equal to 1.5 sigma units where sigma is the common cause process variation.

Assumptions

The following assumptions were made in pursuit of this study:

1. All processes exhibit variation. This variation is composed of variation due to drifts in the mean, unexplained common cause variation and special cause variation due for instance to effects like seasonal variation, and multiple machines performing the same work. Well-intended, but uninformed process control people can increase the variation of the process by adjusting the process when it is exhibiting only common cause variation.
2. Six Sigma processes have at most a 1.5 sigma shift in the process mean.
3. No single analysis method is appropriate for all processes.
4. Process means can be separated from common cause variation using the proper statistical methods.

5. Some processes exhibit stationary mean location, uncorrelated measurements over time, and random variation.
6. Other processes have stationary means but show sudden shifts in the mean, uncorrelated measurements over time and random variation.
7. Additional processes are not stationary, but are uncorrelated over time and have random variation.
8. A few processes will be dominated by large inherent variation making necessary the detection of the change in mean location with CUSUM or EWMA methods to separate the shifts and drifts in the mean from the process random variation.
9. Time series analysis is appropriate for the analysis of process mean shifts.
10. The Autoregressive Integrated Moving Average methods effectively separate the process mean from the white noise variation.
11. The residuals from the ARIMA model are normally distributed, uncorrelated, random variables with zero mean and process sigma.
12. The Exponentially Weighted Moving Average control chart used in conjunction with the Individual X control chart can effectively identify drifts in the mean, special cause and common cause variation.
13. Quality policy of the firm is consistent across all processes.
14. The ARIMA model reflects behavior of the physical process within acceptable error.
15. A sample comprising a fraction of the total processes can represent all the processes for a particular firm.

Delimitations

A delimitation is a boundary, a self-imposed limit on the study. This study will be conducted in view of the following delimitations:

1. A single manufacturing and assembly organization selling off-highway mechanical vehicles in the Midwestern United States.

2. The organization has continuous processes, turning processes, boring processes and assembly processes.
3. Processes to be selected for the study will have at least 100 measurements recorded over at least a six month representative period of time.
4. Process measurements are either continuous or discrete.
5. Data analysis will be done using personal computers.
6. One method will be used to identify the drift in the process average and another will be used to identify the process sigma.

Limitations

Limitations are weaknesses of the study that would limit its generalization. This study will be conducted in view of the following limitations:

1. Continuous flow processes such as used in the chemical process industries are not included in this study.
2. Service or product support activities which are largely procedural in nature are excluded from this study.
3. This study is not conducted with a large group of manufacturers, it is limited to a single manufacturer with a diverse group of processes to make and assemble parts.
4. Only one firm is included in this study, making it more likely that the influence of quality philosophies would have little difference from process to process.
5. Processes with insufficient data or data gathered for too short an interval of time are excluded from this study.
6. Measurements in this study are for the most part made manually by skilled employees and not continuously recorded by automated equipment.

Definition of Terms

The following terms are defined to clarify their use in the context of the study:

1. Analysis of Variance, ANOVA. A technique for comparing means of normal populations assuming the populations have the same variance (NIST, 2006, 7.4.2).

2. Autocorrelation Function, ACF. A plot of the serial correlation coefficients of a time series (Bisgaard & Kulahci, 2005b, p. 481).
3. Autocorrelation. Serial correlation. Correlation of a variable with itself when the measurements are lagged (Bisgaard & Kulahci, 2005b, p. 481).
4. AutoRegressive Integrated Moving Average, ARIMA. A class of time series models for which the d^{th} difference is a stationary mixed autoregressive - moving average series (Box, Jenkins & Reinsel, 1994, p. 89).
5. Autoregressive. A series that can be expressed as a finite, linear aggregate of previous values of the series and a random shock (Box et al., 1994, p. 9).
6. Common cause variation. Variation in observations that are embedded in the system or process itself (Montgomery, 2009, p. 52).
7. Control. The basis for a process being in statistical control is that its joint probability distribution is stationary. It is not, as some mistakenly may think, that the observations are independent (Bisgaard & Kulahci, 2005c, p. 483).
8. CUSUM sequence. Cumulative Sum of the observation deviations from the grand mean of the series. (Lucas, 1985, p. 129).
9. EWMA. Exponentially Weighted Moving Average. A type of moving average in which the entire history of measurements is assigned weights, with weights decreasing as a geometric progression from the most recent point back to the first (Roberts, 1959).
10. F. The distribution of the ratio of sample variances. Named in honor of Sir Ronald A. Fisher geneticist and statistician who lived in the first half of the 20th Century (Snedecor & Cochran, 1980, p. 221).
11. Minimum Mean Square Error, MMSE. The minimum of the squared difference between the predicted value and the observed value. (Box & Paniagua-Quiñones, 2007, p. 98).

12. Monitor. A continuous screen process for detecting assignable (or special) causes of variation (Box et al., 1994, p. 5). A method to describe process management as opposed to controlling the process.
13. Moving average. A time series that is composed of a weighted sum of a finite number of previous random shocks. The series is built from a weighting a finite number of previous errors between the predicted and observed values of the series (Box et al., 1994, p. 10).
14. Normal probability. The distribution of a random variable with probability density, p(x) = $(2\pi)^{-1/2}(\sigma^2)^{-1/2}e^{-(x-\mu)^2/2\sigma^2}$ (Box et al., 1994, p. 280).
15. p. The order of an autoregressive series (Box et al., 1994, p. 9).
16. Partial Autocorrelation Function, PACF. Measures the correlation between z_t and z_{t-1} not accounted for by z_{t-1}, z_{t-2}, $\ldots z_{t-k+1}$(Box et al., 1994, p. 66).
17. PID. Proportional Integral Derivative. The PID controller receives signals from sensors and computes corrective action to the actuators from a computation based on the error (proportional), the sum of all previous errors (integral) and the rate of change of the error (derivative; Box & Luceño, 1997, p. 135).
18. q. The order of a moving average series (Box et al., 1994, p. 10).
19. s-control chart. A Shewhart chart of the standard deviations of the sub-groups of process data (NIST, 2006, 6.3.2.1).
20. Smoothing coefficient, λ_{EWMA}. A constant that determines the depth of memory of the EWMA (Hunter, 1986, p. 206).
21. SPC. Statistical Process Control. A continuous effort to keep processes centered at their target values while maintaining the spread at prescribed values (Ott, Schilling & Neubauer, 2005, p. 195).

22. Special cause variation. Causes of variation that arise from sources that are external to the system or process itself (Montgomery, 2009, p. 52). Special causes are also referred to as assignable causes.
23. Stationary. A process is stationary if it is in equilibrium about a constant mean level (Box et al., 1994, p. 7).
24. Time series. A time oriented or chronological sequence of observations on a variable of interest (Montgomery, Jennings, & Kulahci, 2008, p. 2).
25. White noise. Denote a sequence $\{a_{t,}\}$ of independent identically distributed random variables that are, to an adequate approximation, normally distributed having mean 0 and variance σ_a^2. The individual a_t's are sometimes called innovations and σ_a^2 the innovation variance (Box & Kramer, 1992).

Internal Validity

Internal validity, the degree to which observed differences on the dependent variable (1.5 sigma shift) are directly related to the independent variable (type of process), not to some other (uncontrolled) variable, was low in this method. A particular process may exhibit many forms of variation over time causing shifts and drifts in its process mean.

1. Qualified data, both amount and time interval reflected in the values was a constraint that had to be discussed with the manufacturer.
2. An indication of the organization of the collected data for analysis. The data were arranged in a table with the columns representing the variables and the rows the time sequence of the measurements. The process generating the measurements was recorded in the title of the table.

Statistical or Other Analysis of the Data

The key analysis of this investigation utilized descriptive statistics of mean, standard deviation, and variance. In order to separate variances, ANOVA, control charts: Shewhart, CUSUM, EWMA, and Individuals were employed. In addition, ARIMA time series analysis to separate white noise from process variation was employed.

CHAPTER II

REVIEW OF LITERATURE

This chapter reviews the literature on the application of time series analysis to model autocorrelation, application of deadbands to minimize the condition known as "hunting" or over-adjusting causing instability, alternative models for process variation, autocorrelation effects on process capability and the effect of changing location of the mean on the process capability. The importance of the assumption of independence of process measures is reviewed followed by a conclusion of the literature review that guided the method of this investigation.

Autocorrelation as Time Series Analysis

Hunter (2007a) challenges Black Belts and Quality Engineers to study time series to detect location of the process mean. Shewhart process control charts assume a constant mean with random variation between measurements. The ε_t error epsilon is declared to be Gaussian white noise—in other words, normally distributed, independent with expected value $E(\varepsilon_t) = 0$ and constant variance $Var(\varepsilon_i) = \sigma^2$. The shifts and drifts in the process mean need to be studied as a time series with normality, independence and constant variance as criteria to meet before the sigma of the process can be determined. Hunter goes on to illustrate how to use the Exponentially Weighted Moving Average (EWMA) to judge the drift in the mean and identify the sigma of the process (Hunter, 2007a, 2007b). Others have also advocated the EWMA to detect small drifts in the mean (Alwan & Radson, 1995; Brown, Meyer & D'Escpo, 1961; Hunter, 1986, 1998; MacGregor, 2001; Montgomery & Mastrangelo, 1991). Hunter (1986) points out that the Shewhart chart is one extreme of the EWMA with the smoothing constant equal to 1 and the Cumulative Sum (CUSUM) chart is the other extreme with the constant at zero.

The EWMA is not always a good estimator of the location of the process mean. Faltin and Woodall (1991) point out the limitation of EWMA if the process shows autoregressive dependence of the order of phi, ϕ, less than 0.33. The coefficient, phi, is the autocorrelation coefficient of the time series lagged by one interval. Just like correlation of two variables, the autocorrelation uses the lagged value of the series for the second variable. Box, Jenkins, and Reinsel (1994) show how the autocorrelation of time series can be identified, modeled and used to separate the white noise variation from the process variation. Barnard (1959) showed that processes could be thought of as a series of random disturbances, distributed as Poisson that influenced the level of a process. Some of these disturbances had enduring influence and changed the level of the process while others were fleeting and their effect died after a number of intervals depending on the process. Montgomery, Jennings and Kulahci (2008) and del Castillo (2002) explain time series analysis for process control. These models are termed ARIMA for Autoregressive Integrated Moving Average models because the change in the process mean location can be a result of an autoregressive drift which returns to a stable and constant mean, a drifting, almost linear change in the mean or a moving average where the mean will not return to its previous value unless the process has an intervention by a controller or operator adjustment.

Monitoring and Control with Deadbands

Box and Kramer (1992) showed how to use statistical process monitoring for feedback control. Thus the mean was modeled over time and the feedback was given to either a human for adjustment or a controller to intervene with the process input to control the process output. In a series of articles and finally a book, Box and Luceño (1994, 1995, 1997, 2000) illustrate the similarity between process monitoring with statistical methods and process control with engineering PID, Proportional Integral Derivative, controllers. O'Shaughnessy and Haugh (2002) show the use of EWMA for process monitoring and bounded adjustment to prevent over-adjusting. This control is only effective if the time series can be modeled and predicted. Box and Paniagua-Quiñones (2007) showed how to construct two control charts, one for the shift and drift in the mean using EWMA with minimum mean square error and a traditional Shewhart chart for the residuals from this model to identify special causes. Lucas (1982) had advocated for the use of two control charts, one a CUSUM and the other a Shewhart chart to detect process shifts and drifts and special causes with few false signals but rapid response. Therefore the 1.5 sigma drift can be controlled within boundaries with use of ARIMA models or PID controllers.

In order to help the Black Belt, Bisgaard and Kulahci (2005a, 2005b, 2005c, 2006a, 2006b, 2007a, 2007b, 2007c, 2007d, 2008) have written a series of articles for the identification and characterization of time series models for process control. In one of these articles, the authors speak of regime changes (Bisgaard & Kulahci, 2007a) which are behavioral changes in the process, a departure from its previous behavior. A separate ARIMA model is needed for each of the regimes. Saniga, Davis and Lucas (2009) provide a simple graphical technique to detect change points.

Alternative Models for Process Variation

What emerges from this discussion is a picture of the process mean changing over time in one of five ways. First, the Shewhart model of constant mean and uniform variation. Second, shifts in the mean with uniform variation and constant variation like we find in Analysis of Variance (ANOVA). The third model would be a process with a continual drift of the mean and uniform and constant variation such as found in regression models. Another variant, the fourth, would be a model of the mean drifting slowly and with a variation much smaller than the common variation underlying the process, modeled best by either a CUSUM or EWMA model. The fifth model would incorporate the ARIMA behavior of the process, some having autoregressive, some moving average, some stationary and some not stationary requiring an integration term to separate the process variation from the white noise variation.

Autocorrelation Effect on Process Capability

Kotz and Johnson (2002a, 2002b) updated an earlier study of process capability indices (PCIs) that are predicated on the Shewhart model of constant mean with random variation about it. In discussion of this article, Bothe (2002) points out weaknesses in current indices that appraise the capability of processes to identify the capability to meet specifications for processes having: inherent tool wear; variation in setup between runs; limited data due to short production runs; autocorrelation; and features with geometric dimensioning and tolerancing. Boyles (2002) also addresses autocorrelation in his discussion: we cannot rule out autocorrelation in our measurements—for example, this would rule out virtually all high-tech manufacturing. If we restrict applications of PCIs to processes in the narrowly-defined state of statistical control, we are essentially saying that PCIs are never applicable." While process capability studies could be used for setting goals, the identification of processes in control with constant mean and random variation is considered to be a small portion of all processes familiar to the authors and discussants (Bothe, 2002; Boyles, 2002; Hubele, 2002; Kotz & Johnson, 2002a, 2002b; Lu & Rudy, 2002; Ramberg, 2002; Rodriguez, 2002; Spiring, Cheng, Yeung & Leung, 2002; Vännan, 2002).

In the second instance of constant variance and shifts in the process mean, de Mast and Roes (2004) propose to discern, apart from outliers, one generic pattern that the control chart should detect, namely shifts in the mean. They argue the importance of this pattern is acknowledged by the extensive literature on CUSUM and EWMA charts. Their procedure would have the investigator follow three steps: (1) estimate the locations of possible shifts and test significance of these shifts, (2) estimate (using robust estimators) the means of the intervals between successive shifts, and (3) based on these estimates determine separate control limits for each interval. They state that the F distribution can be used to identify whether the interval means are statistically different. While they derive a mathematical method to identify the shift points, Saniga, Davis and Lucas (2009) propose a simple graphical method that most practitioners would be able to use. Albin, Kang and Shea (1997) indicate that CUSUM and EWMA charts in conjunction with Shewhart X charts can identify shift points in the mean even when the variation is large in comparison. Deleryd (1998) identifies difficulties with the shape of the distribution of the measures of the process, especially those which are skewed, to summarize with a typical PCI value. An F statistic assumes normally distributed values about the interval means so a transformation is often required before deciding the process capability.

<u>Changing Location of the Mean</u>

Spiring (1991) proposed a method to judge processes subject to continually changing locations in the mean, such as a machining operation with tool wear. He proposes a dynamic model for the PCI with lower bounds established based on the process variation to protect the consumer from defectives. This is the third condition mentioned above. Spiring fits a regression line to interval data and uses the Mean Square Error of regression adjusted for degrees of freedom in the Cpm calculation. Vander Weil (1996) proposes modeling the process as an Integrated Moving Average, IMA(1,1), using regression with a deterministic term to fit the process data. He argues that many industrial processes are controlled with PI (proportional, integral) controllers in a wide range of applications. Braun and Park (2008) examine the effect of contaminated data, undetected shifts or drifts, with a process that has a constant mean and variance. Their examination of 10 ways to determine sigma for an individuals chart lead them to conclude that the method of de Mast and Roes (2004) is a reasonable method of estimating the sigma of the individuals control chart. The method is to test robustly for the significant shift in the mean and estimate sigma for intervals incorporating the mean of the interval in the sigma calculation.

Detecting slowly drifting process means where the process variation is relatively large can be achieved using CUSUM or EWMA control charts to locate the mean with a Shewhart chart to judge occurrence of special causes (Baxley, 1990; Lucas & Saccucci, 1990; Roberts, 1959). Ryan (1991) states that CUSUM and EWMA procedures quickly detect a shift in the mean without many false signals when data are independent. Borrow, Champ, and Rigdon (1998) show the use of the EWMA for Poisson data. They compute the average run length (ARL) using Markov chain simulation for selected lambda and control chart factors. Knowing the ARL the control chart factor for upper and lower control lines can be established. They argue that this permits signaling when the defect level falls significantly below the average when a process improvement occurs. The traditional c chart would not signal under these circumstances due to a lower control limit that is an impossible negative number. Crowder (1989) gives methods for constructing EWMA control charts for selected ARLs. Once the ARL is decided, the proper lambda and K, control chart constant for detecting the drifts in the process mean with acceptable alpha and beta risks of false alarms (alpha) and lack of response (beta) when the mean has drifted. Hunter (1986, 1998) shows the similarity of the EWMA monitoring and the PID controlling of the process. He also proposes that a dead band interval may be used to prevent over-adjusting.

Measures that are not Independent

Wardell, Moskowitz and Plante (1992) point out that the restriction of independence of process measures is not true for many processes, specifically machining and forging operations For this reason, more complicated models accounting for autocorrelation are needed for process control. Thus, the fifth type of process variation, time series behavior of the process, differs from those above by replacing the idea of smoothing the series to identify the shift and drift of the independent mean with the ARIMA model incorporating the auto correlated behavior in the model. This method is used for data that show autocorrelation, previously assumed nonexistent. The shifts and drifts are then isolated with the time series variation and the sigma of the residuals is treated separately. Jiang, Tsui and Woodhall (2000) propose an ARMA chart instead of the EWMA chart. This chart uses the ARIMA coefficients directly instead of the EWMA smoothing coefficient for predicting the period-ahead value of the process. The ARMA is superior to the EWMA for autoregressive moving average processes. Jones (2002) addresses the issue of chart design, suggesting that ARL be the criteria for smoothing parameter selection. Box and Paniagua-Quiñones (2007) suggest using the smoothing constant that gives minimum mean square error of the period-ahead prediction. Lu and Reynolds (1999a, 1999b) investigate the behavior of the mean and standard deviation with an ARMA process. While no optimal performance exists across a wide variety of situations, they recommend an EWMA chart of observations used with a Shewhart chart of the residuals for process monitoring and control. Therefore the consensus of these authors is to track the shift and drift in the process mean using EWMA methods and identify the common cause variation of the residuals as the sigma as referred to in Six Sigma™. MacGregor (1991) suggests using engineering feedback control, similar to ARIMA process monitoring, and applying SPC charts to the residuals of this control.

Summary of Literature Recommendations

To distill the literature, an approach that would satisfy the cautions of the various authors would be a method of fitting an ARIMA model, using a method to identify the shift points in the series, confirming the location of the shift points with either CUSUM or EWMA methods, identifying separate control limits for each regime identified, and then employing Shewhart charts to the residuals to detect special causes that may need to be investigated and removed from the model. The relative shift of the mean compared to the variation of the process could be determined from ANOVA using the shift points to group the series. The ratio of the variation of the between group means to the process sigma would be the same as the mean square between to mean square error.

CHAPTER III

METHODOLOGY

In this chapter the methodology for this study is explained. The explanation begins with a description of the sample selection by giving the reader reasons for selection of the particular sample from the alternative methods available. The reasoning behind the purposive selection the study within a process and the systematic nature of recording observations for analysis is presented. A flow chart of the method is used to explain how the analysis was performed. Following a description of the method an example of an assembly process study is presented.

Sample Selection

The procedure for this study was as follows:

The design of this study was quantitative research in the causal comparative group of designs. The researcher attempted to determine the cause, or reason, for pre-existing differences in groups. It is sometimes called an "ex post facto" study because both the effect and the alleged cause have already occurred and must be studied in retrospect. These studies usually involve two (or more) groups, one independent variable and involve making comparisons. Subjects are not randomly selected but selected because they belong to groups. The researcher cannot manipulate the independent variable. The independent variable has already occurred and cannot be manipulated. The random sample was selected from already-existing populations. The researcher used a variety of descriptive and inferential statistics (Fraenkel & Wallen, 2003). In this study, the causal comparative experiment examined the type of manufacturing process and the time series models for explanation of the size of the shift or drift in the process mean over time.

The response was the magnitude of the process shift compared to the inherent, common cause, process variation. The processes studied are shown in Table 1. The choice of a sample depended on the research question. A completely random sample was not used in this research. In this investigation the research question was focused on a single company trying to identify the sigma shift in its processes. While a random sample would represent the mix of processes by count, the investigation would probably miss some small, but very important processes.

An alternative to random sampling, stratified sampling, was chosen because the investigator wanted to make certain each strata or process was represented in the investigation. Consistent with this method is the assurance that the count of these processes in the company would not be as consequential to the research as having a representation of all processes.

Table 1. Processes to be studied for the 1.5 sigma mean shift and drift.

Process	Studies
Assembly	18
Foundry	6
Heat Treatment	3
Machining	10
Matching	1
Shaving	14
Test Machine	5
Grinding	4
Turning	57
Warranty	4
Yield	3

A sample is taken in order to gather information on a population. Most research is based on a sample because a population is broad in scope, spread geographically or incomplete in the sense that more members are being added to the population as the research is being conducted. For instance, a company making product on a continuing basis would be generating more product while the research is being conducted on the sample.

In order to decide upon the proper sample, the population must be defined. In this study, a company that makes off-highway vehicles was studied to evaluate whether the processes important to its success have shifts and drifts in the mean equal to one and a half times the variation of the process. This is referred to as the 1.5 sigma shift in terms used by Six Sigma practitioners. So the investigator defined this population as those processes that were important to the commercial success of the company that produced off-highway vehicles. The processes that this company believed were important to its success were grouped from least to most value added: primary metal casting, machining of gears and shafts, machining of casings and housings, post-machining processing, testing and assembly. Key business metrics such as customer experience while the product is covered by warranty, warranty costs per vehicle and first pass yield were also key business processes.

Fraenkel and Wallen (2003) refer to the target population, the population to which the researcher wishes to generalize, and the accessible population, the population to which the researcher is able to generalize. The target population would be the ideal choice, the accessible would be the researcher's realistic choice. This study would ideally apply to all processes used in the world. A more realistic population for this research is all processes used in the United States of America. Still more realistic would be processes measured with interval data. A still more realistic is a single company that has a variety of processes measured with interval data. While this investigator realized the differences between the ideal and the accessible population, the value of the research was not diminished. No studies have been reported on a large company with a diverse set of processes (the accessible population) measured with interval data.

Another expectation of sampling is that the sample will be representative of the accessible population. The company made discrete parts and purchased others for assembly, test, evaluation and sale. The economics of the processes led to several reasons that the measurements for process control were not interval data. Some processes were not measured. These processes were controlled by tool geometry or tool size. For instance, a broach that produced an internally splined hub was ground to a specific size. As the broach wore the spline size grew. Appearance of the spline surface and the power required to produce the spline were indicators that it was time to sharpen the broach. Interval values of the spline size were not economic to gather. The 1.5 sigma shift is of no interest to the company because economy of operation dictated when to sharpen the tool. The same considerations applied to other processes when a part feature changed size with tool wear. The economic behavior was not to rotate the insert to maintain a targeted size, but rather utilize the insert for the maximum number of pieces by setting the minimum size on an external feature or maximum size on an internal feature so that the total allowed size range was obtained before rotation of the insert. For these reasons, a stratified sample was used in this investigation.

For the reasons just listed the investigator did not draw a random sample within each strata or process. The investigator considered obtaining representative samples within each strata by gathering either systematic, convenience or purposive samples. A combination of these sampling methods was used in this study. A purposive sample was used to select the process for study within each strata because it more appropriately fit the purpose of the investigation. Within a process the samples were drawn by operators of the processes for quality control at equally spaced intervals. Typically a sample was taken at equal time spacing with a rational grouping that was to minimize short term variation while maximizing the ability to identify the long term variation. This concept was inherent in the Six Sigma method by allowing for the 1.5 sigma shift between samples while controlling within variation to plus or minus three sigma.

A word about why convenience samples were not used in this study. Convenience sampling selects subjects to be sampled when conveniently available. These samples have very restrictive generalizability. In industry, product opinions were often gathered where customers come into contact with the sales group. Market forecasts have been made based on "customer feedback" that often came from those who recently purchased the product. All potential buyers were not surveyed so companies' products tended to be more attractive to current customers rather than to competitors' customers. The convenience sampling method was therefore inappropriate for this investigation.

This investigation drew a purposive sample of the processes used within the company with systematic recording of observations while the processes were running.. Fraenkel and Wallen (2003) report that purposive samples were based on previous knowledge of the population and the purpose of the research employing the investigator's personal judgment to gather the sample. Purposive sampling was different than convenience sampling because researchers did not study whoever was available, but used their judgment to select a sample that they believed, based on prior information, would provide the data they needed. Of course, the disadvantage was the possibility of bias in the sample that could affect the generalizability of the results.

To address the issue of bias, the investigator in this study assembled a group of company experts who represented operations, manufacturing engineering and processing, quality engineering and measurement systems. The investigator acquainted the group with the intended purpose of the research and solicited suggestions on the strata to include in the sample and the measurements to be studied. In this manner the knowledge of the population was represented by the participants who were recognized by the company as most knowledgeable in the processes it used. Additionally, managers of the company participated in selection of the key performance metrics to be studied. They believed the methods used in this study would permit them to employ the results in similar operations they managed. They based this opinion on their knowledge of similar processes at other employers and other facilities familiar to them within the company. If the groupings they identified had not been included in the study, they felt that the results would be too specialized to be of good use to their company.

The group helped create a map of the processes for this study.

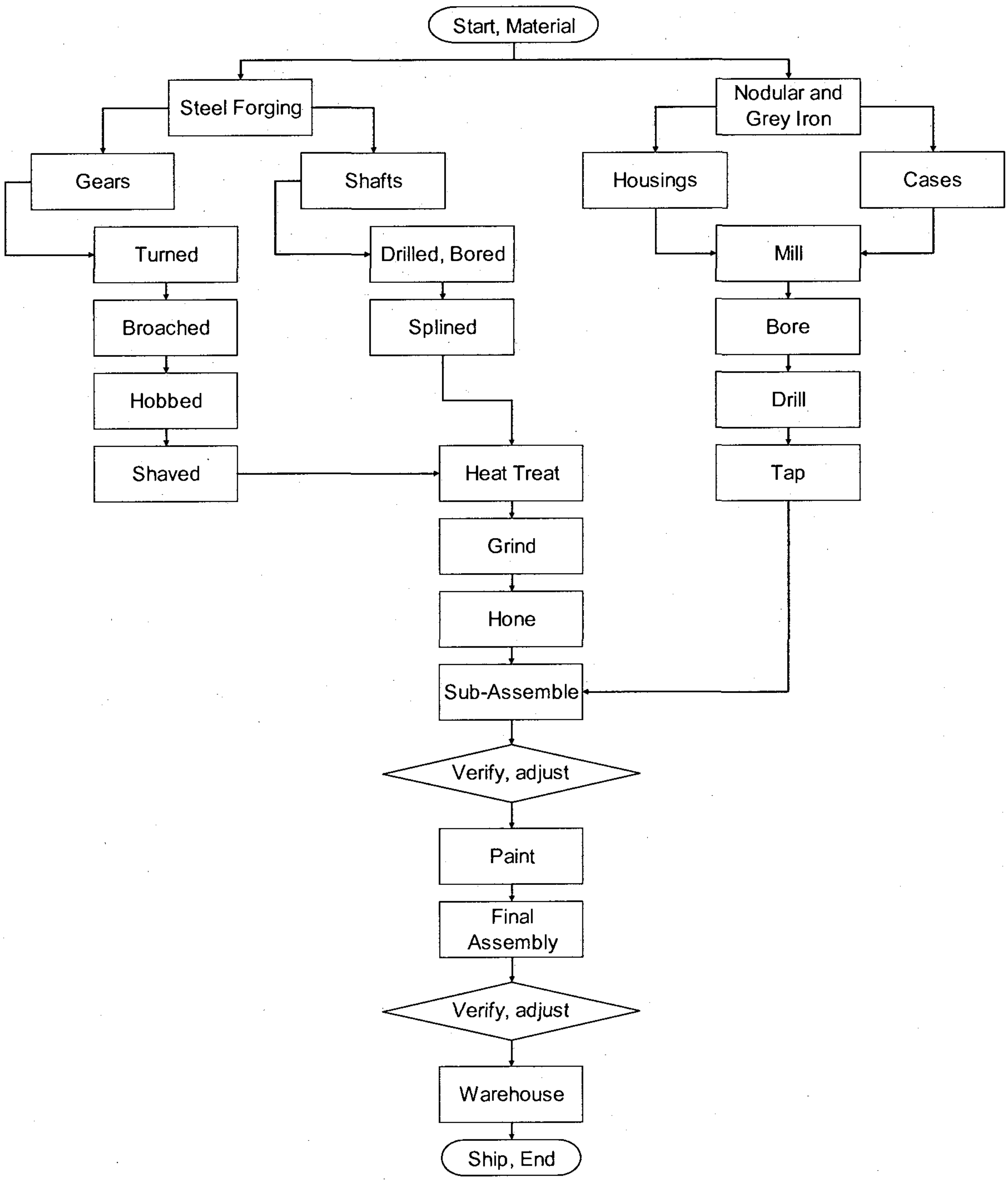

Figure 1. Process flow diagram used to identify cases for the purposive sample.

Figure 2 shows the groupings of the processes which aided in selection of the specific cases for this study.

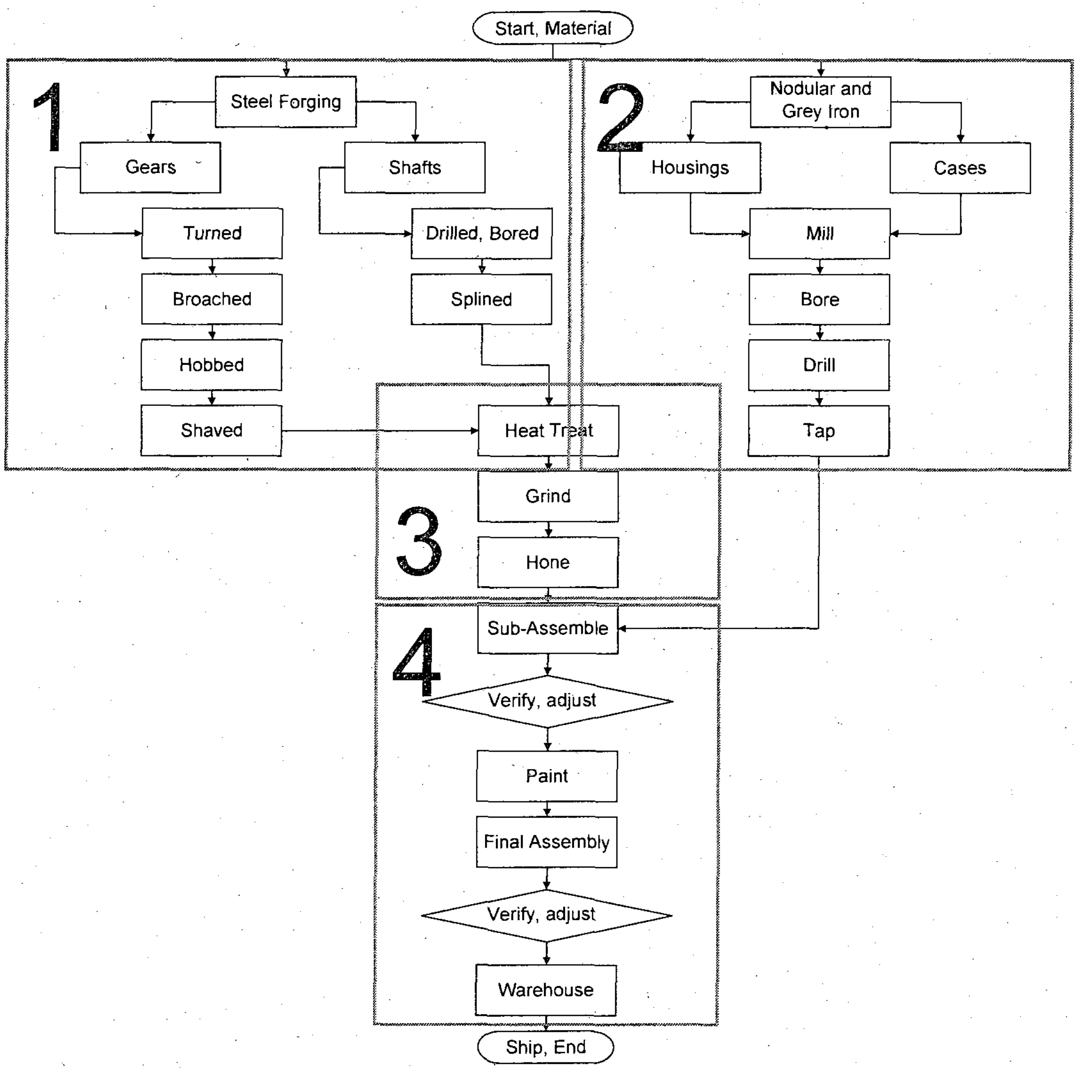

Figure 2. Grouping of processes into strata for case selection.

Method of Analysis

The analysis of process data in this study followed the flow shown in Figure 3. After a purposeful sample of the process was gathered there followed a systematic recording of observations over a representative period of time. The investigator plotted the data on a normal probability scale to judge whether the data were reasonably normal. If the data passed through this review, a plot of the data in time order was constructed. This gave the investigator an appreciation of the behavior of the process data over time. Trends and cycles were sometimes evident in this review.

Time Series Model

A series was considered stationary if there were no upward or downward drifts in the plotted series. If there appeared to be a drift in the series, the model was constructed with a first order difference, d, value equal to unity. A further check was made at the time the Autocorrelation Function (ACF) plot was made. Box, Jenkins and Reinsel (1994) recommend 15 lags were sufficient to judge the time-variant behavior of the process. A plot of the first 15 lags of the ACF was made and examined for appearance of autoregressive or moving average behavior. Once the ACF plot was examined, a Partial Autocorrelation Function (PACF) plot was made. This permitted the investigator to determine whether the model should be constructed as an autoregressive (AR) only or a moving average (MA) or a combination of ARMA. The model order for AR is designated p and for moving average, q. The AutoRegressive Integrated Moving Average (ARIMA) model was now tentatively described as ARIMA(p,d,q) and ready for the examination for seasonal or cyclical patterns in the residuals.

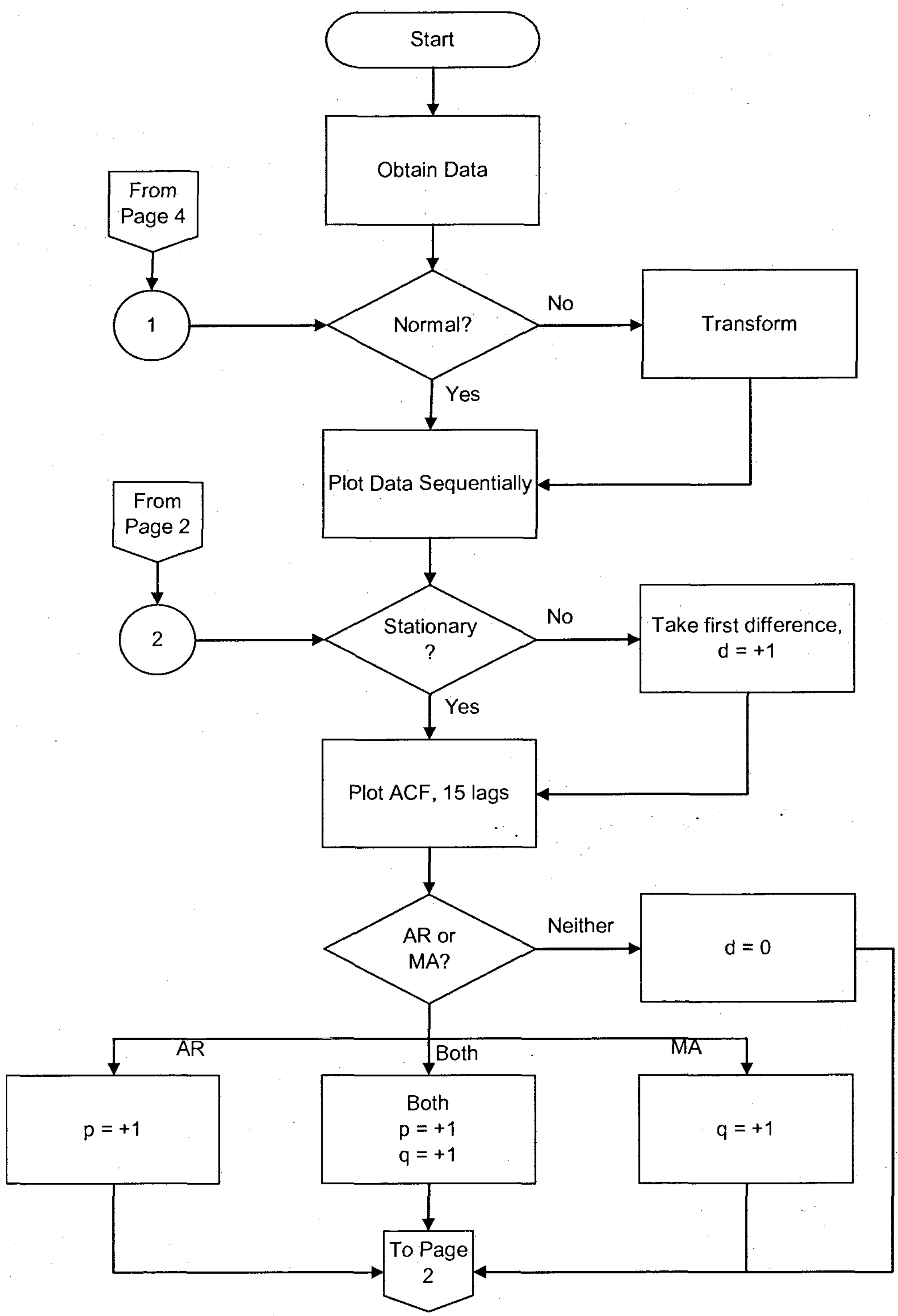

Figure 3. Flow of analysis activities for process data. (figure continues)

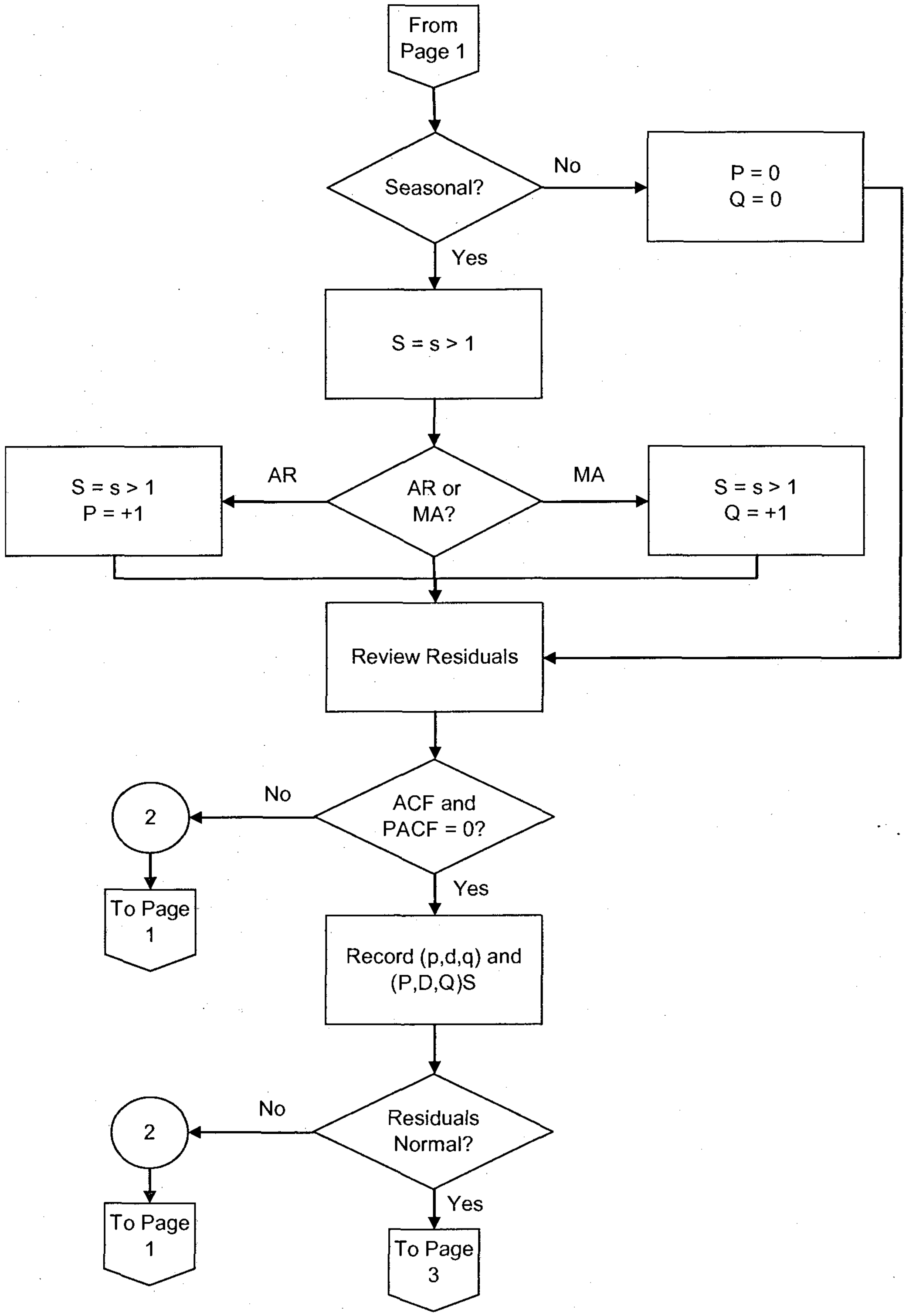

Figure 3. Flow of analysis activities for process data. (figure continues)

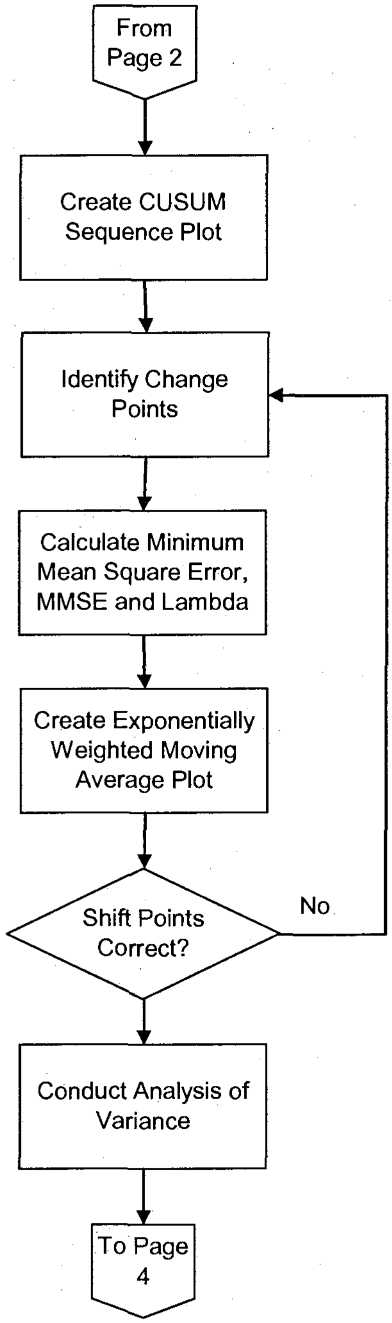

Figure 3. Flow of analysis activities for process data. (figure continues)

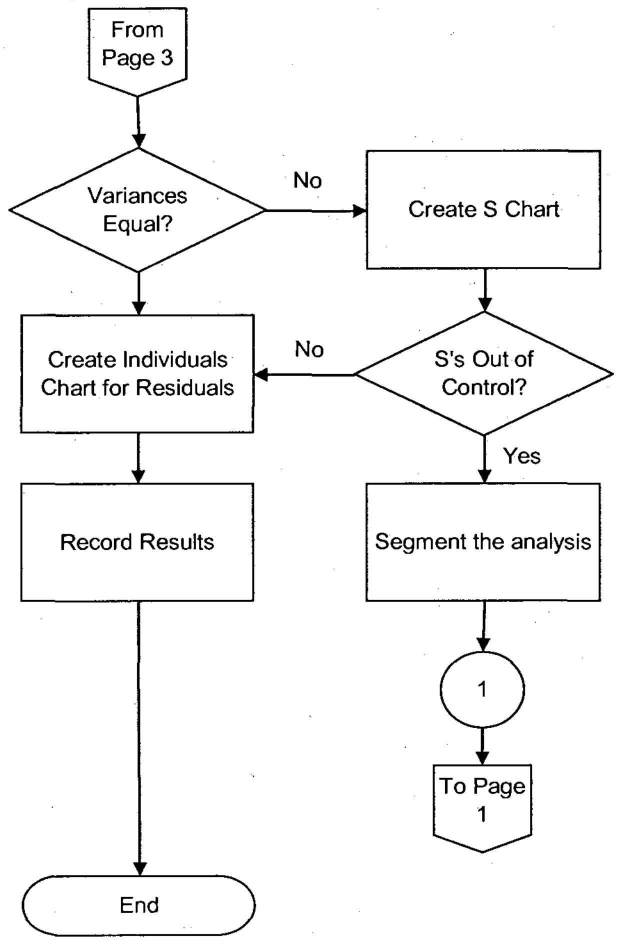

Figure 3. Flow of analysis activities for process data.

Seasonality in the Series

Beginning on the second page of Figure 3, the residuals of the preliminary ARIMA model were examined for periodic significance in the ACF and PACF of the residuals. When the plots suggested seasonality the appropriate seasonal AR or MA orders were added to the model. Re-examining the residuals from the revised model would confirm that the model order terms were complete. Capital letters were used to designate the seasonal terms, P, D, Q with the periodicity of the pattern designated by S. ARIMA(0,1,1)(0,1,1)4 was a time series that was first difference, moving average with a repeating cycle every fourth observation that had a first difference with moving average on these quarterly observations superimposed. Residuals from this proposed model were then examined for centering around zero, normal distribution of values, lack of autocorrelation and constant variance. Once this examination was complete, the parameters were recorded. The time series was specified and recorded.

Change Points in the Mean

Beginning on the third page of Figure 3, the CUSUM sequence plot of Lucas (1985) and later Saniga, Davis and Lucas (2009) was constructed in order to identify change points in location of the process mean. This was a manual activity using a straight edge and eyeball. Whether the correct points had been identified would be confirmed later with the Exponentially Weighted Moving Average (EWMA) plot. The Minimum Mean Square Error (MMSE) was now found by doing a grid search that changed the smoothing coefficient, λ, in uniform increments and judged whether the average squared difference between the smoothed value and the observed value was the smallest of all those calculated. This value was used as the coefficient for the EWMA plot. After plotting the EWMA with the MMSE lambda, the plot was examined for any evidence of shifts in the mean. When signals were found, the shift point location was revised and the EWMA regenerated. While the change points were now believed to be correct, the Analysis of Variance (ANOVA), needed to be performed to ensure the changes in the mean were statistically significant.

Variation in the Process Average

As shown on the concluding page of Figure 3, the ANOVA was performed and the residuals from that analysis were examined for average of zero, normal distribution, and constant variance. Bartlett and Levene tests were conducted on the variances of the sub-groups identified in the CUSUM sequence plots. If the Bartlett and Levene tests indicated non-constant variance then an s-control chart was constructed to identify which variances were unusual. The unusual variances were removed and the overall variance recomputed. If the recomputed variances altered the results from the ANOVA analysis, the new value for the within variance was used. In no cases were the results altered by the recomputed variances. The analysis was performed to generate the relative variation, in sigma units, in the process mean compared to the sigma or process variation. In this manner the two differing sigma were compared. The ANOVA analysis yielded ratio of the between group variances to the within group variances.

Sigma for the Six Sigma Evaluation

During these analyses, other estimates of process sigma were gathered. The root mean square error of the fitted ARIMA model was an independent estimate of sigma. Using the residuals from the ARIMA model a Shewhart chart was constructed giving a second estimate of sigma from the ARIMA model. A third estimate was the root mean square error of the MMSE of the exponentially smoothed series. In all, four estimates of sigma were obtained: two from the ARIMA model, one from the MMSE of the exponentially smoothed series and one from the ANOVA. These estimates were very close in value for the processes in this study.

A1.1 as an Example

To illustrate the method, we will show the analysis steps for the A1.1 Assembly. These data are deficiencies on a machine that either need to be remedied or reviewed, cataloged and evaluated for corrective action. The data are counts of deficiencies per machine. In all, 129 machines were in this study.

Figure 4 shows a pattern in the normal probability plot suggesting a transformation of the measurement units was needed. A logarithmic transformation was performed.

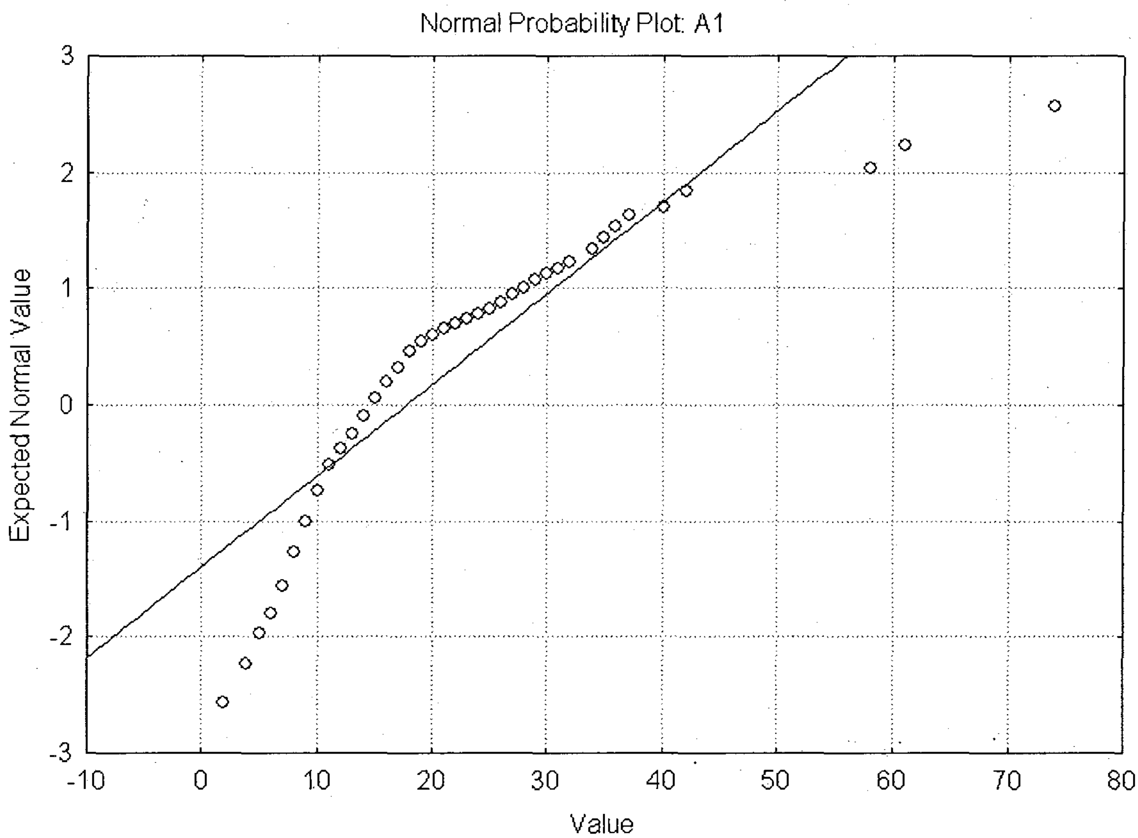

Figure 4. Normal probability plot of A1.1 in original measurement units.

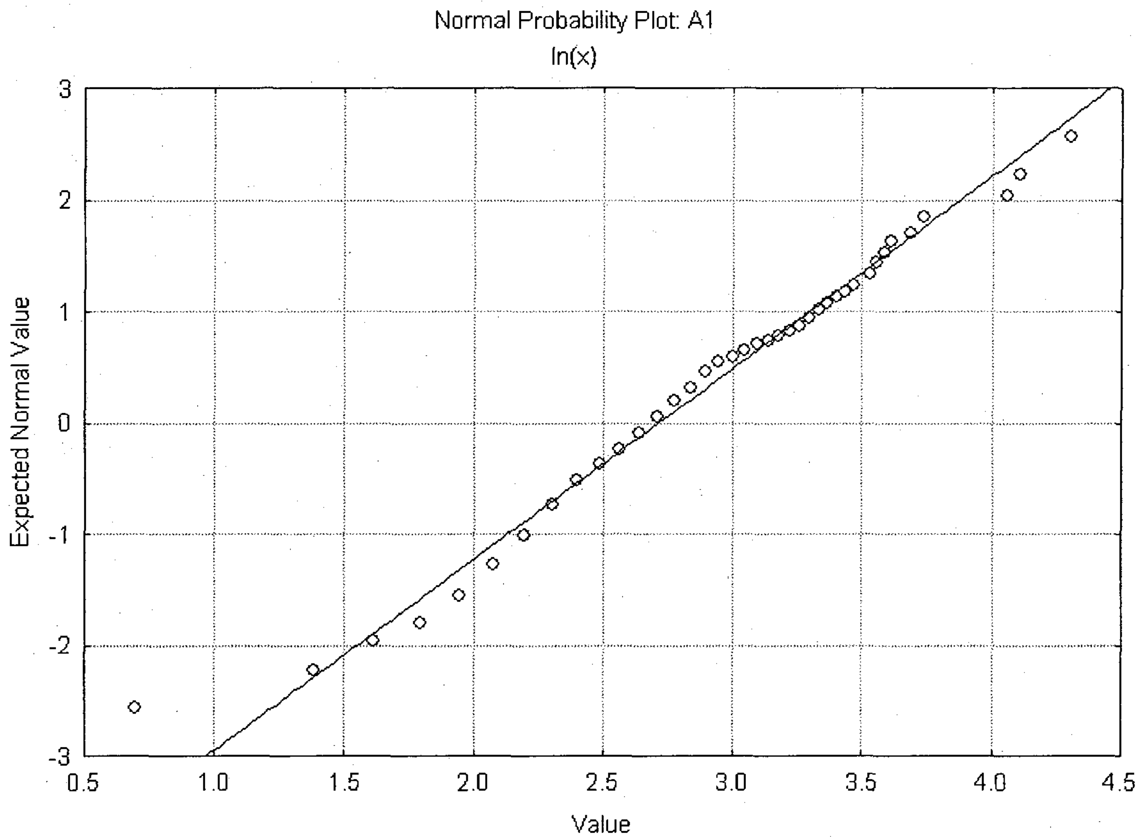

Figure 5. Normal probability plot after logarithmic transformation of A1.1.

A time series model was then created to fit the behavior of the A1.1 data.

Autocorrelation Function

A1 : ln(x)

(Standard errors are white-noise estimates)

Lag	Corr.	S.E.	Q	p
1	+.703	.0870	65.21	.0000
2	+.701	.0867	130.5	0.000
3	+.676	.0863	191.9	0.000
4	+.615	.0860	243.0	0.000
5	+.628	.0857	296.7	0.000
6	+.570	.0853	341.3	0.000
7	+.554	.0850	383.7	0.000
8	+.546	.0846	425.3	0.000
9	+.484	.0843	458.4	0.000
10	+.478	.0839	490.8	0.000
11	+.421	.0836	516.2	0.000
12	+.426	.0832	542.4	0.000
13	+.442	.0829	570.8	0.000
14	+.392	.0825	593.4	0.000
15	+.393	.0821	616.3	0.000

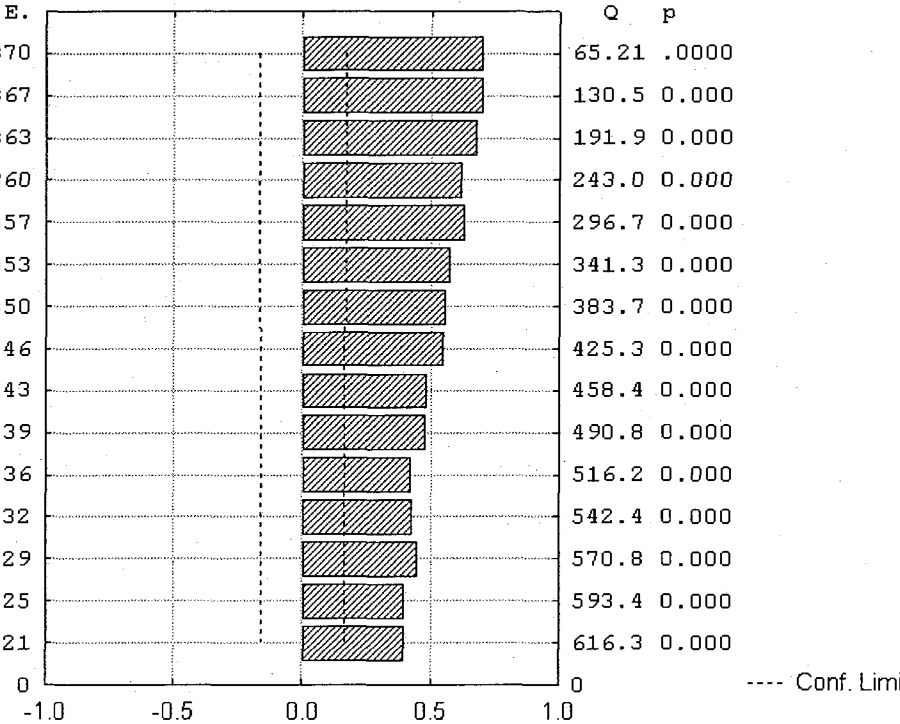

Figure 6. Autocorrelation Function for transformed A1.1 measurements.

The slow decay in the value of the autocorrelation function indicated that the series was not stationary. So a first order difference was included in the model.

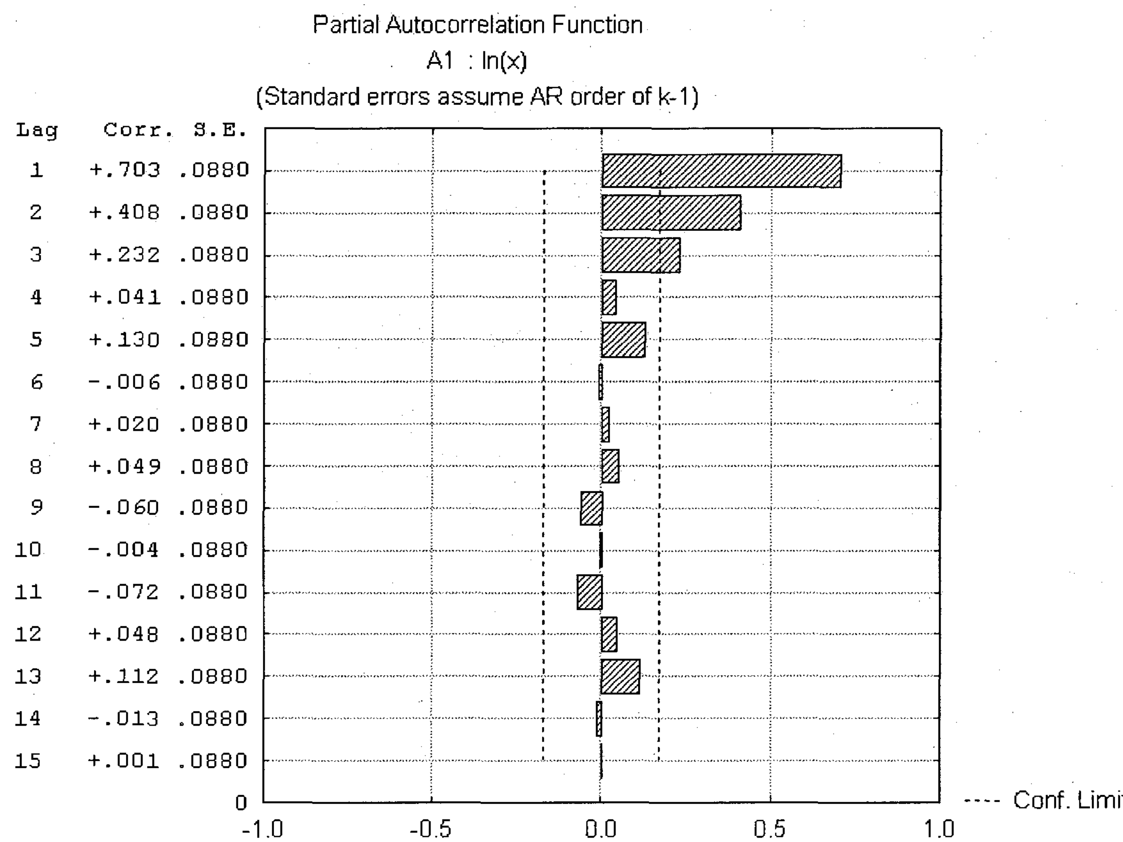

Figure 7. Partial Autocorrelation Function for transformed A1.1 measurements.

The partial autocorrelation function showed that three terms could possibly be in the model. It also suggested that a moving average order should be in the model due to the rapid drop in the value of the coefficients for each of the initial three lags. The first difference was applied to A1.1 data and a moving average of order one was included in the model.

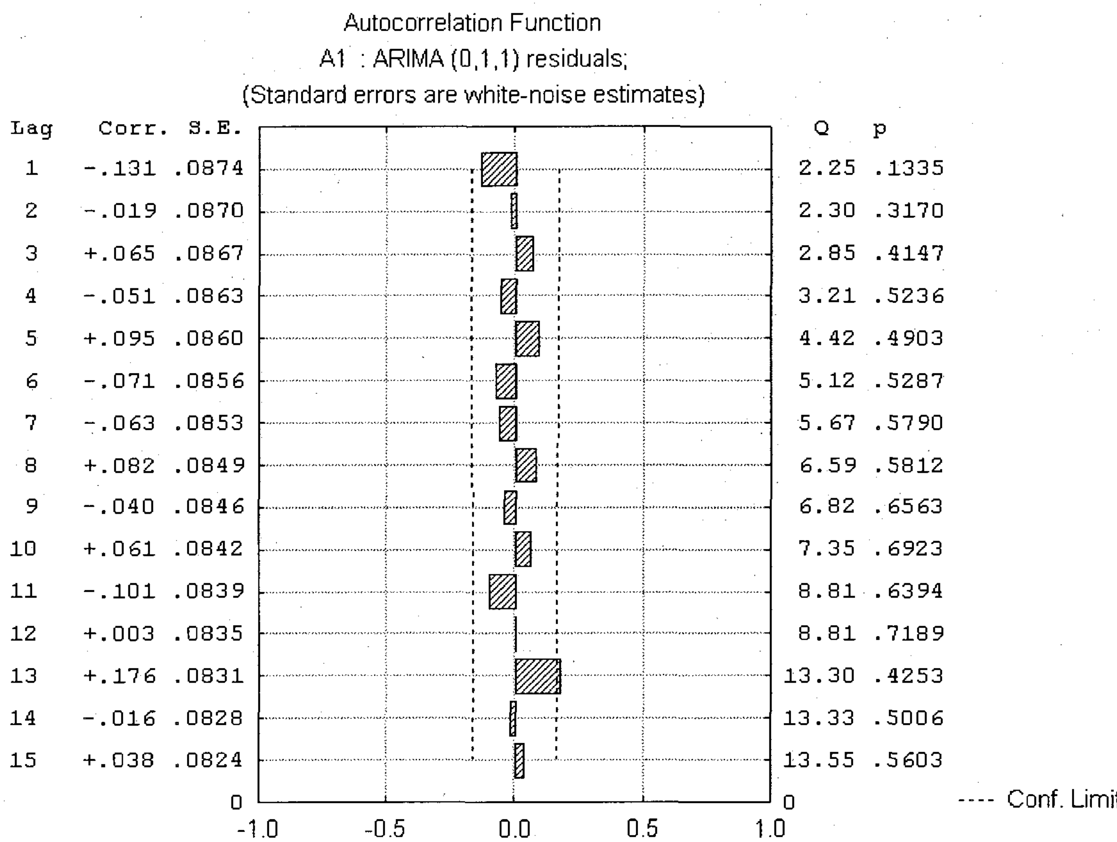

Figure 8. Autocorrelation Function of residuals after fitting the ARIMA model to the transformed A1.1 data.

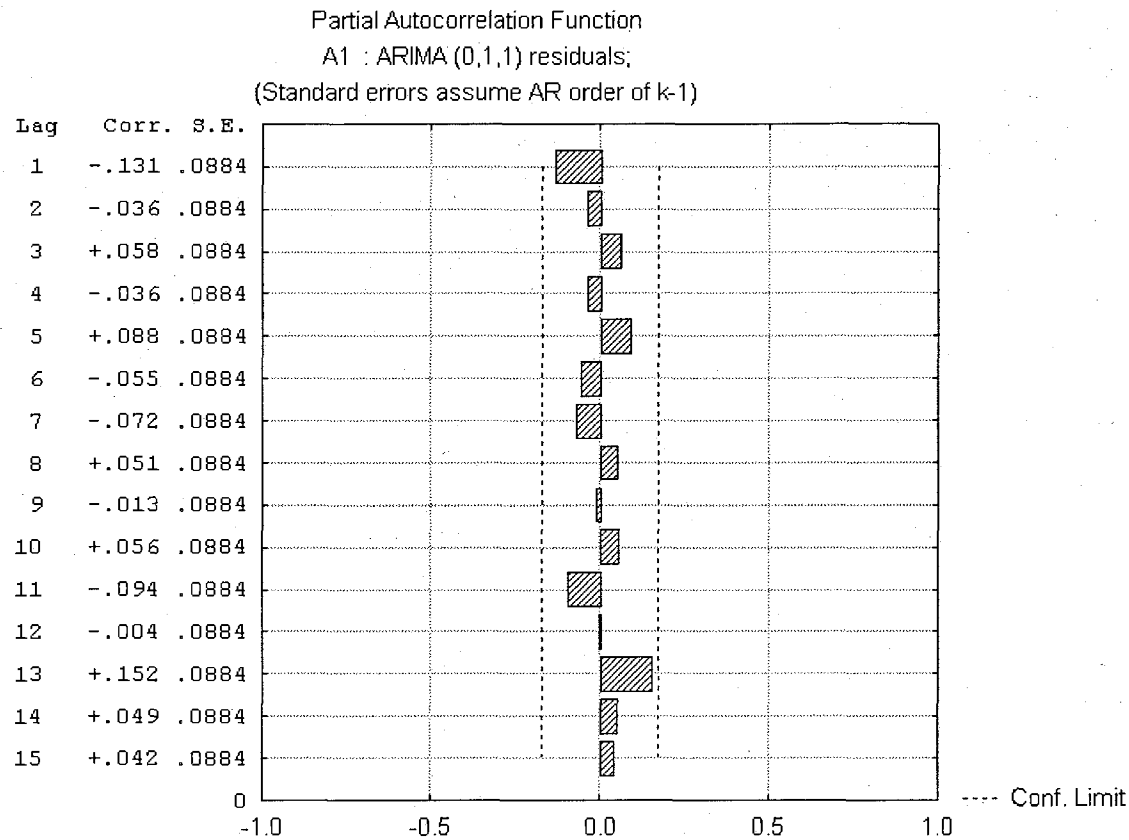

Figure 9. Partial Autocorrelation Function of residuals after fitting the ARIMA model to the transformed data.

The ARIMA(0,1,1) model residuals indicate no remaining autocorrelation. This was a good indication the model fit the data well. The model coefficients and their statistical confidence intervals are in Table 2. The equations converting the ARIMA parameters to original measurement units follows the table.

Table 2. Parameters of the ARIMA model for A1.1 transformed data.

	Input: A1.1 Transformations: ln(x),D(1) Model:(0,1,1) MS Residual= .09391					
Paramet.	Param.	Asympt. Std.Err.	Asympt. t(127)	p	Lower 95% Conf	Upper 95% Conf
q(1)	0.688248	0.057067	12.06040	0.000000	0.575323	0.801173

The method was to first transform the data into logarithmic units, take the difference between neighboring values, then fit the time series order. Once the order was known, the coefficients were calculated. The equations are shown here for A1.1 example.

$$
\begin{aligned}
&z = \ln(x) \\
&z_t - z_{t-1} = (1 - \theta_1 B^1)a_t \\
&z_t = z_{t-1} + a_t - \theta_1 a_{t-1} \\
&\ln(x_t) = \ln(x_{t-1}) + a_t - \theta_1 a_{t-1} \\
&\ln(x_t) = \ln(x_{t-1}) + a_t - 0.688248 a_{t-1} \\
&x_t = x_{t-1} e^{a_t - 0.688248 a_{t-1}}
\end{aligned}
$$

The variance is converted from logarithmic units to measurement units by this formula:

$$\sigma_x = \mu_x (e^{\sigma_{\ln(x)}} - 1)$$

The partial autocorrelation of the ARIMA(0,1,1) model indicated good model fit. The ACF of the residuals showed no correlation at any lag period and the same was true for the PACF of the residuals. The residuals were examined for centering on zero and normal distribution. These were assumptions that the model would have errors that were as much above the true value as below it. The expectation of normality stemmed from the assumption that the prediction errors were clustered close to zero, but random errors would contribute to normality in the error distribution. Box, Jenkins and Reinsel (1994) give a more complete explanation.

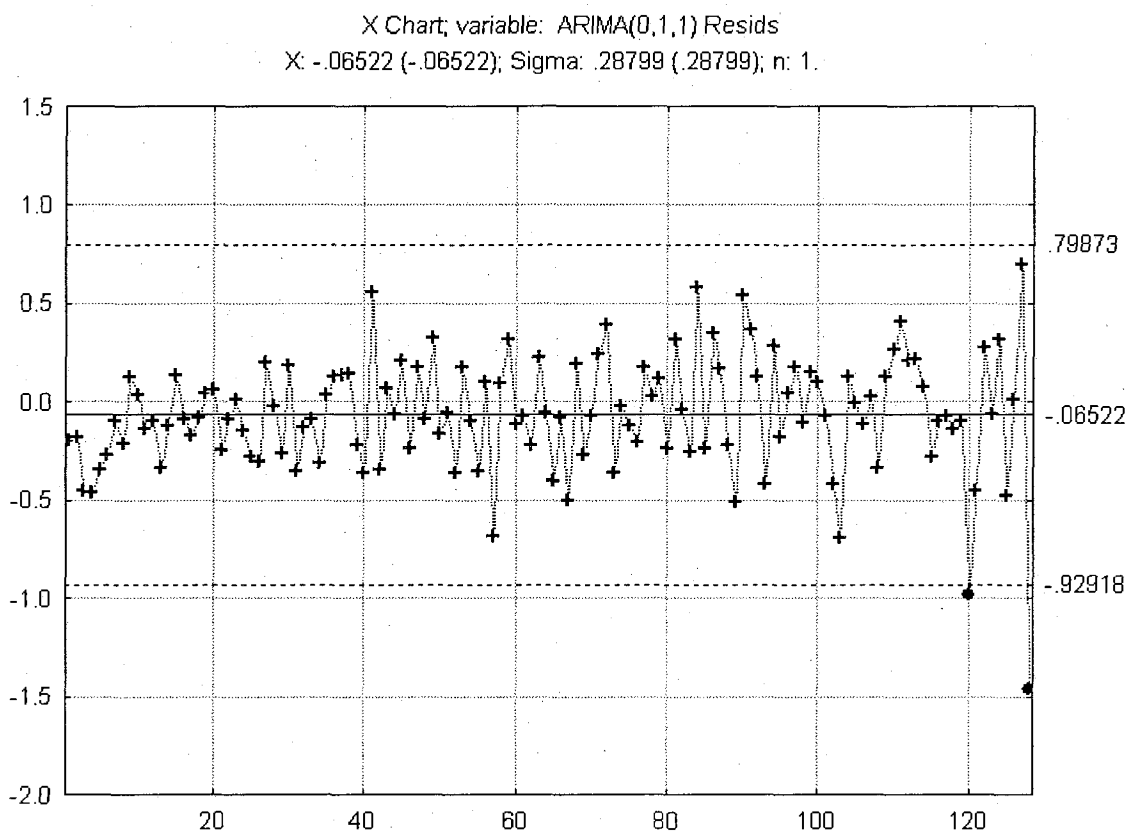

Figure 10. Shewhart control chart of ARIMA residuals from the ARIMA model of A1.1 data.

The Shewhart chart indicated that two observations did not fit the model well. These were observations 120 and 129. When this happened it was taken as a signal from the chart that special causes were present and needed to be investigated.

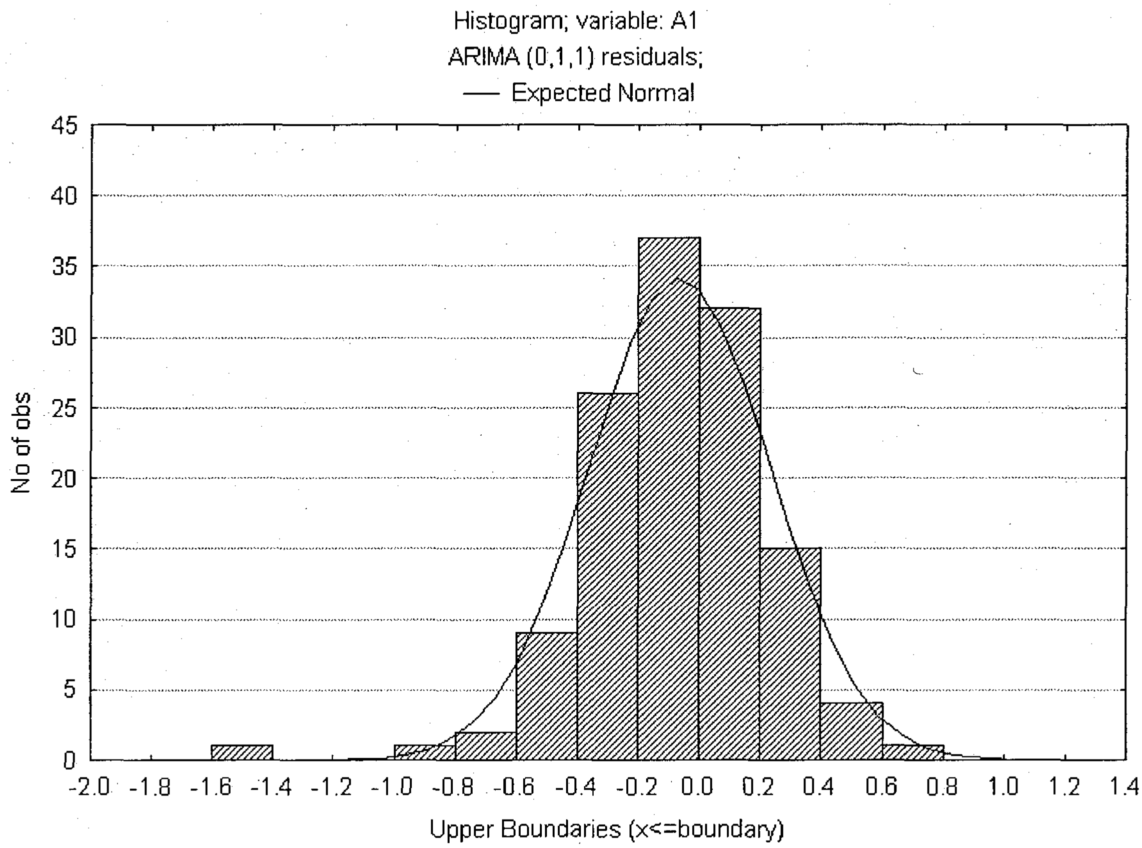

Figure 11. Histogram of A1.1 ARIMA model residuals. Verification of zero mean and reasonably normal shape. Note the location of observation 129 at -1.5.

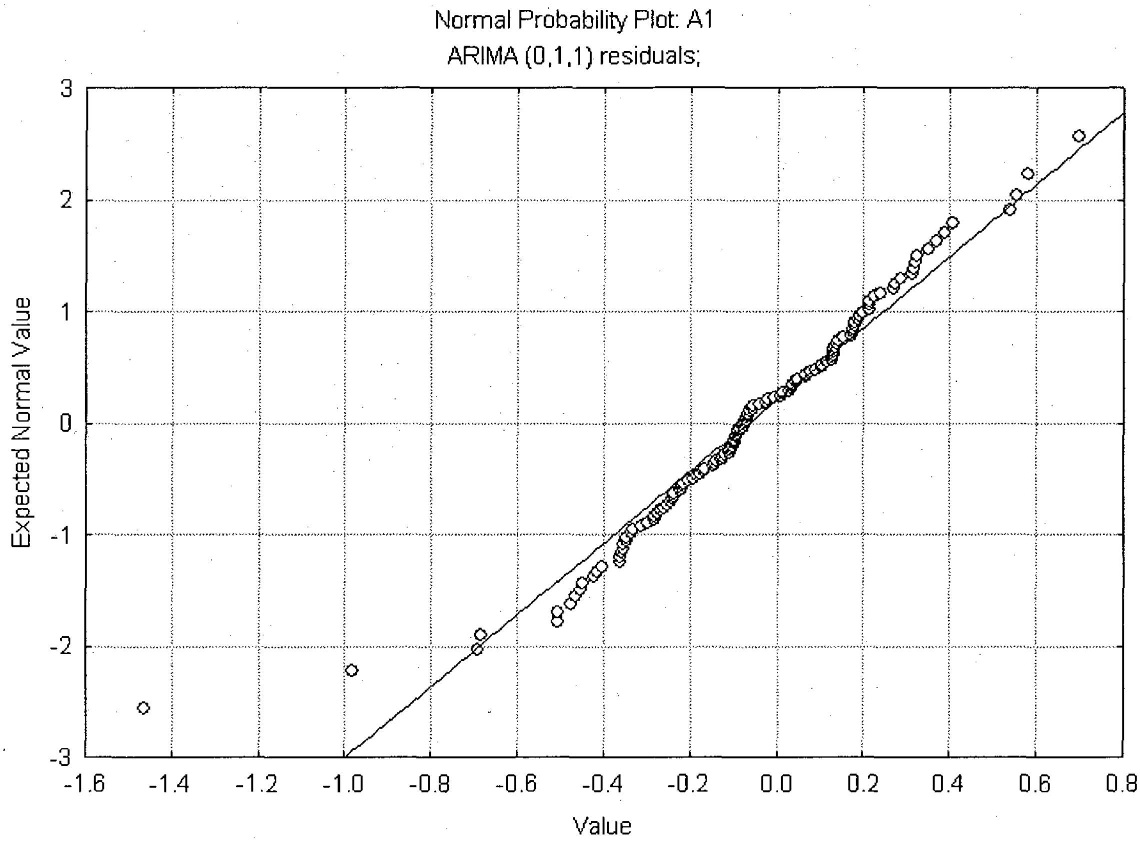

Figure 12. Normal probability plot of A1.1 ARIMA model residuals. Additional check of distribution of the residuals. Points 120 (-1.0) and 129 (-1.5) appear unusual and poorly fit the distribution.

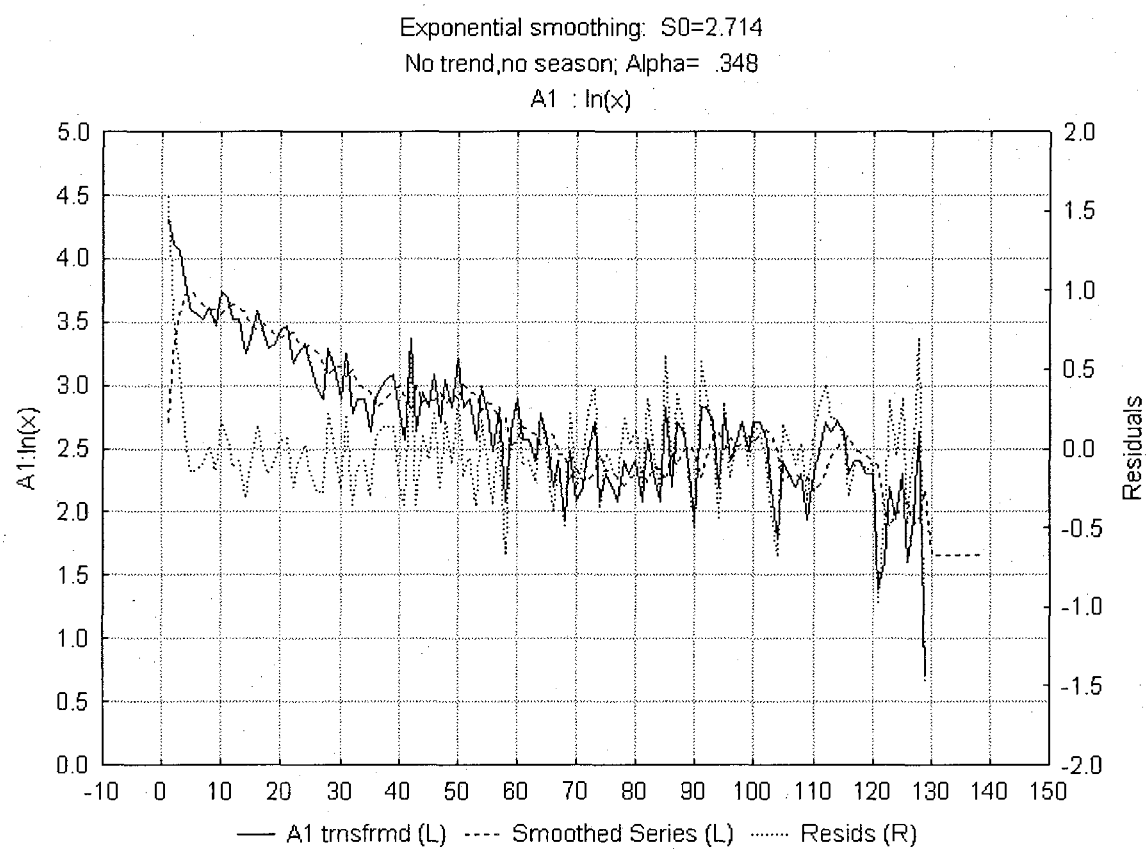

Figure 13. Exponential smoothing of logarithmic transformation of A1.1 data. Transformed, smoothed and residuals shown on plot.

The minimum mean square error was determined by preparing an exponentially weighted moving average chart and searching the smoothing coefficients through a grid search of values in order to find the value that yielded the minimum mean square error. The value of 0.348 reported for alpha is the same value identified as lambda for the EWMA chart.

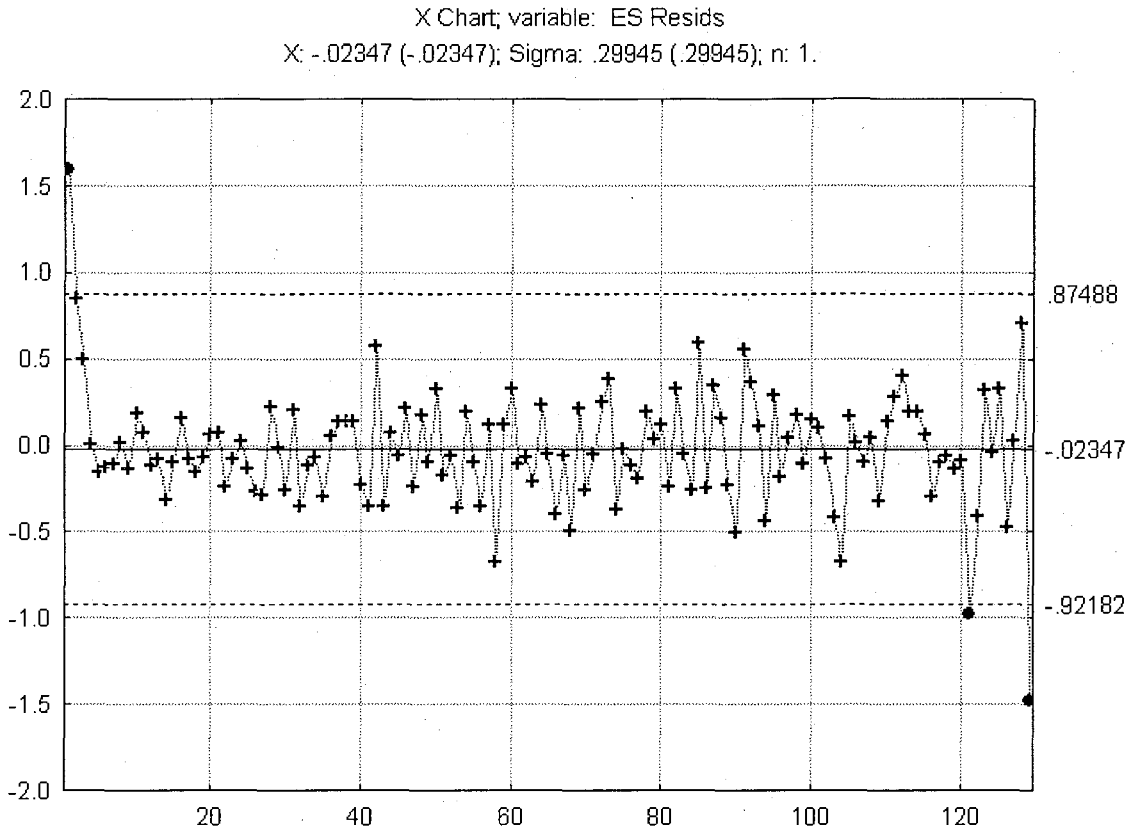

Figure 14. Shewhart chart of the exponentially smoothed residuals for the transformed A1.1 data.

There were three points identified by the exponential smoothing algorithm: 1, 120 and 129. The first is due to the start up of the model. The initial value is often used as the average, target or simply the first observation. After a few calculations the initial conditions have no effect. For this reason reading one would not be considered to have a special cause.

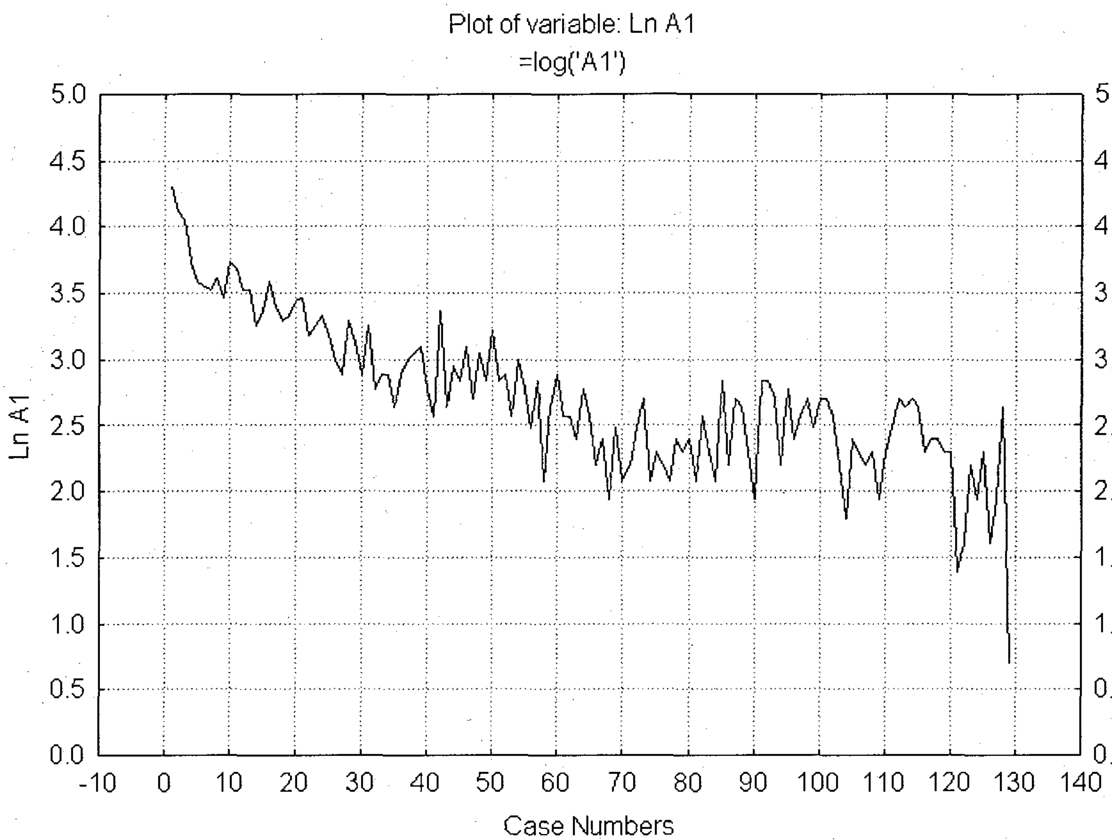

Figure 15. Transformed A1.1 plot in time order. The first observation of the series is much larger than expected. Also, observations 120 and 129 are much smaller than expected.

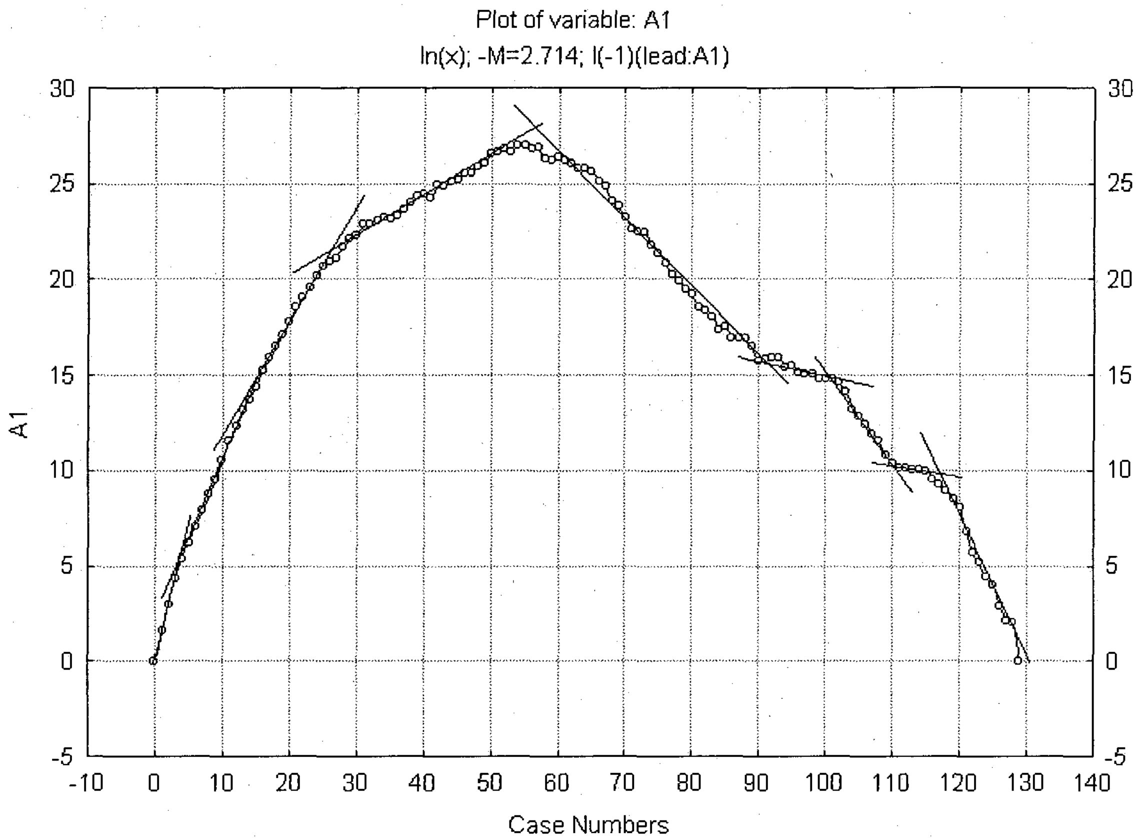

Figure 16. CUSUM sequence plot of transformed A1.1 data to detect shift points in the mean.

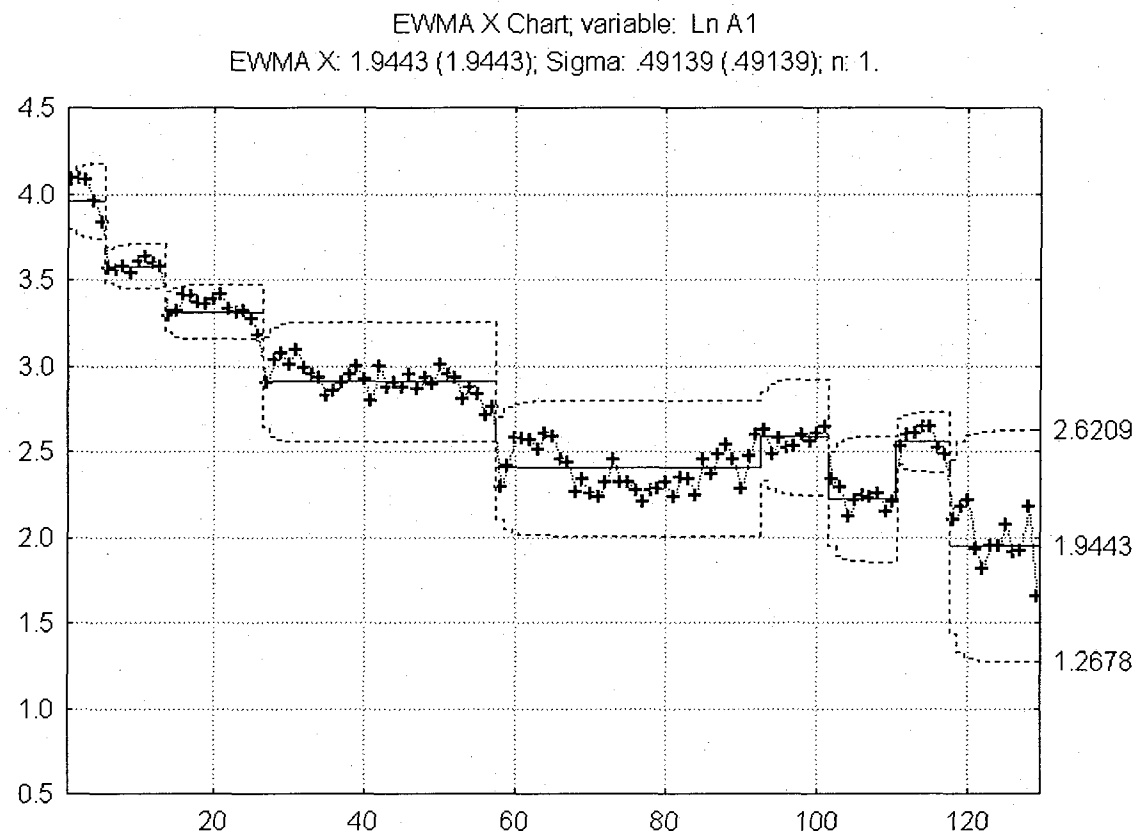

Figure 17. EWMA of transformed A1.1 data confirming the shift point locations

Table 3. ANOVA of transformed A1.1 means in CUSUM sequence groups.

	Univariate Tests of Significance for A1.1 Sigma-restricted parameterization Effective hypothesis decomposition					
Effect	SS	Degr. of Freedom	MS	F	p	s_w
Intercept	711.6477	1	711.6477	9686.978	0.00	
A1.1 Group	32.6034	8	4.0754	55.475	0.00	
Error	8.8157	120	0.0735			0.2710

The ANOVA indicates significant differences in the process group means for the transformed A1.1 data. The 0.2710 value was compared to the other estimates of sigma using the ARIMA(0,1,1) fit, the ARIMA(0,1,1) model error residuals and the exponential smoothing fit residuals. In this example those values were 5.4097, 5.0360, 5.2688 and 4.6968 after converting to original measurement units from the logarithmic units used in the analysis.

The value of M/s_w can be calculated using the following formula and the data from Table 4. Since M/s_w is the maximum movement in the mean from its target, M, divided by the root mean square error, s_w, calculation proceeded as follows:

$$M = Maximum|\bar{x}_i - \bar{x}_0|$$

Table 4 Descriptive Statistics for Ln A1.1

Level of Factor	N	Ln A1.1 Mean	Ln A1.1 Std.Dev.	Ln A1.1 Std.Err	Ln A1.1 -95.00%	Ln A1.1 +95.00%
Group	129	2.714	0.569	0.050	2.615	2.813
0	5	3.959	0.293	0.131	3.596	4.323
1	8	3.580	0.093	0.033	3.502	3.657
2	13	3.314	0.149	0.041	3.224	3.404
3	31	2.906	0.218	0.039	2.826	2.986
4	35	2.401	0.277	0.047	2.306	2.496
5	9	2.583	0.191	0.064	2.437	2.730
6	9	2.223	0.232	0.077	2.044	2.401
7	7	2.554	0.160	0.061	2.406	2.703
8	12	1.944	0.543	0.157	1.600	2.289

M/s_w = Max{Abs(3.959–2.714), Abs(1.944 – 2.714)}/ 0.2710=

Max{1.245, 0.770}/0.2710 = 1.245/0.2710 = 4.595

The initial value of M/s_w was recorded for comparison to a revised value pending the verification of the assumption of constant variance for the ANOVA.

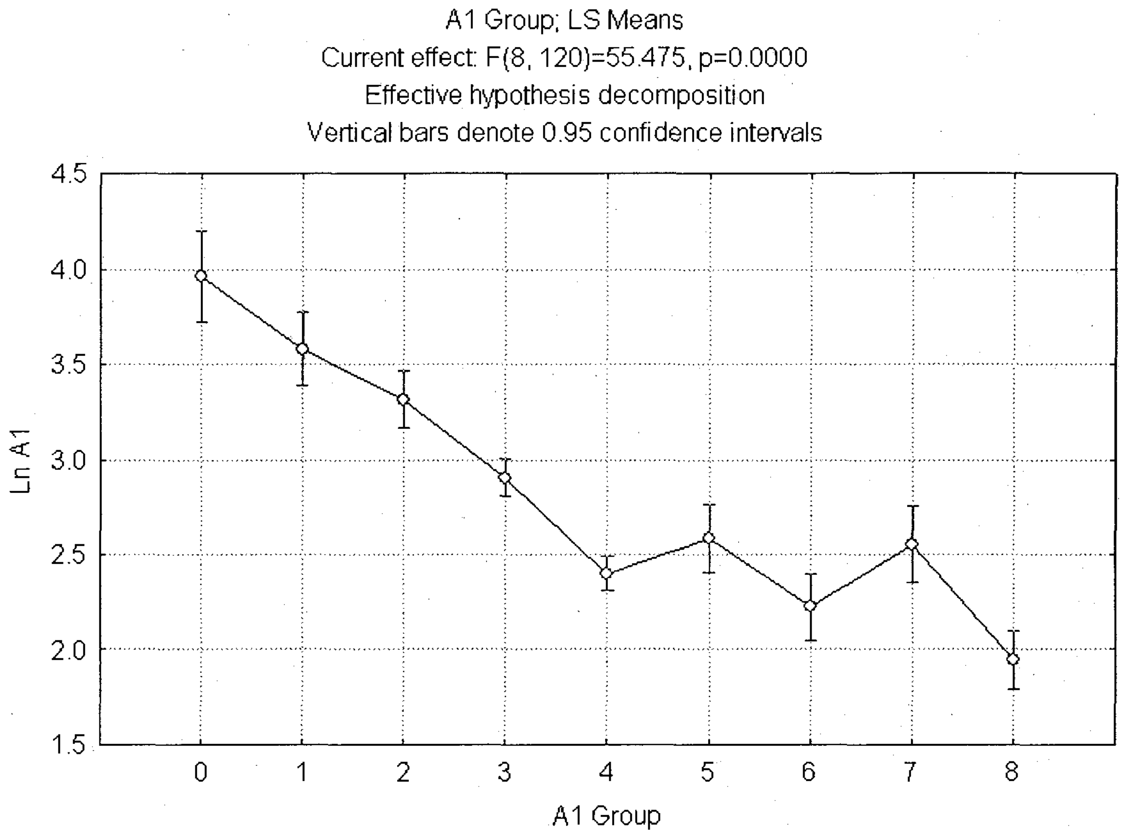

Figure 18. Plot of transformed A1.1 group means showing 95% confidence intervals. This plot illustrates the non-stationary behavior of the process. A first difference was used in the model to remove this trend.

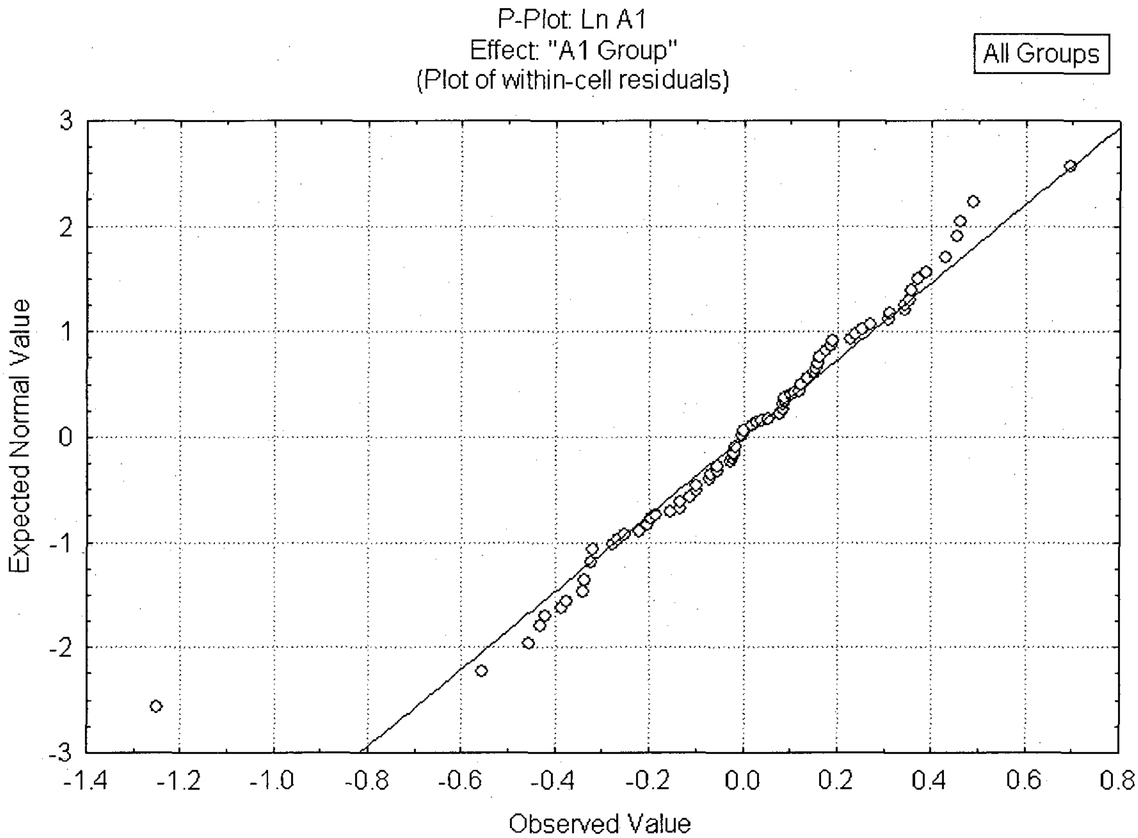

Figure 19. Normal probability plot of transformed A1.1 residuals from ANOVA groups. Plots are within group deviations from group average. This verifies that the within group observations are normally distributed.

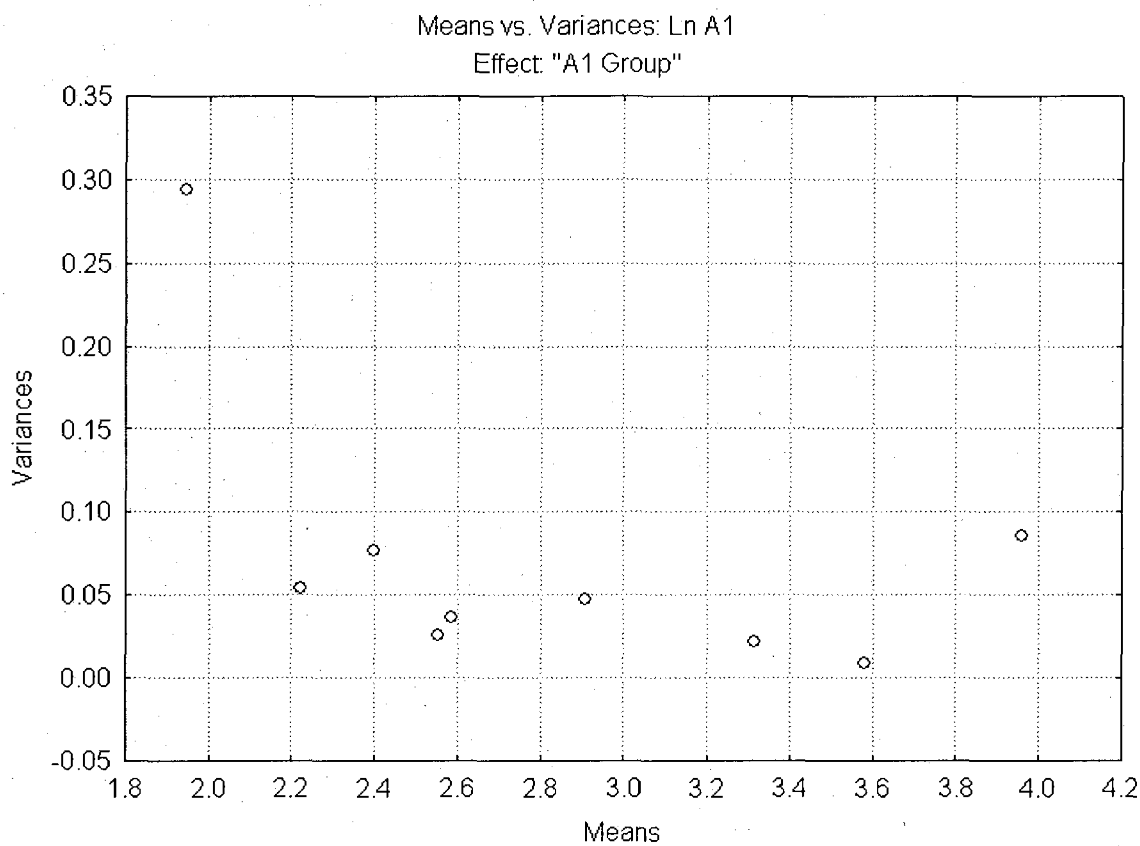

Figure 20. Plot of variance of transformed A1.1 data by CUSUM group. Checking assumption of uniform variance of the transformed A1.1 grouped data. The spread is greater for small means due to the effect of observations 120 and 129.

Table 5 and Table 6 show that the assumption of equal variances is not true for these data. An investigation of the variances was then conducted to identify the impact of unequal variance on the analysis.

Table 5. Tests for homogeneity of variances of transformed A1.1 CUSUM sequence groups

	Tests of Homogeneity of Variances Effect: "A1.1 Group"				
	Hartley F-max	Cochran C	Bartlett Chi-Sqr.	df	p
Ln A1.1	34.36802	0.452289	37.86794	8	0.000008

Table 6. Levene's test for homogeneity of variances of transformed A1.1 CUSUM sequence groups. A second check for constant variance assumption of ANOVA.

	Levene's Test for Homogeneity of Variances Effect: "A1.1 Group" Degrees of freedom for all F's: 8, 120			
	MS Effect	MS Error	F	p
Ln A1.1	0.113207	0.024249	4.668606	0.000054

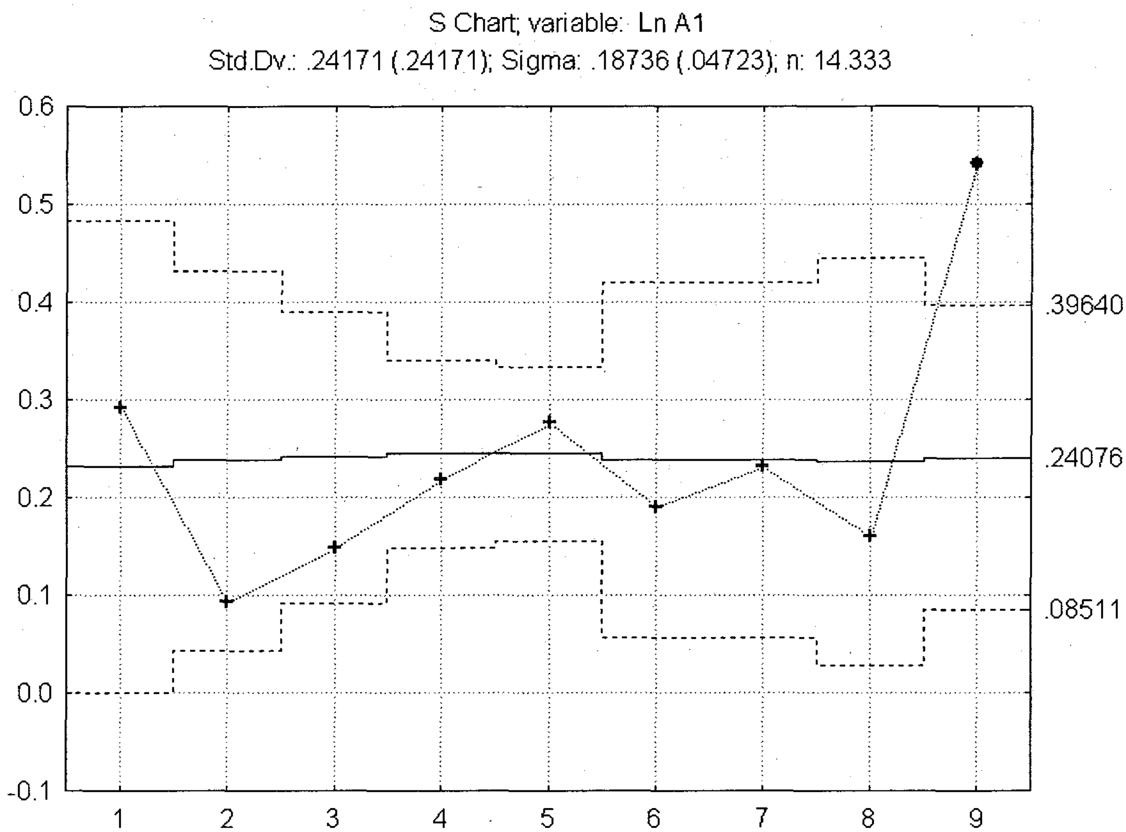

Figure 21. Shewhart s-chart showing control limits for transformed A1.1 CUSUM sequence group standard deviations. This is done to identify which variances were different from the others.

The s chart shows that Group 9 variance is larger than the others. After removing Group 9 from the calculations, Group 2 was then much smaller than the others. Therefore, both Group 2 and 9 were removed and the calculation of M/s_w was repeated.

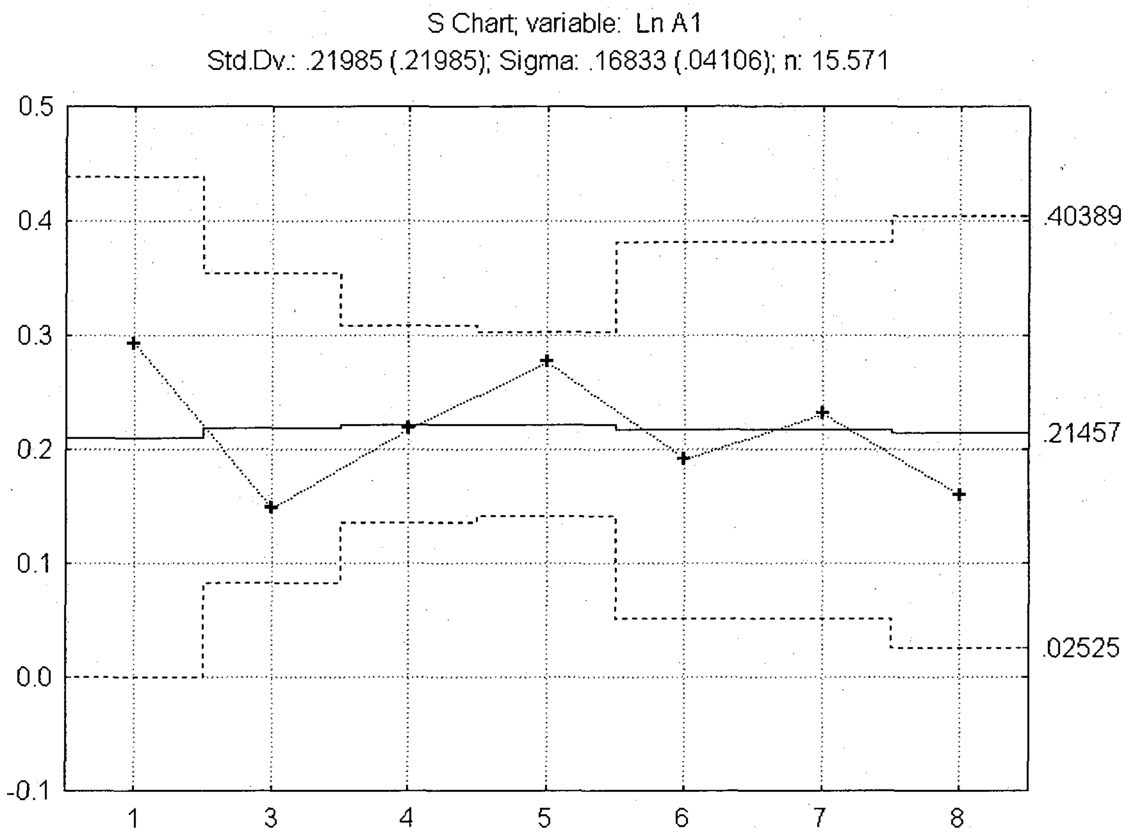

Figure 22. Shewhart s-chart after removing the largest and smallest standard deviations in A1.1 Groups 9 and 2 respectively.

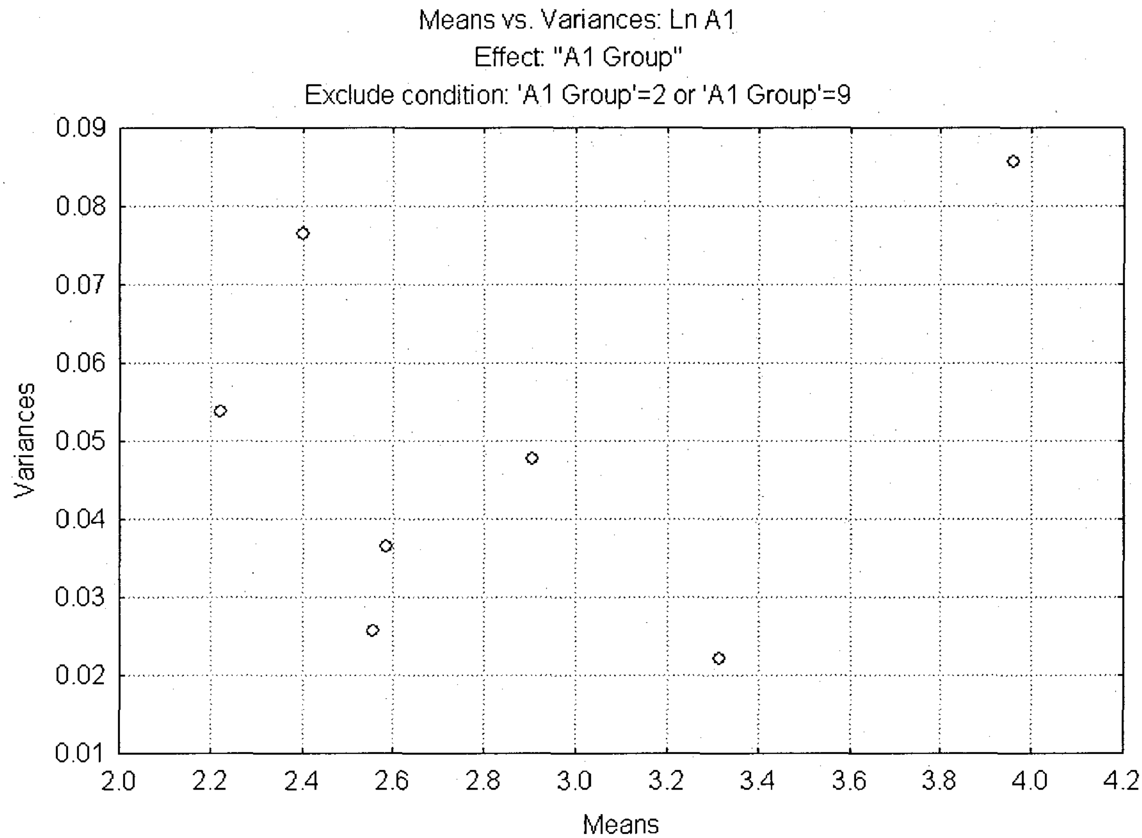

Figure 23. Variance versus mean for the revised analysis of the transformed A1.1 CUSUM sequence groups. Variances appeared more uniform so the ANOVA was repeated without Groups 2 and 9.

The revised estimate of s_w was now 0.2326 as shown in Table 7. Table 8 and Table 9 confirm the variances are reasonably equal.

Table 7. Revised ANOVA with highest and lowest variance groups excluded.

Univariate Tests of Significance for Ln A1.1
Sigma-restricted parameterization
Effective hypothesis decomposition
Exclude condition: 'A1.1 Group'=2 or 'A1.1 Group'=9

Effect	SS	Degr. of Freedom	MS	F	p	s_w
Intercept	565.7289	1	565.7289	10458.95	0.00	
A1.1 Group	19.4506	6	3.2418	59.93	0.00	
Error	5.5172	102	0.0541			0.2326

Table 8. Variances are judged to be homogeneous after excluding Groups 2 and 9.

	Tests of Homogeneity of Variances Effect: "A1.1 Group" Exclude condition: 'A1.1 Group'=2 or 'A1.1 Group'=9				
	Hartley F-max	Cochran C	Bartlett Chi-Sqr.	df	p
Ln A1.1	3.861572	0.246005	8.143495	6	0.227778

Table 9. Levene's test revised for the exclusion of Groups 2 and 9 from the analysis also fails to reject the hypothesis of equal variances for the CUSUM sequence groups.

	Levene's Test for Homogeneity of Variances Effect: "A1.1 Group" Degrees of freedom for all F's: 6, 102 Exclude condition: 'A1.1 Group'=2 or 'A1.1 Group'=9			
	MS Effect	MS Error	F	p
Ln A1.1	0.033437	0.016710	2.001101	0.072278

The repeated analysis now indicated that the assumptions required for a valid analysis of variance were valid. All tests, Hartley's, Cochran's, Bartlett's and Levene's failed to reject the null hypothesis of equality of variances.

Using sw = 0.2326 and the values for the group averages in Table 10 the movement of the process mean in units of the root mean square error was re-computed as shown here.

$$M/s_w = \text{Max}\{\text{Abs}(3.959–2.735), \text{Abs}(2.554 – 2.735)\}/ 0.2326 =$$

$$\text{Max}\{1.224, 0.181\}/0.2326 = 1.224/0.2326 = 5.262$$

Table 10 Descriptive Statistics for Ln A1.1 without Groups 2 and 9.

Level of Factor	N	Ln A1.1 Mean	Ln A1.1 Std.Dev.	Ln A1.1 Std.Err	Ln A1.1 -95.00%	Ln A1.1 +95.00%
Group	129	2.714	0.569	0.050	2.615	2.813
1	5	3.959	0.293	0.131	3.596	4.323
3	13	3.314	0.149	0.041	3.224	3.404
4	31	2.906	0.218	0.039	2.826	2.986
5	35	2.401	0.277	0.047	2.306	2.496
6	9	2.583	0.191	0.064	2.437	2.730
7	9	2.223	0.232	0.077	2.044	2.401
8	7	2.554	0.160	0.061	2.406	2.703

This difference, 0.667 (5.262 – 4.595), was considered to be essentially zero for these two values. The question was whether the shift in the mean exceeded 1.5 s_w and clearly this was the case regardless which value was used. Because the original value utilized the spread in all the averages, and it was more conservative than the 5.262 ratio, the 4.595 value was reported in Table A3.

The remaining 124 analyses were done in the same manner as this one. In order to conserve space, only the summary of the results will be used for the remainder of this report.

CHAPTER IV

RESULTS

The studies of this investigation are summarized in the tables in the appendix. This was done in order to make the manuscript more readable. After describing the studies, time series analysis and its application to these studies is summarized. The estimates of sigma are summarized by time series model and process. Duration of the mean shift is summarized and compared by process and time series model. The smoothing coefficient, λ_{EWMA}, closes the chapter with its distribution and relationships to process and time series model. With all measures the Kruskal-Wallis nonparametric analysis was performed in order to decide whether relationships were statistically significant.

Description of the Studies

The studies were conducted at different times and later organized into Table A1. As the reader will notice, there are several studies on product, four groups with different counts of studies within the group. Most of these studies required a transformation of the data due to the underlying behavior of count data as not normal but easily transformed with either the square root (SqrRt) or logarithm (Ln) of the counts. Assembly is followed by foundry where two identical machines were measured for three properties each. There were no (None) transformations or groupings to these data prior to analysis. The heat treatment studies were done with sub-grouped data. Two had subgroup sizes of seven (SG7) and one with size three (SG3). Machining dimensions, generally linear distances from a reference surface or location followed in the listing. The same dimensional distance was generated on the machine but the part geometry changed so the studies are listed as M4.1 through M4.5 to distinguish the studies from one another. Machine dimension M5 was studied for an unusually long interval with 1011 total observations. The measure of fit in the MM1 study involved mounting two mating parts on a machine and spinning the parts while moving the position of one part closer to the back of the other. A good fit was judged when the vibration of the machine was minimal. This value was then recorded and marked on the parts for later use by the product assemblers. As an amusement a study was conducted on the reference data set supplied by the Automotive Industry Action Group, AIAG, for capability study analysis. This was done to see if the data from an external process appeared any differently than a data set gathered in this study. The conclusion was that it fit nicely with the other data in this study. However, the study on the reference data was not included in analysis reported in this paper. Shaved gears were studied for dimensions on different parts and different dimensions on the different parts. Study S5 was continued for 4743 observations to judge whether there would be more terms needed in the model as a result of greater opportunity for special causes to exhibit themselves. A few shaved gear studies required transformations of logarithm and grouping to normalize the values for the study.

A form of gauge analysis was performed on the test machine, TM, studies. A sample of five of the more than 100 values taken on each major component examined by the machine were studied. These were key characteristics of the component. In this study, the same test machine with the same component was studied over an extended period of time. The cycle time for gathering the data was approximately twenty minutes. Each shift, approximately two per day, would connect the same component to the test stand. The belief was that the differences in the measured values would be due to the behavior of the test stand and the connection to the test machine as well as instrumentation drift. Each time the component was connected, the test machine went through a calibration routine. Following the calibration, the machine commanded pressures and flows that would result in desired behavior of the component. Energy, time, and pressures were gathered and recorded. Those recordings were then used in this study. A total of 106 repeated measures were made for each of the five key characteristics.

Grinding operations are often subject to lobing due to the turning of the wheel while removing material from the part surface. For this reason, these operations often have multiple measurements made on a part feature with data being recorded in sub-group format. Turning operations provided a variety of options for this study. For study TU3.1 through 5.3 three dimensions were measured three different part numbers, same machine and same operator. In studies TU6 through TU10 the same machine, operators and part geometry were used and the dimensions were the same. Only one measurement was made per part, but several dimensions were recorded. Each dimension became a separate study for this investigation.

Beginning with TU19 studies were done by stratifying the data by operator. The reason for this study developed while performing the analysis on the turned dimension. During construction of the normal probability plot, the data pattern appeared very unusual with two cross over points on the graph. Upon further investigation, the researcher discovered that the information stream was not just one stream but the confluence of three streams, one from each operator. By stratifying by operator number, the data were found to be three normal distributions with slightly different means, but greatly different standard deviations. The differences in standard deviation were believed to be due to the operating philosophy of the operators. The operator with the largest standard deviation, unknown to the operator, operated the machine without adjustment in order to get the maximum number of pieces before adjusting the process. Operator two believed in holding as close to the mean as possible by adjusting frequently, almost as often as a difference could be discerned with either measurement or visual examination. Operator three was between the other two, adjusting more often than operator one but not as often as operator two. The data reflected their philosophies because the probability plots were consistently ordered by standard deviation by operator.

In studies TU24.0 through TU25.3 the data required further stratification. A sequential plot of the data showed changes in the process regime as Bisgaard and Kulahci (2007a) refer to changes in the behavior of a process from one time period to another. In TU24.1 and TU24.2 only one regime change was noticed. In TU25.1.1 through TU25.1.3 two regime changes were noted and the analysis was divided by sequence number of the data.

Important measures for management are often gathered from the customer or customer experience is recorded through a warranty or product return policy. For the business in this study product is warranted for a specific use period. Expenses made to correct problems within this period are recorded and management analyzes these data to discover trends or pockets of unusual behavior. This process differs from a manufacturing process because there are few responses that can be taken ahead of time to prevent the occurrence. In a machining operation an examination of trends and previous product leads to an adjustment before the product is out of specification. In a warranty environment the trend of analysis lasts through months and years with few known inputs that need to be "adjusted" before the next month's report. The product is produced one month, waits a period of months before being put into service, then has to accumulate some wear before the defect becomes evident. This large lag would be reflected in the time series of the data. One would expect higher order coefficients in the ARIMA models and greater smoothing coefficients for the EWMA since there are few measurements but large time between measurements.

The last studies listed in Table A1 are for internal assembly efficiency called Yield. The value is the number of units produced divided by the number scheduled. This gives management an idea of how the system is performing. When fewer than expected are produced an investigation is launched. Seldom were more than expected produced because the parts for the assembly would generally not be available.

Time Series Analysis

Table A2 can be thought of as an extension of columns in Table A1. The studies were listed by code from Table A1. The ARIMA, autoregressive integrated moving average, model coefficients were listed in the format ARIMA(p, d, q)(P, D, Q)S. The population parameters associated with these values are generally listed as ARIMA(ϕ, ∇, θ)(Φ, ∇, Θ)S so these symbols are shown in the headings as well. In all cases, only sample values were available to estimate the population parameters. For that reason, the table contains only ARIMA(p, d, q)(P, D, Q)S values. As one might expect, the parameter values were specific to each analysis. No general value could describe any one process. The intended use of Table A2 is to permit the reader to view the variety of the values and compare processes for similarities and differences. Also, if another study is contemplated by the reader, the values may serve as a guide for reasonableness of results.

None of the studies were higher order than order two. This was stated by Box, Jenkins, and Reinsel (1994), Montgomery, Jennings and Kulahci (2008) and Box and Luceño (1997) who indicated that most series were of first order and rarely above third order. Three studies had more than 1000 observations: M5 (1011), S5 (4743) and Y3 (1706). The order of these studies was 2, 2 and 1 respectively. In all, there were seven studies of order two: M4.1, M5, S2.5, S5, G2.2, TU10 and TU17.

Estimates of Sigma

Table A3 lists the value for the exponentially weighted moving average, EWMA, smoothing coefficient, four estimates of sigma and the mean shift in sigma units. The EWMA smoothing coefficient ranged from zero to 0.601 for W1 study. Generally the values were below 0.300. The larger the constant the greater the emphasis on current values. When a smoothing coefficient was zero, a CUSUM chart was used to verify the shift points in the process mean. If the smoothing coefficient had been equal to one a Shewhart control chart would have been used.

The estimates of sigma were done from four sources. The first one listed is s'a which is computed from the square root of the mean square error of the ARIMA model. The value sa is a close likeness to s'a because it is calculated by using the Shewhart range chart to calculate the average nearest neighbor range and divide by the d2 factor 1.128 using the residuals from the ARIMA model. The third estimate listed is s_{ES} which is the sigma from the exponential smoothing of the series. It is also found by using the d2 factor with the nearest neighbor range. The last estimate of sigma was calculated from the analysis of variance, ANOVA, of the shifted mean groups. It is the root mean square error of the ANOVA. There is generally good agreement among the sigma estimates. Where there were large differences it was primarily with either a poor measurement resolution such as count data which is in whole numbers or complex time series models where the EWMA is a poor approximation to the time series behavior of the process.

The last column gives the process mean shift in sigma units. This is the number that is at the heart of this investigation. This number is shown as M/s_w because it is calculated from the mean square error from the ANOVA. The value in the table is the absolute value of the difference of the largest sub-group mean and the overall process average divided by the root mean square error from the ANOVA to put it in units of sigma. The ratio was generally small, however, the MM1 process was unusually large due to the nature of the process of matching two parts and relying on machine vibration at its minimum to identify the proper fit. The M/s_w for this process was 5.241. We were looking for values less than 1.5 if the mean varied within the +/- 1.5 sigma limits specified in the Six Sigma™ philosophy.

Process Groupings

The last table with summary information is Table A4. This table was created to help with the analysis and grouping of influencing factors that the investigator could possibly contribute to the differences in mean shift magnitude, Barnards lambda or the EWMA smoothing constant. The table is created in order of the codes used for the studies listed in the other three tables discussed above. The description is repeated to assist the reader to remember what the code referred to. The ARIMA model is listed in ARIMA(p, d, q)(P, D, Q)S notation to help identify the order of the model and to assist with assessing its complexity. The ARIMA type is a textual description of the ARIMA model. If there is a non-zero value for d or D, the series was non-stationary. If P, D, Q and S were all zero, the ARIMA type was non-seasonal. If all the ARIMA values were zero, the series was said to be constant as Shewhart assumed in his model of constant mean with random disturbances and constant variance.

Following the identification of the ARIMA type, the process group was listed. This is the general grouping of the study and follows the alphabetic character in the code designation of the study. These designations were later used to obtain relationships in the results as well as to refer to the process as data were being collected.

The remaining parts of this chapter will focus on the results as related to the mean shift, M/s_w, the Barnard lambda, λ_B and the EWMA smoothing constant, λ_{EWMA}.

Shifts of the Mean in Sigma Units, M/s_w

A histogram of the mean shift sizes for the studies in this investigation is shown in Figure 24. The majority of the studies are shown to be below three which means that the process mean generally moves less than three sigma or +/- 1.5 sigma from its set point.

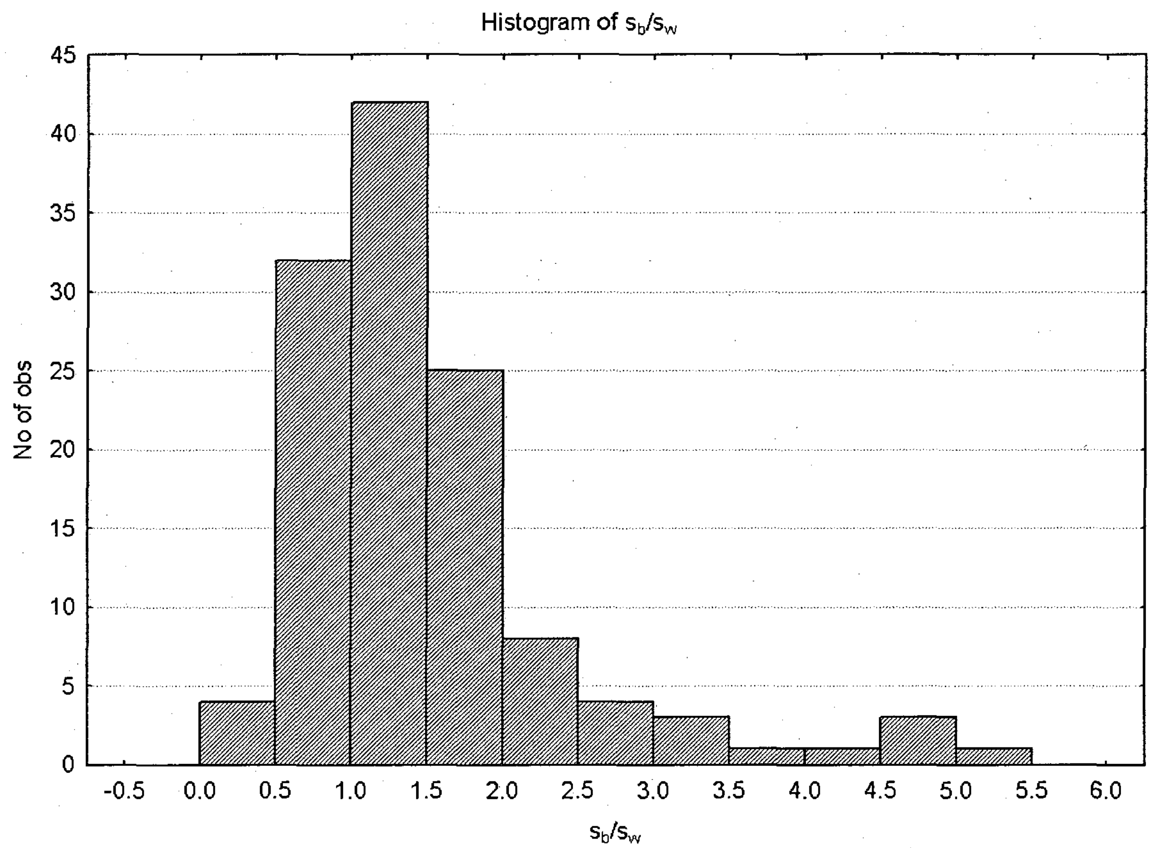

Figure 24. Distribution of mean shifts, M/s_w.

The values were found to group into three sizes: less than 1.5, between 1.5 and 2.25 and more than 2.25. This was found by using the normal probability plot of Figure 25. These were natural break points in the plot and were selected on the basis of this behavior.

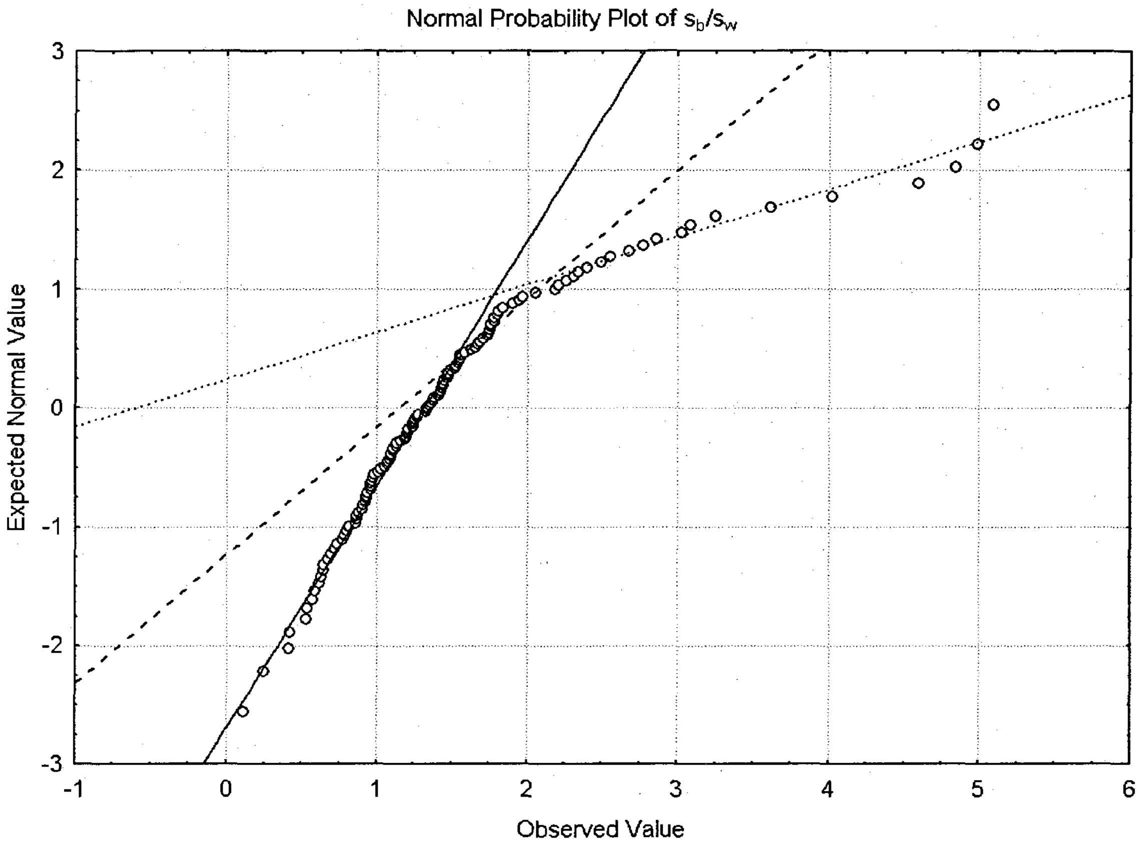

Figure 25. Normal probability plot of mean shift, M/s_w, showing three natural groups: less than 1.5, 1.5 to 2.25 and beyond 2.25

gives more information on the values of the graph above. Listed in the table are the group title indicating the amount of movement in the mean, the number of studies in the category and the 25th quantile, the median and the 75th quantile. The bottom row summarizes the investigation values.

Table 11. Breakdown Table of Descriptive Statistics for mean shift, M/s_w

M/s_w Groups	M/s_w Means	M/s_w N	M/s_w Q25	M/s_w Median	M/s_w Q75
Less than 1.5	1.024	78	0.811	1.060	1.271
1.51 to 2.25	1.757	28	1.608	1.748	1.825
Greater than 2.25	3.287	18	2.495	2.943	4.023
All Grps	1.518	124	0.959	1.336	1.754

The investigator verified that the categories were not overlapping by constructing Figure 26. This shows how much the medians of the three groups differ and also gives an indication of the spread of the values in the three groups.

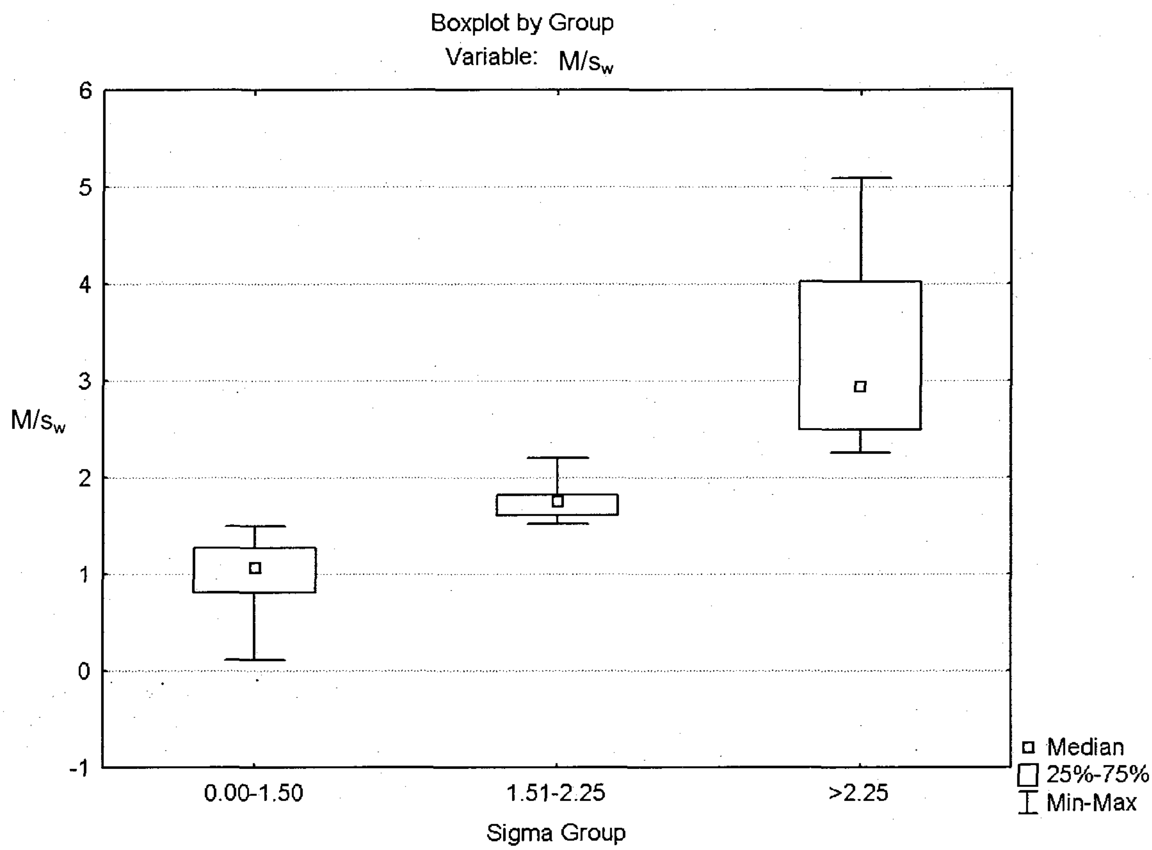

Figure 26. Mean shift, M/s_w, separation by group.

Table 12. Relationship between smallest and medium mean shifts, M/s_w. No relationship between medium and largest.

	Depend.: M/s_w Multiple Comparisons p values (2-tailed); M/s_w Independent (grouping) variable: Sigma Groups Kruskal-Wallis test: H (2, N= 124) =90.59923 p =0.000		
	Less than 1.5	1.51 to 2.25	Greater than 2.25
Less than 1.5		0.00	0.00
1.51 to 2.25	0.00		0.10
Greater than 2.25	0.00	0.10	

The values in Table 12 indicate that there is no relationship between the values of the less than 1.5 and the 1.51 to 2.25 mean shift or the greater than 2.25 groups. The 1.51 to 2.25 and greater than 2.25 groups are not independent. This would indicate that perhaps there are statistically two groups, below 1.5 and above 1.5.

The question now in most minds is whether the process type might influence the movement of the mean. The graphic of Figure 27 and the data of Table 13 were constructed to answer that question.

Table 13. Breakdown Table of Descriptive Statistics for M/s_w

Process Group	M/s_w Means	M/s_w N	M/s_w Q25	M/s_w Median	M/s_w Q75
Assembly	1.577	18	0.975	1.406	1.778
Foundry	1.726	6	1.381	1.609	2.338
Heat Treatment	1.314	4	1.170	1.350	1.458
Machining	2.140	3	1.095	2.552	2.773
Shaving	2.777	10	1.707	2.121	4.839
Test Machine	1.201	14	0.796	1.231	1.491
Grinding	1.252	5	1.244	1.334	1.371
Turning	1.195	57	0.864	1.110	1.522
Warranty	3.258	4	2.698	3.348	3.819
Yield	1.936	3	1.656	1.756	2.396
All Grps	1.518	124	0.959	1.336	1.754

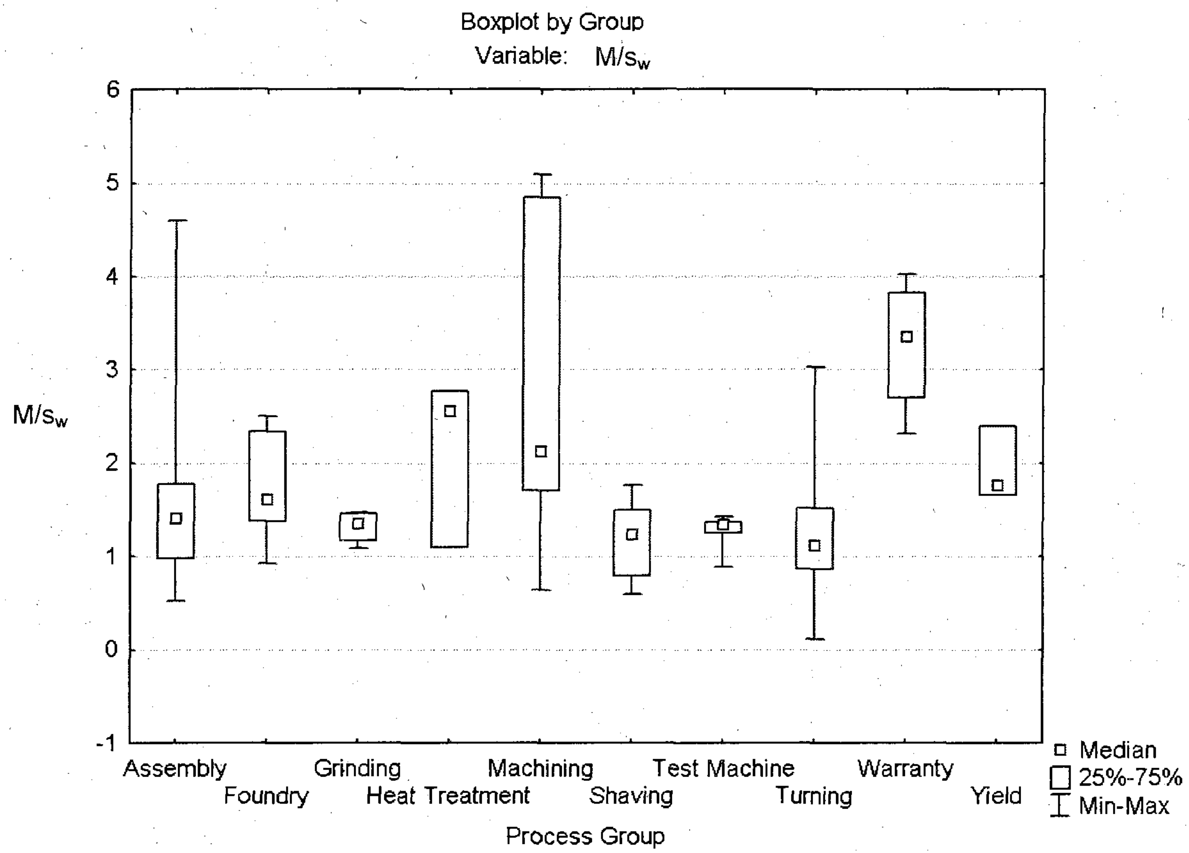

Figure 27. Boxplot of sigma of mean shift, M/s_w, for each process group.

Table 13 shows the average value for each process type. Also shown are the number and the lower, median and upper quartile values to give an indication of the spread in the summarized values. Figure 27 gives a graphic representation of that information.

A fair way to evaluate the difference among groups that are not necessarily expected to be normally distributed is to use a non-parametric method of Kruskal and Wallis that is similar to ANOVA but works with ranks rather than measurements and makes no assumptions about constant variance. The Kruskal-Wallis table is shown as Table 14. This table is the p-value for the comparison of each process to the others. The p-value of 0.0002 indicates that a significant difference exists for shifts in the mean between processes. Processes in this investigation had a behavior that lead to greater variability. A p-value below 0.05 is conventionally taken to be significant.

Table 14. P values for Kruskal-Wallis test showing that classification by process significantly relates to mean shift, M/s_w. No two processes compared to each other are different.

	Depend.:M/s_w Multiple Comparisons p values (2-tailed); M/s_w (ARIMA) Independent (grouping) variable: Process Group Kruskal-Wallis test: H (9, N= 124) =32.61482 p =.0002									
	Asmbly	Fdry	Grind	Heat Treat	Mach	Shaving	Test Machine	Turning	Warranty	Yield
Assembly		1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.49	1.00
Foundry	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Grinding	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00
Heat Treatment	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00
Machining	1.00	1.00	1.00	1.00		0.11	1.00	**0.01**	1.00	1.00
Shaving	1.00	1.00	1.00	1.00	0.11		1.00	1.00	0.07	1.00
Test Machine	1.00	1.00	1.00	1.00	1.00	1.00		1.00	0.55	1.00
Turning	1.00	1.00	1.00	1.00	**0.01**	1.00	1.00		**0.02**	1.00
Warranty	0.49	1.00	1.00	1.00	1.00	0.07	0.55	**0.02**		1.00
Yield	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Values less than 0.05 are shown in Table 14 in boldface type. The relationship between the column and row processes are significantly different if the p value falls below 0.05. Turning, machining and warranty are therefore significantly different in the amount of movement of the mean in sigma units. So process influences shifts in the mean and certain processes differ from one another in a significant manner.

The ARIMA type was next examined for explanation of shifts in the mean. Again a table of the values, Table 15, was created to compare. Again the average value for the mean shift is listed and the count, lower quartile, median and upper quartile are shown. A visual representation that gives a fairer visualization of the spread about these values is in Figure 28. Here we note that the non-stationary ARIMA types show much more spread than the constant or stationary series. In other words, we are less certain about the size of the mean shifts once we realize the ARIMA type is non-stationary.

Table 15. Breakdown Table of Descriptive Statistics of mean shift, M/s_w, by ARIMA Type.

ARIMA Type	M/s_w Means	M/s_w N	M/s_w Q25	M/s_w Median	M/s_w Q75
Non-stationary, Non-seasonal	1.650	47	1.075	1.430	2.207
Non-stationary, Seasonal	1.815	35	1.254	1.556	1.813
Non-stationary AR(2)	1.933	5	1.751	1.948	1.969
Non-stationary D(2)	1.745	1	1.745	1.745	1.745
Constant (Shewhart)	0.974	25	0.788	0.907	1.166
Stationary, Non-seasonal	1.208	6	1.086	1.217	1.251
Stationary, Seasonal	0.828	5	0.537	0.731	0.938
All Grps	1.518	124	0.959	1.336	1.754

One could argue that the mean shift is close to 1.50 (actually 1.518 in this investigation) and so the Six Sigma prescription of 1.5 holds well. As stated in our introduction, these studies were not randomly selected and may be biased toward the interest of the management to improve as well as control processes. Since this is an early study into these behaviors one must proceed with caution and not generalize to too broad a conclusion concerning the mean shift and the process behavior.

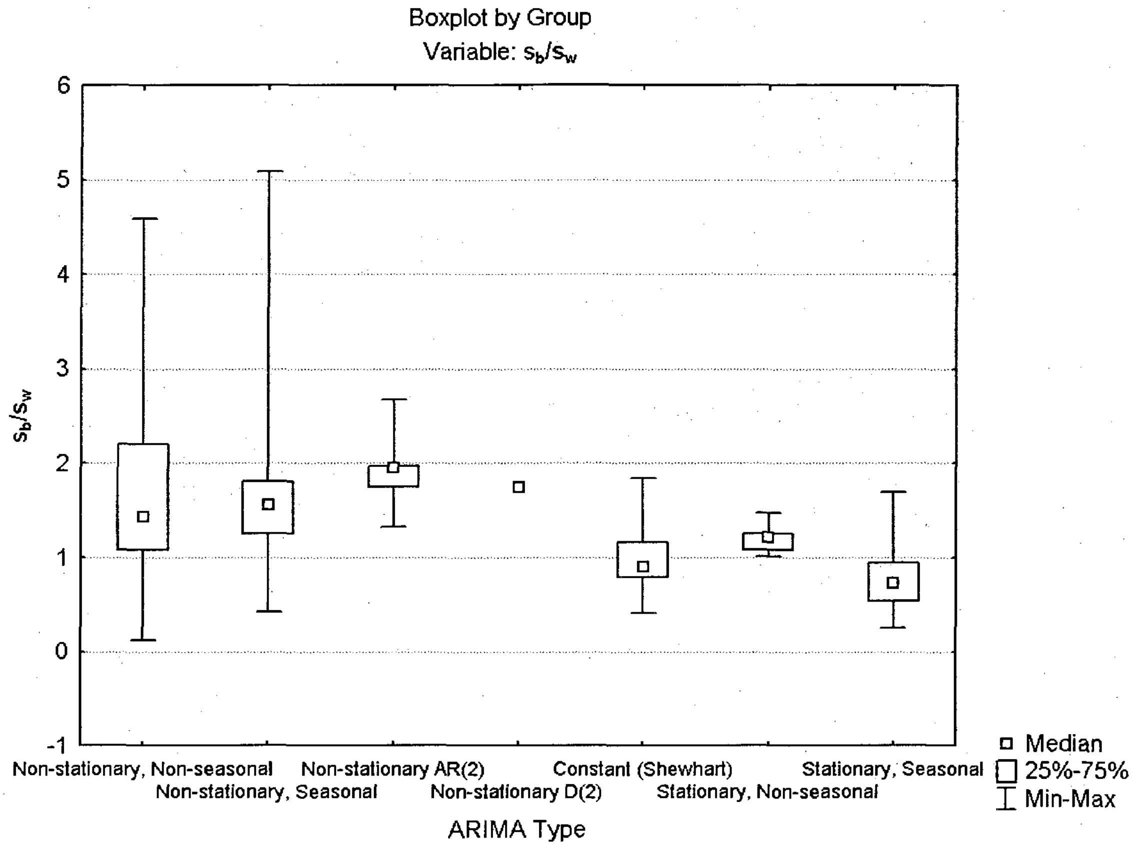

Figure 28. Size of mean shift by ARIMA type classification. Non-stationary series exhibit greater movement in the mean.

In Table 16 an attempt was made to employ the Kruskal-Wallis non-parametric ANOVA to test whether ARIMA type influenced the mean shift. The p-value is 0.0000 which is well below the conventional 0.05 to declare the effect significant. When looking at the table a little closer, we notice that the constant (Shewhart) differs significantly from the three non-stationary classifications. No other significant results were found.

Table 16. P values for the relationship between the mean shift size and the ARIMA type. The Constant (Shewhart) mean shifts differ significantly from the Non-stationary series.

Multiple Comparisons p values (2-tailed); M/s_w
Independent (grouping) variable: ARIMA Type
Kruskal-Wallis test: H (6, N= 126) =43.53060 p =.0000

Depend. M/s_w	Non-stationary, Non-seasonal	Non-stationary, Seasonal	Non-stationary AR(2)	Non-stationary D(2)	Constant (Shewhart)	Stationary, Non-seasonal	Stationary, Seasonal
Non-stationary, Non-seasonal		1.00	1.00	1.00	0.00	1.00	0.38
Non-stationary, Seasonal	1.00		1.00	1.00	0.00	1.00	0.14
Non-stationary AR(2)	1.00	1.00		1.00	**0.01**	1.00	0.08
Non-stationary D(2)	1.00	1.00	1.00		1.00	1.00	1.00
Constant (Shewhart)	**0.00**	**0.00**	**0.01**	1.00		1.00	1.00
Stationary, Non-seasonal	1.00	1.00	1.00	1.00	1.00		1.00
Stationary, Seasonal	0.38	0.14	0.08	1.00	1.00	1.00	

Duration of Mean Shifts, Barnard's Lambda, λ_B

Similar results to the mean shift, M/s_w, will now be presented for the duration of the shift in the process mean. This was first proposed by Barnard (1959) so the investigator refers to Barnard's lambda although he never created that title for this behavior.

A histogram of the mean shift sizes for the studies in this investigation is shown in Figure 29. The majority of the studies are shown to be below twenty which means that the process mean shift lasts typically less than 20 sampling intervals. The shape of the distribution also cautions the reader that these data are very right skewed and the average is not an entirely trustworthy characterization of these results.

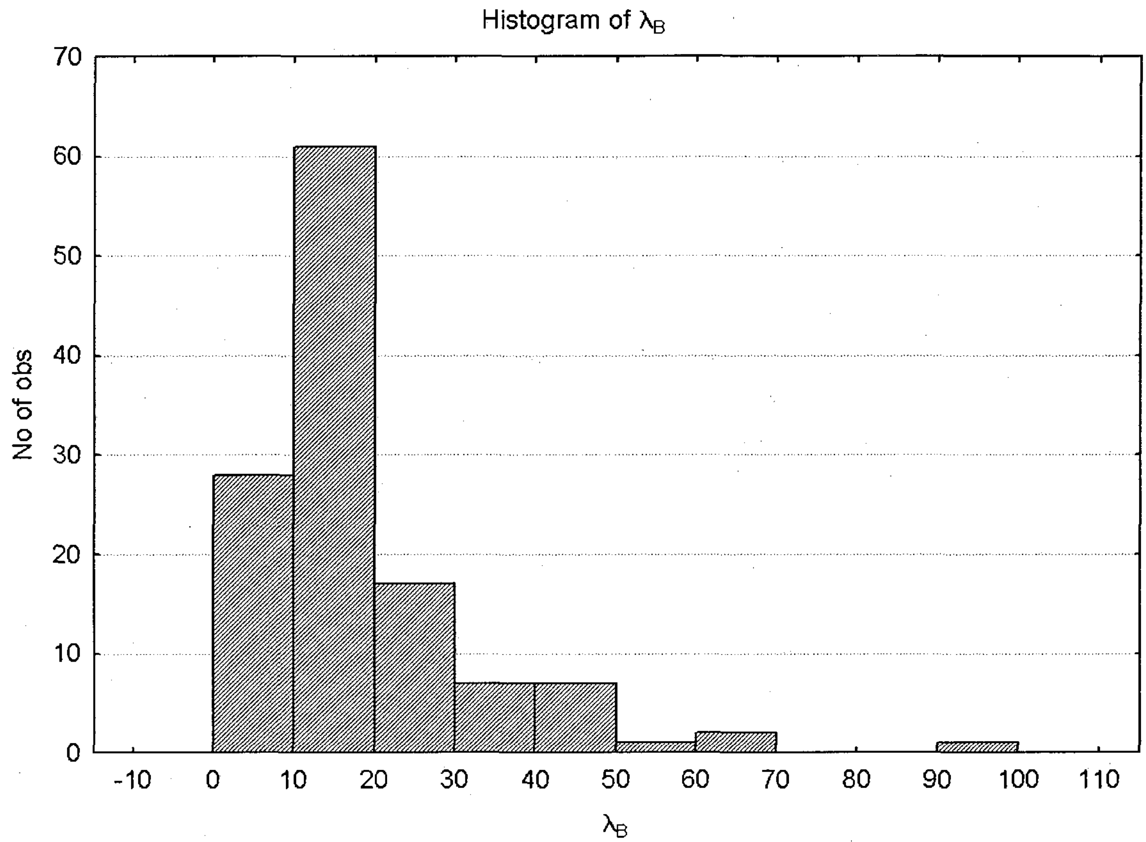

Figure 29. Distribution of Barnard's lambda, λ_B

The values were found to group into three sizes: less than 11, between 11 and 23 and more than 23. This was found by using the normal probability plot of Figure 30. These were natural break points in the plot and were selected on the basis of this behavior.

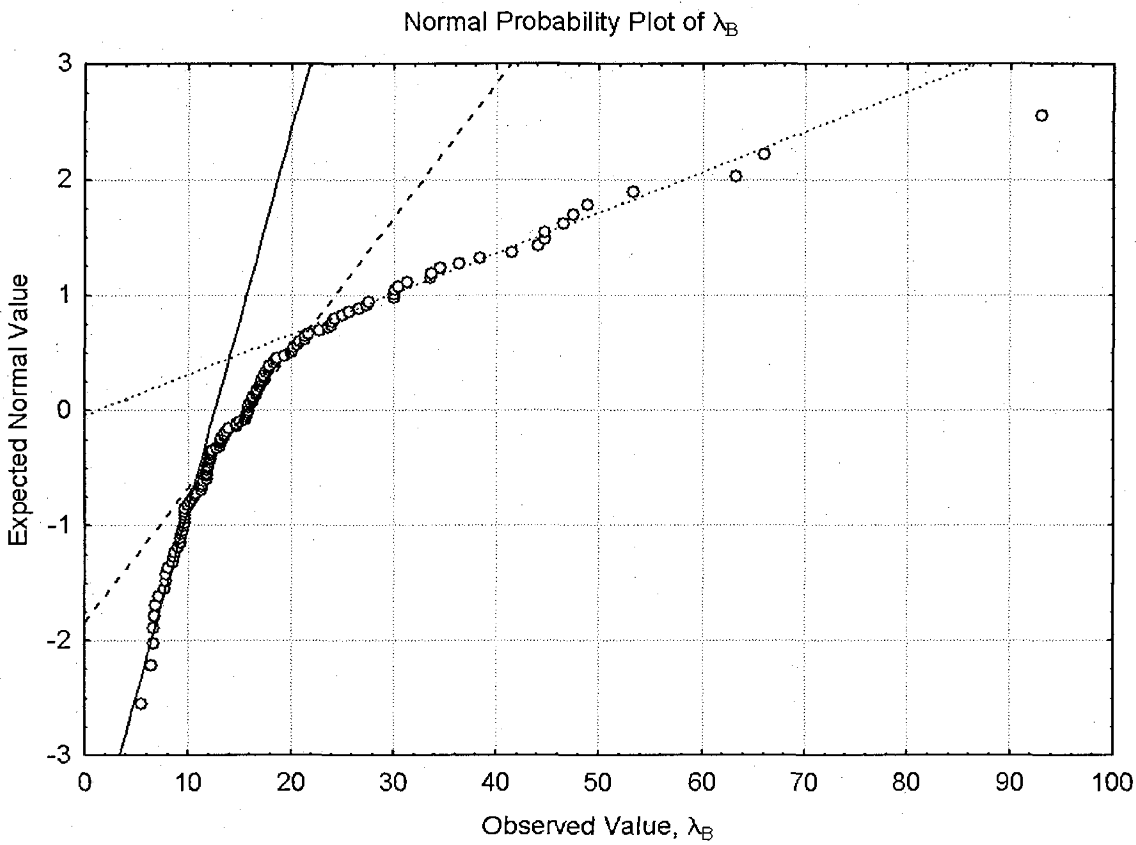

Figure 30. Three groups for Barnard's lambda. One less than 11, the second from 11 to 23 with the last above 23 intervals.

Table 17 gives more information on the values of the graph above. Listed in the table are the group title indicating the amount of movement in the mean, the number of studies in the category and the 25th quantile, the median and the 75th quantile. The bottom row summarizes the investigation values.

Table 17. Breakdown Table of Descriptive Statistics for Barnard's lambda, λ_B

λ_B Group	λ_B Means	λ_B N	λ_B Q25	λ_B Median	λ_B Q75
Less than 11	8.6	30	7.8	9.1	9.6
11 to 23	15.6	64	12.5	15.8	17.7
Greater than 23	38.3	30	27.3	33.6	44.7
All Grps	19.4	124	11.3	15.8	22.2

The investigator verified that the categories were not overlapping by constructing Figure 31. The values in Table 18 indicate that there is no relationship between the values of any of the classifications of Barnard's lambda. These are three distinct categories and there is not overlap.

Table 18. Kruskal-Wallis test for relationship between Groups for λ_B. Table shows that each group is independent of the other.

	Multiple Comparisons p values (2-tailed); λ_B Independent (grouping) variable: λ_B Group Kruskal-Wallis test: H (2, N= 124) =102.6152 p =0.000		
Depend.:λ_B	Less than 11	11 to 23	Greater than 23
Less than 11		0.00	0.00
11 to 23	0.00		0.00
Greater than 23	0.00	0.00	

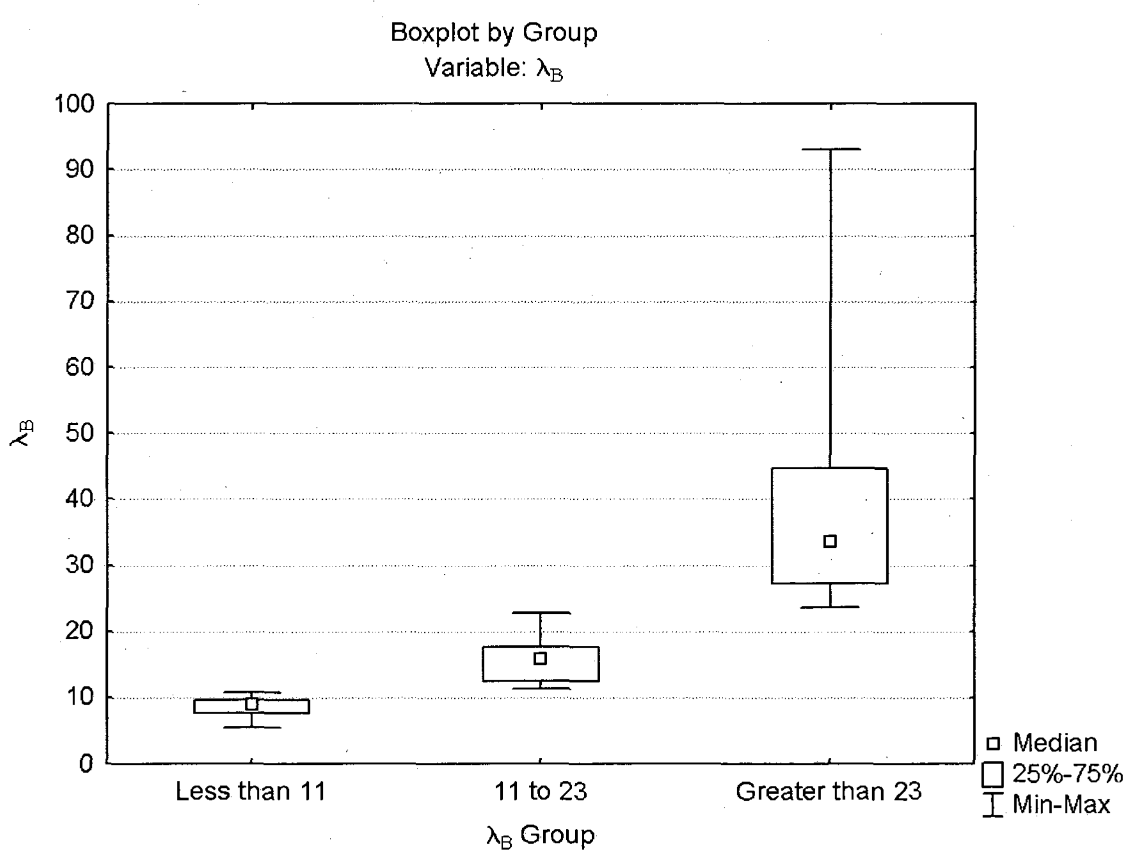

Figure 31. Separation of Barnard's lambda. This is the average number of sampling intervals for each mean shift of the process.

Figure 31 shows how much the medians of the three groups differ and also gives an indication of the spread of the values in the three groups.

Does the process influence the duration of the mean shift? To answer this question we first summarized the Barnard lambda values for each process group. This was done in Table 19. It gives the means for each process type, the number in the group and the first, median and fourth quartile values. The table gives the impression that the duration of the mean shifts is fairly consistent about an overall median value of 16.

Table 19. Breakdown Table of Descriptive Statistics for λ_B

Process Group	λ_B Means	λ_B N	λ_B Q25	λ_B Median	λ_B Q75
Assembly	25	18	16	18	34
Foundry	17	6	15	17	17
Heat Treatment	18	3	14	16	24
Machining	21	10	11	17	21
Shaving	19	14	9	12	19
Test Machine	11	5	10	12	12
Grinding	26	4	19	22	34
Turning	18	57	10	15	21
Warranty	8	4	7	8	9
Yield	23	3	15	25	30
All Grps	19	124	11	16	22

The graphic of Figure 32 displays these same data but also gives the reader an appreciation for the spread of the values around the median. Now one sees that the variation for the shaving operation is much greater than for the warranty process. The interval length for these two processes is not equal. The shaving process interval is measured in hours whereas the warranty interval is months.

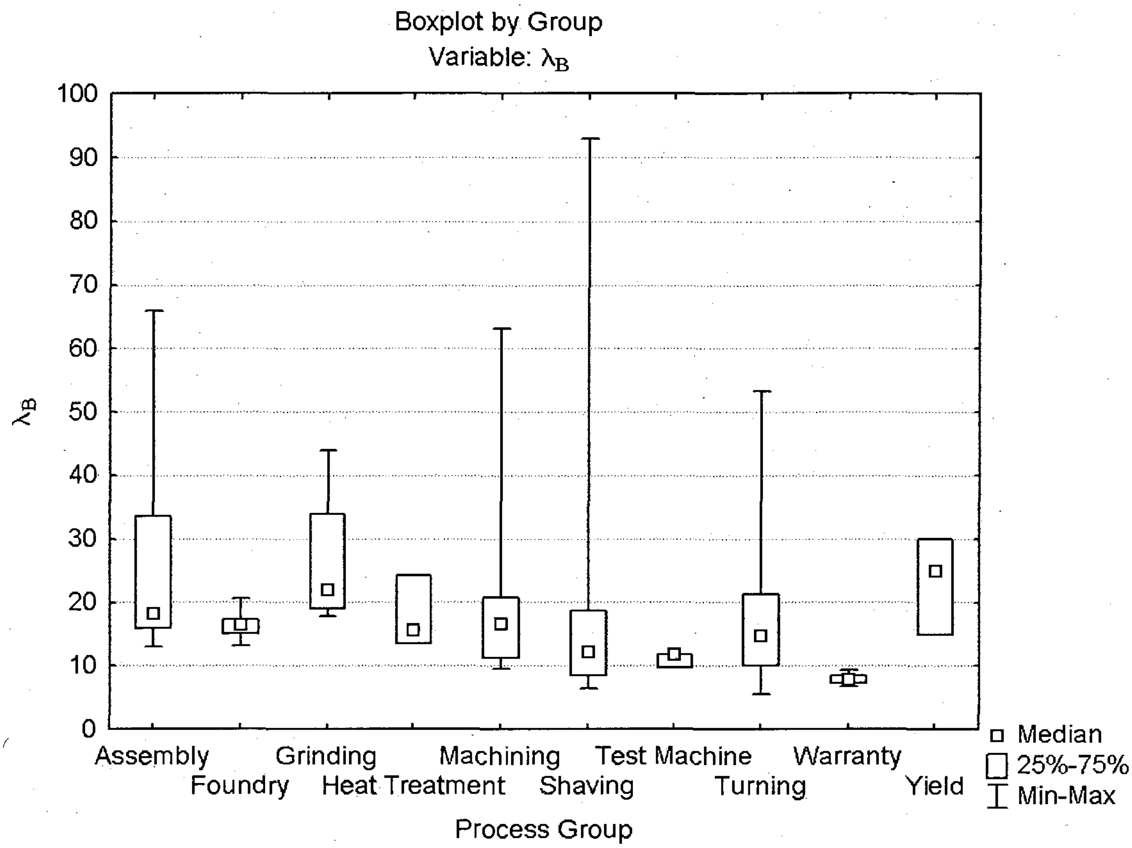

Figure 32. Boxplot of Barnard's average duration of mean shift for each process group.

The plots of Figure 33 show that the median value varies by assembly group. The long whiskers of assembly, machining, shaving and turning show that median can be as much as an order of magnitude within the classification. For this reason conclusions drawn on λ_B by process grouping must be done cautiously. The investigator should not expect a specific value for these processes but should understand process behavior before concluding a value for Barnard's lambda.

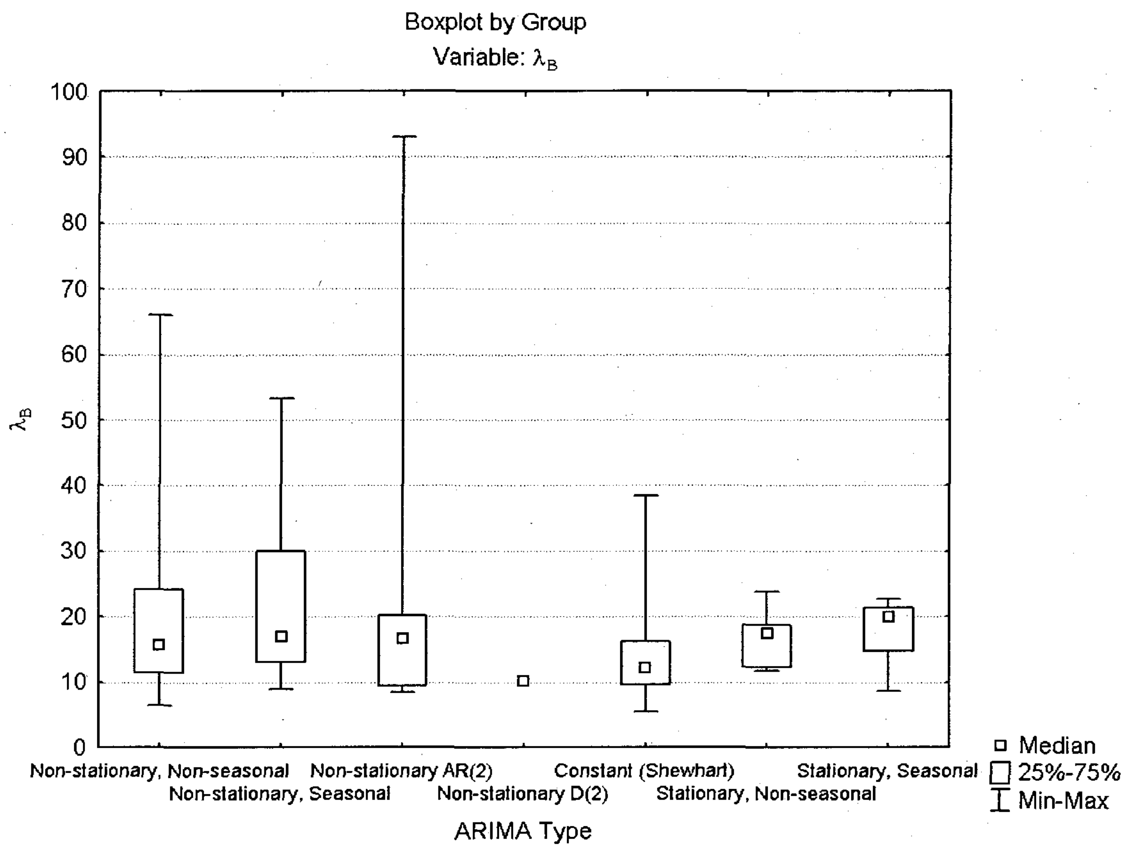

Figure 33. Boxplot of Barnard's average duration of mean shift for each process group.

The Kruskal-Wallis table is shown as Table 20. This table is the p-value for the comparison of each process to the others. The p-value of 0.0018 indicates that a significant difference exists for duration in mean shift between processes. While the process group does significantly influence the duration of the mean shift, the investigator also wanted to know if there were differences between processes. Table 20 indicates that the warranty process differs from assembly and grinding but no other process groups were different from any others. A median value could be a fairly good characterization of the duration of the mean shift for most processes except assembly, grinding and warranty. This information would lead a quality engineer to plan the frequency of sampling to capture the expected frequency of mean shifts and to space the sampling far enough apart to decrease the autoregressive behavior of the data taken at narrow sampling intervals.

Table 20. P values for Kruskal-Wallis test showing that classification by process significantly relates to Barnard's lambda, λ_B. Warranty differs from assembly and grinding. Others do not differ.

Multiple Comparisons p values (2-tailed); λ_B
Independent (grouping) variable: Process Group
Kruskal-Wallis test: H (9, N= 124) =26.37593 p =.0018

	Asmbly	Fdry	Grind	Heat Treat	Mach	Shaving	Test Machine	Turning	Warranty	Yield
Assembly		1.00	1.00	1.00	1.00	0.35	0.16	0.42	0.01	1.00
Foundry	1.00		1.00	1.00	1.00	1.00	1.00	1.00	0.50	1.00
Grinding	1.00	1.00		1.00	1.00	1.00	0.38	1.00	0.04	1.00
Heat Treatment	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00
Machining	1.00	1.00	1.00	1.00		1.00	1.00	1.00	0.27	1.00
Shaving	0.35	1.00	1.00	1.00	1.00		1.00	1.00	1.00	1.00
Test Machine	0.16	1.00	0.38	1.00	1.00	1.00		1.00	1.00	1.00
Turning	0.42	1.00	1.00	1.00	1.00	1.00	1.00		0.36	1.00
Warranty	0.01	0.50	0.04	1.00	0.27	1.00	1.00	0.36		0.22
Yield	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.22	

There could be a relationship between the number of shifts in the mean, the inverse of the duration of the mean shift, and the size of the movement in the mean. To better understand if there is a relationship, Kruskal-Wallis analysis was performed on the M/s_w and λ_B values. The results are shown in Table 21. If there were a relationship between the size of the mean shift and the duration of that shift, then one would expect that the Kruskal-Wallis non-parametric analysis to be significant. Table 21 summarizes the Barnard lambda values by mean shift groups, M/s_w Groups. Generally we see that the smaller mean shift group is also the smaller λ_B mean or median.

Table 21. Breakdown Table of Descriptive Statistics for Barnard's lambda, λ_B

M/s_w Groups	λ_B Means	λ_B N	λ_B Q25	λ_B Median	λ_B Q75
0.00-1.50	19.67	78	11.78	16.46	21.61
1.51-2.25	21.71	28	10.95	15.99	26.14
>2.25	14.29	18	7.85	13.37	16.13
All Grps	19.35	124	11.26	15.84	21.47

The Kruskal-Wallis results are in Table 22. While the summary table led us to believe that there may be a relationship between the size of the mean shift and the duration, the differences were not statistically significant. The p-value was 0.0707, very close to 0.05.

Table 22. No relationship between Barnard's lambda, λ_B for mean shift groups.

M/s_w Groups	Multiple Comparisons p values (2-tailed); λ_B Independent (grouping) variable: S/M/L Groups Kruskal-Wallis test: H (2, N= 124) =5.297900 p =.0707		
	Less than 3.0	3.0 to 4.5	Greater than 4.5
0.00-1.50		1.00	0.07
1.51-2.25	1.00		0.18
>2.25	0.07	0.18	

The box plots by sigma group in Figure 34 also show the variability about the median value for each group. The groups are less than 1.5, 1.51 to 2.25 and greater than 2.25. Their plots appear almost identical.

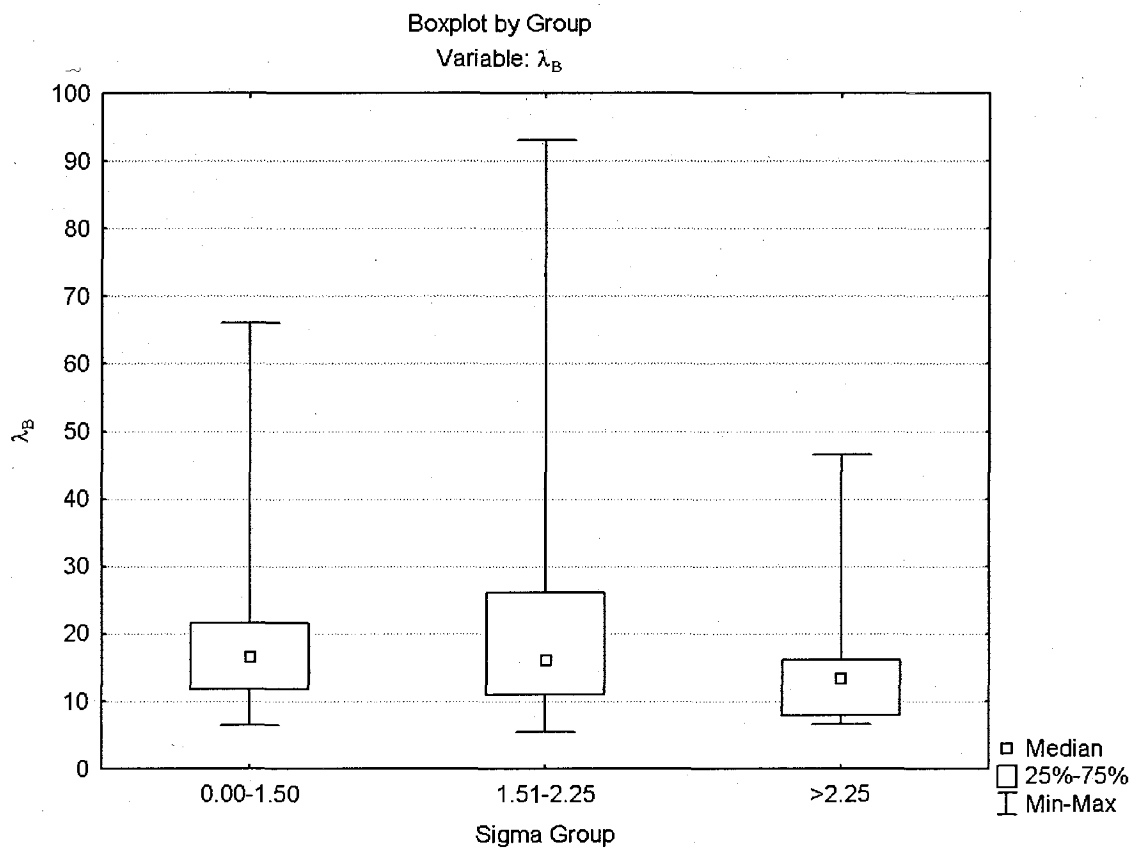

Figure 34. No change in Barnard's lambda, duration of a mean shift, with level of mean shift. The two measures are independent.

We next examine the behavior of the duration of the mean shift, λ_B and the ARIMA type. The values in Table 23 are the results of the Kruskal-Wallis analysis using the ARIMA type as the grouping variable. We quickly see that the p-value for the test is 0.1280 meaning that there is no statistically valid relationship between the various time series models and the duration of the shift in the mean of the processes in this study.

Table 23. P values for the relationship between Barnard's lambda and the ARIMA type. No significant differences were found.

	Multiple Comparisons p values (2-tailed); λ_B Independent (grouping) variable: ARIMA Type Kruskal-Wallis test: H (6, N= 124) =9.921321 p =.1280						
Depend.: λ_B	Non-stationary, Non-seasonal	Non-stationary, Seasonal	Non-stationary AR(2)	Non-stationary D(2)	Constant (Shewhart)	Stationary, Non-seasonal	Stationary, Seasonal
Non-stationary, Non-seasonal		1.00	1.00	1.00	1.00	1.00	1.00
Non-stationary, Seasonal	1.00		1.00	1.00	0.07	1.00	1.00
Non-stationary AR(2)	1.00	1.00		1.00	1.00	1.00	1.00
Non-stationary D(2)	1.00	1.00	1.00		1.00	1.00	1.00
Constant (Shewhart)	1.00	0.07	1.00	1.00		1.00	1.00
Stationary, Non-seasonal	1.00	1.00	1.00	1.00	1.00		1.00
Stationary, Seasonal	1.00	1.00	1.00	1.00	1.00	1.00	

Exponentially Weighted Moving Average Smoothing Coefficient

In this study it was possible to identify the EWMA smoothing coefficient value because the minimum mean square error sigma was desired in order to compare to other estimates of sigma for the inherent process variability. The λ_{EWMA} values are shown in the form of a histogram in Figure 35. For the studies in this investigation many values were zero. These were values typical of the constant mean model of the Shewhart series.

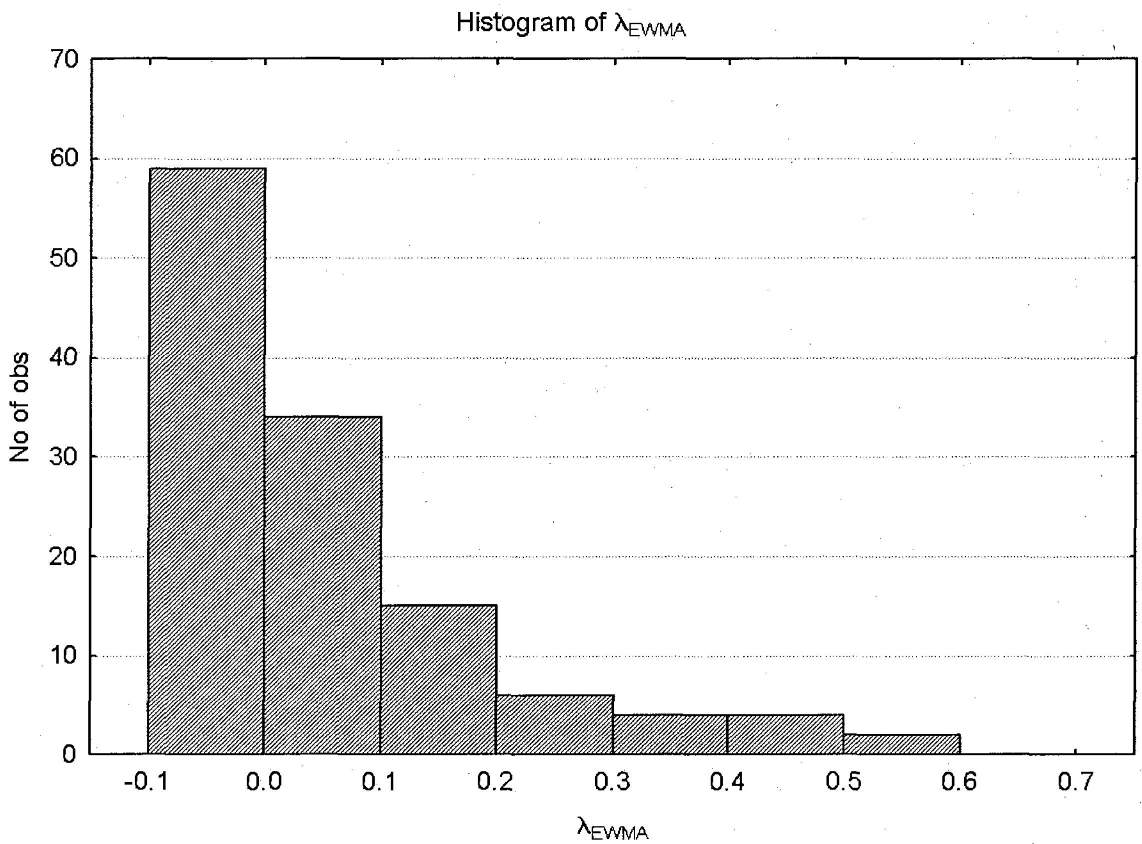

Figure 35. Histogram of exponential smoothing constant for EWMA, λ_{EWMA}

The EWMA smoothing coefficients showed similar behavior to the other measures of mean shifts, M/s_w and λ_B with three natural groups suggested by a normal probability plot. The plot is shown as Figure 36.

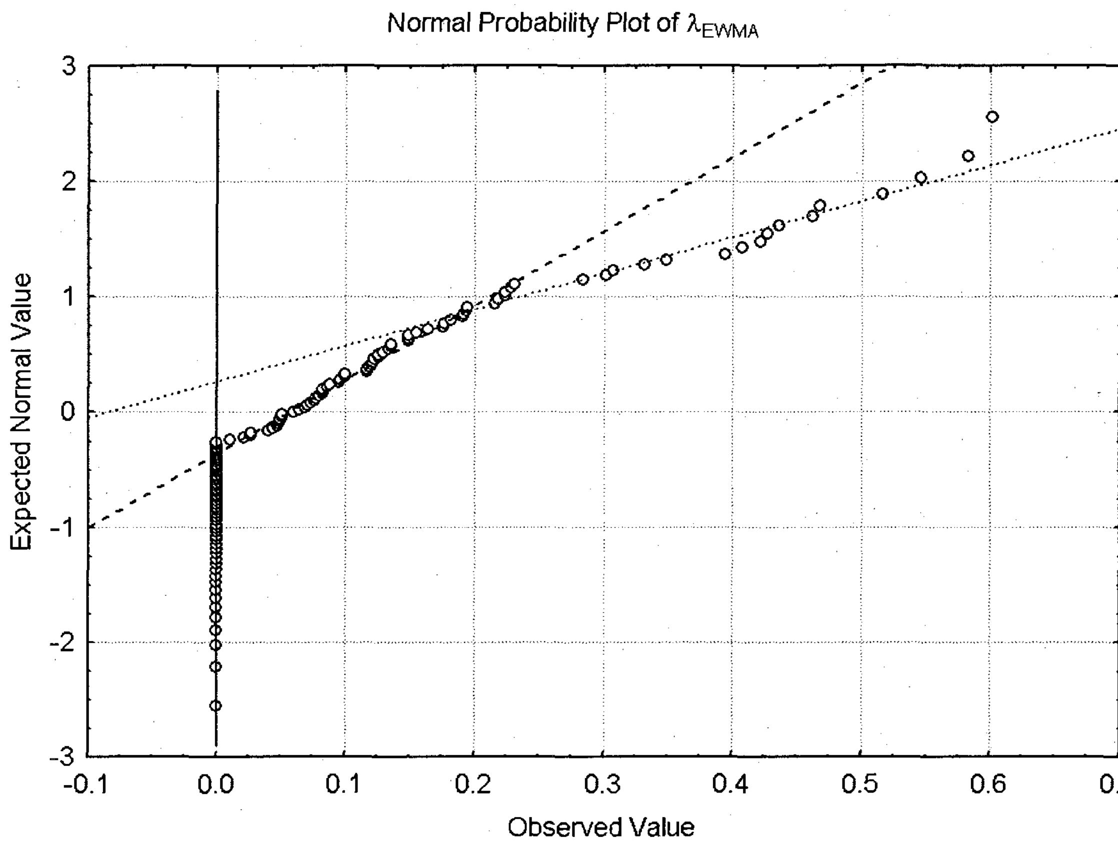

Figure 36. Natural breaks for EWMA coefficient. Break points are zero, less than 0.190 and above 0.190.

Box and Paniagua-Quiñones (2007) and Hunter (2007a) suggest a smoothing constant of 0.2 as a good value if no other information is available on the process. This would be reasonable in this investigation because outside the zero values a good break between the middle and higher values is 0.190 which is very close to 0.20.

A boxplot of these groupings is shown in Figure 37. We note that the groups appear to have good separation indicating that they may be independent. Also, since zero is a separate group in this study no variation is shown in the plot for that group.

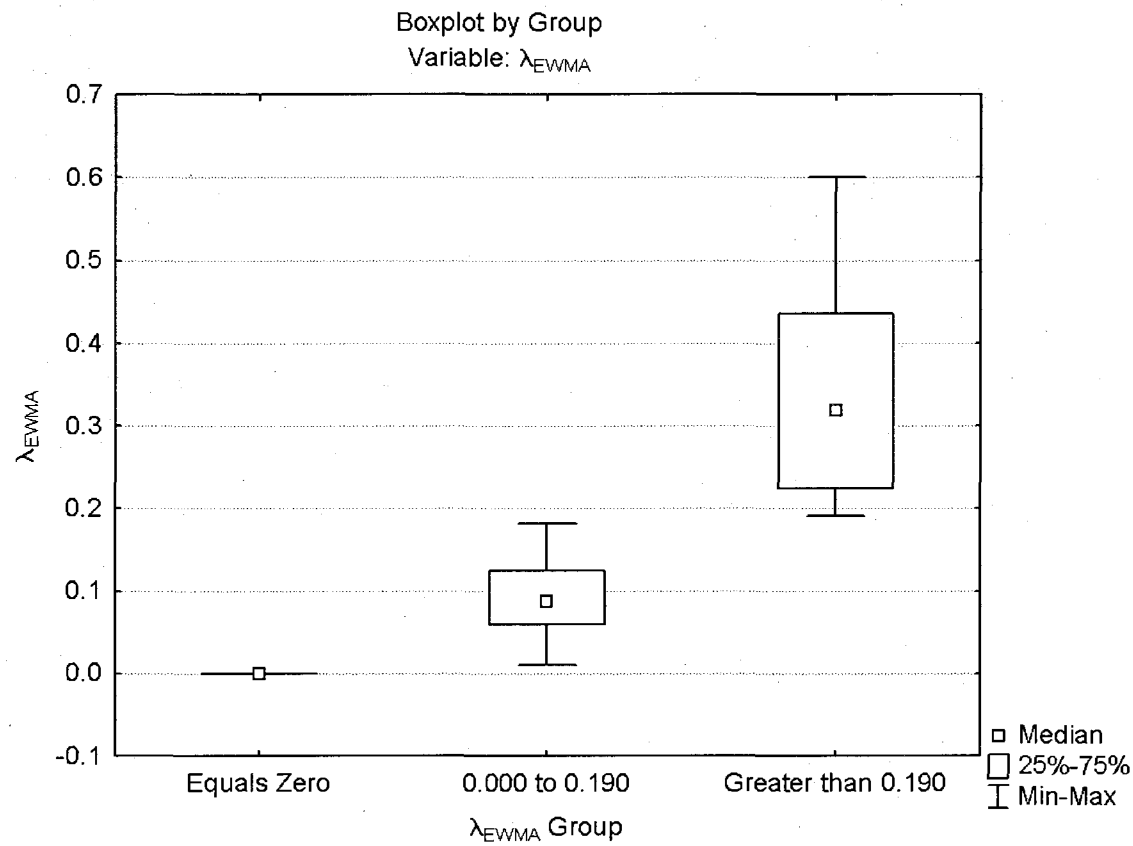

Figure 37. Natural groups for EWMA smoothing coefficient, λ_{EWMA}.

In Table 24 we see a breakdown of the values for the EWMA coefficient. There are equal numbers of zero and 0.000 to 0.190 suggesting that the 0.110 value may be a good estimate for the EWMA smoothing coefficient for the firm whose data are provided in this investigation. As we have done for the other response parameters in this investigation, the values were stratified by

Table 24. Breakdown Table of Descriptive Statistics for EWMA coefficient, λ_{EWMA} by size group.

λ_{EWMA} Group	λ_{EWMA} Means	λ_{EWMA} N	λ_{EWMA} Q25	λ_{EWMA} Median	λ_{EWMA} Q75
Equals Zero	0.000	49	0.000	0.000	0.000
0.000 to 0.190	0.095	49	0.060	0.088	0.125
Greater than 0.190	0.344	26	0.224	0.319	0.436
All Grps	0.110	124	0.000	0.062	0.152

process group and examined for differences between groups. Table 25 shows the breakdown listing the average, group size, and first, median and third quartile values.

Table 25. Breakdown Table of Descriptive Statistics for λ_{EWMA}

Process Group	λ_{EWMA} Means	λ_{EWMA} N	λ_{EWMA} Q25	λ_{EWMA} Median	λ_{EWMA} Q75
Assembly	0.088	18	0.000	0.068	0.120
Foundry	0.120	6	0.051	0.130	0.182
Heat Treatment	0.117	3	0.000	0.133	0.218
Machining	0.255	10	0.051	0.294	0.422
Shaving	0.084	14	0.000	0.022	0.192
Test Machine	0.027	5	0.000	0.000	0.000
Grinding	0.010	4	0.000	0.000	0.020
Turning	0.079	57	0.000	0.027	0.116
Warranty	0.496	4	0.400	0.549	0.591
Yield	0.139	3	0.077	0.149	0.191
All Grps	0.110	124	0.000	0.062	0.155

Figure 38 gives a graphic representation to enable the reader to examine the uncertainty in the median values reported. Machining appears to have a very long range of values, Turning shows a long tail and Warranty shows a high median value with a moderate tail toward smaller values. As shown in Table 26 warranty differs from grinding, test machine and turning. Others do not differ.

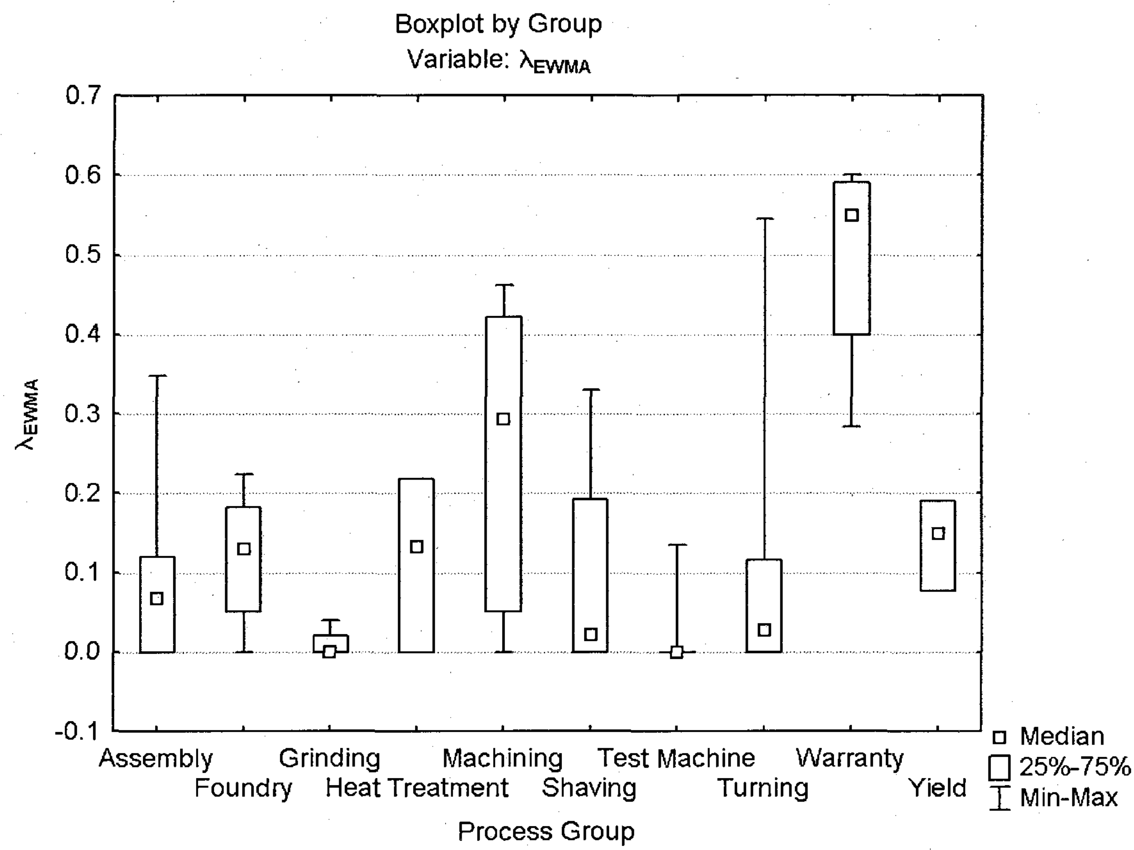

Figure 38. Boxplot of EWMA smoothing coefficient for each process group.

Table 26. P values for Kruskal-Wallis test showing that classification by process significantly relates to EWMA coefficient, λ_{EWMA}. Warranty differs from grinding, test machine and turning. Others do not differ.

Multiple Comparisons p values (2-tailed); λ_{EWMA}
Independent (grouping) variable: Process Group
Kruskal-Wallis test: H (9, N= 124) =28.37851 p =.0008

	Asmbly	Fdry	Grind	Heat Treat	Mach	Shav.	Test Machine	Turning	War-ranty	Yield
Assembly		1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.16	1.00
Foundry	1.00		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Grinding	1.00	1.00		1.00	0.22	1.00	1.00	1.00	**0.03**	1.00
Heat Treatment	1.00	1.00	1.00		1.00	1.00	1.00	1.00	1.00	1.00
Machining	1.00	1.00	0.22	1.00		0.84	0.27	0.16	1.00	1.00
Shaving	1.00	1.00	1.00	1.00	0.84		1.00	1.00	0.10	1.00
Test Machine	1.00	1.00	1.00	1.00	0.27	1.00		1.00	**0.03**	1.00
Turning	1.00	1.00	1.00	1.00	0.16	1.00	1.00		**0.03**	1.00
Warranty	0.16	1.00	**0.03**	1.00	1.00	0.10	**0.03**	**0.03**		1.00
Yield	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

The EWMA smoothing constant was evaluated relative to the size of the mean shift groupings discussed earlier.

Table 27 shows this comparison and Figure 39 gives a graphical representation. It appears that the EWMA coefficient and the size of the mean shift may be related to one another. To verify the separation of the groupings Table 28 was created. This table indicates that the λ_{EWMA} values are significantly different if the M/sw ratio is less than 1.50 or 1.51 to 2.25. The Greater than 2.25 M/sw group had λ_{EWMA} values that were also significant for the Less than 1.5 and also different than the 1.51 to 2.25 λ_{EWMA} values.

Table 27. Breakdown Table of Descriptive Statistics for EWMA constant, λ_{EWMA}

M/s_w Groups	λ_{EWMA} Means	λ_{EWMA} N	λ_{EWMA} Q25	λ_{EWMA} Median	λ_{EWMA} Q75
0.00-1.50	0.038	78	0.000	0.000	0.071
1.51-2.25	0.149	28	0.075	0.123	0.209
>2.25	0.349	18	0.191	0.371	0.467
All Grps	0.109	124	0.000	0.062	0.152

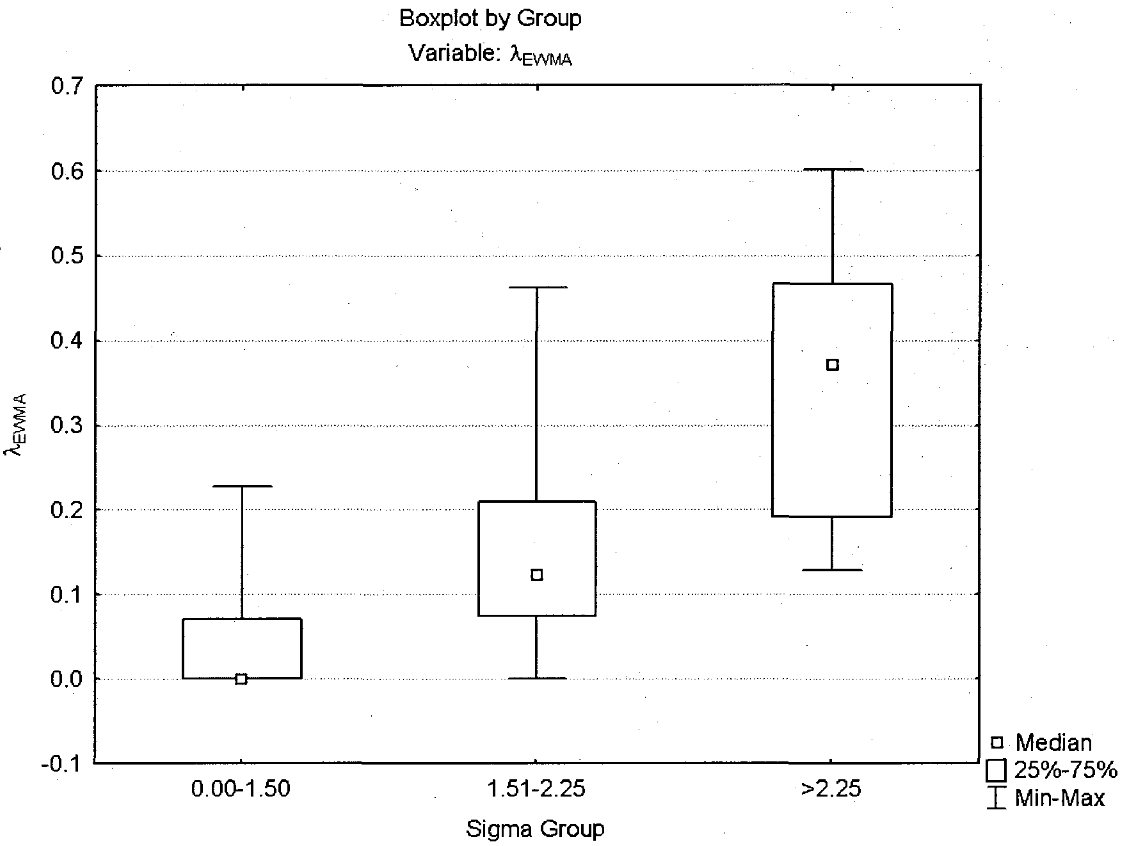

Figure 39. Change in EWMA smoothing coefficient, λ_{EWMA}, with change in size of the mean shift. Middle and large shifts in the mean show about the same level of smoothing coefficient. Small mean changes, Group 1, also appeared to have small smoothing coefficients.

Table 28. Comparison of EWMA smoothing constants by mean shift level. No relationship between medium and largest for λ_{EWMA}.

	Multiple Comparisons p values (2-tailed); λ_{EWMA} Independent (grouping) variable: S/M/L Groups Kruskal-Wallis test: H (2, N= 124) =60.22471 p =.0000		
Depend.: λ_{EWMA}	Less than 1.50	1.51 to 2.25	Greater than 2.25
Less than 1.50		**0.00**	**0.00**
1.51 to 2.25	**0.00**		**0.03**
Greater than 2.25	**0.00**	**0.03**	

The stratification of the λ_{EWMA} by ARIMA type is shown in Table 29. Only three of the constant (Shewhart) group had non-zero λ_{EWMA}: S2.2, S2.4 and TU21.1. So for stationary or constant means we would expect the λ_{EWMA} to be zero. This would also lead us to believe a relationship exists between λ_{EWMA} and ARIMA type. The spread in the values by ARIMA type is shown in Figure 40.

Table 29. Breakdown Table of Descriptive Statistics of EWMA smoothing constant, λ_{EWMA}□ by ARIMA Type.

ARIMA Type	λ_{EWMA} Means	λ_{EWMA} N	λ_{EWMA} Q25	λ_{EWMA} Median	λ_{EWMA} Q75
Non-stationary, Non-seasonal	0.135	47	0.000	0.096	0.193
Non-stationary, Seasonal	0.148	35	0.027	0.099	0.191
Non-stationary AR(2)	0.217	5	0.116	0.194	0.231
Non-stationary D(2)	0.000	1	0.000	0.000	0.000
Constant (Shewhart)	0.009	25	0.000	0.000	0.000
Stationary, Non-seasonal	0.052	6	0.000	0.000	0.082
Stationary, Seasonal	0.058	5	0.000	0.047	0.069
All Grps	0.109	124	0.000	0.062	0.152

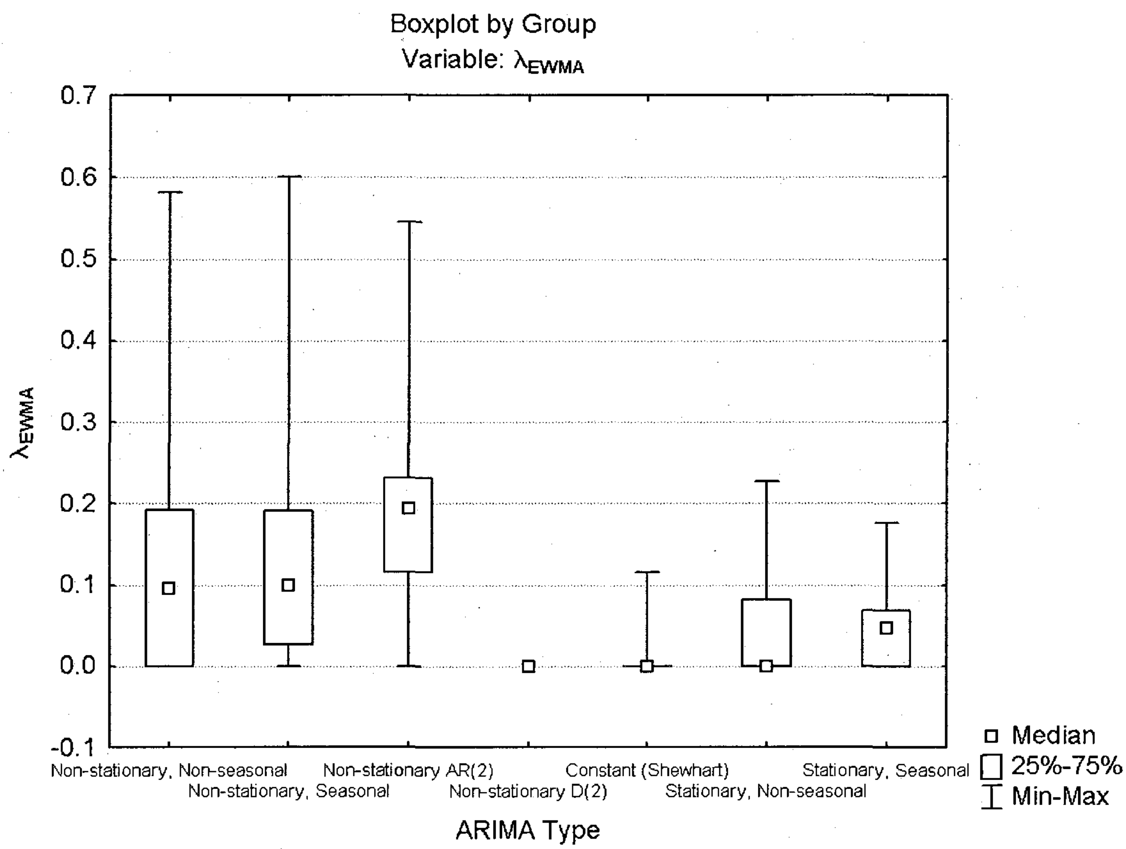

Figure 40. Variation in EWMA smoothing constant with ARIMA type. Greater values relate to more emphasis on recent observations. The greater values were seen for *non*-stationary series.

The non-stationary D(2) category had only one member which explains its lack of variation. The constant (Shewhart) group as explained earlier had few non-zero values. The non-stationary ARIMA types showed considerably more variation in the λ_{EWMA}. Consistent with these observations are the Kruskal-Wallis analysis in Table 30 which shows a difference in λ_{EWMA} due to the ARIMA type grouping. There was a significant difference between the constant (Shewhart) and the non-stationary series.

Table 30. P values for the relationship between EWMA smoothing constant, λ_{EWMA}, and the ARIMA type. The Constant (Shewhart) types differed from the Non-stationary series.

	Multiple Comparisons p values (2-tailed); λ_{EWMA} Independent (grouping) variable: ARIMA Type Kruskal-Wallis test: H (6, N= 126) =34.40174 p =.0000						
Depend.: λ_{EWMA}	Non-stationary, Non-seasonal	Non-stationary, Seasonal	Non-station. AR(2)	Non-station. D(2)	Constant (Shewhart)	Stationary, Non-seasonal	Station., Season.
Non-stationary, Non-seasonal		1.00	1.00	1.00	**0.00**	1.00	1.00
Non-stationary, Seasonal	1.00		1.00	1.00	**0.00**	1.00	1.00
Non-stationary AR(2)	1.00	1.00		1.00	**0.03**	1.00	1.00
Non-stationary D(2)	1.00	1.00	1.00		1.00	1.00	1.00
Constant (Shewhart)	**0.00**	**0.00**	**0.03**	1.00		1.00	1.00
Stationary, Non-seasonal	1.00	1.00	1.00	1.00	1.00		1.00
Stationary, Seasonal	1.00	1.00	1.00	1.00	1.00	1.00	

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Control charts were first introduced by Walter A. Shewhart in 1924 at the Western Electric Hawthorne Works in Chicago, Illinois. His argument was economic, not statistical. Control limits were established on samples of logical subgroups of data. He assumed that the process average was constant and measurements were independent, randomly occurring, normally distributed and centered on the average. In 1959, S. W. Roberts at Bell Labs introduced the geometric moving average chart, now called the Exponentially Weighted Moving Average, to enable more recent observations to carry greater weight for decision making.

If the measurement was beyond the control limits, then Shewhart's assumptions no longer held. Assuming the process was subjected to random shocks with the interval between shocks distributed exponentially the duration of the shocks would be Poisson distributed random events. Barnard (1959) first described this behavior. Montgomery and Mastrangelo (1991) recommended two control charts to monitor the process. The residuals from the EWMA were to be treated with a Shewhart chart analysis. This way Shewhart's assumptions would be valid and gradual, as well as rapid, changes in the mean would be signaled quickly. Jones (2002) confirmed this recommendation as did Lu and Reynolds (1999a, 1999b). Lucas (1985) and Saniga, Davis and Lucas (2009) offered a simple method to detect shifts in the mean.

Caulcutt (1995) indicated that the standard procedure to produce a usable control chart did not always work. Control charts failed to work and shook management's confidence. A new approach was needed. Alwan and Radson (1995) recommended using time-series modeling with Statistical Process Control. Alwan and Roberts (1995) identified autocorrelation as a reason that control chart limits were misplaced and suggested time series analysis to properly place the limits. They stated that 86% of control charts in the literature had undetected autocorrelation.

Albin, Kang and Shea (1997) evaluated the Average Run Length to false alarm and to small shifts in the process mean. They recommended the EWMA and X charts due to few false alarms and to rapid signaling of sudden shifts in the mean. Lu and Reynolds (1999b) based their recommendation on both statistical properties and ease of interpretation to use Shewhart chart of residuals with an EWMA of the observations to control low to moderate autocorrelation processes. de Mast and Roes (2004) distinguished isolated assignable causes from persistent assignable causes and linked these to the Shewhart-type control chart for the first type and charts that accumulate information from successive measurements (CUSUM, EWMA) for the second.

Hunter (1986) showed that the EWMA was the general form of process monitoring charts with CUSUM at one extreme when $\lambda_{EWMA} = 0$, and the Shewhart chart at the other extreme when $\lambda_{EWMA} = 1$. Jones, Champ and Rigdon (2001) and Jones (2002) advocated for modification of sample sizes to obtain the proper ARL when parameters for the EWMA had to be estimated from process data.

Lucas' CUSUM sequence chart is recommended to detect the process mean shifts before deploying the EWMA. Two control charts are recommended. Exponentially Weighted Moving Average with the smoothing coefficient, λ_{EWMA}, chosen to minimize the mean square error of the EWMA prediction and the actual measurement. This chart detects a gradual change in the process mean behavior. A Shewhart chart of the residuals of the EWMA. This chart detects a sudden change in the process quickly especially if the change is large. A signal from either chart indicates an out-of-control condition.

Summary and Conclusions for 1.5 Sigma Shift

In general, the analysis of process data requires the use of many analysis tools in a sophisticated sequence. This investigation demonstrated the use of many commonly used analysis tools in a structured manner to gain insight into the nature of process variation. The hypotheses and conclusions of this study were:

1. The process mean shifts less than 1.5 sigma units from its target during normal operation. The alternative hypothesis is that the mean drifts more than 1.5 sigma units in at least one process. We reject the null hypothesis. In this study we found

that while the average was within the +/- 1.5 sigma shift data we showed there were three groups of M/sw: less than 1.5, 1.51 to 2.25 and greater than 2.25.

2. No process measurements are related to others over time. The alternative hypothesis is that at least one process parameter measurement is related to itself over time. The ARIMA types showed that the non-stationary processes out numbered the constant (Shewhart) and stationary processes. Therefore we reject the null hypothesis.
3. The autoregressive coefficients are zero for all processes. The alternative hypothesis is that the autoregressive coefficients are not zero for at least one process. Only the constant (Shewhart) ARIMA type behaved with zero coefficients. The majority of the processes in this study had autoregressive coefficients that were non-zero. The null hypothesis was rejected.
4. Likewise, the moving average coefficients are zero for all processes. The alternative hypothesis is that the moving average coefficients are not zero for at least one process. Again we reject the null hypothesis based on the ARIMA type that had the general form (p, d, non-zero q). The null hypothesis was rejected.
5. The autoregressive and moving average coefficients are simultaneously zero for all processes. The alternative hypothesis is that these coefficients are not equal to zero for at least one process. While few process had both a p and q component in the ARIMA type, we reject the null hypothesis that all processes are not combinations of autoregressive and moving average behavior. We reject the null hypothesis.
6. ARIMA time series analysis separates the drift in the process average from the common cause variation inherent in the process. The size of the drift would be less than or equal to 1.5 sigma units where sigma is the common cause process variation. The ARIMA time series permitted us to calculate s'a and sa as the

common cause variation inherent in the process. As we concluded in hypothesis 1, there were many time series studies with multiples of s'a or sa that were beyond 1.5. For that reason, we reject the null hypothesis.

Assumptions

The following assumptions were made in pursuit of this study:

1. All processes exhibit variation. This variation is composed of variation due to drifts in the mean, unexplained common cause variation and special cause variation due for instance to effects like seasonal variation, and multiple machines performing the same work. Well-intended, but uninformed process control people can increase the variation of the process by adjusting the process when it is exhibiting only common cause variation. In our studies we found that there were differences in ARIMA type for machines, time, operators and parts as well as the dimension. Tables A1 through A4 list the processes and the values for s'a, sa, s_{ES} and s_w that are non-zero and consistent with this assumption. This assumption was valid for the studies in this investigation.
2. Six Sigma processes have at most a 1.5 sigma shift in the process mean. With three groupings of process variation ratios, M/s_w, we showed that the designation of the process as a Six Sigma process is more a choice of the tolerance than an inherent characteristic of the process group. We compared averages, first, median and third quartiles to show that processes influenced the level of M/s_w, but no process was consistently below 3.0. With the proper choice of tolerance, this assumption held.
3. No single analysis method is appropriate for all processes. The method employed in this investigation was appropriate for all the processes. However, the method of time series analysis would be preferred in some cases because of its inherent accommodation of the process movement with additional terms for autoregression,

moving average, and seasonal patterns. A simpler model using only the Exponentially Weighted Moving Average was not able to provide as good a model. The ARIMA modeling is more complex than needed in some cases so we found no universally appropriate method suitable for all processes. This assumption was valid.

4. Process means can be separated from common cause variation using the proper statistical methods. Using the CUSUM sequence and MMSE EWMA combined, we were able to separate shifts in the means. From the ARIMA and MMSE analysis we generated residuals that could identify occurrence of special causes from common causes. This assumption was valid.
5. Some processes exhibit stationary mean location, uncorrelated measurements over time, and random variation. This assumption held for the constant (Shewhart) studies. This assumption was therefore validated for that case.
6. Other processes have stationary means but show sudden shifts in the mean, uncorrelated measurements over time and random variation. This assumption held for all but the constant (Shewhart) ARIMA types.
7. Additional processes are not stationary, but are uncorrelated over time and have random variation. We found many examples of non-stationary, seasonal and non-seasonal behavior. We validated this assumption.
8. A few processes will be dominated by large inherent variation making necessary the detection of the change in mean location with CUSUM or EWMA methods to separate the shifts and drifts in the mean from the process random variation. Almost all process had sufficiently large shifts in the mean to cause us to use these methods. This assumption was validated.

9. Time series analysis is appropriate for the analysis of process mean shifts. Time series analysis is appropriate for the analysis of process behavior. The mean shifts are more easily detected using the CUSUM sequence plots than the ARIMA models. Simpler models using only the EWMA were appropriate for many studies. This assumption was not valid.
10. The Autoregressive Integrated Moving Average methods effectively separate the process mean from the white noise variation. More appropriately, the white noise variation was separated from the movement of the process means using the ARIMA analysis. This assumption was validated.
11. The residuals from the ARIMA model are normally distributed, uncorrelated, random variables with zero mean and process sigma. The residuals behaved as normally distributed, uncorrelated random variables with zero mean and constant variance. We did not prove this in our investigation, but there was no evidence to doubt this either. This assumption held.
12. The Exponentially Weighted Moving Average control chart used in conjunction with the Individual X control chart can effectively identify drifts in the mean, special cause and common cause variation. We used this method repeatedly. With the MMSE lambda for the EWMA we were able to verify locations of mean shifts after using the CUSUM sequence method. This assumption was valid.
13. Quality policy of the firm is consistent across all processes. The process groups we studied behaved consistent with this assumption. We were able to detect a philosophical difference in operator control behavior, but no policy differences were evident when other departments were engaged in the studies. This assumption was believed valid due to lack of evidence to the contrary.

14. The ARIMA model reflects behavior of the physical process within acceptable error. The ARIMA model yielded sigma estimates that were consistent with other independent estimates of sigma. When there were differences it was usually due to the precision of the measurement (counts in whole units for instance and the sigma differences in tenths of a count) or the presence of the seasonal cycle that single parameter models could not capture.
15. A sample comprising a fraction of the total processes can represent all the processes for a particular firm. Caution needs to be taken here. The findings in our study were from purposeful samples on key characteristics. While we believe we can describe the behavior of the processes within the firm, no general statement could be made about a process group's behavior without performing a study as done here. The results were valid, but the generalizability by process group is limited. Sampling has little validity in these studies for generalizing to the behavior of the population.

Recommendations

For future studies: Sample selection be done on a narrower scope, perhaps a battery of machines, with part features selected at random. Employ Lucas' CUSUM sequence plot and the EWMA to the shift points. Using ANOVA on these groups, identify the mean shift, M/s_w, the duration of the mean shift, λ_B, and the s_w. Using sw as the estimate of the process sigma, compare to specifications. Establish control charts to detect M/s_w size shift in the mean. Compare machine batteries to one another to identify the best and poorest battery sigmas. Work with the process engineers to route parts requiring special tolerance accordingly.

This investigation was conducted at a part manufacturing and assembly firm. Continuous processes would be expected to behave in a similar manner, but the sampling, data recording, frequency of sampling and cost of measurement would be different. The values of M/s_w, λ_B, and

s_w could be used to establish engineering control, automated Proportional, Integral and Derivative (PID) control.

In these studies we found that operator philosophy could affect the behavior of the time series. A study to quantify the effects and economics of alternatives could be conducted. This would be done by setting the three conditions of large variation due to few adjustments, small variation due to frequent adjustment but more costs of adjusting and an intermediate that would be a compromise of the other two. The cost structure would differ for different firms and perhaps processes within the firm, but a pattern may exist that would permit the firm to establish adjustment frequencies based on the more economic method as well as the minimum variation.

Where tolerances are broad relative to process variation, s_w, develop bounded adjustment rules advocated by Box and Luceño (1997), Box and Paniagua-Quiñones (2007) and Hunter (1998). Current rules of thumb could be used as a comparison to judge the effect on the s_w.

Where PID control is already in place, use the ARIMA(p,d,q)(P,D,Q)S models to compare to the parameters in the controller. Often the controllers are set up with judgment and not to minimum variation targets. This feedback could improve product control and reduce variation potentially increasing satisfaction and reducing costs.

This study could be broadened by the firm to include all processes. There was not enough time and resource to study all the processes. However, the method was shown effective in all those studies made. This would give insight into the process behavior with the ARIMA type and the sigmas could be compared. A comparison of sigma estimates would help identify the behavior of complex models that are the result of sampling practices leading to frequent adjustments and high autocorrelation, poor measurement that increases the white noise level and hence the moving average component and seasonal behavior due to special causes undetected and not remedied.

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APPENDIX A

TABLES OF STUDIES AND VALUES FROM ANALYSIS

Table A1. Summary of results for λ_B. Columns are C/M which is Count or Measured, Transformation of data, sample size, number of shifts in the mean, Barnard's average duration of mean shift, λ_B.

Code	Description	C/ M	Transform	n, Number	k, Shifts	λ_B
A1.1	Deficiencies count product group 1	C	Ln	129	9	14.3
A1.2	Deficiencies count product group 1	C	SqrRt	153	11	13.9
A1.3	Deficiencies count product group 1	C	Ln	191	7	27.3
A1.4	Deficiencies count product group 1	C	Ln	279	6	46.5
A1.5	Deficiencies count product group 1	C	SqrRt	198	3	66.0
A2.1	Deficiencies count product group 2	C	SqrRt	197	11	17.9
A2.2	Deficiencies count product group 2	C	SqrRt	167	9	18.6
A2.3	Deficiencies count product group 2	C	SqrRt	269	8	33.6
A2.4	Deficiencies count product group 2	C	SqrRt	269	15	17.9
A2.5	Deficiencies count product group 2	C	SqrRt	184	14	13.1
A2.6	Deficiencies count product group 2	C	SqrRt	240	10	24.0
A2.7	Deficiencies count product group 2	C	SqrRt	159	10	15.9
A3.1	Deficiencies count product group 3	C	SqrRt	436	13	33.5
A3.2	Deficiencies count product group 3	C	SqrRt	582	16	36.4
A3.3	Deficiencies count product group 3	C	SqrRt	292	11	26.5
A3.4	Deficiencies count product group 3	C	SqrRt	183	14	13.1
A3.5	Deficiencies count product group 3	C	SqrRt	151	11	13.7
A4	Deficiencies count major component	C	Ln	179	11	16.3
F1.1	Foundry property 1, line 1	M	None	330	19	17.4
F1.2	Foundry property 1, line 2	M	None	330	19	17.4
F2.1	Foundry property 2, line 1	M	None	330	16	20.6
F2.2	Foundry property 2, line 2	M	None	330	25	13.2
F3.1	Foundry property 3, line 1	M	None	330	22	15.0
F3.2	Foundry property 3, line 2	M	None	330	21	15.7
H1	Heat Treatment Harness, part 1	M	SG7	156	10	15.6
H2	Heat Treatment Harness, part 2	M	SG7	149	11	13.5
H3	Heat Treatment Harness, part 3	M	SG3	242	10	24.2
M1	Machine dimension 1	M	None	203	18	11.3
M2	Machine dimension 2	M	None	203	15	13.5
M3	Machine dimension 3	M	None	243	23	10.6
M4.1	Machine dimension 4, part 1	M	None	783	25	31.3
M4.2	Machine dimension 4, part 2	M	None	447	26	17.2
M4.3	Machine dimension 4, part 3	M	None	696	11	63.3
M4.4	Machine dimension 4, part 4	M	None	439	27	16.3
M4.5	Machine dimension 4, part 5	M	None	144	15	9.6
M5	Machine dimension 5	M	None	1011	50	20.2
M6	Machine dimension 6	M	None	353	17	20.8
MM1	Measure of fit, assembly 1	M	None	300	10	30.0
R	AIAG capability data set	M	None	125	8	15.6
S1.1	Shaved dimension 1, part 1	M	None	126	14	9.0
S1.2	Shaved dimension 1, part 2	M	None	110	16	6.9
S2.1	Shaved dimension 2, part 1	M	None	126	19	6.6
S2.2	Shaved dimension 2, part 2	M	None	109	9	12.1
S2.3	Shaved dimension 2, part 3	M	None	110	17	6.5
S2.4	Shaved dimension 2, part 4	M	None	155	8	19.4

(table continues)

Code	Description	C/M	Transform	n, Number	k, Shifts	λ_B
S2.5	Shaved dimension 2, part 5	M	SG3	102	12	8.5
S3.1	Shaved dimension 3, part 1	M	None	110	9	12.2
S3.2	Shaved dimension 3, part 2	M	None	228	14	16.3
S3.3	Shaved dimension 3, part 3	M	Ln	146	12	12.2
S4.1	Shaved dimension 4, part 1	M	SG4	187	10	18.7
S4.2	Shaved dimension 4, part 2	M	SG4	330	12	27.5
S5	Shaved dimension 5	M	None	4743	51	93.0
S6	Shaved dimension 6	M	SG3	169	10	16.9
TM1	Test machine dimension 1	M	Ln	106	11	9.6
TM2	Test machine dimension 2	M	None	106	9	11.8
TM3	Test machine dimension 3	M	None	106	9	11.8
TM4	Test machine dimension 4	M	None	106	11	9.6
TM5	Test machine dimension 5	M	None	106	9	11.8
G1.1	Ground dimension 1, machine 1	M	SG4	669	28	23.9
G1.2	Ground dimension 1, machine 2	M	SG4	664	33	20.1
G2.1	Ground dimension 2, machine 1	M	SG4	482	27	17.9
G2.2	Ground dimension 2, machine 2	M	SG4	661	15	44.1
TU1	Turned dimension 1	M	SG3	138	12	11.5
TU2	Turned dimension 2	M	SqrRt, SG3	100	10	10.0
TU3.1	Turned dimension 3, part 1	M	SG3	95	8	11.9
TU3.2	Turned dimension 3, part 2	M	SG3	134	8	16.8
TU3.3	Turned dimension 3, part 3	M	SG3	95	2	47.5
TU4.1	Turned dimension 4, part 1	M	SG3	95	8	11.9
TU4.2	Turned dimension 4, part 2	M	SG3	134	11	12.2
TU4.3	Turned dimension 4, part 3	M	SG3	95	6	15.8
TU5.1	Turned dimension 5, part 1	M	SG3	95	11	8.6
TU5.2	Turned dimension 5, part 2	M	SG3	134	14	9.6
TU5.3	Turned dimension 5, part 3	M	SG3	94	8	11.8
TU6.1	Turned dimension 6	M	None	204	26	7.8
TU7	Turned dimension 7	M	None	203	20	10.2
TU8	Turned dimension 8	M	None	202	11	18.4
TU9	Turned dimension 9	M	SG2	760	17	44.7
TU10	Turned dimension 10	M	None	188	20	9.4
TU11.1	Turned dimension 11, part 1	M	None	449	13	34.5
TU11.2	Turned dimension 11, part 2	M	None	390	24	16.3
TU12.1	Turned dimension 12, part 1	M	None	447	10	44.7
TU12.2	Turned dimension 12, part 2	M	None	391	8	48.9
TU13.1	Turned dimension 13, part 1	M	None	449	19	23.6
TU13.2	Turned dimension 13, part 2	M	None	389	18	21.6
TU14.1	Turned dimension 14, part 1	M	None	447	21	21.3
TU14.2	Turned dimension 14, part 2	M	Ln(x+60)	391	23	17.0
TU15	Turned dimension 15	M	None	183	6	30.5
TU16	Turned dimension 16	M	None	132	11	12.0
TU17	Turned dimension 17	M	SG5	50	3	16.7
TU18	Turned dimension 18	M	None	133	8	16.6
TU19.0	Turned dimension 19, all operators	M	SG5	172	11	15.6
TU19.1	Turned dimension 19, operator 1	M	SG5	55	6	9.2

(table continues)

Code	Description	C/ M	Transform	n, Number	k, Shifts	λ_B
TU19.2	Turned dimension 19, operator 2	M	SG5	40	5	8.0
TU19.3	Turned dimension 19, operator 3	M	SG5	77	8	9.6
TU20.0	Turned dimension 20, all operators	M	SG5	172	13	13.2
TU20.1	Turned dimension 20, operator 1	M	SG5	55	6	9.2
TU20.2	Turned dim. 20, operator 2	M	SG5	40	6	6.7
TU20.3	Turned dim. 20, operator 3	M	SG5	77	8	9.6
TU21.0	Turned dim. 21, all operators	M	SG5	182	8	22.8
TU21.1	Turned dim. 21, operator 1	M	SG5	55	10	5.5
TU21.2	Turned dim. 21, operator 2	M	SG5	50	7	7.1
TU21.3	Turned dim. 21, operator 3	M	SG5	77	9	8.6
TU22	Turned dim. 22	M	None	172	14	12.3
TU23	Turned dim. 23	M	None	172	10	17.2
TU24.0	Turned dim. 24, all observations	M	SG3	422	11	38.4
TU24.1	Turned dim 24, operator 1, 1-165	M	SG3	166	4	41.5
TU24.2	Turned dim 24, operator 1, 166-206	M	SG3	40	2	20.0
TU24.3	Turned dim. 24, operator 2	M	SG3	88	6	14.7
TU24.4	Turned dim. 24, operator 3	M	SG3	128	5	25.6
TU25.0	Turned dim. 25, all operators	M	SG3	422	14	30.1
TU25.1.1	Turned dim. 25, operator 1, 1-86	M	SG3	86	8	10.8
TU25.1.2	Turned dim. 25, operator 1, 87-160	M	SG3	73	7	10.4
TU25.1.3	Turned dim. 25, operator 1, 161-206	M	SG3	45	4	11.3
TU25.2	Turned dim. 25, operator 2	M	SG3	88	6	14.7
TU25.3	Turned dim. 25, operator 3	M	SG3	128	10	12.8
TU26.0	Turned dim. 26, all operators	M	SG3	426	8	53.3
TU26.1	Turned dim. 26, operator 1	M	SG3	206	13	15.8
TU26.2	Turned dim. 26, operator 2	M	SG3	88	5	17.6
TU26.3	Turned dim. 26, operator 3	M	SG3	128	6	21.3
W1	Warranty machine 1	M	SqrRt	93	10	9.3
W2	Warranty machine 2	M	SqrRt	94	12	7.8
W3	Warranty machine 3	M	Ln	94	14	6.7
W4	Warranty machine 4	M	Ln	93	12	7.8
Y1	Yield line 1	C	SqrRt	300	12	25.0
Y2	Yield line 2	C	SqrRt	300	10	30.0
Y3	Yield line 3	C	SqrRt	1706	114	15.0

Table A2. Summary of results for ARIMA model values. Upper case letters are for seasonal factors.

Code	p = ϕ	p2	d = ∇	q = θ	q2	P = Φ	D = ∇	Q = Θ	S
A1.1	0	0	1	0.6882	0	0	0	0	0
A1.2	0	0	1	0.8954	0	0	0	0	0
A1.3	0	0	1	0.8897	0	0	0	0.2107	14
A1.4	0	0	1	0.8439	0	0	0	0	0
A1.5	0	0	1	0.8447	0	0	0	0	0
A2.1	0	0	1	0.8132	0	0	0	0	0
A2.2	0	0	1	0.8685	0	0	0	0	0
A2.3	0	0	1	0.9220	0	0	0	0	0
A2.4	0	0	1	0.9210	0	0	0	0	0
A2.5	0	0	1	0.8702	0	0	0	0	0
A2.6	0	0	1	0.9246	0	0	0	0	0
A2.7	0	0	1	0.8958	0	0.1926	0	0	3
A3.1	0	0	1	0.9355	0	0	0	0	0
A3.2	0	0	1	0.9555	0	0.1085	0	0	4
A3.3	0	0	1	0.9020	0	0	0	0	0
A3.4	0	0	1	0.7620	0	-0.3987	1	0	8
A3.5	0	0	1	0.9306	0	0	0	0	0
A4	0	0	1	0.6624	0	0	0	-0.2062	14
F1.1	0	0	1	0.8418	0	0	0	0.1780	9
F1.2	0	0	1	0.9193	0	0	0	0	0
F2.1	0.2816	0	1	0.9250	0	0	0	-0.1865	6
F2.2	0.2366	0	1	0.9411	0	0	0	0	0
F3.1	0.2785	0	1	0.9055	0	-0.1709	0	0	4
F3.2	0.3482	0	1	0.9975	0	0	0	-0.1463	2
H1	0	0	1	0.8196	0	0	0	0	0
H2	0	0	1	0.8518	0	0	0	0	0
H3	0.1746	0	1	0.9878	0	0	0	0	0
M1	0.1616	0	1	0.9945	0	0	0	0	0
M2	0.4001	0	1	0.9285	0	0	0	0	0
M3	-0.7062	0	1	0	0	0.5751	0	0.9104	2
M4.1	0.2552	0.1042	1	0.9706	0	0.0992	0	0	3
M4.2	0.3537	0	1	0.9588	0	0	0	0	0
M4.3	0.3144	0	1	0.9957	0	0	0	0	0
M4.4	0.2445	0	1	0.7506	0	0	0	-0.0149	3
M4.5	-0.4465	0	1	0	0	0	0	0.3004	3
M5	0.2769	0.1119	1	0.9591	0	0	0	0	0
M6	0	0	1	0.5892	0	0	0	-0.1803	4
MM1	-0.2241	0	1	0.4164	0	0	0	0	0
R	0	0	1	0.9482	0	0	0	0	0
S1.1	0	0	1	0.7144	0	0	0	0.2628	6
S1.2	0	0	1	0.7798	0	0	0	0	0
S2.1	0	0	0	0	0	0	0	0	0
S2.2	0	0	0	0	0	0	0	0	0
S2.3	0	0	1	0.9565	0	0	0	0	0
S2.4	0	0	0	0	0	0	0	0	0
S2.5	-0.8494	-0.7941	1	0	0	0	0	0.9316	3
S3.1	0	0	0	0	0	0	0	0	0
S3.2	0	0	0	0	0	0	0	0	0

(table continues)

Code	$p = \phi$	p2	$d = \nabla$	$q = \theta$	q2	$P = \Phi$	$D = \nabla$	$Q = \Theta$	S
S3.3	0	0	0	0	0	0	0	0	0
S4.1	0	0	0	0.7451	0	0	0	0	0
S4.2	0	0	1	0.8028	0	0	0	0	0
S5	-0.1018	0.8823	1	0.7633	0.9890	0	0	-0.7540	0
S6	0	0	0	0	0	0	0	0	0
TM1	0	0	1	0.8299	0	0	0	0.2083	7
TM2	0	0	1	0.9428	0	0	0	0	0
TM3	0	0	1	0.7706	0	0	0	0	0
TM4	0	0	1	0.8928	0	0.3690	0	0	5
TM5	0.9999	0	0	0.3233	0	0	0	0	0
G1.1	0.1229	0	0	0.0000	0	0	0	0	0
G1.2	0	0	1	0.8726	0	0	1	0.9138	4
G2.1	0.2933	0	0	0	0	0	0	0	0
G2.2	-0.3798	0	1	0.4566	0.4696	0	0	0.1336	11
TU1	-0.2043	0	1	0.9753	0	0	0	0	0
TU2	0	0	0	0	0	0	0	0	0
TU3.1	0	0	0	0	0	0	0	0	0
TU3.2	0.2389	0	1	0.9128	0	0	0	0	0
TU3.3	0	0	1	0.8360	0	-0.3023	0	0	2
TU4.1	0	0	0	0	0	0	0	0	0
TU4.2	0	0	1	0.7145	0	0	0	0.2710	0
TU4.3	0	0	0	0	0	0	0	0	0
TU5.1	0	0	0	0	0	0	0	0	0
TU5.2	0.2289	0	1	0.9998	0	0	0	0	0
TU5.3	0	0	1	0.8369	0	0	0	0	0
TU6.1	0.4519	0	1	0.9841	0	0	0	0	0
TU7	-0.4836	0	2	0.8974	0	0	0	0	0
TU8	0.2913	0	1	0.9541	0	0	0	0	0
TU9	0.2190	0	1	0.9802	0	0	0	0	0
TU10	-0.2524	-0.2953	1	0	0	-0.3568	0	0	6
TU11.1	0.2092	0	1	0.9528	0	0	0	0	0
TU11.2	0.2389	0	1	0.9996	0	0	0	0	0
TU12.1	0	0	1	0.8722	0	0	0	0	0
TU12.2	0	0	1	0.8845	0	-0.1255	0	0	7
TU13.1	0.1579	0	1	0.9946	0	0.1067	0	0	2
TU13.2	0.2462	0	1	0.9656	0	0	0	0	0
TU14.1	0	0	1	0.8782	0	0	0	0.1657	8
TU14.2	0.2539	0	1	0.9999	0	0	0	-0.1708	5
TU15	0	0	0	0	0	0	0	0	0
TU16	0	0	1	0.9167	0	0	0	0	0
TU17	-0.7780	-0.2991	1	0	0	0	0	0	0
TU18	0	0	0	0	0	0	0	0	0
TU19.0	0	0	0	0	0	0	0	0	0
TU19.1	0	0	0	0	0	0	0	0	0
TU19.2	0	0	1	0.6862	0	0	0	0	0
TU19.3	0	0	0	0	0	0	0	0	0
TU20.0	0	0	1	0.9110	0	0	0	0	0
TU20.1	0	0	0	0	0	0	0	0	0
TU20.2	0	0	1	0.6949	0	0	0	0	0

(table continues)

Code	p = ϕ	p2	d = ∇	q = θ	q2	P = Φ	D = ∇	Q = Θ	S
TU20.3	0	0	0	0	0	0	0	0	0
TU21.0	0	0	0		0	0	0	-0.3056	8
TU21.1	0	0	0		0	0	0	0	0
TU21.2	0	0	1	0.7803	0	0	0	0	0
TU21.3	0	0	0	0	0	0	0	0.2191	2
TU22	0	0	0	-0.4251	0	0	0	0	0
TU23	0	0	0	-0.2677	0	0	0	0	0
TU24.0	0	0	0	0	0	0	0	0	0
TU24.1	0	0	1	0.9130	0	0	0	0.1472	8
TU24.2	0	0	0	0	0	0	0	-0.5386	7
TU24.3	0	0	0	0.8382	0	0	0	-0.2252	5
TU24.4	0	0	1	0.9998	0	0	0	0	0
TU25.0	0	0	1	0.9139	0	0	0	0.1904	2
TU25.1.1	0	0	0	0	0	0	0	0	0
TU25.1.2	0	0	1	0.7891	0	-0.3364	0	0	7
TU25.1.3	0	0	0	0	0	0	0	0	0
TU25.2	0	0	1	0.9184	0	0	0	-0.2805	0
TU25.3	0	0	0	0	0	0	0	0	0
TU26.0	0	0	1	0.9421	0	0	0	0.1503	14
TU26.1	0	0	1	0.8592	0	-0.1744	0	0	7
TU26.2	0	0	0	0	0	0	0	0	0
TU26.3	0	0	0	0	0	0	0	-0.2454	10
W1	0	0	1	0.3124	0	-0.3192	0	0	2
W2	0	0	1	0.5104	0	0	0	0	0
W3	0	0	1	0.4402	0	0	0	0	0
W4	0	0	0	0.7291	0	0	0	0	0
Y1	0	0	1	0.9104	0	0	0	-0.1727	3
Y2	0	0	1	0.8559	0	0	0	-0.1311	4
Y3	-0.8311	0	1	0.6566	0	0	1	0.7316	2

Table A3. Summary of results for M/s_w. Exponential smoothing constant, sigma from ARIMA, ARIMA residuals, Exponential Smoothing, ANOVA within, and sigma of mean shifts.

Code	λ_{EWMA}	s'a	sa	s_{ES}	s_w	M/s_w
A1.1	0.348	5.4097	5.0360	5.2688	4.6968	4.595
A1.2	0.118	2.3823	2.3796	2.3959	2.1169	1.075
A1.3	0.120	3.4004	3.3942	3.5639	3.0536	1.673
A1.4	0.129	2.7610	2.6514	2.6033	2.5186	3.249
A1.5	0.096	2.6444	2.6783	2.5929	2.4763	1.210
A2.1	0.149	2.1702	2.1864	2.1335	1.9814	1.476
A2.2	0.049	2.4504	2.3564	2.2362	2.2274	0.975
A2.3	0.071	2.3280	2.3937	2.3790	2.1309	1.474
A2.4	0.000	2.2306	2.1878	2.0768	1.9538	1.239
A2.5	0.073	2.4809	2.6241	2.5434	2.1933	1.778
A2.6	0.064	2.5608	2.4697	2.4476	2.2965	1.559
A2.7	0.048	2.9807	2.9131	2.8967	2.6378	0.962
A3.1	0.000	1.7751	1.7974	1.7307	1.6792	0.528
A3.2	0.000	1.8099	1.7899	1.7432	1.7422	0.695
A3.3	0.000	2.0551	2.0496	1.9293	1.9213	0.859
A3.4	0.000	2.4150	2.5111	1.8819	1.7307	1.801
A3.5	0.000	2.0812	1.8764	1.8166	1.8192	1.338
A4	0.316	0.3881	0.3509	0.3458	0.3339	1.902
F1.1	0.125	1.9200	1.9058	1.8834	1.7427	1.381
F1.2	0.051	2.3748	2.1583	2.1201	2.2066	0.921
F2.1	0.182	0.1402	0.1283	0.1201	0.1320	2.338
F2.2	0.135	0.1620	0.1420	0.1327	0.1484	2.495
F3.1	0.000	0.3305	0.3108	0.2634	0.3153	1.438
F3.2	0.224	0.3245	0.3042	0.2890	0.2926	1.780
H1	0.218	0.6647	0.6393	0.6528	0.6016	1.086
H2	0.133	0.7275	0.6326	0.6292	0.6311	1.446
H3	0.000	1.3808	1.3386	1.2182	1.2760	1.470
M1	0.000	0.0138	0.0127	0.0115	0.0128	1.254
M2	0.407	0.0077	0.0075	0.0072	0.0067	2.773
M3	0.394	0.0107	0.0088	0.0093	0.0088	2.552
M4.1	0.051	0.0107	0.0099	0.0088	0.0098	1.095
M4.2	0.175	0.0159	0.0150	0.0137	0.0150	1.436
M4.3	0.021	0.0171	0.0160	0.0135	0.0175	2.859
M4.4	0.427	0.0037	0.0030	0.0029	0.0029	4.989
M4.5	0.462	0.0039	0.0037	0.0036	0.0037	1.707
M5	0.194	0.0062	0.0059	0.0055	0.0057	2.057
M6	0.422	0.0021	0.0019	0.0019	0.0017	0.643
MM1	0.436	0.0710	0.0527	0.0536	0.0543	4.839
R	0.000	0.1964	0.1830	0.1697	0.1725	2.185
S1.1	0.331	0.0164	0.0164	0.0170	0.0127	1.969
S1.2	0.192	0.0159	0.0140	0.0140	0.0135	5.087
S2.1	0.000	0.0094	0.0094	0.0094	0.0097	5.241
S2.2	0.076	0.0072	0.0068	0.0072	0.0064	0.947
S2.3	0.000	0.0088	0.0070	0.0068	0.0076	1.762
S2.4	0.044	0.0110	0.0098	0.0101	0.0105	1.735
S2.5	0.000	0.0081	0.0074	0.0064	0.0070	0.903
S3.1	0.000	0.0147	0.0147	0.0147	0.0141	1.271
S3.2	0.000	0.0167	0.0172	0.0172	0.0161	1.353

(table continues)

Code	λ_{EWMA}	s'a	sa	s_{ES}	s_w	M/s_w
S3.3	0.000	0.0114	0.0097	0.0097	0.0099	1.491
S4.1	0.228	0.0034	0.0032	0.0032	0.0030	1.191
S4.2	0.193	0.0084	0.0080	0.0080	0.0076	1.136
S5	0.116	0.0105	0.0099	0.0098	0.0106	1.751
S6	0.000	0.0023	0.0021	0.0021	0.0022	0.589
TM1	0.000	0.4613	0.4118	0.3748	0.3874	1.426
TM2	0.000	0.4993	0.5206	0.5022	0.4546	0.882
TM3	0.135	19.5895	18.3850	17.8070	16.1323	1.371
TM4	0.000	6.1775	6.3402	6.3302	5.6843	1.334
TM5	0.000	0.0071	0.0059	0.0039	0.0035	1.244
G1.1	0.000	5.00E-05	4.80E-05	4.50E-05	4.68E-05	1.069
G1.2	0.000	2.50E-05	5.30E-05	4.70E-05	4.74E-05	0.931
G2.1	0.000	2.50E-05	6.00E-05	5.20E-05	5.53E-05	0.864
G2.2	0.040	5.88E-05	5.70E-05	5.40E-05	5.63E-05	0.929
TU1	0.000	0.0021	0.0020	0.0022	0.0020	0.571
TU2	0.000	0.0022	0.0018	0.0018	0.0019	0.969
TU3.1	0.000	0.0089	0.0085	0.0088	0.0083	1.281
TU3.2	0.049	0.0130	0.0136	0.0117	0.0120	0.413
TU3.3	0.086	0.0126	0.0117	0.0123	0.0121	0.864
TU4.1	0.000	0.0048	0.0046	0.0046	0.0044	1.099
TU4.2	0.000	0.0048	0.0047	0.0042	0.0043	2.207
TU4.3	0.000	0.0048	0.0042	0.0042	0.0048	2.260
TU5.1	0.000	0.0050	0.0044	0.0044	0.0047	1.745
TU5.2	0.000	0.0052	0.0048	0.0043	0.0047	1.430
TU5.3	0.149	0.0060	0.0053	0.0052	0.0051	0.620
TU6.1	0.467	0.0085	0.0080	0.0081	0.0065	2.673
TU7	0.000	0.0049	0.0049	0.0035	0.0036	1.113
TU8	0.216	0.0060	0.0059	0.0055	0.0054	1.134
TU9	0.011	0.0039	0.0037	0.0034	0.0038	1.539
TU10	0.545	0.0069	0.0058	0.0057	0.0058	1.587
TU11.1	0.122	0.0114	0.0112	0.0105	0.0108	1.813
TU11.2	0.000	0.0157	0.0144	0.0129	0.0146	1.204
TU12.1	0.125	0.0099	0.0088	0.0088	0.0093	1.217
TU12.2	0.095	0.0102	0.0090	0.0090	0.0094	1.418
TU13.1	0.027	0.0152	0.0138	0.0128	0.0142	0.811
TU13.2	0.088	0.0161	0.0150	0.0133	0.0154	1.628
TU14.1	0.099	0.0124	0.0093	0.0085	0.0116	1.948
TU14.2	0.155	0.0163	0.0141	0.0130	0.0151	0.984
TU15	0.000	0.0081	0.0073	0.0073	0.0080	0.633
TU16	0.082	0.0092	0.0087	0.0088	0.0079	0.956
TU17	0.231	0.0006	0.0005	0.0005	0.0004	1.547
TU18	0.000	0.0081	0.0078	0.0078	0.0075	0.645
TU19.0	0.000	0.0030	0.0028	0.0028	0.0030	1.023
TU19.1	0.000	0.0019	0.0018	0.0018	0.0018	1.363
TU19.2	0.307	0.0006	0.0006	0.0006	0.0005	3.026
TU19.3	0.000	0.0041	0.0044	0.0044	0.0041	0.907
TU20.0	0.080	0.0029	0.0029	0.0028	0.0027	1.491
TU20.1	0.000	0.0022	0.0020	0.0020	0.0021	1.191
TU20.2	0.302	0.0006	0.0005	0.0006	0.0005	1.136
TU20.3	0.000	0.0027	0.0027	0.0027	0.0026	1.751

(table continues)

Code	λ_{EWMA}	s'a	sa	s_{ES}	s_w	M/s_w
TU21.0	0.069	0.0039	0.0034	0.0036	0.0038	0.938
TU21.1	0.116	0.0033	0.0029	0.0029	0.0026	1.837
TU21.2	0.224	0.0006	0.0005	0.0005	0.0004	1.517
TU21.3	0.000	0.0047	0.0041	0.0042	0.0048	0.537
TU22	0.000	0.0103	0.0102	0.0089	0.0104	1.008
TU23	0.082	0.0109	0.0107	0.0097	0.0105	1.251
TU24.0	0.000	0.0034	0.0030	0.0030	0.0033	1.522
TU24.1	0.082	0.0012	0.0011	0.0011	0.0011	0.772
TU24.2	0.000	0.0034	0.0025	0.0028	0.0033	0.244
TU24.3	0.176	0.0008	0.0008	0.0008	0.0007	1.688
TU24.4	0.000	0.0047	0.0047	0.0046	0.0047	0.116
TU25.0	0.060	0.0038	0.0038	0.0038	0.0037	0.962
TU25.1.1	0.000	0.0009	0.0007	0.0007	0.0008	1.052
TU25.1.2	0.164	0.0021	0.0020	0.0020	0.0019	1.410
TU25.1.3	0.000	0.0030	0.0029	0.0029	0.0027	0.678
TU25.2	0.099	0.0008	0.0007	0.0007	0.0007	1.556
TU25.3	0.000	0.0055	0.0054	0.0054	0.0049	1.166
TU26.0	0.027	0.0026	0.0026	0.0025	0.0026	0.418
TU26.1	0.121	0.0023	0.0023	0.0022	0.0021	1.110
TU26.2	0.000	0.0012	0.0010	0.0010	0.0010	1.201
TU26.3	0.047	0.0033	0.0033	0.0035	0.0033	0.731
W1	0.601	$145	$137	$139	$106	3.615
W2	0.516	$232	$217	$220	$188	2.315
W3	0.582	$415	$314	$317	$210	4.023
W4	0.284	$462	$441	$445	$348	3.081
Y1	0.077	1.9969	1.8667	1.8576	1.7869	1.656
Y2	0.149	2.5290	2.5764	2.5837	2.2490	1.756
Y3	0.191	2.8281	2.4710	2.0474	2.0862	2.396

Table A4. Summary of ARIMA and process classifications for each study.

Code	Description	ARIMA	ARIMA Type	Process Group
A1.1	Deficiencies count product group 1	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A1.2	Deficiencies count product group 1	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A1.3	Deficiencies count product group 1	(0,1,1)(0,0,1)14	Non-stationary, Seasonal	Assembly
A1.4	Deficiencies count product group 1	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A1.5	Deficiencies count product group 1	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.1	Deficiencies count product group 2	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.2	Deficiencies count product group 2	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.3	Deficiencies count product group 2	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.4	Deficiencies count product group 2	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.5	Deficiencies count product group 2	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.6	Deficiencies count product group 2	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A2.7	Deficiencies count product group 2	(0,1,1)(1,0,0)3	Non-stationary, Seasonal	Assembly
A3.1	Deficiencies count product group 3	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A3.2	Deficiencies count product group 3	(0,1,1)(1,0,0)4	Non-stationary, Seasonal	Assembly
A3.3	Deficiencies count product group 3	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A3.4	Deficiencies count product group 3	(0,1,1)(1,1,0)8	Non-stationary, Seasonal	Assembly

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
A3.5	Deficiencies count product group 3	(0,1,1)	Non-stationary, Non-seasonal	Assembly
A4	Deficiencies count major component	(0,1,1)(0,0,1)14	Non-stationary, Seasonal	Assembly
F1.1	Foundry property 1, line 1	(0,1,1)(0,0,1)9	Non-stationary, Seasonal	Foundry
F1.2	Foundry property 1, line 2	(0,1,1)	Non-stationary, Non-seasonal	Foundry
F2.1	Foundry property 2, line 1	(1,1,1)(0,0,1)6	Non-stationary, Seasonal	Foundry
F2.2	Foundry property 2, line 2	(1,1,1)	Non-stationary, Non-seasonal	Foundry
F3.1	Foundry property 3, line 1	(1,1,1)(1,0,0)4	Non-stationary, Seasonal	Foundry
F3.2	Foundry property 3, line 2	(1,1,1)(0,0,1)2	Non-stationary, Seasonal	Foundry
H1	Heat Treatment Harness, part 1	(0,1,1)	Non-stationary, Non-seasonal	Heat Treatment
H2	Heat Treatment Harness, part 2	(0,1,1)	Non-stationary, Non-seasonal	Heat Treatment
H3	Heat Treatment Harness, part 3	(1,1,1)	Non-stationary, Non-seasonal	Heat Treatment
M1	Machine dimension 1	(1,1,1)	Non-stationary, Non-seasonal	Machining
M2	Machine dimension 2	(1,1,1)	Non-stationary, Non-seasonal	Machining
M3	Machine dimension 3	(1,1,0)(1,0,1)2	Non-stationary, Seasonal	Machining
M4.1	Machine dimension 4, part 1	(2,1,1)(1,0,0)3	Non-stationary, Seasonal	Machining
M4.2	Machine dimension 4, part 2	(1,1,1)	Non-stationary, Non-seasonal	Machining

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
M4.3	Machine dimension 4, part 3	(1,1,1)	Non-stationary, Non-seasonal	Machining
M4.4	Machine dimension 4, part 4	(1,1,1)(0,0,1)3	Non-stationary, Seasonal	Machining
M4.5	Machine dimension 4, part 5	(1,1,0)(0,0,1)3	Non-stationary, Seasonal	Machining
M5	Machine dimension 5	(2,1,1)	Non-stationary AR(2)	Machining
M6	Machine dimension 6	(0,1,1)(0,0,1)4	Non-stationary, Seasonal	Machining
MM1	Measure of fit, assembly 1	(1,1,1)	Non-stationary, Non-seasonal	Matching
R	AIAG capability data set	(0,1,1)	Non-stationary, Non-seasonal	Reference
S1.1	Shaved dimension 1, part 1	(0,1,1)(0,0,1)6	Non-stationary, Seasonal	Shaving
S1.2	Shaved dimension 1, part 2	(0,1,1)	Non-stationary, Non-seasonal	Shaving
S2.1	Shaved dimension 2, part 1	Shewhart	Constant (Shewhart)	Shaving
S2.2	Shaved dimension 2, part 2	Shewhart	Constant (Shewhart)	Shaving
S2.3	Shaved dimension 2, part 3	(0,1,1)	Non-stationary, Non-seasonal	Shaving
S2.4	Shaved dimension 2, part 4	Shewhart	Constant (Shewhart)	Shaving
S2.5	Shaved dimension 2, part 5	(2,1,0)(0,0,1)3	Non-stationary AR(2)	Shaving
S3.1	Shaved dimension 3, part 1	Shewhart	Constant (Shewhart)	Shaving
S3.2	Shaved dimension 3, part 2	Shewhart	Constant (Shewhart)	Shaving
S3.3	Shaved dimension 3, part 3	Shewhart	Constant (Shewhart)	Shaving
S4.1	Shaved dimension 4, part 1	(0,0,1)	Stationary, Non-seasonal	Shaving
S4.2	Shaved dimension 4, part 2	(0,1,1)	Non-stationary, Non-seasonal	Shaving

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
S5	Shaved dimension 5	(2,1,2)(0,0,1)	Non-stationary AR(2)	Shaving
S6	Shaved dimension 6	Shewhart	Constant (Shewhart)	Shaving
TM1	Test machine dimension 1	(0,1,1)(0,0,1)7	Non-stationary, Seasonal	Test Machine
TM2	Test machine dimension 2	(0,1,1)	Non-stationary, Non-seasonal	Test Machine
TM3	Test machine dimension 3	(0,1,1)	Non-stationary, Non-seasonal	Test Machine
TM4	Test machine dimension 4	(0,1,1)(1,0,0)5	Non-stationary, Seasonal	Test Machine
TM5	Test machine dimension 5	(1,0,1)	Stationary, Non-seasonal	Test Machine
G1.1	Ground dimension 1, machine 1	(1,0,0)	Stationary, Non-seasonal	Grinding
G1.2	Ground dimension 1, machine 2	(0,1,1)(0,1,1)4	Non-stationary, Seasonal	Grinding
G2.1	Ground dimension 2, machine 1	(1,0,0)	Stationary, Non-seasonal	Grinding
G2.2	Ground dimension 2, machine 2	(1,1,2)(0,0,1)11	Non-stationary, Seasonal	Grinding
TU1	Turned dimension 1	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU2	Turned dimension 2	Shewhart	Constant (Shewhart)	Turning
TU3.1	Turned dimension 3, part 1	Shewhart	Constant (Shewhart)	Turning
TU3.2	Turned dimension 3, part 2	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU3.3	Turned dimension 3, part 3	(0,1,1)(1,0,0)2	Non-stationary, Seasonal	Turning
TU4.1	Turned dimension 4, part 1	Shewhart	Constant (Shewhart)	Turning
TU4.2	Turned dimension 4, part 2	(0,1,1)(0,0,1)	Non-stationary, Seasonal	Turning
TU4.3	Turned dimension 4, part 3	Shewhart	Constant (Shewhart)	Turning

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
TU5.1	Turned dimension 5, part 1	Shewhart	Constant (Shewhart)	Turning
TU5.2	Turned dimension 5, part 2	(1,1,1)	Non-stationary, Non-seasonal	Turning

(table continues)

TU5.3	Turned dimension 5, part 3	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU6	Turned dimension 6	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU7	Turned dimension 7	(1,2,1)	Non-stationary D(2)	Turning
TU8	Turned dimension 8	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU9	Turned dimension 9	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU10	Turned dimension 10	(2,1,0)(1,0,0)6	Non-stationary AR(2)	Turning
TU11.1	Turned dimension 11, part 1	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU11.2	Turned dimension 11, part 2	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU12.1	Turned dimension 12, part 1	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU12.2	Turned dimension 12, part 2	(0,1,1)(1,0,0)7	Non-stationary, Seasonal	Turning
TU13.1	Turned dimension 13, part 1	(1,1,1)(1,0,0)2	Non-stationary, Seasonal	Turning
TU13.2	Turned dimension 13, part 2	(1,1,1)	Non-stationary, Non-seasonal	Turning
TU14.1	Turned dimension 14, part 1	(0,1,1)(0,0,1)8	Non-stationary, Seasonal	Turning
TU14.2	Turned dimension 14, part 2	(1,1,1)(0,0,1)5	Non-stationary, Seasonal	Turning

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
TU15	Turned dimension 15	Shewhart	Constant (Shewhart)	Turning
TU16	Turned dimension 16	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU17	Turned dimension 17	(2,1,0)	Non-stationary AR(2)	Turning
TU18	Turned dimension 18	Shewhart	Constant (Shewhart)	Turning
TU19.0	Turned dimension 19, all operators	Shewhart	Constant (Shewhart)	Turning
TU19.1	Turned dimension 19, operator 1	Shewhart	Constant (Shewhart)	Turning
TU19.2	Turned dimension 19, operator 2	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU19.3	Turned dimension 19, operator 3	Shewhart	Constant (Shewhart)	Turning
TU20.0	Turned dimension 20, all operators	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU20.1	Turned dimension 20, operator 1	Shewhart	Constant (Shewhart)	Turning
TU20.2	Turned dimension 20, operator 2	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU20.3	Turned dimension 20, operator 3	Shewhart	Constant (Shewhart)	Turning
TU21.0	Turned dimension 21, all operators	(0,0,0)(0,0,1)8	Stationary, Seasonal	Turning
TU21.1	Turned dimension 21, operator 1	Shewhart	Constant (Shewhart)	Turning
TU21.2	Turned dimension 21, operator 2	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU21.3	Turned dimension 21, operator 3	(0,0,0)(0,0,1)2	Stationary, Seasonal	Turning
TU22	Turned dimension 22	(0,0,1)	Stationary, Non-seasonal	Turning
TU23	Turned dimension 23	(0,0,1)	Stationary, Non-seasonal	Turning
TU24.0	Turned dimension 24, all observations	Shewhart	Constant (Shewhart)	Turning
TU24.1	Turned dim 24, operator 1, 1-165	(0,1,1)(0,0,1)8	Non-stationary, Seasonal	Turning
TU24.2	Turned dim 24, operator 1, 166-206	(0,0,0)(0,0,1)7	Stationary, Seasonal	Turning

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
TU24.3	Turned dimension 24, operator 2	(0,0,1)(0,0,1)5	Stationary, Seasonal	Turning
TU24.4	Turned dimension 24, operator 3	(0,1,1)	Non-stationary, Non-seasonal	Turning
TU25.0	Turned dimension 25, all operators	(0,1,1)(0,0,1)2	Non-stationary, Seasonal	Turning
TU25.1.1	Turned dimension 25, operator 1, 1-86	Shewhart	Constant (Shewhart)	Turning
TU25.1.2	Turned dimension 25, operator 1, 87-160	(0,1,1)(1,0,0)7	Non-stationary, Seasonal	Turning
TU25.1.3	Turned dimension 25, operator 1, 161-206	Shewhart	Constant (Shewhart)	Turning
TU25.2	Turned dimension 25, operator 2	(0,1,1)(0,0,1)	Non-stationary, Seasonal	Turning
TU25.3	Turned dimension 25, operator 3	Shewhart	Constant (Shewhart)	Turning
TU26.0	Turned dimension 26, all operators	(0,1,1)(0,0,1)14	Non-stationary, Seasonal	Turning
TU26.1	Turned dimension 26, operator 1	(0,1,1)(1,0,0)7	Non-stationary, Seasonal	Turning
TU26.2	Turned dimension 26, operator 2	Shewhart	Constant (Shewhart)	Turning
TU26.3	Turned dimension 26, operator 3	(0,0,0)(0,0,1)10	Stationary, Seasonal	Turning
W1	Warranty machine 1	(0,1,1)(1,0,0)2	Non-stationary, Seasonal	Warranty
W2	Warranty machine 2	(0,1,1)	Non-stationary, Non-seasonal	Warranty
W3	Warranty machine 3	(0,1,1)	Non-stationary, Non-seasonal	Warranty
W4	Warranty machine 4	(0,1,1)	Non-stationary, Non-seasonal	Warranty
Y1	Yield line 1	(0,1,1)(0,0,1)3	Non-stationary, Seasonal	Yield

(table continues)

Code	Description	ARIMA	ARIMA Type	Process Group
Y2	Yield line 2	(0,1,1)(0,0,1)4	Non-stationary, Seasonal	Yield
Y3	Yield line 3	(1,1,1)(0,1,1)2	Non-stationary, Seasonal	Yield