SYNTHESIS OF MECHANICAL ERROR IN RAPID PROTOTYPING PROCESSES USING STOCHASTIC APPROACH

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Abstract

A synthesis procedure for allocating tolerances and clearances in rapid prototyping (RP) processes has been developed, using a unified method based on stochastic approach, as developed by the authors, to study the mechanical error in RP processes. The tolerances and clearances that cause mechanical error have been assumed to be random variables, and are optimally allocated so as to restrict the mechanical error within the specified limits. Using the synthesis procedure, the allocation is done for the Fused Deposition Modeling (FDM) and the Stereolithography (SL) processes.

1. INTRODUCTION

Rapid Prototyping (RP) is emerging as a key technology with its ability to produce complicated parts within hours. RP systems use a solid model of a part as an input and make a physical model or prototype layer by layer without using tools or fixtures. This technology has also been referred to as layered manufacturing technology, free-form fabrication, model making, desktop manufacturing, 3D Printing, etc.

There are several accuracy issues pertaining to the layer manufacturing technologies, such as limitations in CAD to RP translation which generally take place in the CAD system itself, e.g., tolerance for tessellation, convex boundary error, flipped normal, mid-line node etc. (Fadel et al, 1996; Kai et al, 1997). RP parts usually exhibit a staircase effect on slanted and curved surfaces because of the layering process (Dolenc et al, 1994). Orientation of the build is to be determined considering several factors, such as, surface finish, build time, distortion, etc (Cheng et al, 1995).

The tool of an RP system traces the contour of the slice of a part on a platform. There are several mechanisms, comprising of links and hinges, responsible for the motion of tool and platform (Fig. 1). The error at a point on the contour depends upon the error in the elements of these mechanisms. RP systems consist of mechanisms to move optics, build head, elevator platform, Z-stage platen, etc. The links of a mechanism are manufactured with some tolerances on the link lengths and clearances at the joints. The tolerances and clearances cause mechanical error in the desired position of the tool or the platform, and is a significant accuracy issue in RP processes. The stochastic model developed by Agrawal is the first attempt to analyze the mechanical error in RP processes through a unified approach (Agrawal, 2001).

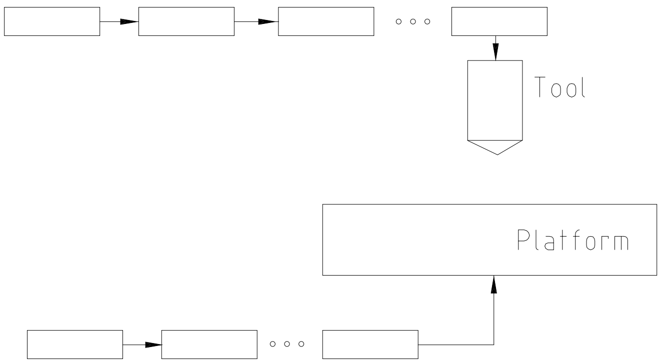

Figure 1: An RP process

While designing a mechanism it is necessary to take into account both the structural and the mechanical errors. Mechanical error in mechanisms has been dealt with considerably in the past literature. Several attempts have been made to analyze path and function generating mechanisms. There are two distinct approaches — deterministic and stochastic. The deterministic approaches are based upon worst-case analysis of individual tolerances and give highly conservative estimates, not reflecting the overall behaviour of the mechanisms. Besides in most of these studies, either tolerances or clearances are considered. In contrast stochastic approaches have been found to be more suitable for both analysis and synthesis of mechanical error.

Dhande et al (1973) and Chakraborty (1975) used a stochastic model for mechanical error analysis of function-generators. Mallik et al (1987) used a stochastic approach for mechanical error analysis of path-generating mechanisms. Agrawal (2001) has extended the concept of stochastic modeling of tolerances and clearances to RP processes, such as FDM and SL.

A significant development in the treatment of tolerances and clearances is their optimal allocation so as to restrict the mechanical error within specified limits. Dhande et al (1973) and Chakraborty (1975) used a stochastic model and an equivalent linkage model to allocate tolerances and clearances in four-bar function-generators. Sutherland (1975) has given a method for synthesis of mechanisms, taking into account structural and mechanical error due to tolerances. The dimensions and tolerances of a mechanism can be obtained for a given maximum allowable function generating error while minimizing the manufacturing cost. Bakthavachalam et al (1975) have considered synthesis of four-bar path-generating mechanisms as an optimization problem under inequality constraints.

Equality constraints are modified by introducing tolerances and clearances and thereby the difficulty in satisfying the equality constraints exactly is eliminated. Through this modification, the objective function is also changed. The penalty function approach is used.

Rao (1978) has suggested an iterative method for the synthesis of mechanisms, taking into account the effect of link deformations, tolerances and joint clearances. However, tolerances and joint clearances were specified prior to the synthesis of mechanisms. Choubey et al (1982) have suggested a method for minimizing the structural error together with the mechanical error due to manufacturing tolerances on the link dimensions. Nominal link lengths are obtained prior to tolerance allocation. The mechanical error is treated as a deviation of structural errors. Tolerances are then allocated with reference to the position of maximum error to limit the mechanical error below a specified value. Sharfi et al (1983) proposed a method for tolerance and clearance allocation in multi-loop planar mechanisms based on the output sensitivity with respect to the link lengths. Tolerances and clearances are allocated as a result of the synthesis of mechanical error.

Mall k et al (1987) developed a stochastic model for the synthesis of mechanical error in four-bar path-generating linkages. They analyzed the mechanical error in the path of a coupler point for the three-sigma band of confidence level and developed a synthesis procedure to allocate tolerances and clearances so as to restrict the output error in the path of coupler point within specified limits. They found that the mechanical error of the coupler-point path is dependent on whether one considers the original mechanism or its cognate mechanisms. Rhyu et al (1988) have presented a procedure for optimal stochastic design of mechanisms considering tolerances and clearances. A weighted sum of the mechanical error and the manufacturing cost is minimized for the optimal allocation of tolerances and clearances. Fenton et al (1989) and Cleghorn et al (1993) have presented a method for error analysis and tolerance synthesis of multi-loop planar mechanisms.

In the present paper, a synthesis procedure for allocating tolerances and clearances in RP processes has been developed. Tolerances and clearances are assigned such that the maximum error on the work surface is within the specified limits. This method was used to allocate tolerances and clearances in the FDM and SL processes.

2. PROBLEM FORMULATION

Agrawal (2001) has expressed the coordinates of a point on the work surface traced by the laser beam (SL) or the tip of the extruder head (FDM) as a function of random variables involved in the process. If there are n random variables $V_1, V_2,\dots,V_n$ involved in the RP process under consideration, then the coordinates of the point are given by $x(V_1, V_2,\dots,V_n)$, $y(V_1, V_2,\dots,V_n)$ and $z(V_1, V_2,\dots,V_n)$

Since the dependent variables x, y and z are functions of random variables, any given range of a dependent variable may be associated with the corresponding probability if the probability densities or at least certain numerical characteristics, such as means *m[V_i]* and variances *D[V_i]* of the random variables V_i are known. The means and the variances of the dependent variables are given by

$$
\begin{aligned}
m[x] &= x(m[V_i], i = 1,2,...,n) \\
m[y] &= y(m[V_i], i = 1,2,...,n) \\
m[z] &= z(m[V_i], i = 1,2,...,n) \\
D[x] &= \sum_{i=1}^{n} (\partial x / \partial V_i)_m^2 D[V_i] \\
D[y] &= \sum_{i=1}^{n} (\partial y / \partial V_i)_m^2 D[V_i] \\
D[z] &= \sum_{i=1}^{n} (\partial z / \partial V_i)_m^2 D[V_i]
\end{aligned}
\tag{1}
$$

It is known that if the functions x, y and z are linear and if the number of random variables $n > 5$, then the dependent variables x, y and z may as well be taken as normal (Ventsel, 1964).

To evaluate the variances D[x], D[y] and D[z], the partial derivatives of x, y and z, respectively, must be evaluated with respect to V_i. Once the variance of a dependent variable has been found for the RP process to trace a particular point on the work surface, the range of dependent variable is evaluated for the three-sigma band of confidence level (with probability 0.9973).

Agrawal (2001) performed analyses of mechanical error on FDM and SL processes and tabulated the variances D[x], D[y] and D[z] at the mean values of random variables at several points on the work surface. The three-sigma bands of mechanical errors in tracing several curves on the work surface for given tolerances and clearances were plotted. This forms the analysis part of the problem. The synthesis part is the inverse of the above problem. In synthesis, the designer has to decide about the levels of tolerances on V_i's for certain allowable tolerance limits on x, y and z. One solution is to keep very strict tolerances on V_i's so that the output errors on x, y and z will be less than the specified limits. However, such a solution is impractical since it is very expensive to manufacture any component or assembly with allowances close to zero. It is desirable to give as much of tolerance as poss ble to keep the manufacturing costs low.

Consider an RP process with n random variables V_i involved in its stochastic model. The random variables V_i have variances *D[V_i]*. The production cost C is assumed to be inversely proportional to the variances *D[V_i]*.

The problem of optimal allocation of tolerances and clearances can be stated as follows:

$$\text{Minimize} \quad C = \sum_{i=1}^{n} 1/\rho_i \tag{2}$$

where

$$\rho_i = D[V_i]$$

subject to

$$\sum_{i=1}^{n} a_i \rho_i = D[x] \tag{3}$$

$$\sum_{i=1}^{n} b_i \rho_i = D[y] \tag{4}$$

$$\sum_{i=1}^{n} c_i \rho_i = D[z] \tag{5}$$

where $a_i = (\partial x/\partial V_i)_m^2$, $b_i = (\partial y/\partial V_i)_m^2$ and $c_i = (\partial z/\partial V_i)_m^2$

The minimization problem is stated at that point P(x,y,z) on the work surface where the errors in x, y and z have been observed to be critical while conducting error analysis. In other words, some study in analyzing mechanical errors should be made prior to undertaking the synthesis. On the basis of this study values of variances D[x], D[y] and D[z] should be specified at a particular point P(x,y,z) on the work surface.

3. OPTIMATIZATION USING LAGRANGE MULTIPLIER TECHNIQUE

The coefficients $(\partial x/\partial V_i)_m^2$ in the above minimization problem are evaluated at the mean value of V_i. The expressions for the mean values of the random variables *m[V_i]* discussed earlier do not depend upon the tolerances, ε_i or the clearances, c_{ij} and hence on ρ_i. So ρ_i and a_i are independent. Hence the constraints are linear functions of design variables for the optimization of ρ_i's. The derivatives can be taken easily. Hence the optimization problem is solved using Lagrange multiplier technique.

Using Lagrange multiplier technique, the modified cost function can be written as

$$M = C. + \lambda_1 \left\{ \sum_{i=1}^{n} a_i \rho_i - D[x] \right\}$$
$$+ \lambda_2 \left\{ \sum_{i=1}^{n} b_i \rho_i - D[y] \right\} \qquad (6)$$
$$+ \lambda_3 \left\{ \sum_{i=1}^{n} c_i \rho_i - D[z] \right\}$$

Where $$C = \sum_{i=1}^{n} 1/\rho_i$$

and λ_1, λ_2 and λ_3 are Lagrange multipliers (Rao, 1984; Reklaitis, 1983). The optimality conditions for minimum M are

$$\partial M/\partial \rho_i = 0 \qquad \text{i = 1, 2,..., n}$$
$$\partial M/\partial \lambda_1 = \partial M/\partial \lambda_2 = \partial M/\partial \lambda_3 = 0$$

Applying the optimality conditions to the above problem gives

$$1/\rho_i^2 = \lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i$$

i = 1,2,…,n

$$\sum_{i=1}^{n} a_i \rho_i - D[x] = 0$$

$$\sum_{i=1}^{n} b_i \rho_i - D[y] = 0 \qquad (7)$$

$$\sum_{i=1}^{n} c_i \rho_i - D[z] = 0$$

Let $\sigma_1 = \lambda_2/\lambda_1$, $\sigma_2 = \lambda_3/\lambda_1$

$K_1 = D[x]/D[y]$, and

$K_2 = D[x]/D[z]$

Then σ_1 and σ_2 can be obtained by solving the following two equations:

$$f(\sigma_1, \sigma_2) = K_1, \quad g(\sigma_1, \sigma_2) = K_2$$

where

$$f(\sigma_1, \sigma_2) = \frac{\sum_i a_i \rho_i}{\sum_i b_i \rho_i} = \frac{\sum_i a_i \big/ \sqrt{\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i}}{\sum_i b_i \big/ \sqrt{\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i}}$$

Or,

$$f(\sigma_{1,} \sigma_2) = \frac{\sum_i \sqrt{a_i^2 \big/ (a_i + \sigma_1 b_i + \sigma_2 c_i)}}{\sum_i \sqrt{b_i^2 \big/ (a_i + \sigma_1 b_i + \sigma_2 c_i)}} \qquad (8)$$

$$g(\sigma_{1,} \sigma_2) = \frac{\sum_i \sqrt{a_i^2 \big/ (a_i + \sigma_1 b_i + \sigma_2 c_i)}}{\sum_i \sqrt{c_i^2 \big/ (a_i + \sigma_1 b_i + \sigma_2 c_i)}} \qquad (9)$$

For σ_1 and σ_2 to be real, it is necessary that

$$a_i + \sigma_1 b_i + \sigma_2 c_i > 0, \qquad i = 1, 2, \ldots, n \qquad (10)$$

The region given by the expression (10) is plotted in Fig. 2 for $a_i < 0$ and $a_i > 0$. Feasible region of σ_1 and σ_2 is where the expression (10) is satisfied for all i.

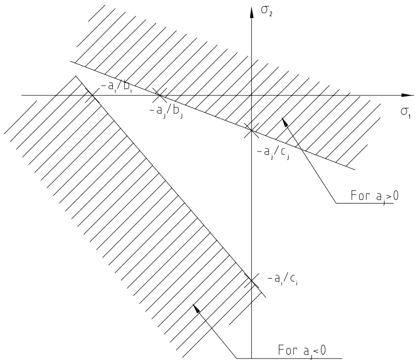

Figure 2: The region $a_i + \sigma_1 b_i + \sigma_2 c_i = 0$

The feasible region is bounded by two convex hulls (Fig. 3). One convex hull is formed by the straight lines $a_i + \sigma_1 b_i + \sigma_2 c_i = 0$, for those values of i where $a_i > 0$. The other convex hull is formed by $a_i < 0$.

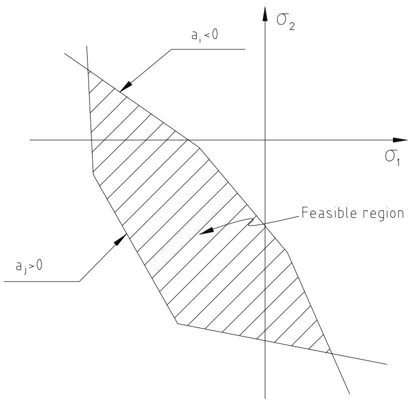

Figure 3: Feasible region of σ_1 and σ_2

The function $f(\sigma_1,\sigma_2)$ and $g(\sigma_1,\sigma_2)$ can be plotted in the feasible region of (σ_1,σ_2). This will give allowable ranges for K_1 and K_2. For given values of K_1 and K_2 in this range, σ_1 and σ_2 can be found by solving Eqs. (8) and (9).

There is a feasible region of σ_1 and σ_2. Arbitrary choice of K_1 and K_2 is not allowed because a real solution of (λ_1, λ_2, λ_3) cannot be obtained for K_1 and K_2 chosen in the infeasible region. But if a real solution is obtained then the expression (10) is satisfied. The solution satisfies the first of equations (7). This gives

$$a_i + \sigma_1 b_i + \sigma_2 c_i = 1/\lambda_1 \rho_i^2 , \qquad \text{i = 1, 2, ..., n} \qquad (11)$$

Expression (10) is satisfied if λ_1 obtained in the solution is positive. The verification that λ_1 is positive avoids plotting $f(\sigma_1,\sigma_2)$ and $g(\sigma_1,\sigma_2)$ to find the allowable range of K_1 and K_2. The first of equations (7) can be rewritten as

$$\rho_i = 1/(\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{1/2} \qquad \text{i = 1, 2,…, n} \qquad (12)$$

Substituting the above expression for ρ_i in the last three equations of (7) gives three non-linear equations in three unknowns λ_1, λ_2 and λ_3 as follows:

$$\begin{aligned} h_1(\lambda_1,\lambda_2,\lambda_3) &= \sum_{i=1}^{n} a_i (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-1/2} - D[x] = 0 \\ h_2(\lambda_1,\lambda_2,\lambda_3) &= \sum_{i=1}^{n} b_i (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-1/2} - D[y] = 0 \qquad (13) \\ h_3(\lambda_1,\lambda_2,\lambda_3) &= \sum_{i=1}^{n} c_i (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-1/2} - D[z] = 0 \end{aligned}$$

The above set of non-linear equations can be solved by Newton-Raphson method. The elements of the Jacobian [J] can be derived as follows

$$\frac{\partial h_1}{\partial \lambda_1} = \sum_{i=1}^{n} a_i \left(-\frac{a_i}{2}\right) (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-3/2}$$

$$\frac{\partial h_1}{\partial \lambda_2} = \sum_{i=1}^{n} a_i \left(-\frac{b_i}{2}\right) (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-3/2}$$

$$\frac{\partial h_1}{\partial \lambda_3} = \sum_{i=1}^{n} a_i \left(-\frac{c_i}{2}\right) (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-3/2}$$

Similarly the expressions for $\partial h_2/\partial \lambda_i$, i = 1,2,3 and $\partial h_3/\partial \lambda_i$, i = 1,2,3 can be obtained. Then the Jacobian [J] can be written as follows:

$$[J] = \sum_{i=1}^{n} \left(-\frac{1}{2}\right) (\lambda_1 a_i + \lambda_2 b_i + \lambda_3 c_i)^{-3/2} \begin{bmatrix} a_i^2 & a_i b_i & a_i c_i \\ b_i a_i & b_i^2 & b_i c_i \\ c_i a_i & c_i b_i & c_i^2 \end{bmatrix} \qquad (14)$$

The analytical expression for Jacobian [J] is available and it is not needed to find the numerical derivatives of h_i, i = 1,2,3. On solving Eq. (13) by Newton-Raphson method the values of λ_1, λ_2 and λ_3 can be obtained. The ρ_i's are obtained from expression (12).

4. OPTIMATIZATION USING GENETIC ALGORITHMS

Genetic algorithms (GAs) are computerized search and optimization algorithms based on the mechanics of natural genetics and natural selection. GAs mimic the survival-of-the-fittest principle of nature to do a search process (Deb, 1995a). The variables in GA are coded in some string structures, where mostly binary coded strings are used. The length of the string is usually determined according to the desired solution accuracy.

In general, a fitness function F(x) is first derived from the objective function and used in successive genetic operation. For maximization problems, the fitness function can be considered to be the same as the objective function or $F(x) = f(x)$. For minimization problems the following function is often used.

$$F(x) = 1/(1 + f(x))$$

The fitness function value of a string is known as the string's fitness.

The operation of GAs starts with a population of random strings representing design or decision variables. Thereafter, each string is evaluated to find the fitness value. The population is then operated by three main operators — reproduction, crossover and mutation — to create a new population of points. The new population is further tested for termination. If the termination criterion is not met, the population is iteratively operated by the above three operators, and evaluated. This procedure is continued until the termination criterion is met. One cycle of these operations and the subsequent evaluation procedure is known as a generation in GA's terminology. Because there are more than one strings being processed simultaneously, it is l kely that the solution obtained may be a global optimum. GAs are used to find global optimum for complex engineering optimization problems.

As GAs use a coding of variables, they work with a discrete search space. Even though the underlying objective function is a continuous function, GAs convert the search space into a discrete set of points. GAs have also been developed to work directly with continuous variables (instead of discrete variables). In those GAs binary strings are not used. Instead, the variables are directly used. These are called real-coded GA (Deb, 1995a; Deb et al, 1995b). A code developed by Deb et al (1995b) for real-coded GA is used in the present work.

5. OPTIMUM TOLERANCES AND CLEARANCES IN FDM

Agrawal (2001) finds the coordinates of a point on the contour of a slice traced by the nozzle tip Q. The geometric model of the FDM process is given in Fig. 4. There are fifteen random variables $V_1, V_2, \ldots, V_{15}$ involved in the stochastic model of the FDM process. He gives the expressions for the influence coefficients a_i, b_i and c_i and the partial derivatives needed for them. The variances of the dependent variable x, y and z and their sum at several points on the work surface is tabulated. The plots of three-sigma bands of error in tracing several curves by the nozzle tip are also given. From the error analysis it is found that the maximum value of the sum of variances DSUM occurs at l_1 = 0.25 m and l_3 = 0.25 m.

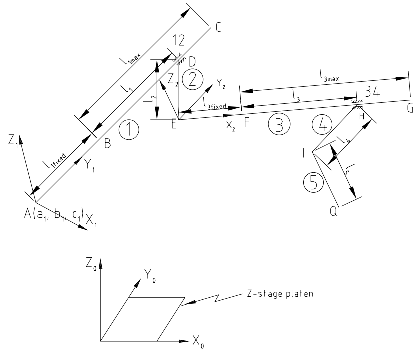

Figure 4: Geometric model of the FDM process

The optimal allocation of tolerances and clearances is done at the mean values of variables and other input values mentioned below:

a_1	=	−0.15 m	b_1	=	−0.10 m	c_1	=	0.22 m			
l_{1fixed}	=	0.09 m	l_{1max}	=	0.25 m	l_2	=	0.07 m			
l_{3fixed}	=	0.12 m	l_{3max}	=	0.25 m	l_4	=	0.01 m	l_5	=	0.15 m
α_1	=	1.0°	β_1	=	0.0°	α_2	=	0.0°	β_2	=	−1.0°

The optimal allocation is based on the location of nozzle tip where DSUM is maximum, that is, at l_1 = 0.25m and l_3 = 0.25 m. Assuming D[x] = 6.5 x $10^{-8}m^2$, D[y] = 3.0 x $10^{-7}m^2$ and D[z] = 19.5 x $10^{-8}m^2$, the optimal values of variances of variables V_i are obtained using Lagrange multiplier technique. Since a real solution for the Lagrange multipliers (λ_1, λ_2, λ_3) is obtained with positive λ_1, therefore, the ratio of variances, K_1 and K_2 are in the allowable range and the solution for (λ_1, λ_2, λ_3) exists. The solutions obtained for the Lagrange multipliers are as follows:

$$\lambda_1 = 1.0396 \times 10^{15} \quad \lambda_2 = 1.2301 \times 10^{18} \quad \lambda_3 = 9.6734 \times 10^{14}$$

The optimal values of variances ρ_i of variables V_i obtained from the Lagrange multiplier technique are listed in Table 1. The derivatives $\partial[q_2]/\partial V_i$, $\partial[T_A]/\partial V_i$

and $\partial[T_B]/\partial V_i$, and hence $\partial[q]/\partial V_i$ are identical for (i = 2,7). They are also identical for the indices i in each of the sets (i = 3,6), (i = 4,8) and (i = 5,9,10,11). Therefore, from Eq. (7) the optimum variances ρ_i are identical for the indices i in each of these sets as obtained by Lagrange multiplier technique in Table 1.

The optimum values of variances are also obtained using real-coded genetic algorithm optimization method. The input values taken for GA runs are as follows:

Population size = 150, Number of variables = 15, Bounds on variables = rigid,

Crossover probability = 0 90, Muta ion probability = 0 05, Random seed number = 0.15.

Optimization is done by varying the number of generation and the number of runs. The optimum values of variances ρ_i on variables V_i, obtained for 600 generations and a single run, are listed in Table 1 along with those obtained from Lagrange multiplier technique.

Table 1: Optimum Values of Variances ρ_i in FDM from Lagrange Multiplier Technique and from Real-Coded GA

Optimum Variances from Lagrange		Optimum Variances from Real GA	
i	**Variance ρ_i**	**i**	**Variance ρ_i**
1	9.018e−10	**1**	8 533e−10
2	3.215e−08	**2**	2 953e−08
3	3.101e−08	**3**	3.078e−08
4	9.018e−10	**4**	7 374e−10
5	2.730e−08	**5**	2.873e−08
6	3.101e−08	**6**	3 123e−08
7	3.215e−08	**7**	2 912e−08
8	9.018e−10	**8**	7.405e−10
9	2.730e−08	**9**	2 920e−08
10	2.730e−08	**10**	2.809e−08
11	2.730e−08	**11**	2.799e−08
12	6.531e−09	**12**	1.445e−08
13	8.058e−08	**13**	7 328e−08
14	6.005e−09	**14**	1 545e−08
15	6.940e−08	**15**	7.611e−08
Production Cost, C from Lagrange		**Production Cost, C from Real GA**	
3.946+09		4.312+09	

Real GA give a near global optimum solution. The solution can be improved by using Cauchy's steepest descent method. The optimum values obtained by real GA compare quite closely with those obtained by the Lagrange method. The minimum production cost, C obtained from real GA is larger than that obtained using Lagrange as real GA give a solution in the vicinity of the global solution. So, the

Lagrange method which gives a closed form solution has given the global optimum. The optimum values of ρ_{12} and ρ_{14} obtained by real GA differ from those obtained by the Lagrange method. This needs to be investigated further. It has been observed that when the optimum values differ by several orders, then some of the optimum variances obtained by GA differ from those obtained by the Lagrange method.

The tolerances and clearances are computed from the optimal values of variances ρ_i of variables V_i obtained from the Lagrange method. Both ρ_6 and ρ_7 give the clearance c_{12} in pair 12. The smaller of the two values is chosen for the clearance c_{12} because narrower clearance will ensure that the variances on the dependent variables do not exceed the specified limit. Similarly, the smaller of the two values of clearances obtained from the variances ρ_8 and ρ_9 is chosen for the clearances c_{34} in pair 34. The optimal values of tolerances and clearances are listed in Table 2.

Table 2: Optimal Allocation of Tolerances in FDM

Optimal Allocation of Tolerances			
		Tolerance	**Units**
Absolute tolerance in positioning member 2 along link 1	ε_1	= 9.009e−05	m
Tolerance per unit length on link 2	ε_2	= 7.685e−03	m/m
Absolute tolerance in positioning member 4 along link 3	ε_3	= 5.283e−04	m
Tolerance per unit length on link 4	ε_4	= 9.009e−03	m/m
Tolerance per unit length on link 5	ε_5	= 3.305e−03	m/m
Radial Clearance in pair 12	c_{12}	= 3.522e−04	m
Radial Clearance in pair 34	c_{34}	= 6.006e−05	m
Absolute tolerance in Z_2 direction at the head attachment	ε_{10}	= 4.957e−04	m
Absolute tolerance in Z_2 direction at the nozzle tip attachment	ε_{11}	= 4.957e−04	m
Absolute tolerance on angle α_1	ε_{12}	= 2.424e−04	rad
Absolute tolerance on angle β_1	ε_{13}	= 8.516e−04	rad
Absolute tolerance on angle α_2	ε_{14}	= 2.325e−04	rad
Absolute tolerance on angle β_2	ε_{15}	= 7.903e−04	rad

Converting the tolerance coefficients into absolute tolerances on the respective links and comparing the tolerances on links we find that stricter tolerances are demanded on links 1 and 4. It may be noted that these two links are the only links in Y_0-direction. Stricter clearance is demanded in pair 34. Stricter tolerances are demanded on angles α_1 and α_2, i.e., the angles about X_0-direction. Therefore, the error is most sensitive to V_1, V_4, V_8, V_9, V_{12} and V_{14}.

6. OPTIMUM TOLERANCES IN SL

Agrawal (2001) finds the position vector $q(u, w) = [\ x\ y\ z\ 1\ ]$ of the point

Q on the contour of a slice in SL. The laser beam in SL draws the contour on the resin surface. The geometric model of the FDM process is given in Fig. 5. There are eleven random variables $V_1, V_2, \ldots, V_{11}$ involved in the stochastic model of the process. Agrawal gives the expressions for the influence coefficients a_i, b_i and c_i and the partial derivatives needed for them. The variances of the dependent variable x, y and z and their sum are tabulated. The plots of three-sigma bands of error in tracing several curves on the resin surface are also given. The maximum value of the sum of variances DSUM occurs at $(q_x = 0, q_y = .3)$.

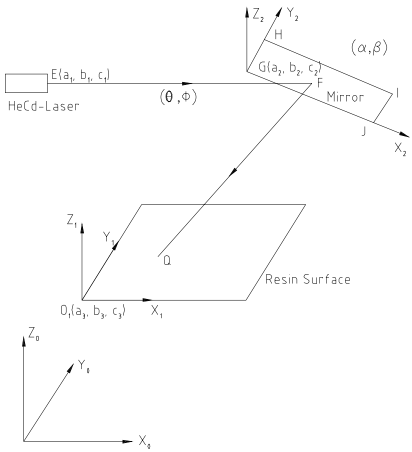

Figure 5: Geometric model of the SL process

The optimal allocation of tolerances is done at the mean values of variables and the other input values mentioned below:

a_1	=	0 0 *m*	b_1	=	0 15 m	c_1	=	1 0 m	θ	=	90 0°	φ	=	0 0°
a_2	=	0 25 m	b_2	=	0 05 m	c_2	=	1 02 m						
a_3	=	0 0 m	b_3	=	0 0 m	c_3	=	0 5 m						

The optimal allocation is based on the point at which the sum of variances DSUM is maximum, that is, at $(q_x = 0, q_y = .3)$ on the resin surface. Assuming $D[x] = 8.0 \times 10^{-8} m^2$, $D[y] = 12.0 \times 10^{-8} m^2$ and $D[z] = 5.0 \times 10^{-8} m^2$, the optimal values of variances of the variables V_i are obtained using Lagrange multiplier technique as discussed in Section 3. Since a real solution for the Lagrange multipliers $(\lambda_1, \lambda_2, \lambda_3)$ is obtained with positive λ_1, therefore, the ratio of variances, K_1 and K_2 are in the allowable range and the solution for λ_1, λ_2 and λ_3 exists. No real solution was found when $K_2 \approx 5.0$. The solution obtained for the Lagrange multipliers are as follows:

$$\lambda_1 = 7.740 \text{ x } 10^{15} \quad \lambda_2 = 3.164 \text{ x } 10^{14} \quad \lambda_3 = -2.034 \text{ x } 10^{15}$$

The optimal values of variances ρ_i are listed in Table 3.

The optimum values of variances are also obtained using real-coded genetic algorithm optimization method as discussed in Section 3. Since the error at the point Q does not depend upon the variable V_1, therefore, only the other ten variables are considered for optimization using GA. The influence coefficients with respect to the variable V_1 are smaller by many orders compared to those for other variables. From Eq. (7) it can be seen that the terms corresponding to V_1 contribute negligibly to the constraints. The input values taken for GA runs are as follows:

Population size = 100 , Number of variables = 10 , Bounds on variables = rigid,

Crossover probability = 0.85 , Mutation probability = 0.05 , Random seed number = 0.51.

Optimization is done by varying number of generation and number of runs. The optimum values of variances ρ_i on variables V_i, obtained for 600 generations and single run, are listed in Table 3 along with those obtained from Lagrange multiplier techniques.

Table 3: Optimum Values of Variances ρ_i in SL from Lagrange Multiplier Technique and from Real Coded GA

Optimum Variances from Lagrange		Optimum Variances from Real GA	
I	Variance ρ_i	i	Variance ρ_i
1	6 730e+00		
2	4 279e−08	2	4.463e−08
3	1.003e−08	3	1 147e−08
4	1.152e−08	4	1 300e−08
5	4 915e−08	5	4.807e−08
6	1.137e−08	6	1 300e−08
7	6 596e−08	7	7.647e−08
8	1.979e−08	8	2 280e−08
9	3 153e−08	9	3 253e−08
10	8 341e−09	10	9 546e−09
11	5.000e−08	11	2 239e−08
Production Cost, C from Lagrange		**Production Cost, C from Real GA**	
5.555+08		5 214+08	

Real GA gives a near global optimum solution. The optimum values of variances obtained by real GA compare very closely to those obtained by Lagrange multiplier techniques. This verifies that the optimum values obtained are global optima. The minimum production cost, C obtained from real GA is smaller than that obtained with Lagrange methods, which gives a closed-form solution. In the vicinity of the closed-form solution the objective function should always be larger. This small difference is due to the machine accuracy of the system. Since the order of influence coefficient and optimum values are very small and eleven variables were used in the Lagrange method compared to ten in real GA, therefore, this much difference is introduced.

The tolerances on coordinates and angles, obtained from the optimal values of variances ρ_i, are listed in Table 4. The tolerance allocated on the variable V_1 is many orders higher compared to those on other variables. This means that the error at the point Q is not depended on the variable V_1. It can be easily seen that changing the x-coordinate of the laser beam does not affect the coordinates of the point Q. The absolute tolerances on other variables are of the same order for this set of input values. Among them, stricter tolerance is demanded on the z-coordinate of the source E and the x-coordinate of mirror. Stricter tolerance is demanded on the angle β. Therefore the error is most sensitive to V_3, V_6 and V_{10}.

Table 4: Optimal Allocation of Tolerances in SL

Optimal Allocation of Tolerances			
		Tolerance	**Units**
Absolute tolerance on coordinate a_1	ε_1	= 7.782e+00	m
Absolute tolerance on coordinate b_1	ε_2	= 6.205e−04	m
Absolute tolerance on coordinate c_1	ε_3	= 3.004e−04	m
Absolute tolerance on angle α_1	ε_4	= 3.220e−04	rad
Absolute tolerance on angle β_1	ε_5	= 6.651e−04	rad
Absolute tolerance on coordinate a_2	ε_6	= 3.198e−04	m
Absolute tolerance on coordinate b_2	ε_7	= 7.705e−04	m
Absolute tolerance on coordinate c_2	ε_8	= 4.220e−04	m
Absolute tolerance on angle α_2	ε_9	= 5.327e−04	rad
Absolute tolerance on angle β_2	ε_{10}	= 2.740e−04	rad
Absolute tolerance on coordinate c_3	ε_{11}	= 6.708e−04	m

The synthesis procedure in the present work enables the designer to allocate tolerances and clearances optimally. The synthesis procedure clearly shows that the allowances on different variables are different depending upon the influence of a particular variable.

7. CONCLUSION

A methodology for optimal allocation of tolerances and clearances in RP processes is presented. For optimal allocation of tolerances and clearances in RP processes, the constraints are linear functions of design variables, and hence, the optimization is done using the Lagrange multiplier technique. The optimal allocation is done at that point on the work surface where the sum of variances DSUM is found to be maximum while conducting error analysis. The optimization is also done using real-coded Genetic Algorithms (real-coded GA). Using the synthesis procedure, the allocation is done for the FDM and SL processes.

The synthesis procedure in the present work enables the designer to allocate tolerances and clearances optimally. The synthesis procedure clearly shows that the allowances on different variables are different, depending upon the influence of that variable. In FDM, stricter tolerances are demanded on links 1 and 4, i.e., the links in Y_0-direction and on angles α_1 and α_2, i.e., the angles about X_0-direction. In SL, stricter tolerances are demanded on the z-coordinate of the source of the laser beam, the x-coordinate of the mirror and the angle β of the mirror. It has been found that the error in SL is not sensitive to the location of the source of the laser beam along the direction of the laser beam.

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