Proceedings of the Iowa Academy of Science

Volume 83 | Number

Article 8

1976

Multiplicative and Additive Processes in the Subjective Evaluation of Travel Expense

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Multiplicative and Additive Processes in the Subjective Evaluation of Travel Expense[1]

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HENSLEY, VALERIE S. and IRWIN P. LEVIN (Department of Psychology, University of Iowa, Iowa City, IA 52242). Multiplicative and Additive Processes in the Subjective Evaluation of Travel Expense. Proc. Iowa Acad. Sci. 83(1): 35-39, 1976.

Subjects were asked to make absolute judgments and relative ratings of travel expense for a series of hypothetical trips described by varying levels of distance to be travelled, gasoline price, and expected gas mileage. In Experiment 1, intuitive estimates of cost in dollars followed a multiplicative model analogous to the "rational" model but allowing individual differences in evaluating and weighting stimulus factors. In Experiment 2, subjective ratings of relative expense followed an additive model. An additive model implies that an extreme value of one factor will be balanced by more neutral values of other factors, whereas a multiplicative model implies that a single extreme value will have an exaggerated effect. Two interpretations of these disparate findings were considered: either the underlying information integration process differed as a function of how the information was to be used, or response differences were due to transformations of the internal responses to the overt response scale. Experiment 3, in which subjects were required to make both kinds of evaluations, ruled out the response transformation interpretation.

One way in which experimental psychologists can address the current societal concern about energy consumption is to develop behavioral models to describe consumer decisions. For example, simple algebraic models have been developed by Corry and Levin (1975) and Norman and Louviere (1974) to describe how a variety of factors affect transportation decisions (e.g., choice of car vs. bus). These models can accomplish the following goals: (1) describe how relevant factors combine to determine consumer decisions; (2) determine the relative influence of each factor on the decision-making process; and (3) relate the perceived (psychological) value of each piece of information to its objective value. Since travel expense is such an important determinant of travel decisions and gasoline consumption, and since estimates of travel expense are usually arrived at subjectively or "intuitively," the present study examined factors affecting subjective judgements of travel expense.

Two classes of models which have gained support in past research on subjective judgmental processes are additive models and multiplicative models. The present study will show that each type of model is applicable when subjects combine various categories of information concerning travel expense. The particular type of model supported depends on how the information is to be used.

Additive and multiplicative models differ in formal respects that allow them to be discriminated statistically using analysis of variance tests. This will be illustrated later. More importantly, the two types of models differ in their conceptualizations of information usage. The most commonly supported form of additive model, the equal-weight averaging model, implies that an extreme value of one information variable will be balanced (averaged) by more neutral values of the other variables, whereas a multiplying model implies that an extreme value of one variable amplifies the effects of other variables.

Recent support for multiplying models has been reported by Anderson and Butzin (1974), Graesser and Anderson (1974), and Shanteau and Anderson (1972). Anderson and Butzin (1974) found support for the equation, Judged Performance = Motivation X Ability; and Graesser and Anderson (1974) found support for the equation, Estimated Gift Size = Generosity X Income. Shanteau and Anderson (1972) found support for a multiplicative model when the judged worth of an added piece of information in a decision task was investigated as a function of the amount of prior information, the informativeness of the added information and the payoff for a correct decision.

Support for averaging models in judgmental tasks is widespread and includes Anderson's (1962) study of personality impression formation, Shanteau's (1972) study of sequential decision making, Levin's (1975) study of simulated shopping decisions and Levin, Kim, and Corry's (1976) study of judged performance of students on the basis of Midterm and Final Exam scores.

The present series of three experiments examines evaluations of travel expense for a series of hypothetical trips as a function of distance to be travelled (D, in miles), price of gasoline (P, in cents per gallon), and expected average gas mileage (M, in miles per gallon). The "rational" model for combining these three categories of information is a multiplying model of the form, $D \times P \times M^{-1}$. In Experiment 1 subjects were asked to make subjective or "intuitive" estimates of the cost in dollars of each hypothetical trip. This experiment thus represents a test of whether the subjects' intuitive processes parallel the rational model. In Experiment 2 subjects were asked to rate the relative expense of each trip in comparison to the other trips. Since this response mode is completely subjective and not subject to an "accuracy" criterion, responses need not conform to the rational model. The pattern of responses did, in fact, differ between Experiments 1 and 2. Experiment 3 was designed to test alternative interpretations of these differences.

Experiment 1

Subjects in Experiment 1 were asked to make judgments of the cost of each of a series of hypothetical trips described by varying levels of D, P, and M. The pattern of responses (R) was compared to that generated by the following equation:

$$R = D \times P \times M^{-1}, \qquad (1)$$

which is the formulation that would give accurate responses if paper-and-pencil calculations of travel expense were made. Such calculations were not, however, permitted. Rather, subjects were asked to make subjective estimates of travel expense in lieu of precise calculations.

Method. Twelve undergraduate volunteers from the 1974 summer session at the University of Iowa were each paid $2.00 for their participation in the hour long experimental session. Each subject was given booklets containing information about 27 hypothetical au-

[1]This study was supported in part by Grant GY-11294 from the National Science Foundation's Undergraduate Research Participation Program — the first author was a participant and the second author was project director — and was supported in part by funds from the University of Iowa's Institute of Urban and Regional Research. An oral version of this paper was presented at the 1975 meeting of the Iowa Academy of Science, Ames, Iowa.

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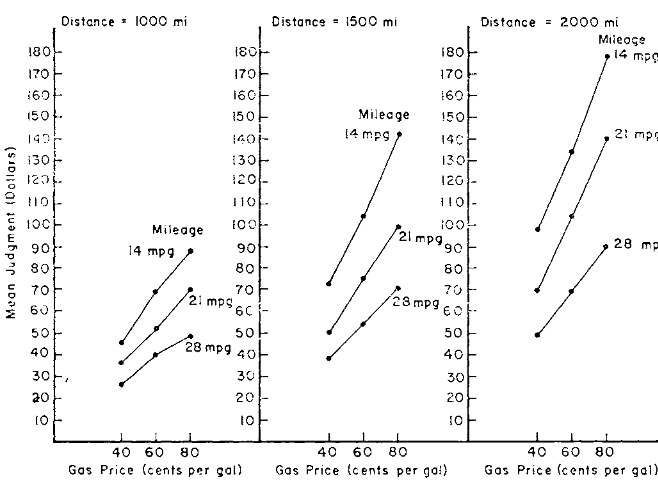
tomobile trips. These trips represented all possible combinations of three levels of D (1000, 1500, and 2000 miles), three levels of P (40, 60, and 80 cents per gallon of gasoline) and three levels of M (14, 21, and 28 miles per gallon of gasoline). Each subject received three replications of this 3 x 3 x 3 factorial design, with each replication appearing in a different booklet with a different random order of presentation of the 27 hypothetical trips.

Subjects were told to make rough numerical estimates of travel expense for each of a series of hypothetical summer vacation trips. They were told to make these estimates in the natural way that they would use if they were actually thinking about a particular trip. They were told to use only the information given to them and to not consider additional expenses such as food and lodging costs. No hint was given as to a correct model for combining the information given.

Three hypothetical trips were described on each page of the test booklet. An answer space was provided next to each description for the subject to write his estimate, expressed in dollars. The task was self-paced, with the restriction that a subject must proceed through the trips in the given sequence without referring to or changing previous answers.

RESULTS AND DISCUSSION

The data from Replications 2 and 3 were combined for statistical analysis, with Replication 1 considered as a practice set. There were, however, two exceptions. Each of two subjects made one response that was so discrepant from the rest of his responses (e.g., off by a factor of 3) that inclusion of these two (out of 648) responses gave a somewhat distorted view of the group data. In each of these two cases, the corresponding response from the other replication was substituted for the deviant response. Resulting means are plotted in Figure 1.



1. Mean numerical judgments of travel cost for Experiment 1.

Tests of multiplying model — According to a multiplying model such as given in Equation 1, all the interactions of the factorial design should be significant and these interaction effects should be concentrated in their linear components. (See Graesser & Anderson, 1974, Appendix, for a more detailed description of the relevant statistical tests.) Graphically, this translates into predicting a diverging fan of straight lines for each panel of Figure 1. It can be seen that this was the case. In the analysis of variance tests, the main effect of each variable (D, P, and M) and all interactions between variables were statistically significant at or near the .01 level. Furthermore, as predicted, the interaction effects were linear.

Ordinarily, in illustrating these tests graphically, spacing along the horizontal axis has to be readjusted to reflect subjective spacing of stimulus values before the diverging fan of straight lines is exhibited. However, no such adjustment was required in the present case because the marginal mean responses of the factorial design were linearly related to the actual stimulus values. Nevertheless, responses of individual subjects did not merely duplicate values that would be obtained with precise paper-and-pencil calculations.

Stimulus levels of each variable were originally chosen so that the highest value was two times the lowest value and that, therefore, each variable would contribute equally to precise calculations of travelling cost. The analysis of group data indicated that, averaged over all 12 subjects, the three variables (P, D, and M) did have an approximately equal effect on subjective cost estimates. However, single-subject analyses revealed that for most subjects one of the variables D, P, or M had a disproportionate influence on cost estimates. For 3 subjects Gasoline Price was the most important variable, while 4 subjects placed greatest weight on Gas Mileage and another 4 subjects placed greatest weight on Distance. For one subject, Mileage and Distance were of equal importance while the effect of Price was slightly depressed. Moreover, 11 out of the 12 subjects showed a tendency to overestimate the cost of the less expensive trips and underestimate the cost of the more expensive trips. The 12th subject consistently overestimated by a factor of approximately 10, indicating a misplaced decimal point. Although there were individual differences in specific response tendencies, almost all subjects showed a pattern of results (linear interactions) characteristic of a multiplicative process. Thus the multiplying model holds for individuals as well as for the group as a whole.

In summary, Experiment 1 showed that subjective estimates of travel cost based on distance to be travelled, gasoline price and gasoline mileage could be described as a multiplicative process. However, rather than duplicating precise arithmetic calculations, subjective estimates showed the effects of individual idiosyncracies. These findings can be readily incorporated into the multiplying model given in Equation 1 by allowing each factor to take on subjective scale values for individual subjects and by including an exponent with each factor that defines the weight or importance of that factor. The expanded model now reads as follows:

$$R = d^{wD} \times p^{wP} \times m^{-wM}, \qquad (2)$$

where d, p, and m represent the subjective correspondents of stimulus values D, P, and M; and ^{w}D, ^{w}P and ^{w}M represent the weights of the different categories of information. In a multiplying model, the weight parameter is equivalent to the unit of the corresponding subjective scale. Thus, while Equation 2 looks more complex than Equation 1 because of the added parameters, it does not represent a more complex cognitive process. On the contrary, it allows the subject to impose his own units on the scales instead of being constrained to match the physical units of the three scales.

The model expressed in Equation 2 captures the subjective nature of the information integration process and illustrates the distinction between *descriptive models* that indicate how subjects actually respond and *normative models* that indicate how subjects *should* respond. This distinction has also been made in number-averaging tasks (Levin, 1974a, b; 1975, 1976).

Experiment 2

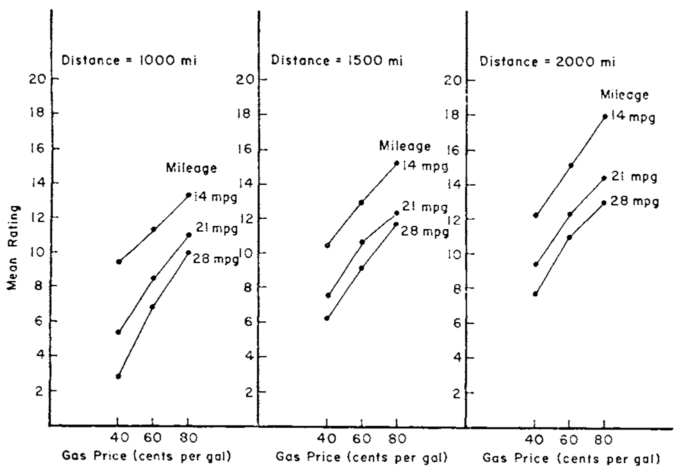
In Experiment 2 subjects were asked to rate the comparative expense of each of a series of hypothetical trips rather than estimating the absolute cost of each trip. It was of primary interest to see if the same model that describes cost estimates also describes comparative ratings.

METHOD

Each of the 21 subjects in Experiment 2 received three replications of the same 27 stimulus combinations used in Experiment 1. Subjects were asked to rate the expense of each hypothetical trip in relationship to the other trips. For each trip description the test booklet contained a 20 cm. line marked "very expensive" at one end and "very inexpensive" at the other end. Subjects were instructed to place an "X" somewhere along this line to indicate their rating of each trip. The more expensive a trip seemed in comparison to the other trips described in the booklet, the closer to the "very expensive" end should be their response. Responses were converted to a scale from 0 to 20, with higher numbers representing more expensive ratings. In order to allow relative ratings, subjects were given 2 min. to examine a sheet summarizing the 27 hypothetical trips. They were then given 10 sec. to respond to each trip. The first page of the first booklet contained two stimulus combinations more extreme than the rest to illustrate examples of "very expensive" and "very inexpensive" trips. Subjects were told that the 27 trips on the subsequent pages would be somewhere between these two extremes of travelling cost.

RESULTS AND DISCUSSION

Mean responses for Replications 2 and 3 combined are graphed in Figure 2. The pattern of results seen here supports an additive rather than a multiplicative model. Whereas the lines in each panel of Figure 1 (Experiment 1) diverge, the lines are approximately parallel in each panel of Figure 2.



2. Mean ratings of relative expense for Experiment 2.

Parallelism in the graphs corresponds to lack of interaction effects in the analysis of variance tests. While the main effects for the three variables D, P, and M were highly significant, $Fs > 100$, none of the two-way interactions approached statistical significance, $F < 1$ in each case. The three-way interaction M x P x D, was of borderline significance at the .05 level, due to a slight convergence of the lines in the left-hand panel of Figure 2. This pattern is not seen in any of the other panels of Figure 2 or in Figure 1. Analyses of the data for individual subjects supported an additive model in almost all cases.

Analyses for individual subjects also revealed that Price was the most important factor for 11 subjects, Gas Mileage was the most important factor for 8 subjects, and Distance was the most important factor for the remaining 2 subjects. Averaged over subjects, Gasoline Price had the largest effect. Also, as in Experiment 1, the psychophysical function relating marginal means of the factorial design to actual stimulus values was approximately linear for D and P. However, the psychophysical function for M is negatively accelerated as can be seen by noting that the curves for 21 and 28 mpg. are closer together than the curves for 14 and 21 mpg.

Either an adding or an averaging form of the additive model might be applicable. The additive model can be stated as follows:

$$R = w_D d + w_P p + w_M m \qquad (3)$$

where the symbols are as defined in Equation 2 but where subjective values of m in terms of expense ratings are inversely related to physical values of M in mpg. If the weights are constant across levels of a given stimulus dimension and if they sum to one, this model would be called an equal-weight averaging model. However, no discrimination between adding and averaging is possible in the present study. Such discrimination requires variation in the amount of information to be evaluated. Typically, averaging models are supported over adding models in analyzing subjective ratings such as these (Anderson, 1965, Levin, 1974a). A reasonable assumption is that an averaging process describes the ratings of Experiment 2.

More important than the distinction between the two forms of the additive model is the distinction between additive and nonadditive (e.g., multiplicative) models. A nonadditive model was supported in the present Experiment 1 but an additive model was supported in Experiment 2. Experiment 3 was designed to test alternative explanations for the disparate results of these two experiments.

Experiment 3

Two alternative interpretations of the different results in Experiments 1 and 2 were tested in Experiment 3. The results of Experiment 1 are relatively clear-subjects combine the three categories of information multiplicatively to determine cost in dollars. This parallels the rationale rule for combining the stimuli in their physical units. The results of Experiment 2 are not so easy to relate to a particular combination rule. The data were additive in nature when comparative ratings were required. This could be because the stimuli were in fact combined additively, with each piece of information evaluated on the dimension of judgment — i.e., relative expense. This would mean that there was a fundamental difference in the way information was combined in the two experiments. An alternative interpretation is that the information was combined multiplicatively in Experiment 2 as in Experiment 1, but when the cost estimates were subsequently translated into ratings of comparative expense, the multiplicative relationship was transformed into an additive relationship. (For a schematization of how the transformation from an internal response to an overt response can represent an important component of information integration, see Birnbaum, 1974, Fig. 1.) In the present case, as can be seen by comparing Equations 2 and 3, a logarithmic transformation could account for why a multiplicative integration rule leads to additive data.

The subjects in Experiment 3 were required to go through a two-stage process. In Part 1 they were required to estimate the cost in dollars of each hypothetical trip. The stimulus values were then removed and they were asked in Part 2 to rate the relative expense of each trip on the basis of their cost estimates. A comparison of the pattern of responses in Parts 1 and 2 was made to examine the issue of response transformations and how it relates to the alternative explanations of the different results for

Experiments 1 and 2. If the experimentally required transformation from cost estimates to relative expense ratings is linear in form (that is, if the pattern of results does not change from Part 1 to Part 2), then it is not likely that the differences between Experiments 1 and 2 are due to response transformations.

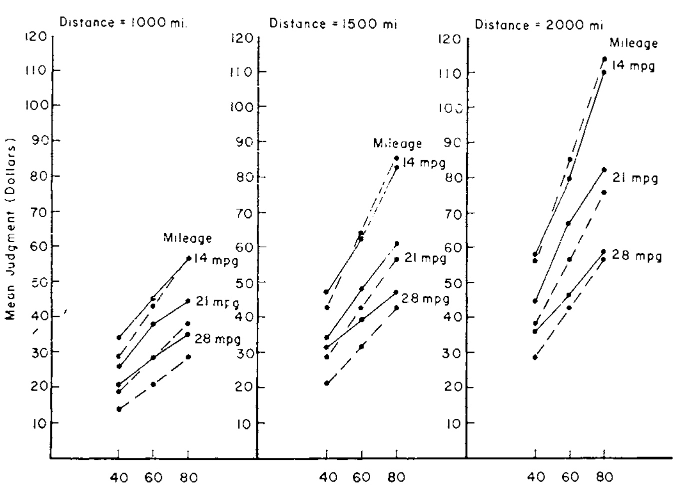
METHOD

The 21 subjects in Experiment 3 were given three replications of the same 27 hypothetical trips used in Experiments 1 and 2. The procedure for Part 1 was the same as in Experiment 1. Subjects were asked to make subjective estimates of the cost in dollars of each hypothetical trip. This time, however, responses were recorded on a sheet separate from the booklet containing the stimulus values. Stimulus materials were collected at the end of Part 1. Part 1 was self-paced.

In Part 2 subjects were given their response sheets for each of the three replications from Part 1 and were then required to rate each cost estimate on a scale labelled "very expensive" at one end and "very inexpensive" at the other end. This is the same 20-point rating scale used in Experiment 2. Part 2 was also self-paced.

RESULTS AND DISCUSSION

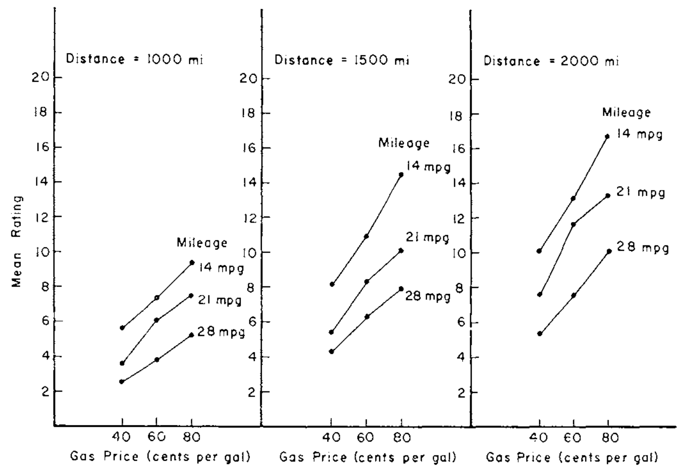
Response means for Part 1-subjective estimates of cost-are shown as solid lines in Figure 3, with the arithmetically correct values plotted as dotted lines. Response means for Part 2 — comparative ratings of expense — are shown in Figure 4. As in the previous experiments, data were combined for Replications 2 and 3. The most important feature to be noted in Figures 3 and 4 is that the curves are similar in form. Specifically, each panel displays the diverging fan of lines representative of a multiplying model. In each case the results are similar to those of Experiment 1. Figure 3 also shows the tendency-previously noted in Experiment 1 but attenuated by one extreme subject — to overestimate the cost of the less expensive trips and underestimate the cost of the more expensive trips.



3. Mean numerical judgments (solid lines) and arithmetically correct values (dotted lines) of travel cost for Part 1 of Experiment 3.

Statistical analyses supported a multiplicative model for each part of Experiment 3. All main effects and interactions were statistically significant, and the interactions were linear in form.

The similarity in the response patterns for Parts 1 and 2 of Experiment 3 indicate that cost estimates were converted to ratings of relative expense through a linear transformation. Such a transformation, of course, cannot account for the different pattern of results found in Experiments 1 and 2. Consequently, it appears that different processes for combining the information were involved in each experiment.



4. Mean ratings of relative expense for Part 2 of Experiment 3.

SUMMARY AND CONCLUSIONS

Experiments 1 and 3 showed that subjective impressions of travel cost as a function of distance to be travelled, gasoline price and gas mileage could be described by a multiplicative model. While the rational or normative model is also multiplicative, the descriptive model did not conform in all respects to the normative model. Results indicated that subjective stimulus scale values did not correspond exactly to the arithmetically correct values and the relative weights of the three stimulus dimensions were not the same for all subjects. These findings add to our understanding of "man as an intuitive statistician" (Peterson & Beach, 1967) by describing how subjective estimates of products are formed. They are formed according to a rational rule but individuals apply their own subjective units and weights to the factors. Previous studies applying descriptive models to judgments of statistical parameters of sets of numbers have dealt primarily with estimates of averages (Anderson, 1964, 1968; Hendrick & Costantini, 1970; Levin, 1974b, 1975, 1976) and have led to similar conclusions.

Experiment 2 showed that ratings of the relative expense of the hypothetical trips could be described by an additive model. Two interpretations of this finding were considered: (1) The information might have been integrated via a multiplicative rule as in Experiment 1 and then the integrated response was converted to the required rating scale by a transformation that led to additive data. (2) The information might have been integrated via an additive rule where the overt responses directly reflect the underlying process. Experiment 3 supported the second interpretation of the additive results in Experiment 2. The mere conversion of absolute cost estimates to comparative ratings did not transform multiplicative data to additive data. The implication is that the additive data in Experiment 2 represent an additive integration rule, where each factor (D, P, and M) is first evaluated along the dimension of judgment (i.e., very inexpensive to very expensive) and then the subjective scale values are balanced to arrive at a final response. With this type of process, the extreme values of each variable do not have the exaggerated effect that they do with a multiplying process.

Why then was there a difference in the underlying integration process for the same stimulus materials in Experiments 1 and 2? A multiplying model was supported in Experiment 1 where subjects might well have imposed an "accuracy of judgment" criterion on their own responses. In estimating cost in dollars a multiplying process was required to achieve accuracy. When comparative (and nonnumerical) ratings were required in Experiment 2, no such "accuracy" criterion could apply. The subjects had to rely on a subjective rule for combining the three categories of information. While both additive and multiplicative rules are logical candidates, previous researchers have concluded that subjects often adopt simplified strategies for complex tasks and, in particular, may use an additive rule for stimuli that should, on objective grounds, follow a multiplying rule (Shanteau & Anderson, 1972; Slovic & Lichtenstein, 1968). In this regard, it is of interest that most instances of support for a multiplying model have involved judgments based on only two stimulus factors where one factor acts as a modifier of the potential impact of the other factor. For example, motivation acted as a multiplier of ability to determine judged performance in the Anderson and Butzin (1974) study; adverbs acted as multipliers of adjectives in the Cliff (1959) study; and generosity acted as a multiplier of income to determined judged gift size in the Graesser and Anderson (1974) study.

In the present comparative rating task (Experiment 2) the subjects apparently evaluated each piece of information in terms of relative expense and then used a simple additive rule to arrive at their combined ratings. The additive rule is simpler than the multiplicative rule because subjects can balance the values of the different pieces of information in the same way for each rating (e.g., they can use the same response increment every time gasoline price increases from 60 to 80¢, irrespective of the levels of the other stimulus factors) rather than having to make a different configural judgment for each stimulus combination. It is of additional interest that in forming comparative ratings of travel expense for different trips, subjects tended to give disproportionate weight to Gasoline Price. While the factors Gas Mileage and Distance are equal in importance to Gasoline Price in determining total cost, Gasoline Price is the one factor expressed in dollars and cents and it is also the factor most in the public eye these days. Thus, while the present study was conducted under a restricted set of experimental conditions, there is some indication that the processes captured in such an experimental approach are relevant for understanding contemporary consumer decisions.

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