R & D Exchange in a Duopoly with Strong Patent Protection

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ABSTRACT. One reason firms engage in research and development is to lower production costs. Strong patent protection provides an additional incentive because firms may earn royalties from licensing their discoveries. This paper models the decision to engage in process research and development in a duopoly. A three stage game is posited and the equilibrium quantity and price of knowledge is calculated under various assumptions.

I. Introduction

Schumpeter [1943, 83] argued that a society might benefit in the long run from sacrificing static efficiency and allowing firms to gain market power through research and development (R&D). Since then economists have analyzed the incentives to innovate under several different market structures. A milestone in the literature on research incentives is d'Aspremont and Jacquemin's [1988] comparison of Cournot and Bertrand competition which uses a model that accounted for spillovers. Their model was the basis for subsequent work comparing multiple forms of competition in dynamic and static equilibria, and for both product and process R&D. (In particular see Qui [1997], Hinloopen [1998] and Kamien et al. [1992].)

Spillovers are the free flow of technical knowledge between competing firms. But spillovers are not the only means by which knowledge moves between firms. Strong patent enforcement eliminates spillovers but creates a market for firms to sell their innovations. Firms can allocate their resources between production and research more efficiently if they understand the market for both their output and their innovations.

The structure of the market for knowledge will differ with the number of firms participating and the type of research each is conducting. I will consider two firms conducting process R&D to lower production costs. I assume that in the output market both firms produce identical goods. Under these conditions I will describe the knowledge market and find the equilibrium quantity and price of innovation exchanged.

3

4   *Major Themes in Economics, Spring 2004*

II. The Model

The production process is a three stage game. First, firms decide the appropriate level of R&D to undertake. Second, firms compete in the knowledge market. Third, firms compete in the output market. Notice that each firm produces technical knowledge and a good or service in different stages. For the sake of clarity suppose that the output of both firms is your favorite color balloon.

   The benefit of R&D is the reduction in the marginal costs of producing balloons during stage three. Assume that the production costs for both firms are linear, but not necessarily equal. Thus the marginal cost of production is constant and equal to the average cost.

   Following d'Apremont and Jacquemin [1988] I assume that the R&D expenditure function is quadratic in terms of the value of the innovation. In other words,

$$\text{(Cost of Innovation)} = (\lambda/2) \cdot \text{(Reduction in Marginal Cost)}^2.$$

In this construction the benefit from investing in R&D exhibits diminishing marginal returns. The scalar $(\lambda/2)$ represents the efficiency of the firm at reducing its costs. Large λ implies high costs associated with innovating. As a firm becomes a more efficient innovator, λ decreases. Note that λ is positive.

   Total Revenue for each firm is the quantity of balloons produced times the price of balloons. Both firms face the same market demand function. Assume a linear demand function where price depends only on the quantity produced. Total output, Q, is the sum of the number of balloons that each firm produces. So $P = a - bQ$, where $Q = q_1 + q_2$.

   Firms can select three methods of competition in the output market. First, firms can collude and act as a single price setting monopoly. Alternatively firms can choose either Cournot or Bertrand competition. In Cournot competition a producer chooses the level of output that will maximize profits given the output of the competing firm. In Bertrand competition a firm sets the price that maximizes profits given the price set by the competing firm. The possibility of collusion in the output market will be ignored in the analysis because it is illegal. Given that the firms both produce balloons (perfect substitutes), the two firms will enter Cournot competition in the third stage because profits are unambiguously higher than under Bertrand competition [Singh and Vives, 1984].

III. Analysis

In the second stage of the game both firms have a monopoly on their cost-reducing knowledge. At the same time each is a monopsonist because it is the only willing buyer of the other's knowledge. The market for knowledge is therefore a bilateral monopoly and poses some problems for traditional equilibrium analysis. Henderson and Quandt explain:

> A monopolist does not have a supply function relating price and quantity. He selects a point on his buyers' demand function that maximizes his profit. Similarly, a monopsonist does not have an input demand function. He selects a point on the sellers' supply function that maximizes his profit…It is not possible for the seller to behave as a monopolist and the buyer to behave as a monopsonist at the same time. The seller cannot exploit a demand function that does not exist, and the buyer cannot exploit a supply function that does not exist [Henderson and Quandt, 1971, 244].

Assume that firms supply a fixed amount of innovation when they participate in the R&D market. Since R&D investment takes place in stage one neither can supply more knowledge than they have already acquired. In addition, assume that neither firm can supply a fraction of its innovation. If a firm has found a way to reduce costs by two dollars it cannot sell the knowledge of how to reduce costs by only one dollar. There are three possible outcomes in a bilateral monopoly: the two firms could collude and find a mutually beneficial agreement, one firm could dominate and dictate the exchange price, or the market could break down [Henderson and Quandt, 1971, 244]. Each case will be addressed separately.

Kamien et al. [1992] identify four R&D investment strategies for firms conducting research and competing in the output market. The first is for the two firms to enter an R&D competition. Each firm chooses an R&D investment that maximizes its profits given the investment of the other firm. Neither firm can avoid duplication. Suppose that one firm can lower marginal costs by three dollars while a more innovative firm can lower marginal costs by four dollars. The total reduction from their combined knowledge is only four dollars because the more innovative firm duplicates the research of the other. When the R&D competition

6   *Major Themes in Economics, Spring 2004*

strategy is adopted one firm will dominate the exchange of knowledge or no exchange will be made.

A second strategy for the two firms is to form an R&D cartel. They coordinate their R&D investment to maximize joint profits. Duplication cannot be avoided since the firms coordinate only their research investment and not their research activities. An R&D cartel is one collusion strategy.

A third strategy for the two firms is to enter a Research Joint Venture (RJV) competition in which each firm decides its R&D investment given the actions of the other firm, but duplication is eliminated through the coordination of research activities. Firms who are able to lower marginal costs by three and four dollars independently realize a combined reduction of seven dollars. They both share the results of their individual investment without cost. RJV competition is another collusion strategy.

In the fourth strategy the two firms form an RJV cartel where they coordinate their R&D investment and their R&D activities to maximize joint profits. Duplication is avoided and the results of the research of each firm are shared without cost. This is the last collusion strategy.

A. FIRMS COLLUDE

If the two firms enter an R&D cartel they face a two-step bargaining process. They must first determine the amount of research that maximizes joint profits and then determine the price that evenly distributes the joint profit between them [Henderson and Quandt, 1971, 247].

Entering an R&D cartel allows the two firms to coordinate their research investment but not their research activities. Neither firm can avoid duplication if it independently undertakes any R&D expenditure. Therefore only one firm, the better innovator, conducts research.

The level of R&D investment that maximizes joint profit is a sub-game perfect equilibria found using reverse induction. Given any outcome in the first stage of the production process firms will maximize profits with respect to output in the third stage. Since collusion is prohibited in the output market firms will enter Cournot competition. The profit function for each firm is given by

$$_1 = (a-bQ)q_1 - [A-r]q_1 - (\lambda_1/2)r^2$$
and
$$_2 = (a-bQ)q_2 - [B-s]q_2 - (\lambda_2/2)s^2$$

where A and B are the marginal (and average) cost to firms 1 and 2. r and s are the reduction in marginal cost to each firm resulting from their R&D investment. The Counrot solution to the output market gives the quantity of balloons produced by each firm. See Appendix I for the derivation of this solution.[1]

$$q_1 = (1/3)[B-2A+2r+a-s]/b$$

$$q_2 = (1/3)[A-2B+2s+a-r]/b$$

Let π_j denote the joint profit function where $\pi_j = \pi_1 + \pi_2$. Since the R&D investment is coordinated r is equivalent to s (let X = r = s) and only the better innovator conducts research. In the joint profit function λ is the minimum of λ_1 and λ_2. Thus,

$$\pi_j = aq_1+aq_2-bq_1{}^2-bq_2{}^2-2bq_1q_2-Aq_1-Bq_2+Xq_1+Xq_2-(\lambda/2)X^2.$$

The value of X that maximizes joint profit is

$$X = 2\frac{2a - A - B}{-4 + 9\lambda b}.$$

I assume that a > A,B to guarantee profitable production and assume b > $(4/9\lambda)$ to satisfy second order conditions. See Appendix II for the derivation of this solution.

X is the amount of research undertaken by firms colluding in an R&D cartel. Without loss of generality assume that firm one undertakes the R&D expense (that is, assume $\lambda_1 < \lambda_2$). Under the agreement of an R&D cartel firm 2 must compensate firm 1 so that profits are equal. Firm 2 might be required to pay more than half of the R&D expense. The terms of an R&D cartel stipulate that both firms share profits equally, not merely the R&D expense. The exchange price, derived in Appendix II, is $\lambda_1(8a^2-27A^2\lambda_1b+14A^2-10B^2+27B^2\lambda_1b+54aA\lambda_1b-32aA+16aB-54aB\lambda_1b+4AB)/(9\lambda_1b-4)^2$.

The two firms could also enter RJV competition. The implied exchange price of technical knowledge is zero; the amount of knowledge exchanged, X, remains to be found. In RJV competition each firm decides the R&D expenditure that will maximize profit given the investment of the other firm. Duplication is avoided through a

8   *Major Themes in Economics, Spring 2004*

coordination of research activities. The quantity exchanged in the knowledge market is the sum of the research undertaken by each firm. So, X= r + s where r and s are found by maximizing π_1 and π_2, given q_1 and q_2.

$$X = 4\frac{a+B-2A-s}{-8+9gb} + \frac{4(a-2B+A-r)}{-8+9hb}$$

I assume b > (8/9g), (8/9h) to satisfy second order conditions and A,B > X to guarantee positive costs to both firms. See Appendix III for the derivation of this solution.

The final collusion strategy for the two firms is to form an RJV cartel. Firms choose the amount of research that will maximize joint profits. The R&D investment of the firms is the same as if they entered an R&D cartel but by entering an RJV cartel the two firms can avoid duplication. Instead of one firm conducting research, both firms coordinate their R&D activities to produce the optimal amount of research at the lowest cost. Firms exchange their knowledge without compensation.

In an RJV cartel the firms behave as a multiple plant monopoly. The assumption is that there is no externality associated with coordinating R&D activities. Together each firm is only as good of an innovator as it would be without cooperation. They maximize joint profit by directing the production of technical knowledge between the research facilities of each firm. The multiple-plant solution found in Appendix IV is

$$r = -2\frac{-hba+2a-2A+5hbA-4hbB}{4-10gb-10hb+9hb^2g}$$

$$s = 2\frac{-2a+gba+4gbA+2B-5Bgb}{4-10gb-10hb+9hb^2g}$$

where r and s are the amount of knowledge produced by firm one and firm two, respectively.

B. ONE FIRM DOMINATES IN THE KNOWLEDGE MARKET

The cost structure and innovative efficiency of a firm has no bearing on

its ability to dominate in the R&D market. Forces outside the model determine the ability of either firm to negotiate a beneficial exchange. In the first stage both firms enter R&D competition to decide their research expenditure.

Each firm maximizes profit given the R&D investment of the other. So the amount of knowledge produced by

$$\text{Firm 1} = r = \frac{-a - B + 2A + s}{-8 + 9gb} \quad \text{and} \quad \text{Firm 2} = s = -4\,\frac{-a + 2B - A + r}{-8 + 9hb}.$$

I assume $b > (8/9g)$, $(8/9h)$ to satisfy second-order conditions. See Appendix V for the derivation of this solution.

Once a firm establishes the dominant position it will dictate the terms of the exchange. The exchange price is determined during the negotiations so it cannot be found using this model. Yet there are parameters that the price must fit. The exchange price must be greater than zero. Further, the non-dominant firm cannot be forced to pay more than the value of the knowledge. Duplication cannot be avoided in R&D competition so the maximum benefit of purchasing knowledge is the difference between the amounts of knowledge produced by both firms. R&D lowers the marginal cost of producing balloons, so its value is the amount of the reduction times the quantity of balloons produced.

Suppose that firm 1 is more innovative in stage one. In stage two firm 2 will pay between 0 and $(r-s)q_2$ and receive $(r-s)$ amounts of useful knowledge.

C. THE MARKET BREAKS DOWN

Firms determine their collusion strategy before either begins any R&D activities. Exchange of knowledge is implicit in an agreement to coordinate research efforts or expenses. If the firms adopt any collusion strategy an exchange will be made so long as both firms honor the terms of the agreement.

The potential for market break down arises only if the firms enter R&D competition. Since both firms are producing balloons the more innovative firm may not have an incentive to divulge its technical knowledge since lower production costs make a firm more competitive in the output market. Further investigation is needed to determine the precise

10   *Major Themes in Economics, Spring 2004*

cost and demand circumstances that cause a firm to hoard its technical knowledge.

IV. Conclusion

Technical knowledge can flow throughout an industry by several mechanisms. Under strong patent protection the mechanism is a market. The analysis of the knowledge market here is limited to the case of two firms producing identical goods. Several knowledge markets remain to be studied as the number of firms and the type of goods produced vary. Also, this discussion focused on the conditions under which exchange is made. The economic circumstances that inhibit exchange are just as interesting to explore.

The research expenditure function is assumed to be quadratic so that it exhibits diminishing marginal returns. The ability of a firm to determine its research expenditure function is dependant on its ability to determine the value of λ. If the value of λ cannot be known then the model loses its ability to describe the market for knowledge accurately. When firms coordinate their R&D activities I assumed that they are just as innovative as if they had not cooperated. An examination of how the amount research undertaken changes in response to an innovative externality is needed.

With only two firms in an industry the market for knowledge is a bilateral monopoly with three possible outcomes. Firms can collude, one firm can dominate the exchange, or no exchange will be made. If firms decide to collude they have three strategies for deciding their R&D investment: form an R&D cartel, an RJV cartel, or enter an RJV competition. Without collusion both firms will adopt the R&D competition strategy whether or not an exchange is anticipated.

The model and analysis above provide some predictive ability so that firms can responsibly choose a research strategy and investment. By assuming patent protection the three-stage model allows for an analysis of licensing practices that are common in many industries.

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Sly: R&D Exchange in a Duopoly **11**

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Endnotes

1. Calculations in all appendices were completed using Maple 6™

12   *Major Themes in Economics, Spring 2004*

Calculations in all appendices were completed using Maple 6 software.

Appendix I

> **restart;**

p1 and p2 are the respective profit functions where t=q1, v=q2, g=(lambda 1) and h=(lambda 2).

> **p1:= a*t-b*t^2-b*t*v-A*t+r*t-(g/2)*r^2;**

$$p1 := at - bt^2 - btv - At + rt - \frac{1}{2}gr^2$$

> **p2:= a*v-b*v^2-b*t*v-B*v+s*v-(h/2)*s^2;**

$$p2 := av - bv^2 - btv - Bv + sv - \frac{1}{2}hs^2$$

Take the derivative of each profit function with respect to output.

> **c1:=diff(p1,t);**

$$c1 := a - 2bt - bv - A + r$$

> **c2:=diff(p2,v);**

$$c2 := a - 2bv - bt - B + s$$

Set each derivative equal to zero to find each firm's reaction function. Now solve the system of reaction functions simultaneously for t=q1 and v=q2 to obtain the Cournot solution.

> **with (linalg):**

Warning, the protected names norm and trace have been redefined and unprotected

> **A:=matrix(2,2, [-2*b, -b,-b,-2*b]);**

$$A = \begin{bmatrix} -2b & -b \\ -b & -2b \end{bmatrix}$$

> **Z:=matrix(2,1, [A-r-a, B-a-s]);**

$$Z := \begin{bmatrix} A - r - a \\ B - a - s \end{bmatrix}$$

Sly: R&D Exchange in a Duopoly   **13**

The Maple function linsolve finds the vector x that satisfies the equality Ax=Z. Let x=[q1, q2]

> **linsolve (A,Z);**

$$\begin{bmatrix} -\dfrac{1}{3}\dfrac{1-a+2A-B+s-2r}{b} \\[2ex] \dfrac{1}{3}\dfrac{A-2B+a+2s-r}{b} \end{bmatrix}$$

Then q1= (1/3)(-s-2A+2r+a+B)/b and q2= (1/3)(2s+A-r+a-2B)/b.

14   *Major Themes in Economics, Spring 2004*

Appendix II

> **restart;**

We need to find the combined profit function where r and s are the same, call it x. Even though the firms can collude to innovate they cannot collude in the output market. Thus t and v will be determined according to the Cournot solution found in Appendix I. Now find x which maximizes combined profits, pc=p1 + p2.

> **pc:=a*t+a*v-b*t^2-b*v^2-2*b*t*v-A*t-B*v+x*t+x*v-(1/2)*(lambda)*x^2;**

$$pc = at + av - bt^2 - bv^2 - 2btv - At - Bv + xt + xv - \frac{1}{2}\lambda x^2$$

> **t:=(1/3)*(-2*(A-x)+a+B-x)/b;**

$$t = \frac{1}{3}\frac{-2A+x+a+B}{b}$$

> **v:=(1/3)*(A-x+a-2*(B-x))/b;**

$$v = \frac{1}{3}\frac{A+x+a-2B}{b}$$

> **simplify(pc);**

$$-\frac{1}{18}\frac{4aA-8ax-4a^2+4aB+9\lambda\,x^2b-10A^2+4Ax+16AB-4x^2+4xB-10B}{b}$$

> **diff(pc,x);**

$$\frac{2}{3}\frac{a}{b}-\frac{1}{9}\frac{-2A+x+a+B}{b}-\frac{1}{9}\frac{A+x+a-2B}{b}-\frac{1}{3}\frac{A}{b}-\frac{1}{3}\frac{B}{b}+\frac{\frac{2}{3}x}{b}-\lambda\,x$$

Set the derivative of pc equal to zero to find critical value of x.

> **solve(%=0, x);**

$$2\frac{2a-A-B}{-4+9\lambda\,b}$$

Sly: R&D Exchange in a Duopoly   **15**

> **restart;**

Now we have the value of x that the more efficient innovator will undertake and give to the less innovative firm. WLOG suppose lambda1<lambda2. Then find the price that firm 2 pays firm 1 which equalizes profit.

> **pp1:=a*t-b*t^2-b*t*v-A*t+x*t-((lambda)/2)*x^2;**

$$ppl := at - bt^2 - btv - At + xt - \frac{1}{2} \lambda x^2$$

> **pp2:=a*v-b*v^2-b*t*v-B*v+x*v-P;**

$$pp2 := av - bv^2 - btv - Bv + xv - P$$

>   **subs(t=(1/3)*(-2*(A-x)+a+B-x)/b,   v=(1/3)*(A-x+a-2*(B-x))/b, x=2*(2*a-A-B)/(9*(lambda)*b-4), pp1=pp2);**

$$\frac{1}{3}\frac{a\left(-2A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a+B\right)}{b} - \frac{1}{9}\frac{\left(-2A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a+B\right)^2}{b}$$

$$-\frac{1}{9}\frac{\left(-2A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a+B\right)\left(A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a-2B\right)}{b}$$

$$-\frac{1}{3}\frac{A\left(-2A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a+B\right)}{b}$$

$$+\frac{\dfrac{2}{3}(2a-A-B)\left(-2A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a+B\right)}{(9\lambda b-4)b} - \frac{2\lambda(2a-A-B)^2}{(9\lambda b-4)^2} =$$

$$\frac{1}{3}\frac{a\left(A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a-2B\right)}{b} - \frac{1}{9}\frac{\left(A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a-2B\right)^2}{b}$$

$$-\frac{1}{9}\frac{\left(-2A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a+B\right)\left(A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a-2B\right)}{b}$$

$$-\frac{1}{3}\frac{B\left(A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a-2B\right)}{b}$$

$$+\frac{\dfrac{2}{3}(2a-A-B)\left(A+\dfrac{2(2a-A-B)}{9\lambda b-4}+a-2B\right)}{(9\lambda b-4)b}-P$$

> **simplify(%);**

$(4\,A^2-8\,A\,B+4\,B^2+9\,a^2\,\lambda^2\,b^2-8\,a^2\,\lambda\,b+36\,A^2\,\lambda^2\,b^2-26\,A^2\,\lambda\,b$
 $-14\,B^2\,\lambda\,b+9\,B^2\,\lambda^2\,b^2-36\,a\,A\,\lambda^2\,b^2+20\,a\,A\,\lambda\,b-4\,a\,B\,\lambda\,b$
 $+18\,a\,B\,\lambda^2\,b^2-36\,A\,\lambda^2\,b^2\,B+32\,A\,\lambda\,b\,B)\,/\,(b\,(9\,\lambda\,b-4)^2\,)$
 $=(4\,A^2-8\,A\,B+4\,B^2+9\,a^2\,\lambda^2\,b^2+9\,A^2\,\lambda^2\,b^2-12\,A^2\,\lambda\,b$
 $-24\,B^2\,\lambda\,b+36\,B^2\,\lambda^2\,b^2+18\,a\,A\,\lambda^2\,b^2-12\,a\,A\,\lambda\,b+12\,a\,B\,\lambda\,b$
 $-36\,a\,B\,\lambda^2\,b^2-36\,A\,\lambda^2\,b^2\,B+36\,A\,\lambda\,b\,B-16\,P\,b-81\,P\,b^3\,\lambda^2$
 $+72\,P\,b^2\,\lambda)\,/\,(b\,(9\,\lambda\,b-4)^2\,)$

> **Price:=solve(%, P);**

Price$:=\lambda\,(8\,a^2-27\,A^2\,\lambda\,b+14\,A^2-10\,B^2+27\,B^2\,\lambda\,b+54\,a\,A\,\lambda\,b$
 $-32\,a\,A+16\,a\,B-54\,a\,B\,\lambda\,b+4\,A\,B)\,/\,(9\,\lambda\,b-4)^2$

This is the price that the less innovative firm pays to equalize profits when they collude as an R&D cartel.

Appendix III

> **restart;**

t and v are the output produced in stage 3.

> **t:=(1/3)*(a+B-2*A+2*r-s)/b;**

$$t = \frac{1}{3}\frac{a+B-2A+2r-s}{b}$$

> **v:=(1/3)*(-2*B+A-r+a+2*s)/b;**

$$v = \frac{1}{3}\frac{-2B+A-r+a+2s}{b}$$

In stage one each firm decides its R&D investment given the investment of the other firm and given the output decided according to Cournot competition in stage 3. So each firm maximizes profit given t and v with respect to either r or s.

> **p1:=a*t-b*t^2-b*t*v-A*t+r*t-(g/2)*r^2:**
> **p2:=a*v-b*v^2-b*t*v-B*v+s*v-(h/2)*s^2:**
> **diff(p1, r);**

$$\frac{2}{3}\frac{a}{b}-\frac{2}{9}\frac{-2B+A-r+a+2s}{b}-\frac{2}{3}\frac{A}{b}+\frac{\frac{2}{3}r}{b}-gr$$

> **R:=solve(%=0, r);**

$$R:=4\frac{a+B-2A-s}{-8+9gb}$$

R is the amount of research conducted by firm one.
> **diff(p2, s);**

$$\frac{2}{3}\frac{a}{b}-\frac{2}{9}\frac{a+B-2A+2r-s}{b}-\frac{2}{3}\frac{B}{b}+\frac{\frac{2}{3}s}{b}-hs$$

> **S:=solve(%=0,s);**

$$S: = 4\frac{a-2B+A-r}{-8+9hb}$$

S is the amount of research conducted by firm two.

The amount of knowledge exchanged, X, in stage 2 is R+S.
> **X:=R+S;**

$$X: = 4\frac{a+B-2A-s}{-8+9gb} + \frac{4(a-2B+A-r)}{-8+9hb}$$

Appendix IV

In an RJV cartel firms act as a multiple plant monopoly. pc is the monopoly (combined) profit function.

> **restart;**

> **pc:= a*t+a*v-b*t^2-b*v^2-2*b*t*v-A*t-B*v+r*t+s*v-(g/2)*r^2-(h/2)*s^2;**

$$pc: = at + av - bt^2 - bv^2 - 2btv - At - Bv + rt + sv - \frac{1}{2}gr^2 - \frac{1}{2}hs^2$$

> **t:=(1/3)*(a+B-2*A+2*r-s)/b;**

$$t = \frac{1}{3}\frac{a+B-2A+2r-s}{b}$$

> **v:=(1/3)*(-2*B+A-r+a+2*s)/b;**

$$v = \frac{1}{3}\frac{-2B+A-r+a+2s}{b}$$

> **dpr:=diff(pc, r);**

$$dpr: = \frac{1}{3}\frac{a}{b} + \frac{\frac{1}{9}(a+B-2A+2r-s)}{b} - \frac{2}{9}\frac{-2B+A-r+a+2s}{b} - \frac{2}{3}\frac{A}{b}$$
$$+ \frac{\frac{1}{3}B}{b} + \frac{\frac{2}{3}r}{b} - \frac{1}{3}\frac{s}{b} - gr$$

> **dps:=diff(pc, s);**

$$dps: = \frac{1}{3}\frac{a}{b} - \frac{2}{9}\frac{a+B-2A+2r-s}{b} + \frac{\frac{1}{9}(-2B+A-r+a+2s)}{b}$$
$$+ \frac{\frac{1}{3}A}{b} - \frac{2}{3}\frac{B}{b} - \frac{1}{3}\frac{r}{b} + \frac{\frac{2}{3}s}{b} - hs$$

> **simplify(dpr);**

$$-\frac{1}{9}\frac{-2a-8B+10A-10r+8s+9grb}{b}$$

> **simplify(dps);**

$$-\frac{1}{9}\frac{-2a+10B-8A+8r-10s+9hsb}{b}$$

Solve the system dps=0 and dpr=0 simultaneously to obtain the multiple plant solution.

> **with (linalg):**

> **J:=matrix (2,2, [(10-9*g*b)/(9*b), -8/(9*b), -8/(9*b), (10-9*h*b)/(9*b)]);**

$$J:=\begin{bmatrix} \dfrac{1}{9}\dfrac{10-9gb}{b} & -\dfrac{8}{9}\dfrac{1}{b} \\[2ex] -\dfrac{8}{9}\dfrac{1}{b} & \dfrac{1}{9}\dfrac{10-9hb}{b} \end{bmatrix}$$

> **K:=matrix(2,1, [(-1/(9*b))*(2*a+8*B-10*A), (-1/(9*b))*(2*a-10*B+8*A)]);**

$$K:=\begin{bmatrix} -\dfrac{1}{9}\dfrac{2a+8B-10A}{b} \\[2ex] -\dfrac{1}{9}\dfrac{2a-10B+8A}{b} \end{bmatrix}$$

> **linsolve(J,K);**

$$\begin{bmatrix} -2\dfrac{-hba+2a-2A+5hbA-4hbB}{4-10gb-10hb+9hb^2g} \\[3ex] 2\dfrac{-2a+gba+4gbA+2B-5Bgb}{4-10gb-10hb+9hb^2g} \end{bmatrix}$$

The first entry in the vector above is the amount of research conducted at firm one's facilities. The second entry is the amount of research conducted at firm two's facilities.

Sly: R&D Exchange in a Duopoly   **21**

Appendix V

Each firm maximizes profit with respect to its R&D investment, given t and v.

> **restart;**
> **pr1:= a*t-b*t^2-b*t*v-A*t+r*t-(1/2)*g*r^2;**

$$prl := at - bt^2 - btv - At + rt - \frac{1}{2}gr^2$$

> **t:=(1/3)*(a+B-2*A+2*r-s)/b;**

$$t = \frac{1}{3}\frac{a+B-2A+2r-s}{b}$$

> **v:=(1/3)*(-2*B+A-r+a+2*s)/b;**

$$v = \frac{1}{3}\frac{-2B+A=r+a+2s}{b}$$

> **FOC:=diff(pr1,r);**

$$FOC := \frac{2}{3}\frac{a}{b} - \frac{2}{9}\frac{-2B+A-r+a+2s}{b} - \frac{2}{3}\frac{A}{b} + \frac{\frac{2}{3}r}{b} - gr$$

> **SOC:=diff(FOC, r);**

$$SOC := \frac{8}{9}\frac{1}{b} - g$$

> **R:=solve(FOC=0,r);**

$$R := -4\frac{-a-B+2A+s}{-8+9gb}$$

R the amount of research undertaken by firm 1
> **pr2:=a*v-b*v^2-b*v*t-B*v+s*v-(h/2)*s^2:**
> **FOC2:=diff(pr2,s);**

$$FOC2 := \frac{2}{3}\frac{a}{b} - \frac{2}{9}\frac{a+B-2A+2r-s}{b} - \frac{2}{3}\frac{B}{b} + \frac{\frac{2}{3}s}{b} - hs$$

22   *Major Themes in Economics, Spring 2004*

> **SOC2:=diff(FOC2, s);**

$$SOC2 = \frac{8}{9}\frac{1}{b} - h$$

> **S:=solve(FOC2=0, s);**

$$S = -4\frac{-a + 2B - A + r}{-8 + 9hb}$$

S is the amount of research undertaken by firm two.