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Proton Orbit Calculations for the Iowa State University Cyclotron

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Abstract. This paper describes the equations governing the proton beam in a small cyclotron and reports calculations carried out on the Iowa State University undergraduate cyclotron. These calculations relate the intensity of the proton beam to changes in the magnetic field of the cyclotron and give the beam height as a function of the radius drawn from the center of the dee box. It was found that changes in the magnetic field strength of only a few gauss can result in a large reduction of the beam strength. Calculations of the beam height show that the beam height changes little with changes in the magnetic field, but rather that the beam height is almost completely determined by the derivative, $\frac{\partial B}{\partial r}$, of the magnetic field.

This work was motivated by the presence of the 1.5 Mev undergraduate physics cyclotron at Iowa State University. The machine has been described by McGuire (1) and the experimental program has been discussed by Burns (2).

Two of the main characteristics of the proton beam in a cyclotron are the intensity of the beam, that is, the number of protons striking a target in a given interval of time, and the distribution of these protons in space at the target. The beam in a cyclotron is large only for a critical value of the magnetic field and drops off rapidly as the field departs from the optimum value. The sharpness and shape of this tuning resonance is a critical operating characteristic of the cyclotron. Another property of interest is the dependence of the beam intensity on target radius, since at some radius the beam intensity will fall off rapidly to values too small to be useful. Focusing of the beam by the electric and magnetic fields of the cyclotron causes the beam to become narrower with increasing radius. The vertical extent of the beam or "beam height" at the target is useful in designing targets and in evaluating the effectiveness of the focusing processes in the machine.

The first objective of this research was to predict theoretically the dependence of the beam intensity on the magnetic field and target radius. A second objective was to study the effect of focusing on beam height. The results may be compared with the experimental determinations of beam behavior carried out in this laboratory by Burns (2) and Crosland (3).

[1] This work was assisted by a grant from the National Science Foundation Undergraduate Research Participation Program.
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Many of the calculations described here are based on theoretical formulae first published by Rose (4). The calculations of beam intensity and beam height were carried out on the Iowa State University undergraduate cyclotron which operates under the following conditions:

Magnetic field strength, B_0	17,000 gauss
Diameter of dees	22.5 cm
Peak dee-to-dee voltage, $2V_0$	10 kv
Dee height, 2k	2.4 cm
Dee gap	1.4 cm

Beam Intensity Studies

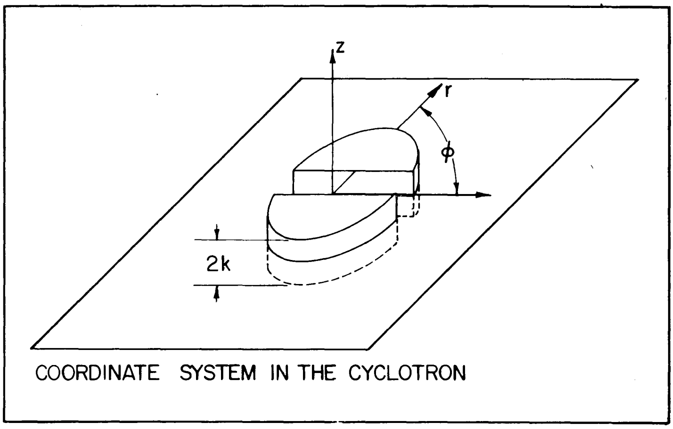

Figure 1. Coordinate system in the cyclotron.

Phase of a Proton

Consider the cylindrical coordinate system shown in Figure 1. Here (r,ϕ,z), as functions of time, give the position of the proton at any instant, where the proton starts from the origin at $t=0$. Now if the voltage across the dees is given by $V = 2V_0 \cos(\omega t + \theta_0)$ where V_0 is the peak r.f. dee-to-ground voltage and ω is the frequency of the r.f. voltage across the dees, θ_0 is defined as the initial phase lag of the proton. The total phase lag, θ, of the proton at any instant is defined by $\theta = \omega t + \theta_0 - \phi$. Its negative is equal to the azimuthal coordinate of the proton when the r.f. voltage reaches its peak value. Hence, the voltage across the dees when the proton is passing through the dee gap is $V = 2V_0 \cos \theta$, and the proton crosses the gap θ electrical radians after the r.f. field is a maximum.

For most phase considerations it is convenient to express θ as a function of r. To obtain such a relation consider the expression $\frac{du}{dr} = \frac{du}{d\theta} \cdot \frac{d\theta}{d\mu} \cdot \frac{d\mu}{dE} \cdot \frac{dE}{dr}$. This method of relating phase to radius was developed by Rose (4). Here $u = \sin\theta$, $\mu = \frac{\phi}{\pi}$, and E is the kinetic energy of the proton.

Now, $\frac{du}{d\theta} = \cos\theta$

Also, $\theta = \omega t + \theta_0 - \phi = \omega t + \theta_0 - \mu\pi$

$$\frac{d\theta}{d\mu} = \omega\frac{dt}{d\mu} - \pi$$

but, $\frac{dt}{d\mu} = \frac{\pi}{d\phi dt} = \frac{\pi m}{eB}$, and $\omega = \frac{eB_1}{m}$

where B_1, called the resonant magnetic field, is defined by the relation $\omega = \frac{eB_1}{m}$. Hence, we arrive at the relation $\frac{d\theta}{d\mu} = \pi\left(\frac{B_1 - B}{B}\right)$.

Now, let $\triangle B = B_1 - B$ so that $\frac{d\theta}{d\mu} = \frac{\pi\triangle B}{B}$.

The kinetic energy, E, is given by $E = \frac{B^2 e^2 r^2}{2m}$ where e and m are the charge and the mass of the proton. Then, $\frac{dE}{dr} = \frac{2E}{r}\left(1 + \frac{r}{B}\frac{\partial B}{\partial r}\right)$. The expression $-\frac{r}{B}\frac{\partial B}{\partial r}$ is defined to be n.

Hence, $\frac{dE}{dr} = \frac{2E}{r}(1-n)$.

Also, $\frac{dE}{d\mu} = \frac{1}{\frac{d\mu}{dE}} = 2eV_0\cos\theta$ so that $\frac{d\mu}{dE} = \frac{1}{2eV_0\cos\theta}$.

Finally, $\frac{du}{dr} = \frac{\pi E\triangle B(1-n)}{eV_0 Br} = \frac{\pi erB\triangle B(1-n)}{2mV_0}$. This equation gives the rate of change of the sine of the phase lag, θ, as a function of r and the magnetic field, B. Integration gives

$$u = \frac{\pi e}{2mV_0}\int_0^r rB\triangle B\,(1-n)\,dr + u_0\,. \quad (1)$$

From this result the phase of the proton can be obtained at any radius if the initial phase lag and the magnetic field are known. For convenience, the quantity $u = \sin\theta$ will sometimes

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be referred to as the phase of the proton where the context makes clear whether u or θ is meant.

The gradient, $\frac{\partial B}{\partial r}$, of the magnetic field of the ISU cyclotron was measured with the field and gradient meter developed by Thoburn (5). The magnetic field, B, was obtained by numerical integration of this gradient. Values of u were obtained by numerical integration of Equation 1. Figure 2 shows u as a function of r for several values of δB where δB is defined as $B_0 - B_1$ and where B_0 is the value of the magnetic field at $r=0$. δB is called

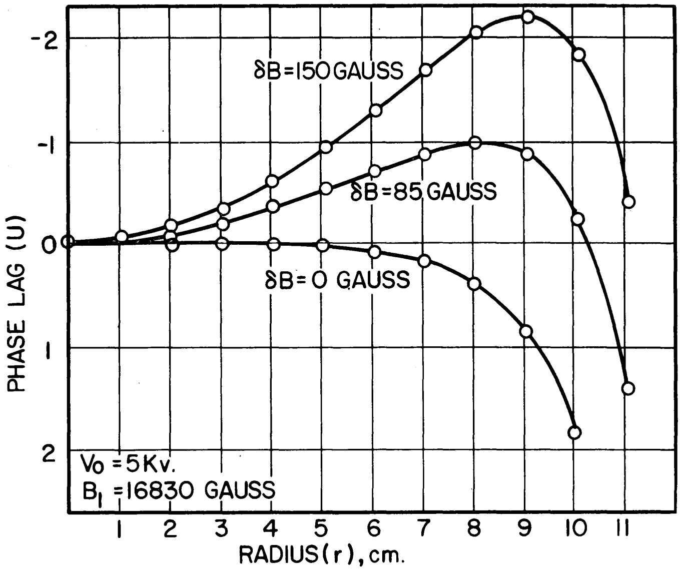

Figure 2. Phase lag as a function of radius for three values of δB.

the tune-down of the magnetic field. In Figure 2, u_0 is taken to be 0. As can be seen from Equation 1 it is only necessary to translate the curves shown in Figure 2 along the vertical axis to obtain u for any given initial phase.

Phase Considerations for a Group of Protons

The relative intensity of the proton beam at any given radius can be calculated by determining which initial phases will allow a proton to reach a given target radius. A proton will be lost from the beam if its phase, θ, becomes greater than $\frac{\pi}{2}$ or less than $-\frac{\pi}{2}$, for in these cases the dee-to-dee voltage, $2\ V_0 \cos \theta$, will be negative and the proton will experience a deceleration. Hence, the problem is to determine which initial phases allow a proton to reach a given target radius with phase such that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. In the following calculations it is assumed that all protons with negative initial phases are lost from the beam because of electric defocusing (4). It is also assumed that for all positive initial phases no protons are lost from the beam because of defocusing, and that the protons are distributed randomly with respect to initial phase θ_0. Thus, a proton with a positive initial phase will be lost from the proton beam if its phase, θ, becomes greater than $\frac{\pi}{2}$ or less than $-\frac{\pi}{2}$. Equivalently the proton will be lost if $u > 1$ or $u < -1$. Figure 3 shows the phase as a function of radius for radii between zero and the target radius r_2. Curves are shown for three protons with different initial phases. (Notice that positive lags are plotted downward). The minimum value of u for curve A, which has zero

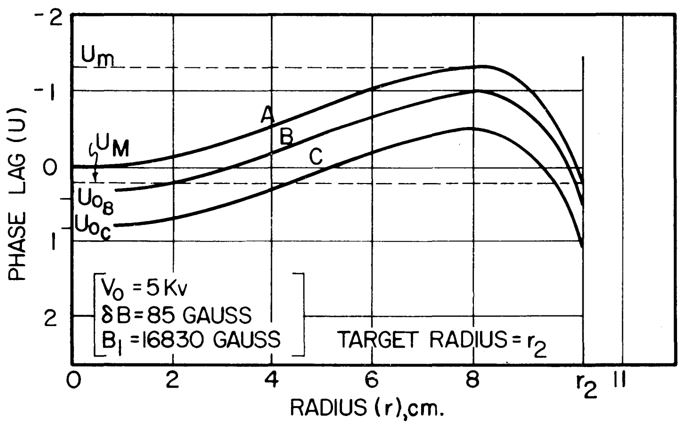

Figure 3. Phase lag as a function of radius for three initial phases.

initial phase, is denoted by u_m and the maximum value of u for this curve is denoted by u_M. A proton having its phase given by curve A would be lost from the beam since $u_m < -1$. If curve A is translated downward along the vertical axis a distance $(-1 - u_m)$, its position corresponds to that of curve B. Hence, the initial phase, θ_{ob}, of curve B is $\sin^{-1}(-1-u_m) = \sin^{-1} u_{ob}$. Now the minimum value of curve B is just -1 so that protons with $\theta_0 < \sin^{-1} u_{ob}$ will be lost from the beam because of excessive phase leads at radii intermediate to 0 and r_2. If curve A is translated downward a distance $1 - u_M$, its position corresponds to that of curve C. Thus the initial phase of curve C is $\sin^{-1}(1-u_M) = \sin^{-1} u_{oc}$. Curve C crosses the line $u=1$ at the target radius $r=$

r_2. Hence all protons with $u_o > u_{oc}$ will cross this line at some radius less than r_2 and will be lost from the beam because of excessive phase lags. Just those protons with initial phases such that $\sin^{-1}(-1-u_m) < \theta_o < \sin^{-1}(1-u_M)$ will reach the target. Thus for the δB shown in Figure 3 the relative intensity, I, of the proton beam at the target radius is given by

$$I = \frac{\sin^{-1}(1-u_M) - \sin^{-1}(-1-u_m)}{\frac{\pi}{2}}, \qquad (2)$$

since this is just the fraction of initial phases, for which $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ for all r. However, in general δB may be such that Equation 2 has to be modified. First of all, Equation 2 is defined only when $-2 \leqq u_m \leqq 0$ and $0 \leqq u_M \leqq 2$. However, if $u_m < -2$, $u_{ob} > 1$ and if $u_M > 1$, $u_{oc} < 0$. In either case the beam intensity at the target radius is zero since in these cases there are no protons with initial phases such that $\sin\theta_o > u_{ob}$ or $\sin\theta_o < u_{oc}$. Also, if $\sin^{-1}(1-u_M) \leqq \sin^{-1}(-1-u_m)$ we see that the beam intensity is again zero at the target radius since here $u_{oc} \leqq u_{ob}$. In other cases, Equation 2 is easily modified. For example, if $u_M < 0$ but $-2 < u_m < 0$, Equation 2 would be changed to $I = \dfrac{\frac{\pi}{2} - \sin^{-1}(-1-u_m)}{\frac{\pi}{2}}$.

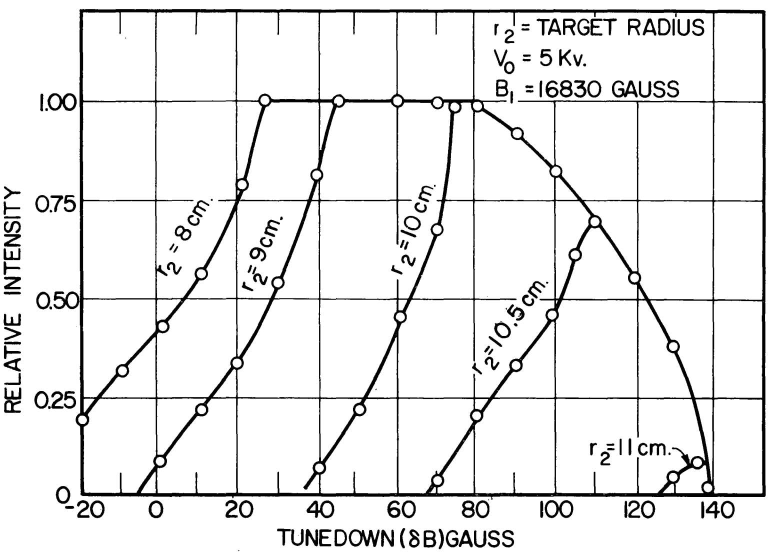

Figure 4. Relative beam intensity in the Iowa State cyclotron.

From Equation 2 and the restrictions stated above, the relative intensity of the proton beam was calculated for values of δB at several different target radii. To do this it was necessary to develop empirical relations for u_m and u_M as functions of δB for various target radii. The results of these calculations are illustrated in Figure 4.

Discussion of Calculations

It is not expected that these intensity curves give an exact quantitative description of the intensity of the proton beam as related to δB at the various radii. Actually, several important phenomena have been neglected in these calculations.

For one, phase grouping, as described by Cohen (6), has not been considered. It was decided that a detailed study of off-center orbits in the cyclotron would have to be made before accurate calculations could be carried out taking phase grouping into account.

Also, the assumption that all protons with negative initial phases would be lost from the beam and that no protons with positive initial phase would be lost from the beam because of defocusing is not entirely justified. It is shown by Rose (4) that the focusing force is, to a good approximation, a linear function of z multiplied by sin θ. Hence, those protons which start out with both small initial phase, θ_o, and large initial height, z_o, would probably be lost from the beam since, as shown in Figure 3, the phase of these protons would become negative at radii small enough that the magnetic focusing effect is much less than the electric defocusing. However, protons having the same initial phase as above might not be lost from the beam if their initial height is small enough. A proton with small initial height and small initial vertical velocity, $\left(\frac{dz}{dt}\right)_o$, may have a fairly large negative initial phase and still reach the target radius. Again, the fact that the electric defocusing force is a linear function of z is the important factor.

When $n = -\frac{r}{B}\frac{\partial B}{\partial r} = 0.2$ a resonance condition occurs between the vertical and radial oscillations of the proton. This resonance which occurs at $r = 10.1$ cm in the ISU cyclotron is possibly responsible for a major loss in beam intensity at large radii. This resonance condition will be mentioned later in connection with a comparison of the above calculations and experimental results obtained by Burns (2).

The somewhat simplified model of the proton orbits in the cyclotron was used in order to simplify the calculations. In obtaining this simplification, however, a certain loss in accuracy of the results of the calculations has necessarily been introduced. The intensity curves shown in Figure 4 are expected to give a correct qualitative description of the beam in the cyclotron and to provide a rough quantitative description.

Comparison of Calculations with Experimental Results

Beam Intensity Curve Shapes. Figure 2 in Burns (2) shows a comparison of the theoretical and experimental intensity curves. It can be noted that the experimental curves are somewhat broader at all target radii. This result is to be expected since, as discussed above, there are probably protons with negative initial phases which contribute to the beam strength.

Also, the theoretical curves are flatter than the experimental curves at maximum intensity. It is believed that this is due to the dependence of defocusing on initial height. In the theoretical calculations there are fairly large intervals of B where no protons with positive initial phase are lost from the beam. However, if the focusing dependence on initial height is considered, some of the protons with large z_0 will be lost from the beam as soon as the phase becomes negative. This will tend to smooth out the tops of the intensity curves so that only one point of maximum intensity is reached.

Decrease of Beam Intensity with Radius. Table 1 of Burns (2) shows the decrease of maximum intensity, I, as a function of the target radius. It is noted that the experimental values of maximum intensity fall off somewhat smoothly with increasing target radius, while the theoretical values do not fall off until a larger target radius is reached and then these theoretical values decrease rapidly. There are several phenomena which may account, at least partially, for this difference.

First, it can be noted in Figure 4 that the interval of δB values for which the relative intensity, I, is equal to 1 decreases with increasing target radius. If it were the case that a maximum value occurred for only one value of δB, as considered above, increasing the target radius might cause the protons with initial phases constituting this maximum to be lost from the beam. In this regard it is noted that the width of the maximum decreases more rapidly with increasing radius in the theoretical curves just as the value of the maximum falls off more rapidly with increasing radius in the experimental case.

The effect of the resonant condition, $n=0.2$, is difficult to determine. The resonant condition does not occur until $r=10.1$ cm. At this point the beam intensity is quite low already; a detailed experimental study is to be carried out later.

It is not at all clear what effect phase grouping might have on this drop-off.

Tune-down at Maximum Intensity. Figure 3 of Burns (2) shows the tune-down at maximum intensity, δB_m, as a function of target radius r_2. It is noted that the theoretical and experimental values differ at small radii but seem to converge at larger radii.

One explanation of this difference is again the possibility of protons with negative initial phases reaching the target.

If phase grouping is affecting the beam in this cyclotron the tendency would be to group values of δB_m near zero. Thus, phase grouping might also account for some of the difference between the theoretical and experimental values of δB_m. However, the effect of phase grouping in the ISU cyclotron is still largely unknown.

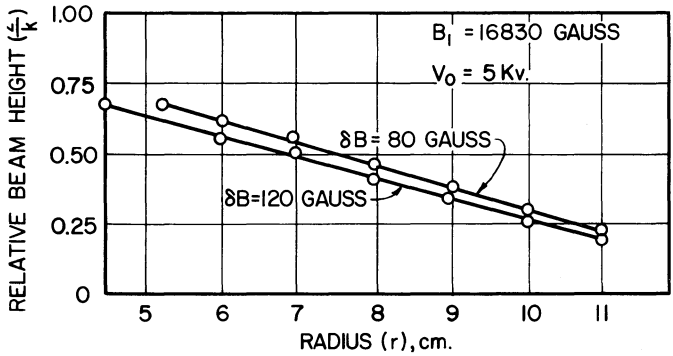

Figure 5. Relative beam height in the Iowa State cyclotron.

Beam Height Studies

The amplitude of the oscillations of a proton in a focusing field is given by Rose (4) as

$$Z \sim A = \left[\frac{\pi e\, V_0 \sin\theta}{E} - \frac{\pi^2 er}{B_Z}\frac{\partial B_Z}{\partial r} \right]^{-1/4} . \qquad (3)$$

The envelope of these oscillations is given by

$$Z = \frac{k\,A}{A_{max}} \qquad (4)$$

where k is one-half the dee height and A_{max} is the maximum value of A. Calculations were carried out on the ISU cyclotron using the above equations. Figure 5 shows beam height as a function of radius for two values of δB. These curves are simply plots of Equation 4 over all initial phases. It can be noted that the beam height does not change considerably with a change in the tune-down. In Equation 3 the second term, $\frac{-\pi^2 er}{B_Z} \frac{\partial B_Z}{\partial r}$, is dominant for radii greater than 6 cm. Hence, the beam height is dependent almost completely on the ratio of the gradient of the magnetic field to the magnetic field. In the ISU cyclotron, which has a relatively large magnetic field gradient, the beam height vs. radius curve is approximately linear for radii greater than 6 cm.

Acknowledgments

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