Proceedings of the Iowa Academy of Science

Volume 45 | Annual Issue Article 44

1938

Fundamental Dimensions and Electrical Units

John A. Eldridge
University of Iowa

FUNDAMENTAL DIMENSIONS AND ELECTRICAL UNITS

JOHN A. ELDRIDGE

Proposition: What dimensions and how many are fundamental is a matter of choice, custom and convenience, not of logical necessity. Limitation: This does not deal with how many and what fundamental entities are required to build a universe; it is simply a question of mensuration; how many and what independent dimensions shall (must) we use to express physical quantities unambiguously, and to make physical equations balance.

(1) Every concept: Length, area, force, work, etc., is to be regarded initially as a different fundamental dimension. Area is not length squared. That is, the number of area units in a rectangle is not the product of the length units except subject to a special assumption (defining equation). Primitive relations are proportions. For circles $A_1/A_2 = r_1^2/r_2^2$ or $A = k_1 r^2$; for squares $A = k_2 L^2$. $k_2 =$ (dimensionally) (A/L^2) and equation checks dimensionally.

(2) A great step is scientific simplicity was introduction of rationalized units. We define the unit of A in terms of L, usually making k_2 equal to unity. The law becomes a definition, k_1 becomes a dimensionless number π. Every independent defining equation reduces the fundamental constants by one.

(3) In mechanics there are such relations as

$$(1)\quad F = k_1 ma$$

$$(2)\quad F = G\frac{m_1 m_2}{r^2}$$

$$(3)\quad F = k_2 \frac{dL}{L} A \text{ (for quartz)}$$

Assuming acceleration with dimensions (L/T^2) dimensions of k_1 are (FT^2/ML).

Such equations including quantities k balance identically; they are also meaningless until it is stated the k is a constant. We have four fundamental mechanical units. Work (defined as Fs) is dimensionally different from kinetic energy ($\frac{1}{2}mv^2$). $W = \frac{1}{2} k_1 mv^2$.

(4) It is conventional to set k_1 equal to a dimensionless unity. F

equals ma becomes a definition instead of a law and F is dimensionally $\frac{ML}{T^2}$. We could alternatively accept a gravitational definition of F setting G equal to unity. This changes the dimensions of force to $\frac{M^2}{L^2}$ and changes dimensions of physical quantities in general. Or setting $k_2 = 1$, $F = (L^2)$; magnitude of the force is the area of a quartz rod which would be doubled in length by it.

(5) Changing the defining equation from (1) to (2) will change the form of physical equations to the extent of changing the proportionality constants (magnitude and dimension) which appear in them. Equations will check dimensionally with any choice. Also any fundamental dimensions can be eliminated by decreeing it dimensionless; the equations still balance.

(6) Mechanical dimensions may be reduced to two or one by additional defining equations. For example we can measure time by the distance light travels: $t = s$; or we can set k_1, G and k_2 all equal to unity. But it is conventional in mechanics to have three fewer defining equations than variables, giving three fundamental variables which in the nomenclature appear as length, time and mass among physicists, as length, time and force among engineers. (In standard engineering practice, k_1 is a dimensionless 1/32 instead of unity.)

(7) In heat the custom is less well defined. T (H) is usually regarded as a fourth fundamental independent of its mechanical interpretations; the quantity of heat is often considered as of the dimensions of mechanical energy (making J dimensionless); this gives entropy the dimensions of $\frac{\text{energy}}{\text{temperature}}$ i. e. $\frac{ML^2}{T^2\Theta}$. This is very inconsistent behavior but one cannot say it is wrong. T (H) is better regarded as energy per degree of freedom, giving entropy dimensions of "degree of freedom" (dimensionless?).

(8) In electricity charge is defined (electrostatically) by the equation $F = q_1 q_2/r^2$. Does this make q dimensionally $\sqrt{(ML^3)/T}$ or should the equation be written $F = q_1q_2/k\, r^2$ where k, the dielectric constant for free space equals unity and is dimensionally $\frac{Q^2T^2}{ML^3}$, charge being another dimension? Although much eloquence and philosophy have been spent on this subject the distinction is meaningless. It leads to questions like this: Electric field equals dynes per unit charge; electric displacement equals 4π charge for square centimeter. In free space these are always equal numerically. But $\frac{F}{Q}$ is dimensionally different from $\frac{Q}{A}$ and we should name one unit Prunes and the other Persimmons unless k is dimen-

sionless and $Q = \sqrt{\frac{(ML^3)}{T}}$. But the number of Prunes always equals the number of Persimmons. So we are back at the beginning. Does k really have dimensions? The thing k really has is the value 1. A more refined experiment on repulsion of charges would not change the value of k but would change the accepted value of q. The unit of charge is numerically dependent upon Fr^2; of course it is not *rcally* Fr^2 any more than force is *really* ma. (This *really* business of course is tail chasing for philosophers.) Dynes per stat-coulomb are identical with coulombs per sq. cm. (subject to $F = \frac{q_1q_2}{r^2}$) exactly as dyne centimeters are equivalent to gm cm^2/sec^2 (subject to $F = ma$).

(9) There are also other independent definitions of q. For example it can be defined from magnetic force between parallel currents:

$F = \frac{2i_1i_2}{r} L$, giving $q_{em}(= it) = q_{es}/c$. With this new defining equation, of course dimensions and constants are charged in all electrical equations.

(10) In practice the "dimensional" unity (dielectric constant) is suppressed. For example: is the dimensionless fine structure constant $2\pi e^2/hc$ or $2\pi e^2/khc$ where $k = 1$?

(11) As opposed to this mathematical formulism which makes all choices of fundamental allowable, we should when possible use natural atomic entities as units in a really absolute sense, thus reducing measurement to counting. Thus charge is the number of electrons. Then Q becomes independent; k a quantity measured in the oil drop experiment. Is Q then a pure number?

REFERENCES

BRIDGMAN'S Dimensional Analysis.
ESCHBACH: Engineering Handbook (Weber).

PHYSICS DEPARTMENT,
UNIVERSITY OF IOWA,
IOWA CITY, IOWA.