Proceedings of the Iowa Academy of Science

Volume 74 | Annual Issue

Article 25

1967

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Recommended Citation

Thomas, Barbara and Tuttle, Sherwood D. (1967) "Differentiation of Drift Topographies By Statistical Analysis of Slope Data," *Proceedings of the Iowa Academy of Science, 74(1),* 147-159.
Available at: https://scholarworks.uni.edu/pias/vol74/iss1/25

Differentiation of Drift Topographies By Statistical Analysis of Slope Data

BARBARA THOMAS [1] AND SHERWOOD D. TUTTLE

Abstract. The purpose of this study is an investigation of slope development by statistical analysis of slope angle, slope length, relief and stream gradient data developed on four different glacial drift deposits in Iowa. Measurement from randomly selected points on 1:24,000 topographic maps provide the data. Analysis of variance is used with both F-ratios and between-group t-tests. Differences from comparisons based on the age of the different glacial drifts were found to be significant for all parameters. Significant differences were also found for some of the variables in comparisons based on physiographic classification, stream order of the nearest stream, and orientation of the slope. Relationships among the variables are indicated by correlation coefficients with significance determined by t-tests.

THE PROBLEM

The purpose of this study is to analyze quantitative data from the different glacial drift areas in Iowa to see if there are differences as a means of investigating mode of slope development. The data treated include various elements of slope and stream gradient measurements.

The factors of climate and bedrock control, though likely to influence slope development are not considered. Climate is assumed to be constant throughout the study area, and glacial drift is assumed to be thick enough to eliminate bedrock control as a factor. The other major assumption is that each glacial drift area was in essentially the same condition initially. Thus, differences in the elements of slope will be attributed to effects of variables considered in this study on post glacial erosion: age, materials, stream order, orientation and distance from base level.

Analysis of variance is used for testing the hypotheses involved. Arithmetic means are used as the test statistic for magnitude and standard deviation as the measure for dispersion. F-ratio, t-tests and Fisher's product-moment coefficient of correlation were computed for the various comparisons. The level of significance was set at 5%.

One of the assumptions in the use of ANOVA is that the data are normally distributed. While some departure is permissible, log and log-log transformations are often used to improve the model in this respect (Krumbein and Miller, 1953, p. 527). Figure 1, a histogram of the slope data for the Iowan sample, is fairly representative of the skewness of the sample distributions.

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[1] This study was completed to fulfill the research in science requirement for the Ph.D. degree in Science Education at The University of Iowa.

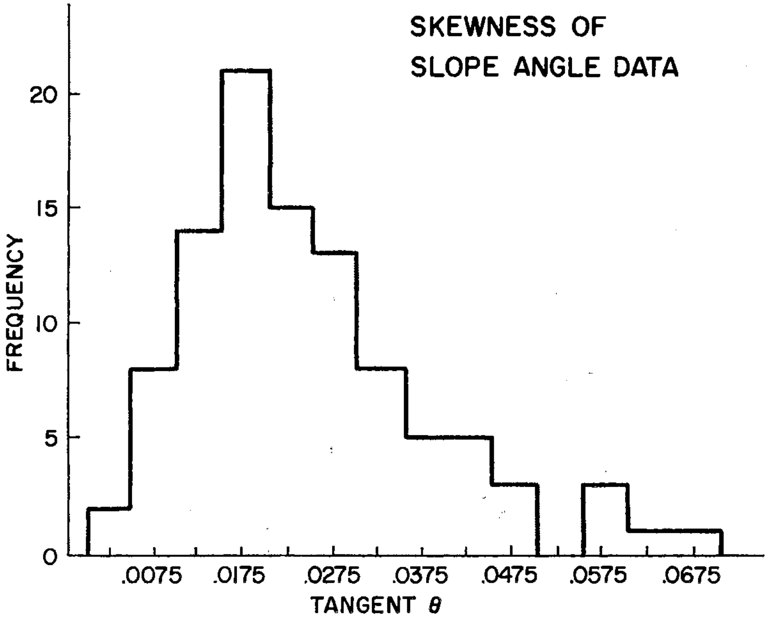

Figure 1. Skewness of the slope angle data for the Iowan sample.

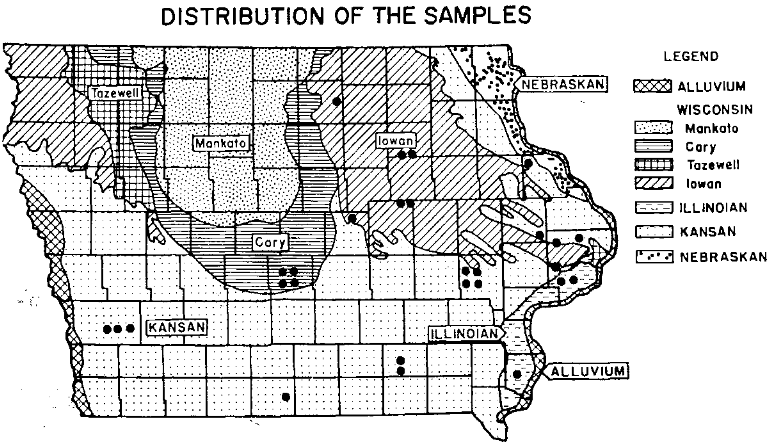

Figure 2. Distribution of the samples.

Cumulative percentage frequency plots on probability paper and on log-probability paper to achieve linearity preceded the use of transformations. The variables, their dimensions and necessary transformations are listed in Table 1. The tangent θ and stream gradient data were multiplied by 10^4 to avoid negative logarithms.

Table 1.

Variable	Dimension	Transformation
Relief	feet	log R
Slope Length	feet	log S. L.
Tangent	dimensionless	log 10^4xtan θ
Stream Gradient	dimensionless	log-log 10^4 S. G.

Another condition for the use of ANOVA involves testing for homogeneity of variance. The validity of this assumption was investigated for the most heterogeneous comparisons by means of $F_{max} = s^2{}_{max}/s^2{}_{min}$ (Hartley, 1950, p. 310). This F_{max} value was found to be significant at the 5% level thereby rejecting the hypothesis of homogeneous variance. However, since in no instance was F_{max} greater than 2.04, the F values reported may be regarded as close approximations and the validity of the analysis of variance is not seriously impaired. If the reader wishes to drop the significance level to 1%, he will find the conclusions to be unaltered.

PROCEDURES

The parameters were measured from 1:24,000 topographic maps (Strahler, A.N., 1956, p. 679). Figure 2 illustrates the distribution of the samples in the Kansan, Illinoian, Iowan and Cary drift areas. The criteria for the non-random initial selection of these maps were the limited number of 1:24,000 maps available and avoiding areas of direct drainage into the Mississippi River. Local base level was anticipated to be too influential to use any quadrangles bordering the Mississippi River. However, the sampling for data on each of the maps was done at random. A grid numbered from 00 to 99 in each direction and a table of random numbers to obtain the coordinates were used to obtain independent random samples from each drift area (Strahler, 1954, p. 4 and Tate, 1955, p. 568-569).

For the materials comparisons, Kay's physiographic classifications were used: loess mantled erosional, unmodified erosional, drift mantled erosional, loess depositional and drift depositional (Kay and Apfel, 1928, p. 37). A generalized soil association classification was also used (Anderson and Welp, 1960, p. 40). These divisions, the topographic maps and the sample size are listed in Table 2.

Table 2. Materials Classification of the Samples

Quadrangle Name	Age of Glacial Drift	Kay's Physiographic Classification	Soil Assoc.*	Sample Size
Lowden	Kan	Loess M. Eros.	F	23
New Vienna	Kan	Loess M. Eros.	F	22
Mechanicsville SE	Kan	Loess M. Eros.	TM	10
Mechanicsville SW	Kan	Loess M. Eros.	F	10
Mechanicsville NE	Kan	Loess M. Eros.	TM	10
Mechanicsville NW	Kan	Loess M. Eros.	F	10
Dixon	Kan	Loess M. Eros.	F	15
Ottumwa N	Kan	Unmod. Eros.	WL	20
Ottumwa S	Kan	Unmod. Eros.	WL	20
Woodland	Kan	Unmod. Eros.	WL	20
Oakland	Kan	Loess Dep.	M	20
Avoca	Kan	Loess Dep.	M	20
Taylor	Kan	Loess Dep.	M	20
Sperry	Ill	Unmod. Eros.	CL	22
Walcott	Ill	Loess M. Eros.	TM	22
W. Davenport	Ill	Loess M. Eros.	TM	22
Eldridge	Ill	Unmod. Eros.	TM	22
Dixon	Ill	Loess M. Eros.	TM	12
Waverly	Iowan	Dr. M. Eros.	CC	20
Mason City	Iowan	Dr. M. Eros.	CC	20
Denver	Iowan	Dr. M. Eros.	CC	20
Buckingham	Iowan	Dr. M. Eros.	CC	20
Eagle Center	Iowan	Dr. M. Eros.	CC	20
Des Moines NW	Cary	Dr. Dep.	CW	15
Des Moines NE	Cary	Dr. Dep.	CW	15
Des Moines SW	Cary	Dr. Dep.	CW	15
Des Moines SE	Cary	Dr. Dep.	CW	15

* F = Fayette
TM = Tama and Muscatine
WL = Weller and Lindley
M = Marshall
CL = Clinton and Lindley
CC = Carrington and Clyde
CW = Clarion and Webster

VARIABLES

The slope parameter used is dimensionless, the tangent of the slope angle, tan θ. The slope measured was along a perpendicular drawn on the map to the nearest second or third order stream from the point randomly selected. The system of ordering follows that of Strahler (1952, p. 1120). This method of slope measurement represents a departure, since most map studies of slope utilize measurements of slope orthogonal to the contour. The slope length is that from the stream to the highest point along this perpendicular line allowing no reversal of slope. This departure, from the usual equal increments, allows the inclusion of slope length and relief as variables in the subsequent analyses.

Maximum slope angle has been widely used with the stated advantage of being more precisely defined and more easily replicated (Strahler, 1950, p. 677). This is probably especially true of field collected data. What one sees and would choose to measure in the field is generally the steepest part of the slope. However, this value may not be as representative of the landscape as a value including the low angle components at each end of the slope. In this study, the map-derived slope data were consistently found to be less steep than that measured in a field check of twelve points on the Mechanicsville NW and SE maps.

In this check, measuring slope perpendicular to the stream was found to be satisfactory in the majority of cases. However, in one instance it was found that a stream turn had been missed on the map and a depression mistakenly included in the measurement. In another, it appeared that a different angle would have provided more representative data. In a third, an error in slope length was found.

Some random points were eliminated from the sample for the following reasons: 1) Culture destroyed the natural slope development. 2)The nearest drainage was fourth order or higher. 3) The point fell in a stream or standing body of water. 4) The point fell at the intersection of two streams so that the perpendicular paralleled one stream. 5) The point fell at the edge of the map. 6) The slope reversed from the nearest stream. 7) There were inadequate contours to provide even an estimate of stream gradient for the stream associated with the slope.

Other variables in addition to relief and slope length are orientation, stream gradient and stream order. Orientation is represented by a dichotomous classification of north-facing or south-facing slopes. Stream gradient is dimensionless since it was measured as the difference in the contour values divided by the stream length between the contours. This was limited to the area immediately adjacent to the measured slope. Improved replicational accuracy would probably be obtained by specifying a certain stream length to be included. In general, the streams included are mapped as intermittent streams. However, streams were drawn in if they fulfilled the nearest second or third order requirement.

The variables for representative comparisons are listed without transformation in Table 3. These data could not be used in the analysis of variance because their distributions were skewed, but are included as being more meaningful for the magnitude of the parameters. Certain anomalous age relationships are apparent. As Ruhe suggested, predominantly depositional areas should not be compared with predominantly erosional areas because of the

Table 3. Variables Without Transformation

		tan θ	R	S.L.	S.G.
Classification	N	$\bar{x}$	$\bar{x}$	$\bar{x}$	$\bar{x}$
Age:					
Kan LME	100	.0495	47.05	1070	.0148
Ill T	100	.0515	39.80	1145	.0110
Ia DME	100	.0247	34.80	1648	.0087
Cary DD	60	.0343	47.05	1653	.0167
Materials:					
Kan LME	100	.0495	47.05	1070	.0148
Kan UE	60	.0674	70.92	1307	.0185
Kan LD	60	.0646	71.42	1242	.0182
Ill LME	56	.0450	38.30	1136	.0108
Ill UE	44	.0596	41.70	1157	.0111
Stream Order:					
Kan LME 2	51	.0543	43.82	878	.0210
Kan. LME 3	49	.0444	50.41	1269	.0083
Ia DME 2	44	.0243	28.75	1295	.0127
Ia DME 3	56	.0251	39.55	1925	.0056
Orientation:					
Kan UE N	28	.0758	72.86	1271	.0208
Kan UE S	32	.0600	69.22	1337	.0166
Ia DME N	47	.0266	34.89	1534	.0095
Ia DME S	53	.0230	34.72	1749	.0080
Base Level:					
Kan LME-Dixon 2	7	.0580	45.71	900	.0116
Ill LME-Dixon 2	8	.0269	20.63	862	.0115
Kan LME-Dixon 3	8	.0416	38.12	1025	.0086
Ill LME-Dixon 3	4	.0311	40.00	1300	.0077

confusion of constructional forms with erosional forms (Ruhe, 1950, p. 438). This is proposed to explain the greater slope angle of the Cary over the Iowan.

RESULTS

The first comparison, for differences due to age, would ideally be made with all other factors affecting slope held constant. This was impossible to do, not only because of the limitations of the available data, but also because of the likelihood of failing to consider all pertinent factors. The alternative of identifying and evaluating certain variables was chosen. The possibility of important factors being omitted should be noted as a limitation of the study.

Table 4 lists the comparisons for the erosional forms for the Kansan, Illinoian and Iowan samples. The Kansan sample is classified as loess mantled erosional, the Illinoian sample includes both loess mantled and unmodified erosional classifications and the Iowan sample is labeled drift mantled erosional. The slope angle parameter, tangent θ, was found to differ at the 5% significance level with the main difference apparent between the Iowan sample and the Kansan and Illinoian samples. The sign of

Table 4. Age of Glacial Drift: Angle of Slope

Age	N	df	F	Between-group t-tests	
		2,297	24.48*	Kan	Ill
Ill	100			1.28	0.00
Kan	100			0.00	
Ia	100			-7.49*	-5.08*

*$P < .05$

the between group t-tests shows which statistic is favored; the positive value is the direction of favor, for example, $1.28 > 0.00$ showing the Illinoian mean to be greater than the Kansan. Both of these values are greater than -7.49 showing them to be greater than the mean of the Iowan data.

The standard deviations for the three samples are 0.29, 0.37, and 0.26 indicating a fairly small dispersion for all. The S.D. for the Illinoian data is probably larger because of proximity to the Mississippi River.

The second comparison for differences due to materials was limited to within-age comparisons of the Kansan and Illinoian data.

Table 5. Kay's Physiographic Classifications: Angle of Slope

	N	df	F	Between-group t-tests	
		2,217	9.99*		
Kan LME	100			0.00	
Kan UE	60			3.11*	0.00
Kan LD	60			4.02*	0.58
		1,098	1.93		
Ill LME	56			0.00	
Ill UE	44			1.39	

*$P < .05$

Differences were found in the Kansan comparison with differentiation between the Kansan LME and the Kansan UE and Kansan LD samples at significant levels. No significant differences were apparent in the Illinoian comparison. Thus, a hypothesis of no differences is only partially rejected. The standard deviations for the Kansan LME, UE and LD samples are 0.29, 0.26 and 0.18 respectively; the standard deviation for the Illinoian LME is 0.35 and for the Illinoian UE is 0.39.

The third comparison, that for stream order, was made by dividing the age-materials samples into their second and third order stream components and comparing these. Table 6 gives the results of testing the effects of stream order associated with the slope.

The only significant results were for the Kansan unmodified erosional comparison.

Table 6. Stream Order 2 vs. 3: Angle of Slope

	N	df	F
Kan LME 2	51		
Kan LME 3	49		
		1,98	1.14
Kan UE 2	25		
Kan UE 3	35		
		1,58	9.10*
Kan LD 2	29		
Kan LD 3	31		
		0.04	1.58
Ill LME 2	29		
Ill LME 3	27		
		1,54	0.99
Ill UE 2	8		
Ill UE 3	36		
		1,42	0.00
Ia 2	44		
Ia 3	56		
		1,98	0.04
Cary 2	31		
Cary 3	29		
		1,58	0.09

*P < .05

The fourth comparison is for orientation. The age-materials samples are divided as to whether the slopes are north-facing or south-facing. Table 7 gives the results of testing the effects of orientation of the slope.

The only significant results are for the Cary sample. This result would lead to retaining a hypothesis of no differences as tenable, a result in agreement with previous findings in other areas (Strahler, 1950, p. 809).

Table 7. Orientation, North vs. South: Angle of Slope

	N	df	F
Kan UE N	28		
Kan UE S	32		
		1,58	0.83
Kan LD N	33		
Kan LD S	27		
		1,58	0.77
Ill UE N	20		
Ill UE S	24		
		1,42	0.34
Ia DME N	47		
Ia DME S	53		
		1,98	0.69
Cary DD N	31		
Cary DD S	29		
		1,58	3.48*

*P < .05

A previously reported anomalous situation in which Illinoian slope was found to be greater than Kansan sample data (McConnell, 1966 p. 722-723) was confirmed in this study (Table 3). The following factors have been found to be influential: 1) Physiographic classification (materials), 2) Stream order for Kansan UE comparisons only, 3) Orientation for Cary comparison only. An additional comparison equating these factors was made. A soil association classification was made on the Kansan LME and Illinoian LME samples to further refine the materials sub-divisions. The F-ratio for this comparison is 0.07 and the means (transformed) are: Kansan 2.40 and Illinoian 2.50, failing to resolve the anomaly.

The second group of data tested involved the concept that relief and slope would be different where different areas were not the same distance above local base level. The Dixon map area contains both Kansan and Illinoian drift, which have the same soil association classification as well as the same Kay physiographic classification. Table 8 gives the comparisons of slopes with both sorted stream orders and mixed stream orders. Here where the distance to baselevel can be assumed to be the same, the expected age relationships are confirmed and the anomaly resolved, at least in part.

Table 8. Kansan Illinoian Anomaly: Base Level

	N	tan $\bar{X}$ (transformed)	df	F
Kan LME Dixon-2	7	2.71		
Ill LME Dixon-2	8	2.38	1,13	6.91*
Kan LME Dixon-3	8	2.54		
Ill LME Dixon-3	4	2.49	1,10	0.11
Kan LME Dixon	15	2.62		
Ill LME Dixon	12	2.42	1,25	4.75*

*$P < .05$

Hypotheses regarding relief and slope length were tested using the same bases for comparison as had been used for angle of slope. For brevity and because these results parallel those for angle of slope, these two parameters are shown together in Tables 9 through 12 with significant differences shown only by the starred F-ratios.

Stream gradient data are considered independently because of the need to separate second and third order stream data. That is, the comparison of stream gradients of the various samples cannot be made with mixed stream order data because the difference in stream gradient with stream order is significant in all cases. Table 8 reports the results.

Table 9 shows that relief and slope length differ at the 5% level of significance for the age comparison of erosional samples.

Table 9. Age of Glacial Drift: Relief and Slope Length

			R	S.L.
Age	N	df	F	F
Kan	100			
Ill	100			
Ia	100			
		2,297	4.13*	18.06*

*P < .05

Table 10 shows differences in the three Kansan samples to be significant at the 5% level for relief. No other comparisons showed significant differences.

Table 10. Kay's Physiographic Classification: Relief and Slope Length

Class	N	df	R	S.L.
			F	F
Kan LME	100			
Kan UE	60			
Kan LD	60			
		2,217	18.07*	2.02
Ill LME	56			
Ill UE	44			
		1,98	1.63	0.13

*P < 0.05

When the samples are compared on the basis of stream order as shown in Table 11, relief comparisons are significant at the 5% level for all except the Cary comparison. All slope length comparisons are significant at this level except for the Cary and Illinoian UE data. Both relief and slope length means are greater for

Table 11. Stream Order 2 vs. 3: Relief and Slope Length

Class	N	df	R	S.L.
			F	F
Kan LME 2	51			
Kan LME 3	49			
		1,98	2.82*	14.96*
Kan UE 2	25			
Kan UE 3	35			
		1,58	2.64*	25.69*
Kan LD 2	29			
Kan LD 3	31			
		1,58	5.45*	5.38*
Ill LME 2	29			
Ill LME 3	27			
		1,54	12.28*	7.48*
Ill UE 2	8			
Ill UE 3	36			
		1,42	2.92*	1.55
Ia 2	44			
Ia 3	56			
		1,98	9.44*	9.19*
Cary 2	31			
Cary 3	29			
		1,58	0.74	2.45

*P < .05

the slopes associated with third order streams than for those associated with second order streams.

Table 12. Orientation N vs. S: Relief and Slope Length

Class	N	df	R	S.L.
			F	F
Kan UE N	28			
Kan UE S	32			
		1,58	0.02	0.59
Kan LD N	33			
Kan LD S	27			
		1,58	0.44	0.01
Ill UE N	20			
Ill UE S	24			
		1,42	0.74	0.01
Ia DME N	47			
Ia DME S	53			
		1,98	0.04	1.26
Cary DD N	31			
Cary DD S	29			
		1,58	3.67*	0.00

* P< .05

Table 12 shows that the only effect significant at the 5% level is the relief parameter for the Cary data. In this comparison, the relief associated with north-facing slopes is greater than that associated with south-facing slopes. The dispersion for these parameters and these comparisons ranged from 0.22 to 0.30, all quite small and quite uniform.

Since stream gradient differs significantly for all comparisons made on the basis of stream order, the age comparison is shown for samples of like stream order.

Table 13 shows the results to be significant at the 5% level for the second order data but not for the third order data.

Table 13. Age: Stream Gradient

Class	N	df	F	Between-group t-tests	
Kan LME 2	51			0.00	
Ill LME 2	29			-2.87*	0.00
Ia DME 2	44			-4.28*	-0.70
		2,121	9.70*		
Kan LME 3	49			0.00	
Ill LME 3	27			-0.70	0.00
Ia DME 3	56			-1.96	-0.94
		2,129	1.97		

*P < .05

The dispersion of stream gradient data as indicated by the standard deviations was extremely small for all comparisons, in the order of magnitude of 0.06 to 0.09.

The relationships among the variables, angle of slope relief, slope length and stream gradient, are shown by the use of the Pearson product-moment coefficient of correlation. The t-test of significance was used to determine those which would not be expected to result from chance at both the 1% and 5% levels (Snedecor, 1956, p. 174).

Slope angle is related positively to relief at a significance level of at least 5% for 78% of the comparisons. The range of the r values was from 0.27 (N = 60) to 0.70 (N = 51).

Slope angle is related negatively to slope length at a significance level of at least 5% for 91% of the comparisons. The range of the r values was from 0.37 (N = 60) to 0.69 (N = 20).

Slope angle is related positively to stream gradient for 54% of the comparisons. The range of r values was from 0.31 (N = 100) to 0.83 (N = 8).

Stream gradient is related positively to relief for 7% of the comparisons. It is related negatively to slope length for 68% of the comparisons. The r values for this latter relationship range from 0.37 (N = 56) to 0.49 (N = 28).

CONCLUSIONS

Conclusions based on the preceding assumptions and our limited data are in general in accord with traditional geologic theory and may be summarized as follows:

1. Angle of slope as measured by arithmetic means differs significantly from one glacial drift area to another. The angle of slope decreases from Kansan, if one accepts comparisons where distance above base level is approximated through Illinoian to Iowan. The angle of slope decreases from Kansan through Illinoian to Iowan. Relief decreases in this same manner while slope length increases in this direction. Stream gradient decreases from Kansan through Illinoian to Iowan both for the total samples and when data of like stream order are compared.
2. Slope angles differ significantly among the physiographic classifications, loess mantled erosional, unmodified erosional and loess depositional of the Kansan data, increasing respectively. The differences were not significant for the loess mantled erosional and unmodified erosional samples of the Illinoian data. Relief comparisons produced exactly the same results. Slope length comparisons produced no significant results.
3. In general, slope angles are greater for slopes associated with second order streams than for those associated with third order streams; however, these differences are significant at the 5% level only for the Kansan unmodified erosional comparison. Slope length means are significantly greater for slopes associated with third order streams than for those

associated with second order streams for nearly all comparisons. These results would be expected since grading of slopes and streams proceeds upstream.

4. Whether the slope was north-facing or south-facing produced significant effects only for the angle and relief parameters of the Cary data. In general, the slope angle and relief means were greater for slopes associated with north-facing slopes while the slope length means were greater for the slopes associated with south-facing slopes.
5. The dispersion as measured by the standard deviations was quite small and quite uniform for all parameters and all comparisons. That the parameters comprise a narrow range clustering around the mean might be interpreted as a tendency to approach a sort of equilibrium.
6. The standard deviation for the stream gradient data was notably smaller than those for the other parameters, slope angle, relief and slope lengh. This might be interpreted as an indication that streams are more advanced toward grade than hillslopes.
7. Slope angle is related positively to relief and stream gradient and negatively to slope length.
8. Stream gradient is related negatively to slope length and positively but much less strongly to relief.

Literature Cited

Anderson, Donald A. and Theodore L. Welp, 1960, An Engineering Report on the Soils, Geology, Terrain and Climate of Iowa: Ames, Iowa. Iowa State Highway Commission.

Harley, N. O., 1950, The Maximum F-Ratio as a Short Cut Test for Heterogeneity of Variance: Biometrika, v. 36-37, p. 308-312.

Kay, George F., and Apfel, Earl T., 1928, The Pre-Illinoian Pleistocene Geology of Iowa: Iowa Geological Survey, annual report, v. 34, p. 1-282.

Krumbein, W. C. and Robert L. Miller, 1953, Design of Experiments for Statistical Analysis of Geological Data: Journal of Geology, v. 61, p. 510-522.

McConnell, Harold, 1966, A Statistical Analysis of Spatial Variability of Mean Topographic Slope on Stream-Dissected Glacial Materials: Annals of the Association of American Geographers, v. 56, p. 712-728.

Ruhe, Robert V., 1950, Graphic Analysis of Drift Topographies: American Journal of Science, v. 240, p. 435-443.

Snedecor, George W., 1956, Statistical Methods: The Iowa State University Press, Ames, Iowa.

Strahler, A. N., 1950, Equilibrium Theory of Erosional Slopes Approached by Frequency Distribution Analysis: American Journal of Science, v. 248, p. 673-696 and p. 800-813.

Strahler, A. N., 1952, Hypsometric Analysis of Erosional Topography: Geological Society of America Bulletin, v. 63, p. 1117-1142.

Strahler, A. N. 1954, Statistical Analysis in Geomorphic Research: Journal of Geology, v. 62, p. 1-25.

Strahler, A. N., 1956, Quantitative Slope Analysis: Geological Society of America Bulletin, v. 67, p. 571-596.

Tate, Merle W., 1955, Statistics in Education: The MacMillan Co., New York.