Time Reversal Violation through B Meson Mixing

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Abstract

Time reversal (T) violation is investigated through B physics. We utilize observables in the process of $\Upsilon(4S) \to B\bar{B}$, which are measurable in B factory experiments. Due to Einstein-Podolsky-Rosen entanglement, the correlated information about $B\bar{B}$ is available.

In this thesis, we introduce methodology to gain observables which are sensitive to T violation. The phenomenon of *neutral meson mixing* enables us to test discrete symmetries. The event rates of two processes, $B_- \to \bar{B}^0$ and $\bar{B}^0 \to B_-$ ($-$ implies a CP eigenvalue), are utilized. These processes are apparently related with flipping time direction so that the event number difference of the processes seems to be a T violating quantity. However, it turns out that the observables are not exact T violating quantities since a genuine time reversed process is unobserved in the experiments.

We construct time reversal-like asymmetries which consist of the event number difference for the mixing processes of B meson. One can clarify how the asymmetries behave under T transformation to demonstrate that the observable is not precisely a T violating quantity. The overall factors of the time dependent decay rates are taken into account in this thesis. The effect of mixing-induced CP violation in K meson system is extracted, which yields $\mathcal{O}(10^{-3})$ contribution to an observable. Some combinations of the asymmetry enable us to constrain parameters for wrong sign decay of B meson, which is suppressed in the standard model. As a probe of physics beyound the standard model, CPT violation is testable via $B^0 - \bar{B}^0$ mixing observables. The constraints on BSM are obtained through the precise measurement in the experiments. Furthermore, we suggest conditions for the asymmetry to be a T-odd quantity. One of such conditions arises due to the difference of overall factors which form the asymmetry.

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Chapter 1

Introduction

The standard model (SM), which unifies weak and electromagnetic interactions [1–3], is a successful model that is consistent with most phenomena observed in experiments. All the elementary particles contained in the SM have been found after the discovery of Higgs boson [4–6] at the Large Hadron Collider (LHC) experiment [7, 8]. The gauge symmetry based on invariance under $SU(3)_c \times SU(2)_L \times U(1)_Y$ transformation leads to comprehensive description of fundamental interactions.

However, the SM cannot explain several facts which are observed in experiments. One of such issues is the origin of neutrino mass. Although neutrinos are massless in the SM, the experimental results of neutrino oscillation [9, 10] demonstrated non-zero mass. Another issue is the hierarchical structure of fermion mass. The observed mass spectra for quarks imply large gaps, which require unnatural fine-tuning of theoretical parameters. To resolve these issues, the SM needs its extension; models of physics beyond the standard model (BSM) are constructed for certain motivation. In this context, a phenomenological evidence of BSM is worth pursuing so that the experimental searches for a signal of BSM are extensively conducted.

To check the validity of a theory, discrete symmetries, which represent characteristic properties of the model, are tested in experiments. Such symmetries are based on the following discrete transformations:

- Charge conjugation (C), which interchanges particle and its anti-particle.
- Parity (P) transformation, which flips the sign of spacial coordinates.
- CP transformation, based on a combined operation of C and P transformation.
- Time reversal (T) transformation, interchanging an initial state and a final state.
- CPT transformation, based on a combined operation of CP and T transformation.

Provided that Lorentz invariance and Hermiticity are satisfied in local quantum field theory, CPT symmetry must be conserved (CPT theorem [11,12]). Consequently, the SM should satisfy CPT invariance. (CPT violating extension of the SM is suggested in Ref. [13].)

To understand what CPT theorem ensures, let us consider a CP conserving theory. This case is depicted as follows:

CP	CPT
conserved	conserved

In this context, the T symmetric property is fixed in the following way:

CP	T	CPT
conserved	conserved	conserved

As shown above, T symmetry is required to conserve. Likewise, if we consider a CP violating theory, T symmetry is determined:

CP	T	CPT
violated	violated	conserved

Hence, T symmetry should be violated in association with CP violation. In this sense, T symmetry is automatically connected with CP symmetry under the presence of CPT invariance. No clear evidence of CPT violation has been observed in experiments [14, 15].

As originally suggested in Ref. [16], CP symmetry is violated through weak interaction in the three-generation standard model. In the quark [17, 18] sector, CP violation is caused by an irreducible phase in Cabibbo-Kobayashi-Maskawa (CKM) [16, 19] mixing matrix, which characterizes the *flavor* changing interaction in the charged current. In this sense, the measurement of flavor changing processes are of particular importance to observe CP violation.

It is well-known that quarks are confined [20] inside hadrons so that we cannot directly observe the interior particles. A bound state for quark and anti-quark is referred to as a *meson*, which enables us to study physics in the quark sector. Therefore, phenomenology of CP violation in the quark sector is discussed in weak decays of hadrons. For typical mesons, the properties including their quantum numbers are shown in Tab. 1.1.

Table 1.1: Properties of neutral mesons. In the second row, contained quarks which form a bound state are exhibited. From the third to fifth row, quantum numbers of strangeness, beauty and spin-parity are shown, respectively.

meson	K^0	$\bar{K^0}$	B^0	$\bar{B}^0$	B_s	$\bar{B}_s$	ψ	Υ(nS)
quark	$\bar{s}d$	$s\bar{d}$	$\bar{b}d$	$b\bar{d}$	$\bar{b}s$	$b\bar{s}$	$c\bar{c}$	$b\bar{b}$
S	$+1$	-1	0	0	-1	$+1$	0	0
B	0	0	$+1$	-1	$+1$	-1	0	0
J^P	0^-	0^-	0^-	0^-	0^-	0^-	1^-	1^-

As an experimental consequence for non-invariance of discrete symmetries, parity violation [21] in weak interaction was detected through beta decays of nuclei [22]. Afterward, CP violation in $K_L \to 2\pi$ decay was first discovered in 1964 [23]. Furthermore, the evidences of CP violation in K meson decay have been also verified in the experiments [24–27]. Subsequently, the result [28, 29] of the B factory experiments confirmed *large* CP violation [30] which is predicted in the SM. The flavor factories have also verified CP violation in $B^0 \to K^+\pi^-$ [31, 32] and $B^0 \to \rho\pi$ [33, 34] decays. The result of the phenomenological analysis [35] demonstrates that the measured CP violating phenomena [36] are consistent with the prediction of the theory, which characterizes one of the most successful aspects of the SM.

Since the presence of CP violation is firmly clarified, T symmetry is expected to be violated due to the CPT theorem. Crosschecking the CP violation and T violation, one can get information about whether CPT symmetry is violated. In this sense, the experimental observation of T violation provides a method to investigate BSM.

To discuss a phenomenological search for T violation, let us denote a probability of the transition as $P[i \to j]$, where i and j indicate some (multi)particle state. Since time reversal flips the direction of time, T symmetry is characterized in the following relations:

$$\begin{cases} P[i \to j] = P[j \to i] & \text{(T symmetry is conserved.)} \\ P[i \to j] \neq P[j \to i] & \text{(T symmetry is violated.)} \end{cases}$$

Consequently, the difference of the probabilities is a probe of T violation, *i.e.*,

$$P[i \to j] - P[j \to i] \propto \text{(T violation)}. \tag{1.1}$$

As shown above, one should prepare the transition probabilities for both $i \to j$ and $j \to i$ to measure the evidence of T violation.

Although the experimental confirmation of CP violation is well-established, the measurement of time reversed processes is still a difficult task. Consider $B^0 \to \psi K_S$ and its time reversed process $\psi K_S \to B^0$. Since $\psi - K_S$ collision is not available in B factory experiments, the process for $\psi K_S \to B^0$ is an undetectable mode.

For the purpose to measure CP and/or T violation in the quark sector, *neutral meson mixing* is a particularly important phenomenon. Neutral meson such as $K^0 = (\bar{s}d)$, is changed into $\bar{K^0} = (s\bar{d})$, due to weak interaction. As a consequence of this phenomenon, the transition from meson to anti-meson occurs through time evolution, *e.g.*, $K^0 \leftrightarrow \bar{K^0}$, $B^0 \leftrightarrow \bar{B^0}$ and $B_s^0 \leftrightarrow \bar{B_s^0}$, where Schrödinger equation is applicable to the description of the mixing system.

In the CPLEAR experiment, K meson system is utilized to investigate T violation. The experiment is conducted by proton-antiproton collision to produce kaons through a process of strong interaction:

$$p\bar{p} \longrightarrow \begin{cases} K^+\pi^-\bar{K^0} \\ K^-\pi^+K^0 \end{cases} \tag{1.2}$$

The kaons produced above are utilized to measure time dependent process rates of $K^0 \to \bar{K^0}$ and $\bar{K^0} \to K^0$ [39]. These processes are related under the discrete transformations in the following way:

$$\begin{array}{ccc} \boxed{K^0 \to \bar{K^0}} & \quad & \boxed{K^0 \to \bar{K^0}} \\ \text{CP} \Updownarrow & & \text{T} \Updownarrow \\ \boxed{\bar{K^0} \to K^0} & & \boxed{\bar{K^0} \to K^0} \end{array}$$

As shown above, these processes are related with both CP and T transformation. Thus, if the transition probability of $K^0 \to \bar{K^0}$ is different from one for $\bar{K^0} \to K^0$, it implies both T violation and CP violation. The measurement of the CPLEAR collaboration in Ref. [40] indicates that non-zero time integrated asymmetry is observed, which results in the first demonstration of T violation. However, this result was not surprising since CP violation in K meson system had been already observed; to extract a T violating observable which is distinguished from CP violation, these processes are irrelevant modes.

To experimentally identify flavor contents of B mesons, the following experimental method is implemented: If $B \to l^+X$ decay is measured, where X represents some accompanying particles, the decaying particle is identified as B^0 since $\bar{B^0} \to l^+X$ is suppressed due to the $\Delta B = \Delta Q$ rule in the SM. In this sense, the state l^+X can filter the flavor content of $B^0 = (\bar{b}d)$.

Likewise, one can identify $\bar{B}^0 = (b\bar{d})$ if the final state is $l^- X$. This method is so-called *flavor tagging* [37], which is broadly implemented in B factory experiments to observe B^0 and $\bar{B}^0$.

As another tagging method, *CP tagging* [38] enables us to identify CP eigenvalues of B meson. If a final state is a CP eigenstate, the decaying particle is filtered as a state which has the same CP value. The final state is taken as $\psi K_{S(L)}$, which is a CP-odd (even) eigenstate in the limit where $K_{S(L)}$ is a CP eigenstate. Consequently, if a $B \to \psi K_S$ decay is observed, the decaying particle is identified as B_-, where $-$ stands for a CP eigenvalue.

In the B factory experiments such as BaBar and Belle, e^+e^- collision is utilized to produce B mesons in the following process,

$$e^+e^- \to \Upsilon(4S) \to B\bar{B}, \tag{1.3}$$

where $\Upsilon(4S)$ is a spin-1 bottomonium resonance. Imposed by Bose statistics, the created pair of B mesons should be a coherent state. In this context, Einstein-Podolsky-Rosen (EPR) entanglement [41] enables us to extract the correlated information about $B\bar{B}$. As a consequence, if one B^0 is filtered by the flavor tagging, another one in a pair of $B\bar{B}$ is determined as $\bar{B}^0$, which is a state orthogonal to B^0. In this way, a time dependent process $\bar{B}^0 \to B^0$ is measurable in B factories. The same identification method applies to CP eigenstate B mesons. If $B_\pm$ is filtered by the CP tagging, the B meson on the opposite side is identified as $B_\mp$ state at the same time. Thus, one can measure the time dependent process for $B_- \to B_+$ and $B_- \to \bar{B}^0$, etc. The implementation of flavor-identification is sketched in Fig. 1.0.1 while the extraction of CP eigenstate B meson is depicted in Fig. 1.0.2

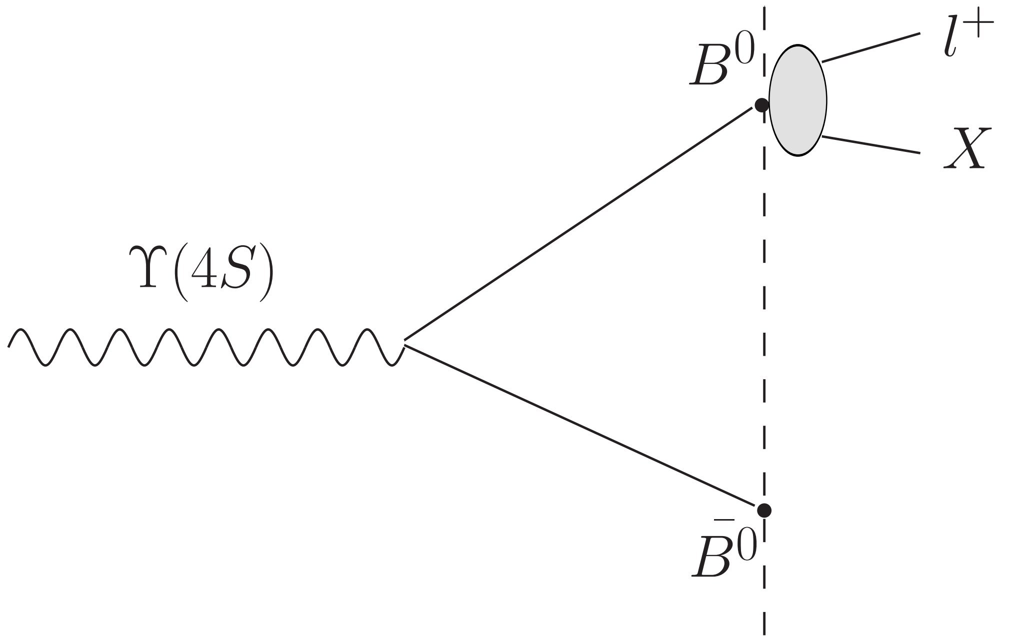

Figure 1.0.1: Flavor-identification in entangled system of $B\bar{B}$. On the upper side, B^0 is filtered by implementing the flavor tagging. B meson on the lower side is determined as $\bar{B}^0$ at the same time.

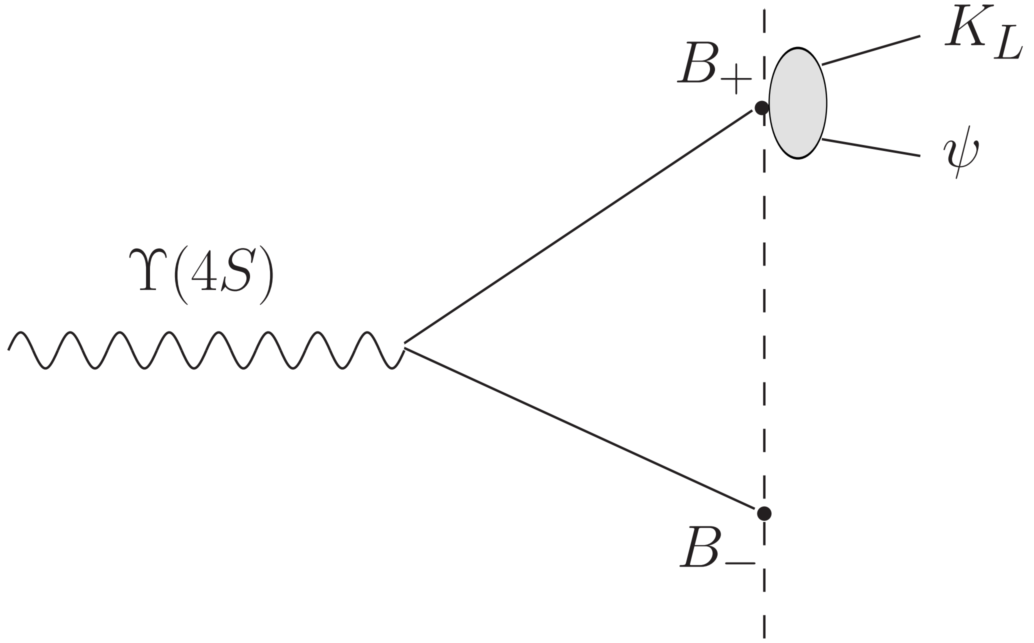

Figure 1.0.2: Identification of a CP eigenvalue in entangled system of $B\bar{B}$. On the upper side, CP eigenvalue of B meson is filtered through the CP value of the final state. B meson on the lower side is determined as B_- at the same time.

As an experimental check of T violation in the B factory experiment, the difference of the event rate for $B^0 \to \bar{B}^0$ and $\bar{B}^0 \to B^0$ are measured by the BaBar collaboration [42]. These mixing processes are related under discrete transformations in the following:

$$\begin{array}{ccc} \boxed{B^0 \to \bar{B}^0} & \qquad & \boxed{B^0 \to \bar{B}^0} \\ \text{CP} \Big\Updownarrow & & \text{T} \Big\Updownarrow \\ \boxed{\bar{B}^0 \to B^0} & & \boxed{\bar{B}^0 \to B^0} \end{array}$$

As depicted above, the processes are connected with both CP and T transformation. Therefore, the difference of the transition probability for $B^0 \to \bar{B}^0$ and one for $\bar{B}^0 \to B^0$ signals both CP violation and T violation. In the BaBar experiment [42], non-zero asymmetry was observed to demonstrate T violation in B physics. As analogous to the context of the CPLEAR experiment, the T violating result was not surprising since CP violation had been already measured in the B factory experiments.

To observe T violation distinguished from CP violation, methodology is suggested in Ref. [43], further discussed in Refs. [44–48] and reviewed in Ref. [49]. Their idea is based on mixing processes of B meson for $B_- \to \bar{B}^0$ and $\bar{B}^0 \to B_-$, where B_- is a CP-odd eigenstate. These processes are related under discrete transformations in the following:

$$\begin{array}{ccc} \boxed{\bar{B}^0 \to B_-} & \qquad & \boxed{\bar{B}^0 \to B_-} \\ \text{not CP} \Big\Updownarrow\!\!\!\!\!\times & & \text{T} \Big\Updownarrow \\ \boxed{B_- \to \bar{B}^0} & & \boxed{B_- \to \bar{B}^0} \end{array}$$

As one can see above, these two processes are not related with CP transformation. If the transition probability for $\bar{B}^0 \to B_-$ is different from one for $B_- \to \bar{B}^0$, it apparently implies T violation which is distinguished from CP violation. Following this idea, the BaBar collaboration reported [50] that they measured non-zero asymmetry. In this sense, the principle aim to measure T violation has been accomplished in the processes which are not related with CP transformation. (See review in the literature [51].)

However, it is suspicious that the measurement of the BaBar collaboration exactly indicates T violation. In Ref. [52], it is pointed out that there exist subtleties in the BaBar measurement since an inverse decay, a genuine time reversed process such as $l^+X \to B^0$, is not observed in the experiment. In their study, the BaBar observables are written in terms of an expression which includes inverse decay amplitudes to clarify how the asymmetry is deviated from a T-odd quantity. They demonstrated that BaBar asymmetry is identical to a T-odd quantity if and only if the following conditions are satisfied (these conditions are derived by assuming that ψK_S and ψK_L are exact CP eigenstates):

(1) the absence of the wrong sign semi-leptonic B meson decays

(2) the absence of the wrong strangeness B meson decays

(3) the absence of CPT violation in the strangeness changing decays

Under the presence of wrong sign decay amplitudes of $\bar{B}^0 \to l^+X$ and/or $B^0 \to l^-X$, which violate the $\Delta B = \Delta Q$ rule, the final state l^+X does not tell us exact information about the flavor content of B meson. In this sense, the wrong sign semi-leptonic decay amplitude gives rise to uncertainty to the flavor taggings. Such careful argument of T violation is discussed in Refs. [53–56] for K meson system.

In this thesis, we analyze the time dependent asymmetry of the processes for $\bar{B}^0 \to B_-$ and $B_- \to \bar{B}^0$. Our investigation is the extension of the work in Ref. [52], incorporating the difference of overall constants for the rates that form the asymmetry. The contribution from CP and CPT violation in K meson mixing is taken into account. The asymmetry is written in terms of parameters which are independent of redefinition of phases of quarks. We specify how the asymmetries behave under T, CP and CPT transformation. It is shown that T conserving terms also contribute to the observables although the original idea suggested a way to extract T violation. Furthermore, in the latter part of this thesis, we show that some combinations of the observables enable us to extract theoretical parameters of interest, *e.g.,* wrong sign decay amplitudes of B meson. CPT violating parameters are also extracted from the observables, as investigated in B meson system in Refs. [57–59]. Our formulation is applicable to the measurement in a future experiment, such as Belle II, which is expected to collect 50ab^{-1} data sample. As a final remark in this thesis, we discuss the T conserving parts of the asymmetry. One can find that the asymmetry is a T violating quantity when several conditions are satisfied.

This thesis is organized as follows: In Chap. 2, the system of neutral meson mixing is briefly introduced. The time dependence of B^0 and $\bar{B}^0$ states is derived in the simplified description governed by the Weisskopf-Wigner approximation [60]. In Chap. 3, we discuss a time dependent decay rate in entangled B meson system. The time dependent asymmetry is defined to gain observables which are sensitive to non-invariance of the discrete symmetries. In Chap. 4, we define theoretical parameters to express the asymmetry. In our notation, one can argue unambiguous discrete transformation properties of the observables. The relation between the notation in Ref. [52] and ours is also discussed. It turns out that the defined parameters are phase convention independent quantities. The contribution from indirect CP violation in K meson system is extracted. In Chap. 5, the event number asymmetry is analyzed in terms of the parameters defined in Chap. 4. We show that the constructed asymmetry consists of not only the T-odd part but also T-even part. In Secs. 5.1-5.3, some parameters of interest, which include CPT violation and wrong sign decay amplitude, are extracted from the observables. In Chap. 6, we suggest the conditions that T-even parts of the asymmetry vanish. As an extension of the discussion in Ref. [52], we suggest the intuitive reason why these conditions are imposed. These conditions are categorized as two types: The first one requires the B meson state, which appears in the diagram of $B_- \to \bar{B}^0$, being equivalent to a state in the genuine time reversed process. The second condition accounts overall constant which forms the asymmetry. We find that the second condition is needed when one takes account of the difference of overall constant of the two rates. Chapter 7 is devoted to summary and future prospects.

This thesis is based on the published paper [61] and proceedings [62].

Chapter 2

Neutral Meson Mixing

In this chapter, the system of neutral meson mixing is introduced. Such mixing phenomenon occurs through weak interaction. The time evolution of the mixing system is governed by Schrödinger equation [63]. The formulation of the neutral meson mixing is found in the literature [64–74]. In the following, the system for the $B^0\bar{B}^0$ is addressed. The states for neutral meson are transformed under CP as,

$$\mathrm{CP}\,|B^0\rangle = -\,|\bar{B}^0\rangle\,, \quad \mathrm{CP}\,|\bar{B}^0\rangle = -\,|B^0\rangle\,, \tag{2.1}$$

where flavor-definite states are denoted as $|B^0\rangle$ and $|\bar{B}^0\rangle$. Hereafter, we adopt simplified formalism [60, 75] for the system in which the wave function is given as,

$$|\psi(t)\rangle = c_1(t)\,|B^0\rangle + c_2(t)\,|B^0\rangle\,. \tag{2.2}$$

The time dependence of wave function for neutral mesons is described by the differential equation,

$$i\frac{\mathrm{d}}{\mathrm{d}t}\,|\psi(t)\rangle \;=\; \mathcal{H}\,|\psi(t)\rangle\,, \tag{2.3}$$

$$\mathcal{H} = M - \frac{i}{2}\Gamma \;=\; \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}, \tag{2.4}$$

where t denotes proper-time for neutral mesons. Hamiltonian in Eq. (2.4) is given as a non-Hermitian matrix to account decay of $B^0\bar{B}^0$ system. In Eq. (2.4), M and Γ are Hermitian matrices which stand for off-shell and on-shell intermediate states, respectively. The diagonal part in Hamiltonian, *i.e.,* $\mathcal{H}_{ii}(i = 1, 2)$, expresses the transitions of $B^0 \to B^0$ and $\bar{B}^0 \to \bar{B}^0$ while the off-diagonal part given as $H_{ij}(i \neq j)$ is associated with the transitions of $B^0 \leftrightarrow \bar{B}^0$.

The discrete symmetries relate the matrix element,

$$M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22}. \quad \text{(CPT limit)} \tag{2.5}$$

In the following, the formulation including CPT violation [52] is adopted. Effective Hamiltonian in Eq. (2.4) is diagonalized as,

$$X^{-1}\mathcal{H}X = \mathrm{diag}(\omega_H, \omega_L), \qquad X = \begin{pmatrix} p\sqrt{1+z} & p\sqrt{1-z} \\ -q\sqrt{1-z} & q\sqrt{1+z} \end{pmatrix}, \tag{2.6}$$

where ω_H and ω_L denote complex eigenvalues of Hamiltonian,

$$\omega_{H(L)} = M_{H(L)} - \frac{i}{2}\Gamma_{H(L)}. \tag{2.7}$$

We introduce parameters of the eigenvalues,

$$\Delta m = M_H - M_L, \qquad \Delta\Gamma = \Gamma_H - \Gamma_L, \tag{2.8}$$

$$m = \frac{M_H + M_L}{2}, \qquad \Gamma = \frac{\Gamma_H + \Gamma_L}{2}. \tag{2.9}$$

The above eigenvalues are written in terms of the matrix elements of effective Hamiltonian. In particular, the squared difference of the eigenvalues satisfies,

$$(\omega_H - \omega_L)^2 = \left[(M_{11} - M_{22}) - \frac{i}{2}(\Gamma_{11} - \Gamma_{22})\right]^2 + 4\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right). \tag{2.10}$$

The real and imaginary parts in the above equation lead to relations,

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 + (M_{11} - M_{22})^2 - \frac{1}{4}(\Gamma_{11} - \Gamma_{22})^2, \tag{2.11}$$

$$\Delta M \Delta\Gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}^*) + (M_{11} - M_{22})(\Gamma_{11} - \Gamma_{22}). \tag{2.12}$$

Furthermore, the mixing parameters are written in terms of the matrix element of effective Hamiltonian,

$$\left(\frac{p}{q}\right)^2 = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}, \tag{2.13}$$

$$z = -\frac{M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22})}{\Delta m - \frac{i}{2}\Delta\Gamma}. \tag{2.14}$$

As shown in Eq. (2.14), z implies CPT violation in mixing since $M_{11} - M_{22}$ (or $\Gamma_{11} - \Gamma_{22}$) vanishes in the CPT limit. For B_d system, the experimental constraint on z is obtained by the BaBar collaboration [76]. In the following, we consider the system in which the incoming mass eigenstates are given as,

$$|B_L^{\rm in}\rangle = p\sqrt{1-z}\,|B^0\rangle + q\sqrt{1+z}\,|\bar{B}^0\rangle\,, \tag{2.15}$$

$$|B_H^{\rm in}\rangle = p\sqrt{1+z}\,|B^0\rangle - q\sqrt{1-z}\,|\bar{B}^0\rangle\,. \tag{2.16}$$

In the above equations, the state of B_L (B_H) is associated with a lighter (heavier) mass eigenstate. As solution of Schrödinger equation, time evolution of the mass eigenstates is,

$$|B_{H(L)}(t)\rangle = e^{-i\omega_{H(L)}t}\,|B_{H(L)}(0)\rangle\,. \tag{2.17}$$

For an initial condition for the system, we take pure eigenstates for strong interaction, setting,

$$|B^0(0)\rangle = |B^0\rangle\,, \quad |\bar{B}^0(0)\rangle = |\bar{B}^0\rangle\,. \tag{2.18}$$

In this circumstance, the time evolution of the definite flavor states is determined as,

$$\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = X{\rm diag}(e^{-i\omega_H t}, e^{-i\omega_L t})X^{-1}\begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix}\,, \tag{2.19}$$

or, equivalently,

$$|B^0(t)\rangle = (g_+(t) + zg_-(t))\,|B^0\rangle - \frac{p}{q}\sqrt{1-z^2}g_-(t)\,|\bar{B}^0\rangle\,, \tag{2.20}$$

$$|\bar{B}^0(t)\rangle = -\frac{q}{p}\sqrt{1-z^2}g_-(t)\,|B^0\rangle + (g_+(t) - zg_-(t))\,|\bar{B}^0\rangle\,, \tag{2.21}$$

$$g_\pm(t) = \frac{1}{2}(e^{-i\omega_H t} \pm e^{-i\omega_L t}). \tag{2.22}$$

The initial condition is accounted in the time dependent factor in Eq. (2.22) since,

$$(g_+(0), g_-(0)) = (1, 0), \tag{2.23}$$

is satisfied.

As for outgoing states of neutral mesons, the reciprocal basis is used for the system with non-Hermitian Hamiltonian, as discussed in the literature [77–81]. The mass eigenstates satisfy orthogonal conditions,

$$\langle B_H^{\rm out}|B_H^{\rm in}\rangle = 1, \langle B_H^{\rm out}|B_L^{\rm in}\rangle = 0, \langle B_L^{\rm out}|B_L^{\rm in}\rangle = 1, \langle B_L^{\rm out}|B_H^{\rm in}\rangle = 0. \tag{2.24}$$

As states which satisfy the conditions in Eq. (2.24), outgoing mass eigenstates are defined,

$$\langle B_H^{\rm out}| = \frac{1}{2pq}(q\sqrt{1+z}\,\langle B^0| - p\sqrt{1-z}\,\langle \bar{B}^0|), \tag{2.25}$$

$$\langle B_L^{\rm out}| = \frac{1}{2pq}(q\sqrt{1-z}\,\langle B^0| + p\sqrt{1+z}\,\langle \bar{B}^0|). \tag{2.26}$$

In K meson system, $K_{L(S)}$ is associated with a long (short)-lived mass eigenstate. As discussed in Refs. [82, 83], we account CPT non-invariance in K meson mixing. The incoming mass eigenstates for K mesons are given as,

$$|K_L^{\rm in}\rangle = p_K\sqrt{1+z_K}\,|K^0\rangle - q_K\sqrt{1-z_K}\,|\bar{K}^0\rangle\,, \tag{2.27}$$

$$|K_S^{\rm in}\rangle = p_K\sqrt{1-z_K}\,|K^0\rangle + q_K\sqrt{1+z_K}\,|\bar{K}^0\rangle\,. \tag{2.28}$$

The time dependence of K meson system is obtained with replacement of $(z, p, q) \to (z_K, p_K, q_K)$. and $(\omega_H, \omega_L) \to (\lambda_L, \lambda_S)$ in Eqs. (2.20-2.22), where λ_L and λ_S stand for the eigenvalues of Hamiltonian in K meson system. The outgoing mass eigenstates are obtained as,

$$\langle K_L^{\rm out}| = \frac{1}{2p_Kq_K}(q_K\sqrt{1+z_K}\,\langle K^0| - p_K\sqrt{1-z_K}\,\langle \bar{K}^0|), \tag{2.29}$$

$$\langle K_S^{\rm out}| = \frac{1}{2p_Kq_K}(q_K\sqrt{1-z_K}\,\langle K^0| + p_K\sqrt{1+z_K}\,\langle \bar{K}^0|). \tag{2.30}$$

As for estimation of the transition amplitude of neutral mesons, the analysis in the SM is given in Ref. [84].

Chapter 3

Time Dependent Decay Rate for Entangled System of $B\bar{B}$

In this chapter, the wave function for the entangled system of $B\bar{B}$ is introduced. Subsequently, the time dependent asymmetry is constructed to obtain the observable sensitive to violation of the discrete symmetries.

3.1 Entangled State of $B\bar{B}$

As mentioned previously, B factory experiments are based on the process of $\Upsilon(4S) \to B\bar{B}$, where $\Upsilon(4S)$ has spin-1. Bose statistics requires that created pairs of $B\bar{B}$ should be a CP symmetric state. In this context, the measurement of B and $\bar{B}$ is correlated with each other, which is associated with a coherent state. Due to the angular momentum conservation, the produced pair of mesons is P-wave so that $B\bar{B}$ should be a parity-odd state. Combining the requirements of the P-odd and CP-even property, one should demand that the $B\bar{B}$ pair is a C-odd state. Therefore, the structure of the entangled wave function is,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|B^0(\mathbf{k},t)\rangle \otimes |\bar{B}^0(-\mathbf{k},t)\rangle - |\bar{B}^0(\mathbf{k},t)\rangle \otimes |B^0(-\mathbf{k},t)\rangle), \tag{3.1}$$

where $\mathbf{k}$ denotes a momentum carried by the neutral meson at the rest frame of $\Upsilon(4S)$. The relative sign in Eq. (3.1) represents the C-odd property of the wave function. The time dependence of the definite flavor states in Eq. (3.1) results from neutral meson mixing, as shown in Eqs. (2.20, 2.21). The EPR correlation of the flavors are measured for $K\bar{K}$ [85,86] and $B\bar{B}$ [87] system, both of which reported the result consistent with the prediction of quantum mechanics. Consequently, no clear evidences of decoherence [88] have been observed so far in the flavor factory experiments. For K meson system, EPR correlation and decoherence are reviewed in the literature [89]. Under the presence of CPT violation in some quantum gravity model, the coherence is weakened (ω-effect) as discussed in Refs. [90–92].

Since any orthogonal basis is available as an entangled $B\bar{B}$ state, the wave function is also written in terms of the CP eigenstates,

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|B_+(\mathbf{k},t)\rangle \otimes |B_-(-\mathbf{k},t)\rangle - |B_-(\mathbf{k},t)\rangle \otimes |B_+(-\mathbf{k},t)\rangle). \tag{3.2}$$

Hereafter, the momentum of neutral mesons is omitted for simplicity. We set $t=0$ at the time when pair-creation of $B\bar{B}$ occurs. Let us denote f_1 and f_2 as final states observed at t_1 and t_2 ($t_1 < t_2$). (f_1 and f_2 are referred to as a *tagging side* and a *signal side*, respectively.) The transition amplitude for the correlated observation of the $B\bar{B}$ pair is,

$$\langle f_1; f_2| T |\psi\rangle \tag{3.3}$$

The squared quantity of the amplitude in Eq. (3.3) leads to time dependent decay rate, which is in principle measured in B factory experiments. In Ref. [52], a general formula for the time dependent decay rate of the entangled $B\bar{B}$ system is given,

$$\begin{aligned}\Gamma_{(f_1)\perp,f_2} = e^{-\Gamma(t_1+t_2)} N_{(1)\perp,2}\kappa_{(1)\perp,2}[\cosh(y\Gamma t) + \frac{\sigma_{(1)\perp,2}}{\kappa_{(1)\perp,2}}\sinh(y\Gamma t) \\ + \frac{\mathcal{C}_{(1)\perp,2}}{\kappa_{(1)\perp,2}}\cos(x\Gamma t) + \frac{\mathcal{S}_{(1)\perp,2}}{\kappa_{(1)\perp,2}}\sin(x\Gamma t)].\end{aligned} \tag{3.4}$$

where t is defined as $t_2 - t_1$ and,

$$x = \frac{m_H - m_L}{\Gamma}, \quad y = \frac{\Gamma_H - \Gamma_L}{2\Gamma}. \tag{3.5}$$

As shown in Eq. (3.4), the rate is proportional to the factor of $e^{-\Gamma(t_1+t_2)}$, which accounts the time after $B\bar{B}$ creation. The time evolution of the signal side is represented as the hyperbolic and trigonometric functions in Eq. (3.4) with time interval of $t_2 - t_1$. Note that x is an $\mathcal{O}(1)$ quantity for B^0 system while y is suppressed in the SM. The coefficient of the time dependent functions in Eq. (3.4) are calculated in Ref. [52]. For completeness, we list the expressions of these parameters in App. A.

3.2 Time Dependent Asymmetry

In this section, we give an asymmetry for the entangled decays of B mesons, including overall factor $N_{(1)\perp,2}\kappa_{(1)\perp,2}$ in Eq. (3.4). A generic formula for the event number asymmetry of the two sets for final states: (f_1, f_2) versus (f_3, f_4) is written as,

$$A = \frac{\Gamma_{(f_1)\perp,f_2} - \Gamma_{(f_3)\perp,f_4}}{\Gamma_{(f_1)\perp,f_2} + \Gamma_{(f_3)\perp,f_4}}. \tag{3.6}$$

In Chap. 5, the time dependent asymmetry in Eq. (3.6) is analyzed with specific final states which are utilized in the BaBar experiment [50]. Using the master formula in Eq. (3.4), we can rewrite the asymmetry,

$$A = \frac{\left(\frac{1}{\sqrt{N_R}} - \sqrt{N_R}\right)\cosh(y\Gamma t) + \Delta\sigma\sinh(y\Gamma t) + \Delta\mathcal{S}\sin(x\Gamma t) + \Delta\mathcal{C}\cos(x\Gamma t)}{\left(\frac{1}{\sqrt{N_R}} + \sqrt{N_R}\right)\cosh(y\Gamma t) + \hat{\sigma}\sinh(y\Gamma t) + \hat{\mathcal{S}}\sin(x\Gamma t) + \hat{\mathcal{C}}\cos(x\Gamma t)}, \tag{3.7}$$

where,

$$N_R \equiv \frac{N_{(3)\perp,4}\kappa_{(3)\perp,4}}{N_{(1)\perp,2}\kappa_{(1)\perp,2}}, \tag{3.8}$$

$$\Delta X \equiv \frac{1}{\sqrt{N_R}}\frac{X_{(1)\perp,2}}{\kappa_{(1)\perp,2}} - \sqrt{N_R}\frac{X_{(3)\perp,4}}{\kappa_{(3)\perp,4}}, \qquad (\text{for } X = \sigma, \mathcal{C}, \mathcal{S}) \tag{3.9}$$

$$\hat{X} \equiv \frac{1}{\sqrt{N_R}}\frac{X_{(1)\perp,2}}{\kappa_{(1)\perp,2}} + \sqrt{N_R}\frac{X_{(3)\perp,4}}{\kappa_{(3)\perp,4}}. \qquad (\text{for } X = \sigma, \mathcal{C}, \mathcal{S}) \tag{3.10}$$

N_R in Eq. (3.8) stands for the ratio of overall normalization factors for a time dependent decay rate in Eq. (3.4). In Eqs. (3.7, 3.9, 3.10), the contribution from overall factors are taken into account. If one takes the limit,

$$N_R \to 1, \quad y \to 0, \quad \hat{S} \to 0, \text{ and } \hat{C} \to 0, \tag{3.11}$$

the asymmetry defined in Eq. (3.7) becomes one used in the BaBar experiment [50]. In Eq. (3.9, 3.10), $\Delta\mathcal{S}$ ($\Delta\mathcal{C}$) is identical to ΔS_T^+ (ΔC_T^+) defined in Ref. [52] if one takes the limit of $N_R \to 1$.

In practice, we only need to consider the time difference t within the interval which is shorter than the life-time of B meson so that the approximation,

$$\sinh(y\Gamma t) \simeq y\Gamma t, \qquad \cosh(y\Gamma t) \simeq 1, \tag{3.12}$$

is valid since $y \ll 1$ for B^0 meson system [93–96]. Thus, the time dependent asymmetry is expanded,

$$A \simeq \frac{-\frac{\Delta N_R}{2} + \frac{\Delta\sigma}{2} y\Gamma t + \frac{\Delta\mathcal{S}}{2}\sin(x\Gamma t) + \frac{\Delta\mathcal{C}}{2}\cos(x\Gamma t)}{1 + \frac{\hat{\sigma}}{2} y\Gamma t + \frac{\hat{\mathcal{S}}}{2}\sin(x\Gamma t) + \frac{\hat{\mathcal{C}}}{2}\cos(x\Gamma t)}, \quad (3.13)$$

$$N_R = 1 + \Delta N_R. \quad (3.14)$$

Non-zero value of ΔN_R in the above equation indicates that overall normalization ratio of decay rates are slightly deviated from unity.

Chapter 4

Definition of Parameters with Definite Flavor States

In this chapter, we introduce parameters that appear in the event number asymmetry in Eq. (3.13). In the time dependent decay rate, final states of B decay are given as the same ones used for the BaBar experiment [50]. The neutral meson mixing parameters, (p, q, z, p_K, q_K, z_K) which are defined in the previous chapter, lead to the transformation property for the discrete symmetry as,

$$p \overset{\text{CP or T}}{\leftrightarrows} q, \quad p \xrightarrow{\text{CPT}} p, \quad q \xrightarrow{\text{CPT}} q, \tag{4.1}$$

$$z \xrightarrow{\text{CP}} -z, \quad z \xrightarrow{\text{T}} +z, \quad z \xrightarrow{\text{CPT}} -z. \tag{4.2}$$

The transformation properties of the parameters in K meson system (p_K, q_K, z_K) are the same as (p, q, z), respectively.
Following Ref. [52], we introduce B meson decay amplitudes and inverse decay amplitudes,

$$A_f \equiv \langle f| T |B^0\rangle, \quad \bar{A}_f \equiv \langle f| T |\bar{B}^0\rangle, \quad A_f^{\text{ID}} \equiv \langle B^0| T |f^T\rangle, \quad \bar{A}_f^{\text{ID}} \equiv \langle \bar{B}^0| T |f^T\rangle, \tag{4.3}$$

where f^T is the time reversed state of f, *i.e.*, the state with flipped momenta and spins. Note that A_f ($\bar{A}_f$) and A_f^{ID} ($\bar{A}_f^{\text{ID}}$) are interchanged under T transformation. Using notation in Eq. (4.3), one denotes the following parameters,

$$\lambda_{\psi K_{S,L}} \equiv \frac{q}{p}\frac{\bar{A}_{\psi K_{S,L}}}{A_{\psi K_{S,L}}}\sqrt{\frac{1+\theta_{\psi K_{S,L}}}{1-\theta_{\psi K_{S,L}}}} = \frac{q}{p}\frac{A^{\text{ID}}_{\psi K_{S,L}}}{\bar{A}^{\text{ID}}_{\psi K_{S,L}}}\sqrt{\frac{1-\theta_{\psi K_{S,L}}}{1+\theta_{\psi K_{S,L}}}}, \tag{4.4}$$

$$\theta_{\psi K_{S,L}} = \frac{A_{\psi K_{S,L}}A^{\text{ID}}_{\psi K_{S,L}} - \bar{A}_{\psi K_{S,L}}\bar{A}^{\text{ID}}_{\psi K_{S,L}}}{A_{\psi K_{S,L}}A^{\text{ID}}_{\psi K_{S,L}} + \bar{A}_{\psi K_{S,L}}\bar{A}^{\text{ID}}_{\psi K_{S,L}}}. \tag{4.5}$$

Note that ψK_L and ψK_S are not exact CP eigenstates. For the description of time dependent asymmetries, the notation G_f, S_f and C_f are introduced with λ_f as,

$$G_f = \frac{2\mathrm{Re}\lambda_f}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathrm{Im}\lambda_f}{1+|\lambda_f|^2}, \quad C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \tag{4.6}$$

$$G_f^2 + S_f^2 + C_f^2 = 1. \tag{4.7}$$

The parameters in Eqs. (4.5, 4.6) explicitly appear in coefficients of the master formula (A.1-A.5). In Eq. (4.4), $\lambda_{\psi K_{S,L}}$ is written in terms of the decay amplitude whose final state is the mass eigenstate $\psi K_{S,L}$. The strangeness changing decay amplitudes can be expanded with respect to amplitudes of flavor definite states, which are exhibited in App. B.

Note that the wrong strangeness decay amplitudes,

$$A_{\psi \bar{K}^0}, \quad A^{\mathrm{ID}}_{\psi \bar{K}^0}, \quad \bar{A}_{\psi K^0}, \quad \bar{A}^{\mathrm{ID}}_{\psi K^0}, \tag{4.8}$$

are numerically smaller than the right strangeness decay in the SM. The model independent analysis is given in Ref. [97] for the two body B meson decays in which wrong sign kaons are involved. The experimental constraints on wrong strangeness decay amplitudes are obtained for $\bar{B}^0 \to \psi K^{*0}$ and $B^0 \to \psi \bar{K}^{*0}$ [98]. The right strangeness decay amplitudes are given as,

$$A_{\psi K^0}, \quad A^{\mathrm{ID}}_{\psi K^0}, \quad \bar{A}_{\psi \bar{K}^0}, \quad \bar{A}^{\mathrm{ID}}_{\psi \bar{K}^0}. \tag{4.9}$$

We treat wrong sign decay amplitudes as perturbation of small number. Using Eqs. (B.1-B.8), one can obtain CP and CPT violating parameters in decays,

$$\theta_{\psi K_S} \simeq \theta_K - z_K, \quad \theta_{\psi K_L} \simeq \theta_K + z_K, \tag{4.10}$$

$$\theta_K = \frac{A_{\psi K^0} A^{\mathrm{ID}}_{\psi K^0} - \bar{A}_{\psi \bar{K}^0} \bar{A}^{\mathrm{ID}}_{\psi \bar{K}^0}}{A_{\psi K^0} A^{\mathrm{ID}}_{\psi K^0} + \bar{A}_{\psi \bar{K}^0} \bar{A}^{\mathrm{ID}}_{\psi \bar{K}^0}}, \tag{4.11}$$

where θ_K indicates CP and CPT violation in right strangeness decays of B meson, associated with $\hat{\theta}_{\psi K}$ in Ref [52]. The CPT violating parameter in K meson mixing, z_K, is taken into account in this study. When deriving Eq. (4.10), we treated z_K, θ_K and wrong strangeness decay amplitudes as perturbation and ignored higher order contributions. Within this approximation, $\lambda_{\psi K_{S,L}}$ is,

$$\lambda_{\psi K_S} \simeq \lambda(1 - \Delta\lambda_{\mathrm{wst}}), \quad \lambda_{\psi K_L} \simeq -\lambda(1 + \Delta\lambda_{\mathrm{wst}}), \tag{4.12}$$

$$\lambda \equiv \frac{q}{p}\frac{p_K}{q_K}\frac{\bar{A}_{\psi \bar{K}^0}}{A_{\psi K^0}}\sqrt{\frac{1+\theta_K}{1-\theta_K}} = \frac{q}{p}\frac{p_K}{q_K}\frac{A^{\mathrm{ID}}_{\psi K^0}}{\bar{A}^{\mathrm{ID}}_{\psi \bar{K}^0}}\sqrt{\frac{1-\theta_K}{1+\theta_K}}, \tag{4.13}$$

where $\Delta\lambda_{\rm wst}$ consists of the wrong strangeness decays,

$$\Delta\lambda_{\rm wst} = \lambda^{\rm wst}_{\psi\bar{K}^0} - \bar{\lambda}^{\rm wst}_{\psi K^0}, \tag{4.14}$$

$$\lambda^{\rm wst}_{\psi\bar{K}^0} \equiv \frac{p_K}{q_K}\frac{A_{\psi\bar{K}^0}}{A_{\psi K^0}}\sqrt{\frac{1+\theta_{\psi K^0}}{1-\theta_{\psi K^0}}} = \frac{p_K}{q_K}\frac{\bar{A}^{\rm ID}_{\psi K^0}}{\bar{A}^{\rm ID}_{\psi\bar{K}^0}}\sqrt{\frac{1-\theta_{\psi K^0}}{1+\theta_{\psi K^0}}}, \tag{4.15}$$

$$\bar{\lambda}^{\rm wst}_{\psi K^0} \equiv \frac{q_K}{p_K}\frac{\bar{A}_{\psi K^0}}{\bar{A}_{\psi\bar{K}^0}}\sqrt{\frac{1+\bar{\theta}_{\psi\bar{K}^0}}{1-\bar{\theta}_{\psi\bar{K}^0}}} = \frac{q_K}{p_K}\frac{A^{\rm ID}_{\psi\bar{K}^0}}{A^{\rm ID}_{\psi K^0}}\sqrt{\frac{1-\bar{\theta}_{\psi\bar{K}^0}}{1+\bar{\theta}_{\psi\bar{K}^0}}}, \tag{4.16}$$

$$\theta_{\psi K^0} \equiv \frac{A_{\psi K^0}\bar{A}^{\rm ID}_{\psi\bar{K}^0} - A_{\psi\bar{K}^0}\bar{A}^{\rm ID}_{\psi K^0}}{A_{\psi K^0}\bar{A}^{\rm ID}_{\psi\bar{K}^0} + A_{\psi\bar{K}^0}\bar{A}^{\rm ID}_{\psi K^0}}, \quad \bar{\theta}_{\psi\bar{K}^0} \equiv \frac{\bar{A}_{\psi\bar{K}^0}A^{\rm ID}_{\psi K^0} - \bar{A}_{\psi K^0}A^{\rm ID}_{\psi\bar{K}^0}}{\bar{A}_{\psi\bar{K}^0}A^{\rm ID}_{\psi K^0} + \bar{A}_{\psi K^0}A^{\rm ID}_{\psi\bar{K}^0}}. \tag{4.17}$$

In Eq. (4.17), non-zero values of $\theta_{\psi K^0}$ and $\bar{\theta}_{\psi\bar{K}^0}$ imply CPT violation in wrong strangeness decays. Parameters including wrong strangeness decay amplitudes are defined as,

$$\hat{\lambda}_{\rm wst} = \lambda^{\rm wst}_{\psi\bar{K}^0} + \bar{\lambda}^{\rm wst}_{\psi K^0}. \tag{4.18}$$

Since the wrong sign semi-leptonic decay amplitudes and CPT violation are small, we expand Eqs. (4.15, 4.16) as,

$$\lambda^{\rm wst}_{\psi\bar{K}^0} \simeq \frac{p_K}{q_K}\frac{A_{\psi\bar{K}^0}}{A_{\psi K^0}} \simeq \frac{p_K}{q_K}\frac{\bar{A}^{\rm ID}_{\psi K^0}}{\bar{A}^{\rm ID}_{\psi\bar{K}^0}}, \quad \bar{\lambda}^{\rm wst}_{\psi K^0} \simeq \frac{q_K}{p_K}\frac{\bar{A}_{\psi K^0}}{\bar{A}_{\psi\bar{K}^0}} \simeq \frac{q_K}{p_K}\frac{A^{\rm ID}_{\psi\bar{K}^0}}{A^{\rm ID}_{\psi K^0}}. \tag{4.19}$$

As shown in Eq. (4.12), $\lambda_{\psi K_{S,L}}$ is composed of the leading part λ and the sub-leading part suppressed by wrong strangeness decay amplitude. If one takes the CPT conserving limit in Eq. (4.12), the relation in Ref. [99] is obtained. Note that λ has the definitive transformation property of T, CP and CPT, *i.e.*,

$$\lambda \xrightarrow{\rm T} (\lambda)^{-1}, \quad \lambda \xrightarrow{\rm CP} (\lambda)^{-1}, \quad \lambda \xrightarrow{\rm CPT} \lambda. \tag{4.20}$$

We introduce G, S and C in the notation analogous to Eq.(4.6) by replacing λ_f with λ,

$$G = \frac{2{\rm Re}\lambda}{1+|\lambda|^2}, \quad S = \frac{2{\rm Im}\lambda}{1+|\lambda|^2}, \quad C = \frac{1-|\lambda|^2}{1+|\lambda|^2}. \tag{4.21}$$

In Eq. (4.22), $|\lambda|$ is close to 1 since direct CP violation in strangeness changing decays and mixing-induced CP violation in K and B system are small. Consequently, we can find that C

is a small parameter. The parameters in Eq. (4.21) are transformed under T as,

$$G \xrightarrow{\text{T}} \frac{2\text{Re}(1/\lambda)}{1+|1/\lambda|^2} = \frac{2\text{Re}\lambda^*}{|\lambda|^2+1} = +G, \tag{4.22}$$

$$S \xrightarrow{\text{T}} \frac{2\text{Im}(1/\lambda)}{1+|1/\lambda|^2} = \frac{2\text{Im}\lambda^*}{|\lambda|^2+1} = -S, \tag{4.23}$$

$$C \xrightarrow{\text{T}} \frac{1-|1/\lambda|^2}{1+|1/\lambda|^2} = \frac{|\lambda|^2-1}{|\lambda|^2+1} = -C. \tag{4.24}$$

One can verify that the CP transformation property of G, S, C is the same as Eqs. (4.22-4.24). Thus, the CPT transformation property is also determined as,

$$G \xrightarrow{\text{CPT}} +G, \quad S \xrightarrow{\text{CPT}} +S, \quad C \xrightarrow{\text{CPT}} +C. \tag{4.25}$$

One can also derive the transformation property of the parameters for wrong strangeness decays in Eqs. (4.15, 4.16) as,

$$\lambda^{\text{wst}}_{\psi\bar{K}^0} \xrightarrow{\text{T}} \bar{\lambda}^{\text{wst}}_{\psi K^0}, \quad \lambda^{\text{wst}}_{\psi\bar{K}^0} \xrightarrow{\text{CP}} \bar{\lambda}^{\text{wst}}_{\psi K^0}, \quad \lambda^{\text{wst}}_{\psi\bar{K}^0} \xrightarrow{\text{CPT}} \lambda^{\text{wst}}_{\psi\bar{K}^0}. \tag{4.26}$$

Therefore, the parameters in Eqs. (4.14, 4.18) are transformed as,

$$\Delta\lambda_{\text{wst}} \xrightarrow{\text{T}} -\Delta\lambda_{\text{wst}}, \quad \hat{\lambda}_{\text{wst}} \xrightarrow{\text{T}} \hat{\lambda}_{\text{wst}}. \tag{4.27}$$

The CP transformation property of the parameters (4.14, 4.18) is the same as Eq. (4.27). $G_{\psi K_{S,L}}, S_{\psi K_{S,L}}$ and $C_{\psi K_{S,L}}$ are related with the parameters G, S and C as,

$$G_{\psi K_S} \simeq G + S\Delta\lambda^I_{\text{wst}}, \quad G_{\psi K_L} \simeq -(G - S\Delta\lambda^I_{\text{wst}}), \tag{4.28}$$

$$S_{\psi K_S} \simeq S - G\Delta\lambda^I_{\text{wst}}, \quad S_{\psi K_L} \simeq -(S + G\Delta\lambda^I_{\text{wst}}), \tag{4.29}$$

$$C_{\psi K_S} \simeq C + \Delta\lambda^R_{\text{wst}}, \quad C_{\psi K_L} \simeq C - \Delta\lambda^R_{\text{wst}}, \tag{4.30}$$

where we used notation for a complex number A, $A^R \equiv \text{Re}A, A^I \equiv \text{Im}A$. When deriving Eqs. (4.28-4.30), we ignored higher order terms of C and $\Delta\lambda_{\text{wst}}$. One can find that Eqs. (4.10, 4.28-4.30) lead to the relations given as,

$$\theta_{\psi K_S} + \theta_{\psi K_L} = 2\theta_K, \quad \theta_{\psi K_S} - \theta_{\psi K_L} = -2z_K, \tag{4.31}$$

$$G_{\psi K_S} - G_{\psi K_L} = 2G, \quad S_{\psi K_S} - S_{\psi K_L} = 2S, \quad C_{\psi K_S} + C_{\psi K_L} = 2C, \tag{4.32}$$

$$G_{\psi K_S} + G_{\psi K_L} = 2S\Delta\lambda^I_{\text{wst}}, \quad S_{\psi K_S} + S_{\psi K_L} = -2G\Delta\lambda^I_{\text{wst}}, \quad C_{\psi K_S} - C_{\psi K_L} = 2\Delta\lambda^R_{\text{wst}}. \tag{4.33}$$

In Eq. (4.13), we have included the contribution of indirect CP violation of K meson system, as carefully discussed in Refs. [100–103]. The expressions of G, C and S in Eqs. (4.22-4.24) are invariant under the arbitrary large rephasing of,

$$\langle K^0| \to e^{-i\alpha_K} \langle K^0| , \quad \langle \bar{K^0}| \to e^{i\alpha_K} \langle \bar{K^0}| . \tag{4.34}$$

Nevertheless, the mixing parameter ratio given as,

$$\frac{p_K}{q_K} = \frac{1+\epsilon_K}{1-\epsilon_K} \simeq 1 + 2\epsilon_K, \qquad (|\epsilon_K| \ll 1) \tag{4.35}$$

allows only the small rephasing $\alpha_K \ll 1$. In the following, we show how the correction arises from ϵ_K. Keeping only the terms which are linear with respect to the parameter of mixing-induced CP violation in K meson system, we expand G, S and C,

$$\begin{aligned} G &= G' - 2S'\epsilon_K^I, \\ S &= S' + 2G'\epsilon_K^I, \\ C &= C' - 2\epsilon_K^R, \end{aligned} \tag{4.36}$$

where G', S' and C' are obtained by taking the limit $(p_K/q_K) \to 1$ in G, S and C: the parameters in Eq. (4.36) are defined by replacing λ with λ' in the expression for G, S and C,

$$\lambda' = \frac{q}{p}\frac{\bar{A}_{\psi\bar{K^0}}}{A_{\psi K^0}}\sqrt{\frac{1+\theta_K}{1-\theta_K}}, \quad G' = \frac{2\mathrm{Re}\lambda'}{1+|\lambda'|^2}, \quad S' = \frac{2\mathrm{Im}\lambda'}{1+|\lambda'|^2}, \quad C' = \frac{1-|\lambda'|^2}{1+|\lambda'|^2}. \tag{4.37}$$

If one takes the limit where $\epsilon_K \to 0$, (G', S', C') is identical to $(\hat{G}_{\psi K}, \hat{S}_{\psi K}, \hat{C}_{\psi K})$ defined in Ref. [52]. The difference of the notations between ours and one given in Ref. [52] is summarized in Tab. 4.1. For the CPT violation parameter of strangeness changing decay, one can show that $\theta_{\psi K1}$ and $\theta_{\psi K2}$ are identical to θ_K in our notation, which leads to the relation of,

$$\hat{\theta}_{\psi K} = \theta_K, \qquad \Delta\theta_{\psi K} = 0. \tag{4.38}$$

Table 4.1: Relation of the parameters in this thesis and ones in Ref. [52]. The first column shows the quantities defined for the K meson mass eigenstates (K_L, K_S). From the third row to the eighth row in the second column, the quantities in the first column are expanded up to the first order of ϵ_K, and written in terms of the quantities for the CP eigenstates K_1, K_2 in their notation. In the third and fourth column, we show how the quantities in their notation are related to ones defined in this thesis.

Notation in this thesis	Notation in Ref [52]	Notation in Ref. [52]	Notation in this thesis
$\lambda_{\psi K_S}$	$\frac{p_K}{q_K}\lambda_{\psi K_1}$	$\lambda_{\psi K_1}$	$\lambda'(1-\Delta\lambda_{\rm wst})$
$\lambda_{\psi K_L}$	$\frac{p_K}{q_K}\lambda_{\psi K_2}$	$\lambda_{\psi K_2}$	$-\lambda'(1+\Delta\lambda_{\rm wst})$
$G_{\psi K_S}$	$G_{\psi K_1} - 2S_{\psi K_1}\epsilon_K^I$	$\hat{G}_{\psi K} = \frac{G_{\psi K_1} - G_{\psi K_2}}{2}$	G'
$S_{\psi K_S}$	$S_{\psi K_1} + 2G_{\psi K_1}\epsilon_K^I$	$\hat{S}_{\psi K} = \frac{S_{\psi K_1} - S_{\psi K_2}}{2}$	S'
$C_{\psi K_S}$	$C_{\psi K_1} - 2\epsilon_K^R$	$\hat{C}_{\psi K} = \frac{C_{\psi K_1} + C_{\psi K_2}}{2}$	C'
$G_{\psi K_L}$	$G_{\psi K_2} - 2S_{\psi K_2}\epsilon_K^I$	$\Delta G_{\psi K} = \frac{G_{\psi K_1} + G_{\psi K_2}}{2}$	$S'\Delta\lambda_{\rm wst}^I$
$S_{\psi K_L}$	$S_{\psi K_2} + 2G_{\psi K_2}\epsilon_K^I$	$\Delta S_{\psi K} = \frac{S_{\psi K_1} + S_{\psi K_2}}{2}$	$-G'\Delta\lambda_{\rm wst}^I$
$C_{\psi K_L}$	$C_{\psi K_2} - 2\epsilon_K^R$	$\Delta C_{\psi K} = \frac{C_{\psi K_1} - C_{\psi K_2}}{2}$	$\Delta\lambda_{\rm wst}^R$
θ_K	$\hat{\theta}_{\psi K} = \frac{\theta_{\psi K1} + \theta_{\psi K2}}{2}$	$\Delta\theta_{\psi K} = \frac{\theta_{\psi K1} - \theta_{\psi K2}}{2}$	0

If one changes the phase convention of the states, the phase of λ' is transformed as follows,

$$\lambda' \to \lambda' e^{2i\alpha_K}. \tag{4.39}$$

Assuming the phase α_K is small, G', S', and ϵ_K^I are changed as,

$$\begin{aligned} G' &\to G' - 2\alpha_K S', \\ S' &\to S' + 2\alpha_K G', \\ \epsilon_K^I &\to \epsilon_K^I - \alpha_K, \end{aligned} \tag{4.40}$$

while C' and ϵ_K^R are invariant, *i.e.*,

$$C' \to C', \quad \epsilon_K^R \to \epsilon_K^R. \tag{4.41}$$

As phase convention independent notation, we use C' and ϵ_K^R instead of C in the following discussion. The numerical significance of ϵ_K^R will be mentioned in the next chapter.

We turn to the definition for parameters including semi-leptonic decay amplitudes. In what follows, from Eq. (4.42) to Eq. (4.47), we adopt the notations in Ref. [52]. Right sign semi-leptonic decay amplitudes are denoted as,

$$\begin{aligned} A_{l^+} &= \langle l^+ X|T|B^0\rangle, \qquad A_{l^+}^{\mathrm{ID}} = \langle B^0|T|(l^+X)^T\rangle, \\ \bar{A}_{l^-} &= \langle l^- X|T|\bar{B}^0\rangle, \qquad \bar{A}_{l^-}^{\mathrm{ID}} = \langle \bar{B}^0|T|(l^-X)^T\rangle, \end{aligned} \tag{4.42}$$

while wrong sign semi-leptonic decay amplitudes are given as,

$$\begin{aligned} A_{l^-} &= \langle l^- X|T|B^0\rangle, \qquad A_{l^-}^{\mathrm{ID}} = \langle B^0|T|(l^-X)^T\rangle, \\ \bar{A}_{l^+} &= \langle l^+ X|T|\bar{B}^0\rangle, \qquad \bar{A}_{l^+}^{\mathrm{ID}} = \langle \bar{B}^0|T|(l^+X)^T\rangle. \end{aligned} \tag{4.43}$$

For the case of the SM, the wrong sign semi-leptonic decay amplitudes are numerically suppressed compared with the right sign decay amplitudes. Thus, we ignore higher powers of the wrong sign decay amplitudes. The parameters including semi-leptonic decay amplitudes are defined as,

$$\lambda_{l^+} \equiv \frac{q}{p}\frac{\bar{A}_{l^+}}{A_{l^+}}\sqrt{\frac{1+\theta_{l^+}}{1-\theta_{l^+}}} = \frac{q}{p}\frac{A_{l^-}^{\mathrm{ID}}}{\bar{A}_{l^-}^{\mathrm{ID}}}\sqrt{\frac{1-\theta_{l^+}}{1+\theta_{l^+}}}, \quad \theta_{l^+} = \frac{A_{l^+}A_{l^-}^{\mathrm{ID}} - \bar{A}_{l^+}\bar{A}_{l^-}^{\mathrm{ID}}}{A_{l^+}A_{l^-}^{\mathrm{ID}} + \bar{A}_{l^+}\bar{A}_{l^-}^{\mathrm{ID}}}, \tag{4.44}$$

$$\lambda_{l^-} \equiv \frac{q}{p}\frac{\bar{A}_{l^-}}{A_{l^-}}\sqrt{\frac{1+\theta_{l^-}}{1-\theta_{l^-}}} = \frac{q}{p}\frac{A_{l^+}^{\mathrm{ID}}}{\bar{A}_{l^+}^{\mathrm{ID}}}\sqrt{\frac{1-\theta_{l^-}}{1+\theta_{l^-}}}, \quad \theta_{l^-} = \frac{A_{l^-}A_{l^+}^{\mathrm{ID}} - \bar{A}_{l^-}\bar{A}_{l^+}^{\mathrm{ID}}}{A_{l^-}A_{l^+}^{\mathrm{ID}} - \bar{A}_{l^-}\bar{A}_{l^+}^{\mathrm{ID}}}, \tag{4.45}$$

where $\theta_{l^\pm}$ stands for CPT violation in semi-leptonic decays of B meson. By the definition in Eqs. (4.44, 4.45), one can find the transformation law of the parameters for the semi-leptonic decays, *i.e.,*

$$\lambda_{l^+} \xrightarrow{\mathrm{T}} (\lambda_{l^-})^{-1}, \quad \lambda_{l^+} \xrightarrow{\mathrm{CP}} (\lambda_{l^-})^{-1}, \quad \lambda_{l^+} \xrightarrow{\mathrm{CPT}} \lambda_{l^+}. \tag{4.46}$$

We assume that CPT violating parameter $\theta_{l^\pm}$ is small and treat it as perturbation. At linear order of $\theta_{l^\pm}$ and wrong sign semi-leptonic decay amplitudes, we obtain,

$$\lambda_{l^+} \simeq \frac{q}{p}\frac{\bar{A}_{l^+}}{A_{l^+}} \simeq \frac{q}{p}\frac{A^{\mathrm{ID}}_{l^-}}{\bar{A}^{\mathrm{ID}}_{l^-}}, \quad \lambda_{l^-}^{-1} \simeq \frac{p}{q}\frac{A_{l^-}}{\bar{A}_{l^-}} \simeq \frac{p}{q}\frac{\bar{A}^{\mathrm{ID}}_{l^+}}{A^{\mathrm{ID}}_{l^+}}, \tag{4.47}$$

where we can find that the contribution from $\theta_{l^\pm}$ is negligible in Eq. (4.47). Following Ref. [52], we also define $G_{l^\pm}, S_{l^\pm}$ and $C_{l^\pm}$ analogous to Eq. (4.6) by replacing λ_f with $\lambda_{l^\pm}$. Equation (4.47) gives approximate expressions for $G_{l^\pm}, S_{l^\pm}$ and $C_{l^\pm}$ as,

$$G_{l^+} = \frac{2\mathrm{Re}\lambda_{l^+}}{1+|\lambda_{l^+}|^2} \simeq 2\mathrm{Re}\lambda_{l^+},\ G_{l^-} = \frac{2\mathrm{Re}\lambda_{l^-}}{1+|\lambda_{l^-}|^2} \simeq 2\mathrm{Re}(\lambda_{l^-}^{-1}),\ C_{l^\pm} = \frac{1-|\lambda_{l^\pm}|^2}{1+|\lambda_{l^\pm}|^2} \simeq \pm 1,$$
$$S_{l^+} = \frac{2\mathrm{Im}\lambda_{l^+}}{1+|\lambda_{l^+}|^2} \simeq 2\mathrm{Im}\lambda_{l^+},\ \ S_{l^-} = \frac{2\mathrm{Im}\lambda_{l^-}}{1+|\lambda_{l^-}|^2} \simeq -2\mathrm{Im}(\lambda_{l^-}^{-1}). \tag{4.48}$$

The parameters of $G_{l^\pm}, S_{l^\pm}$ and $C_{l^\pm}$ explicitly appear in the coefficients of the time dependent asymmetry analyzed in the subsequent chapter. Note that $G_{l^\pm}$ and $S_{l^\pm}$ are small numbers since λ_{l^+} and $\lambda_{l^-}^{-1}$ are suppressed due to the $\Delta B = \Delta Q$ rule. We can find the relations,

$$G_{l^+} + G_{l^-} = 2\hat{\lambda}_l^R, \quad S_{l^+} - S_{l^-} = 2\hat{\lambda}_l^I, \tag{4.49}$$
$$G_{l^+} - G_{l^-} = 2\Delta\lambda_l^R, \quad S_{l^+} + S_{l^-} = 2\Delta\lambda_l^I, \tag{4.50}$$

where $\hat{\lambda}_l$ and $\Delta\lambda_l$ are defined as,

$$\hat{\lambda}_l \equiv \lambda_{l^+} + \lambda_{l^-}^{-1}, \quad \Delta\lambda_l \equiv \lambda_{l^+} - \lambda_{l^-}^{-1}. \tag{4.51}$$

The parameters above are transformed definitively under CP, T and CPT,

$$\hat{\lambda}_l \xrightarrow{\mathrm{T}} (\lambda_{l^-})^{-1} + \lambda_{l^+} = +\hat{\lambda}_l, \quad \Delta\lambda_l \xrightarrow{\mathrm{T}} (\lambda_{l^-})^{-1} - \lambda_{l^+} = -\Delta\lambda_l. \tag{4.52}$$

The CP transformation property of $\hat{\lambda}_l$ and $\Delta\lambda_l$ is the same as Eq. (4.52). Hence, the CPT transformation property of $\hat{\lambda}_l$ and $\Delta\lambda_l$ is also determined as,

$$\hat{\lambda}_l \xrightarrow{\mathrm{CPT}} \hat{\lambda}_l, \quad \Delta\lambda_l \xrightarrow{\mathrm{CPT}} \Delta\lambda_l. \tag{4.53}$$

Furthermore, one defines,

$$R_M \equiv \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}, \quad \xi_l \equiv \frac{\bar{A}_{l^-} A^{\rm ID}_{l^+} - A_{l^+} \bar{A}^{\rm ID}_{l^-}}{\bar{A}_{l^-} A^{\rm ID}_{l^+} + A_{l^+} \bar{A}^{\rm ID}_{l^-}}, \quad C^l_\xi, \equiv \frac{1 - |\lambda^l_\xi|^2}{1 + |\lambda^l_\xi|^2}, \tag{4.54}$$

$$\lambda^l_\xi \equiv \frac{A_{l^+}}{\bar{A}_{l^-}} \sqrt{\frac{1+\xi_l}{1-\xi_l}} = \frac{A^{\rm ID}_{l^+}}{\bar{A}^{\rm ID}_{l^-}} \sqrt{\frac{1-\xi_l}{1+\xi_l}}. \tag{4.55}$$

In Eq. (4.54), R_M implies mixing-induced CP and T violation in B meson system [52]. This parameter is extracted from HFAG data [104] for the average of the experimental results,

$$R_M = (-7 \pm 9) \times 10^{-4}. \tag{4.56}$$

As shown above, indirect CP violation $B^0 - \bar{B}^0$ is small enough to treat it as perturbation. In Eq. (4.54), ξ_l stands for CP and T violation in right sign semi-leptonic decays, which is considered as a small number. We assume direct CP violation in $B^0 \to l^+ X$ is small so that C^l_ξ in Eq. (4.54) is also treated as perturbation. Equations (4.28-4.30, 4.49, 4.50, 4.54) enable one to write the asymmetry in Eq. (3.7) in terms of parameters which are exactly T-odd or T-even.

In the following, we address some significant points of the parameters defined in this chapter. Note that the parameters given as,

$$S, C, G, \theta_K, R_M, z, z_K, \hat{\lambda}_l, \Delta\lambda_l, \xi_l, C^l_\xi, \hat{\lambda}_{\rm wst} \text{ and } \Delta\lambda_{\rm wst}, \tag{4.57}$$

have the definitive transformation properties exhibited in Tab. 4.2. In the processes which are discussed in the subsequent chapter, $K_{S,L}$ is included as a final state, and the contribution of mixing-induced T and CP violation, p_K/q_K, appears in the expressions of $G, S, C, \hat{\lambda}_{\rm wst}$ and $\Delta\lambda_{\rm wst}$. The CP and CPT violation parameter in K meson mixing denoted as z_K, also affects the time dependent asymmetry. In the subsequent chapter, the asymmetry is written in terms of parameters in Eq. (4.57), and explicitly divided into T-odd and T-even parts.

The parameters defined as,

$$p/q, p_K/q_K, \theta_{\psi K^0}, \bar{\theta}_{\psi \bar{K}^0}, \theta_{l^\pm}, \lambda, \lambda^{\rm wst}_{\psi \bar{K}^0}, \bar{\lambda}^{\rm wst}_{\psi K^0}, \lambda_{l^\pm}, \text{ and } \lambda^l_\xi, \tag{4.58}$$

are introduced to keep the definitive transformation property of parameters in Tab. 4.2. The transformation property of the parameters in Eq. (4.58) is exhibited in Tab. 4.3.

The parameters given as,

$$\theta_{\psi K^0}, \bar{\theta}_{\psi \bar{K}^0}, \theta_{l^\pm}, C, \theta_K, R_M, z, z_K, \hat{\lambda}_l, \Delta\lambda_l, \xi_l, C^l_\xi, \hat{\lambda}_{\rm wst} \text{ and } \Delta\lambda_{\rm wst}, \tag{4.59}$$

are all small numbers, and our analysis is based on linear order approximation with respect to the quantities given in Eq. (4.59) throughout this thesis.

Table 4.2: Transformation properties of the parameters under T, CP and CPT.

	S	C	G	θ_K	R_M	z	z_K	$\hat{\lambda}_l$	$\Delta\lambda_l$	ξ_l	C_ξ^l	$\hat{\lambda}_{\text{wst}}$	$\Delta\lambda_{\text{wst}}$
T	$-$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$-$
CP	$-$	$-$	$+$	$-$	$-$	$-$	$-$	$+$	$-$	$-$	$-$	$+$	$-$
CPT	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$+$

Table 4.3: Transformation properties of the parameters which are introduced to keep the definitive transformation property of the quantities in Tab. 4.2

	p/q	p_K/q_K	$\theta_{\psi K^0}$	θ_{l^+}	λ	$\lambda_{\psi \bar{K}^0}^{\text{wst}}$	λ_{l^+}	λ_ξ^l
T	q/p	q_K/p_K	$-\bar{\theta}_{\psi \bar{K}^0}$	θ_{l^-}	$(\lambda)^{-1}$	$\bar{\lambda}_{\psi K^0}^{\text{wst}}$	$(\lambda_{l^-})^{-1}$	λ_ξ^l
CP	q/p	q_K/p_K	$\bar{\theta}_{\psi \bar{K}^0}$	$-\theta_{l^-}$	$(\lambda)^{-1}$	$\bar{\lambda}_{\psi K^0}^{\text{wst}}$	$(\lambda_{l^-})^{-1}$	$(\lambda_\xi^l)^{-1}$
CPT	p/q	p_K/q_K	$-\theta_{\psi K^0}$	$-\theta_{l^+}$	λ	$\lambda_{\psi \bar{K}^0}^{\text{wst}}$	λ_{l^+}	$(\lambda_\xi^l)^{-1}$

Chapter 5

Analysis of Time Dependent Asymmetry

In this chapter, we apply the event number asymmetry defined in Eq. (3.13) to the processes for B meson decays. The time dependent asymmetry in this thesis includes the effect of different normalization for the decay rates, *i.e.,* non-zero value of ΔN_R defined in Eq. (3.14). As the BaBar asymmetry investigated in Ref. [50], the final states f_1, f_2, f_3 and f_4 are assigned with $\psi K_L, l^- X$, $l^+ X$ and ψK_S, respectively. This process is referred to as I, which is associated with the asymmetry for $B_- \to \bar{B}^0$ versus $\bar{B}^0 \to B_-$. We also consider other three processes which can be obtained by interchanging $l^- X$ with $l^+ X$ and ψK_S with ψK_L in the process I. To summarize, we analyze the processes given as,

$$\begin{aligned}
&\text{(I)} \qquad (f_1, f_2, f_3, f_4) = (\psi K_L, l^- X, l^+ X, \psi K_S), \\
&\text{(II)} \qquad (f_1, f_2, f_3, f_4) = (\psi K_S, l^- X, l^+ X, \psi K_L), \\
&\text{(III)} \qquad (f_1, f_2, f_3, f_4) = (\psi K_L, l^+ X, l^- X, \psi K_S), \\
&\text{(IV)} \qquad (f_1, f_2, f_3, f_4) = (\psi K_S, l^+ X, l^- X, \psi K_L).
\end{aligned} \tag{5.1}$$

For all the above processes, we can find that the following parameters are treated as perturbation,

$$\Delta N_R, \quad \Delta\sigma, \quad y\Gamma t, \quad \Delta\mathcal{C}, \quad \hat{\mathcal{S}}, \quad \hat{\mathcal{C}}. \tag{5.2}$$

To analyze the processes in Eq. (5.1), the list for the coefficients of the trigonometric functions in the decay rates is given in App. C. The time dependent asymmetry in Eq. (3.13) are expanded,

$$\begin{aligned} A &\simeq R_T + C_T \cos(x\Gamma t) + S_T \sin(x\Gamma t) \\ &\quad + B_T \sin^2(x\Gamma t) + D_T \sin(x\Gamma t)\cos(x\Gamma t) + E_T(y\Gamma t)\sin(x\Gamma t), \end{aligned} \tag{5.3}$$

where the coefficients of the time dependent trigonometric functions are given as,

$$R_T = -\frac{\Delta N_R}{2} + \frac{\Delta\sigma}{2} y\Gamma t \simeq -\frac{\Delta N_R}{2}, \tag{5.4}$$

$$C_T = \frac{\Delta\mathcal{C}}{2}, \quad S_T = \frac{\Delta\mathcal{S}}{2}, \tag{5.5}$$

$$B_T = -\frac{\Delta\mathcal{S}}{4}\hat{S}, \quad D_T = -\frac{\Delta\mathcal{S}}{4}\hat{C}, \tag{5.6}$$

$$E_T = -\frac{\Delta\mathcal{S}}{4}\hat{\sigma}. \tag{5.7}$$

In Eqs. (5.4-5.7), we ignored the contribution from $\Delta\sigma y$. Note that $\hat{\sigma}$ and $\Delta\mathcal{S}$ are $\mathcal{O}(1)$ parameters and $\hat{\sigma}y$ gives rise to small contribution. The parametrization in Eq. (5.4-5.3) without the last term can be found in Ref. [52]. In the following, the coefficients in Eqs. (5.4-5.7) are analyzed for each process. We label suffix I − IV on the quantities associated with the processes in Eq. (5.1) to distinguish them.

Below, the asymmetry and the coefficients for the process I are shown. The detailed derivation of the coefficients for the process I is given in App. D. For the other three processes, a simple rule to obtain the coefficients for the processes II-IV from I is considered in App. E. We first investigate ΔN_R in Eq. (3.14) for the process I. Through Eq. (D.8), one can obtain,

$$\Delta N_R^I = 2[-Sz^I + R_M + \hat{\lambda}^R_{\rm wst} - G\hat{\lambda}^R_l - C^l_\xi - \xi^R_l]. \tag{5.8}$$

Using Eqs. (5.8, D.9-D.13), one can derive the coefficients in the time dependent asymmetry,

$$R_T^I = -\frac{\Delta N_R^I}{2} = Sz^I - R_M - \hat{\lambda}^R_{\rm wst} + G\hat{\lambda}^R_l + C^l_\xi + \xi^R_l, \tag{5.9}$$

$$C_T^I = \frac{\Delta\mathcal{C}^I}{2} = C - Sz^I + \theta^R_K + S\Delta\lambda^I_l = C' - 2\epsilon^R_K - Sz^I + \theta^R_K + S\Delta\lambda^I_l, \tag{5.10}$$

$$S_T^I = \frac{\Delta\mathcal{S}^I}{2} = -[S(1 - Gz^R) - G\theta^I_K + GS\Delta\lambda^R_l], \tag{5.11}$$

$$B_T^I = -\frac{\Delta\mathcal{S}^I}{4}\hat{S}^I \simeq S[G(z^I_K - \Delta\lambda^I_{\rm wst}) - z^I + SR_M + S\hat{\lambda}^R_{\rm wst} - SC^l_\xi - S\xi^R_l], \tag{5.12}$$

$$D_T^I = -\frac{\Delta\mathcal{S}^I}{4}\hat{C}^I \simeq S[z^R_K - \Delta\lambda^R_{\rm wst} - Gz^R - S\hat{\lambda}^I_l], \tag{5.13}$$

$$E_T^I = -\frac{\Delta\mathcal{S}^I}{4}\hat{\sigma}^I \simeq GS. \tag{5.14}$$

If one imposes the following conditions,

- No CPT violation
- No wrong sign decays
- No CP violation in $B^0 - \bar{B}^0$ mixing and right sign semi-leptonic decays
- $y \to 0$

it is shown that the asymmetry coincides the function adopted in the experiment [50],

$$A^I = C_T^I \cos(x\Gamma t) + S_T^I \sin(x\Gamma t). \tag{5.15}$$

Under the presence of CPT violation, wrong sign decays, non-zero width difference of B meson mass eigenstates and CP violation in B meson mixing and semi-leptonic decays, the relevant function form is one given in Eq. (5.3).

All of the coefficients in Eqs. (5.9-5.14) are expressed in terms of the phase convention independent parameters defined in the previous chapter. In Eq. (5.10), the contribution from mixing-induced CP violation in K meson system explicitly appears. Assuming that all of mixing induced CP violation of B^0 system, direct CP violation in $B^0 \to \psi K^0$ and CPT violation in strangeness changing decay of B meson are small numbers, we expand C' in Eq. (4.37),

$$C' \simeq 2 - \left|\frac{q}{p}\right| - \left|\frac{\bar{A}_{\psi\bar{K^0}}}{A_{\psi K^0}}\right| - \theta_K^R, \quad \left|\frac{q}{p}\right| \simeq 1 - \frac{1}{2}\mathrm{Im}\left(\frac{\Gamma_{12}^d}{M_{12}^d}\right) \tag{5.16}$$

A theoretical prediction for the B^0 system is given in Ref. [105], which enables us to extract,

$$\mathrm{Im}\left(\frac{\Gamma_{12}^d}{M_{12}^d}\right) \sim \mathcal{O}(10^{-4}). \tag{5.17}$$

As for direct CP violation in $B^0 \to \psi K^0$, theoretical evaluation is obtained in Refs. [74, 106], which results in,

$$1 - \left|\frac{\bar{A}_{\psi\bar{K^0}}}{A_{\psi K^0}}\right| \simeq \mathcal{O}(10^{-3}). \tag{5.18}$$

Consequently, mixing-induced CP violation in K meson system and direct CP violation in $B^0 \to \psi K^0$ are dominant in the coefficient in Eq. (5.10), which predicts $C_T^I \sim \mathcal{O}(10^{-3})$, if CPT violations and the wrong sign decay in $B \to lX$ in Eq. (5.10) are negligible. This prediction of order is valid unless the cancellation between the parameters occurs.

If the coefficients of the time dependent decay rate in Eqs. (5.9-5.13) were genuine T-odd quantities, they would vanish in the limit of T symmetry. In other words, if there remain non-vanishing contributions in the T symmetric limit, the coefficients are not T-odd quantities. From Eqs. (5.9-5.14), one can observe the presence of T-even contributions. Some of them do not vanish in the limit of T symmetry whereas there exist terms quadratic with respect to T-odd quantities, which vanish in the T symmetric limit.

In what follows, we investigate conditions that require the asymmetry being a T-odd quantity. The following relations are needed for T-even terms in each coefficient to vanish,

$$\hat{\lambda}^R_{\text{wst}} = 0, \quad G\hat{\lambda}^R_l = 0, \quad C^l_\xi = 0 \;\; \rightarrow \;\; R^I_T : \mathrm{T-odd}, \tag{5.19}$$

$$\theta^R_K = 0, \quad S\Delta\lambda^I_l = 0 \;\; \rightarrow \;\; C^I_T : \mathrm{T-odd}, \tag{5.20}$$

$$G\theta^I_K = 0, \quad GS\Delta\lambda^R_l = 0 \;\; \rightarrow \;\; S^I_T : \mathrm{T-odd}, \tag{5.21}$$

$$SG\Delta\lambda^I_{\text{wst}} = 0, \quad S^2\hat{\lambda}^R_{\text{wst}} = 0, \quad S^2C^l_\xi = 0 \;\; \rightarrow \;\; B^I_T : \mathrm{T-odd}, \tag{5.22}$$

$$S\Delta\lambda^R_{\text{wst}} = 0, \quad S^2\hat{\lambda}^I_l = 0 \;\; \rightarrow \;\; D^I_T : \mathrm{T-odd}. \tag{5.23}$$

Since both real and imaginary part of λ do not vanish, G and S are non-zero quantities. Thus, the conditions to obtain T-odd coefficients in Eqs. (5.19)-(5.23) are,

$$\theta_K = \Delta\lambda_{\text{wst}} = \Delta\lambda_l = \hat{\lambda}_l = \hat{\lambda}^R_{\text{wst}} = C^l_\xi = 0. \tag{5.24}$$

The above relations except $C^l_\xi = 0$ agree with ones obtained in Ref. [52]. The additional condition is required since we account the overall constants in the time dependent decay rates.

In Tab. 5.1, we show how each coefficient of the asymmetry in Eq. (5.3) depends on a T-odd combination of the parameters. The dependence on the T-even contributions of the parameters is also exhibited. Likewise, the T-odd and even contributions for the processes II-IV are listed in Tabs. 5.2-5.4, respectively.

Table 5.1: Coefficients of the asymmetry for the process I and the sources which give rise to the non-vanishing contribution to the time dependent asymmetry. The sources of the second column correspond to T-odd terms and the others are associated with T-even terms. In the third column, the contribution from CP and CPT violation in right strangeness decays is exhibited. In the fourth column, the contribution from CP and CPT violation in the right sign semi-leptonic decays is shown. In the fifth and the sixth column, T-even contribution from the wrong strangeness decays and the wrong sign semi-leptonic decays are given, respectively.

	T-odd terms	$\theta_K \neq 0$	$C^l_\xi \neq 0$	$A_{\psi \bar{K}^0} \neq 0, \bar{A}_{\psi K^0} \neq 0$	$\bar{A}_{l^+} \neq 0, A_{l^-} \neq 0$
R^I_T	$Sz^I - R_M + \xi^R_l$	0	C^l_ξ	$-\hat{\lambda}^R_{\rm wst}$	$G\hat{\lambda}^R_l$
C^I_T	$C - Sz^I$	θ^R_K	0	0	$S\Delta\lambda^I_l$
S^I_T	$-S[1 - Gz^R]$	$G\theta^I_K$	0	0	$-GS\Delta\lambda^R_l$
B^I_T	$S[Gz^I_K - z^I + SR_M - S\xi^R_l]$	0	$-S^2C^l_\xi$	$S^2\hat{\lambda}^R_{\rm wst} - SG\Delta\lambda^I_{\rm wst}$	0
D^I_T	$S[z^R_K - Gz^R]$	0	0	$-S\Delta\lambda^R_{\rm wst}$	$-S^2\hat{\lambda}^I_l$
E^I_T	GS	0	0	0	0

Table 5.2: The same table as Tab. 5.1 for the process II.

	T-odd terms	$\theta_K \neq 0$	$C^l_\xi \neq 0$	$A_{\psi \bar{K}^0} \neq 0, \bar{A}_{\psi K^0} \neq 0$	$\bar{A}_{l^+} \neq 0, A_{l^-} \neq 0$
R^{II}_T	$-Sz^I - R_M + \xi^R_l$	0	C^l_ξ	$\hat{\lambda}^R_{\rm wst}$	$-G\hat{\lambda}^R_l$
C^{II}_T	$C + Sz^I$	θ^R_K	0	0	$-S\Delta\lambda^I_l$
S^{II}_T	$S[1 + Gz^R]$	$-G\theta^I_K$	0	0	$-GS\Delta\lambda^R_l$
B^{II}_T	$-S[Gz^I_K - z^I - SR_M + S\xi^R_l]$	0	$-S^2C^l_\xi$	$-S^2\hat{\lambda}^R_{\rm wst} + SG\Delta\lambda^I_{\rm wst}$	0
D^{II}_T	$S[z^R_K - Gz^R]$	0	0	$-S\Delta\lambda^R_{\rm wst}$	$-S^2\hat{\lambda}^I_l$
E^{II}_T	GS	0	0	0	0

Table 5.3: The same table as Tab. 5.1 for the process III.

	T-odd terms	$\theta_K \neq 0$	$C_\xi^l \neq 0$	$A_{\psi \bar{K}^0} \neq 0, \bar{A}_{\psi K^0} \neq 0$	$\bar{A}_{l^+} \neq 0, A_{l^-} \neq 0$
R_T^{III}	$Sz^I + R_M - \xi_l^R$	0	$-C_\xi^l$	$-\hat{\lambda}_{\text{wst}}^R$	$G\hat{\lambda}_l^R$
C_T^{III}	$-C - Sz^I$	$-\theta_K^R$	0	0	$S\Delta\lambda_l^I$
S_T^{III}	$S[1 + Gz^R]$	$-G\theta_K^I$	0	0	$-GS\Delta\lambda_l^R$
B_T^{III}	$S[Gz_K^I - z^I - SR_M + S\xi_l^R]$	0	$S^2 C_\xi^l$	$S^2\hat{\lambda}_{\text{wst}}^R - SG\Delta\lambda_{\text{wst}}^I$	0
D_T^{III}	$S[z_K^R - Gz^R]$	0	0	$-S\Delta\lambda_{\text{wst}}^R$	$-S^2\hat{\lambda}_l^I$
E_T^{III}	$-GS$	0	0	0	0

Table 5.4: The same table as Tab. 5.1 for the process IV.

	T-odd terms	$\theta_K \neq 0$	$C_\xi^l \neq 0$	$A_{\psi \bar{K}^0} \neq 0, \bar{A}_{\psi K^0} \neq 0$	$\bar{A}_{l^+} \neq 0, A_{l^-} \neq 0$
R_T^{IV}	$-Sz^I + R_M - \xi_l^R$	0	$-C_\xi^l$	$\hat{\lambda}_{\text{wst}}^R$	$-G\hat{\lambda}_l^R$
C_T^{IV}	$-C + Sz^I$	$-\theta_K^R$	0	0	$-S\Delta\lambda_l^I$
S_T^{IV}	$-S[1 - Gz^R]$	$G\theta_K^I$	0	0	$-GS\Delta\lambda_l^R$
B_T^{IV}	$S[-Gz_K^I + z^I - SR_M + S\xi_l^R]$	0	$S^2 C_\xi^l$	$-S^2\hat{\lambda}_{\text{wst}}^R + SG\Delta\lambda_{\text{wst}}^I$	0
D_T^{IV}	$S[z_K^R - Gz^R]$	0	0	$-S\Delta\lambda_{\text{wst}}^R$	$-S^2\hat{\lambda}_l^I$
E_T^{IV}	$-GS$	0	0	0	0

From Tabs. 5.1-5.4, one can find that each coefficient of the asymmetry is related to one in another process. It is shown that the following relations among the coefficients for four processes are satisfied:

$$
\begin{aligned}
R_T^{IV} &= -R_T^I, \quad R_T^{III} = -R_T^{II}, \\
C_T^{III} &= -C_T^{II}, \quad C_T^{IV} = -C_T^I, \\
S_T^{III} &= S_T^{II}, \quad S_T^{IV} = S_T^I, \\
B_T^{III} &= -B_T^{II}, \quad B_T^{IV} = -B_T^I, \\
D_T^I &= D_T^{II} = D_T^{III} = D_T^{IV}, \\
E_T^I &= E_T^{II} = -E_T^{III} = -E_T^{IV}.
\end{aligned}
$$

As shown above, the ten independent coefficients,

$$R_T^I, R_T^{II}, C_T^I, C_T^{II}, S_T^I, S_T^{II}, B_T^I, B_T^{II}, D_T^I \text{ and } E_T^I, \tag{5.25}$$

are available to constrain the theoretical parameters. In Tab. 5.5, we show how ten independent combination of the coefficients can be written in terms of the CPT-even, CPT-odd and wrong sign decay parameters. Since we have the eighteen parameters for B meson and K meson decay and mixing, the number of the independent coefficients is not enough to determine all the theoretical parameters.

The measurement of the coefficients are useful to obtain constraints on S and G as well as various non-standard interactions, *i.e.,* the wrong sign decay and CPT violation while the asymmetry in Eq. (5.3) is not exactly T-asymmetry; some combinations of the coefficients enable us to extract the theoretical parameters of interest. In the following sections, we investigate how to determine S and G and consider a method to constrain the various non-standard interactions. In Sec. 5.1, we study the general case without any assumption. In the subsequent section, we investigate two interesting cases, one of which is associated with the case that CPT is a good symmetry in Sec. 5.2. For the case without wrong sign decay amplitudes, we also suggest how to constrain the parameters in Sec. 5.3.

Table 5.5: List of combinations of the independent coefficients in the asymmetry. In the first column, the combinations for the experimental observables are shown. In the other columns, we specify how a coefficient is written in terms of theoretical parameters. The parameters of interest are categorized as three types given in each column.

	CPT even parameters	CPT violating parameters	wrong sign decays
$\frac{R_T^I + R_T^{II}}{2}$	$-R_M + \xi_l^R$	C_ξ^l	0
$\frac{R_T^I - R_T^{II}}{2}$	0	Sz^I	$-\hat{\lambda}_{\rm wst}^R + G\hat{\lambda}_l^R$
$\frac{C_T^I + C_T^{II}}{2}$	C	θ_K^R	0
$\frac{C_T^I - C_T^{II}}{2}$	0	$-Sz_I$	$S\Delta\lambda_l^I$
$\frac{S_T^I + S_T^{II}}{2}$	0	SGz^R	$-SG\Delta\lambda_l^R$
$\frac{S_T^I - S_T^{II}}{2}$	$-S$	$G\theta_K^I$	0
$\frac{B_T^I + B_T^{II}}{2}$	$S^2(R_M - \xi_l^R)$	$-S^2C_\xi^l$	0
$\frac{B_T^I - B_T^{II}}{2}$	0	$S(Gz_K^I - z^I)$	$S(S\hat{\lambda}_{\rm wst}^R - G\Delta\lambda_{\rm wst}^I)$
D_T^I	0	$S(z_K^R - Gz^R)$	$-S(\Delta\lambda_{\rm wst}^R + S\hat{\lambda}_l^I)$
E_T^I	GS	0	0
$\frac{B_T^I + B_T^{II}}{R_T^I + R_T^{II}}$	$-S^2$	0	0

5.1 Extracting Parameters of Interest: General Case

Let us first examine how the theoretical parameters are determined by the measurements of the coefficients shown in Tab. 5.5. Note that we can constrain the product of GS through the observation of E_T. Since the coefficient is multiplied by y in Eq. (5.3), one cannot extract E_T solely from the time dependent asymmetry. Therefore, the value of y should be fixed through another experiment. As defined in Eq. (3.5), y is proportional to the width difference of the B meson mass eigenstates. A method to measure the product $y\cos 2\beta \simeq Gy$ is suggested in Ref. [107]. Combining the measurement of the product $E_T^I y \simeq GSy$, one can determine S. The absolute value of G is fixed through the approximate relation,

$$S^2 + G^2 \simeq 1 - \mathcal{O}(C^2). \tag{5.26}$$

where the quadratic term with respect to C is negligible. Consequently, the measurement of E_T determines $(\pm G, S)$ within two-fold ambiguity. This ambiguity is removed if we assume that the standard model contribution is dominant for the width difference. (See Fig. 5.1.1.)

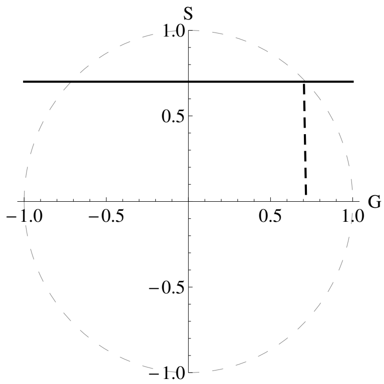

Figure 5.1.1: Determination of G and S. These parameters are on the circle of unit length. Once S is known, G is determined within two-fold ambiguity. This is reproduced from Ref. [61].

As an alternative way, the relation,

$$\frac{B_T^I + B_T^{II}}{R_T^I + R_T^{II}} = -S^2, \tag{5.27}$$

is utilized to fix the absolute value of S. The sign ambiguity for S is removed since at the leading order $2S$ is equal to $S_T^{II} - S_T^I$. Provided that the sub-leading contribution does not change the sign of the leading term, the sign of S is fixed through $S_T^{II} - S_T^I$. Having determined G and S, we can consider constraining the other theoretical parameters of interest.

Note that the following relation is satisfied,

$$\frac{R_T^I - R_T^{II}}{2} + \frac{C_T^I - C_T^{II}}{2} = -\hat{\lambda}_{\rm wst}^R + G\hat{\lambda}_l^R + S\Delta\lambda_l^I. \tag{5.28}$$

Since the right-handed side is independent of CPT violation, non-vanishing combination in l.h.s. implies the unambiguous evidence of wrong sign decays. Furthermore, once S is fixed, one can write the imaginary part of CPT violation in B decays,

$$\theta_K^I = \frac{S_T^I - S_T^{II} + 2S}{2G}. \tag{5.29}$$

However, the real part of θ_K cannot be solely extracted due to the small correction of C,

$$\theta_K^R + C = \frac{C_T^I + C_T^{II}}{2}. \tag{5.30}$$

We stress that the following combination is also convenient,

$$-R_M + \xi_l^R + C_\xi^l = \frac{R_T^I + R_T^{II}}{2}. \tag{5.31}$$

To summarize, if any one of the following combinations,

$$R_T^I - R_T^{II}, \quad C_T^I - C_T^{II}, \quad S_T^I + S_T^{II}, \quad B_T^I - B_T^{II}, \quad D_T^I \tag{5.32}$$

is non-zero, it implies the presence of CPT violation and/or wrong sign decay.

5.2 Extracting Parameters of Interest: CPT Symmetric Limit

In this section, we consider the case in the limit of CPT symmetry. In this circumstance, all the contributions in the third column in Tab. 5.5 vanish. Since the wrong sign decay parameters are CPT-even, the fourth column in Tab. 5.5 is not eliminated. Therefore, the following parameters are fixed,

$$C = \frac{C_T^I + C_T^{II}}{2}, \tag{5.33}$$

$$S = \frac{S_T^{II} - S_T^I}{2}, \tag{5.34}$$

$$R_M - \xi_l^R = -\frac{R_T^I + R_T^{II}}{2}. \tag{5.35}$$

Moreover, the T-odd wrong sign semi-leptonic decay amplitude is extracted,

$$\Delta\lambda_l^I = \frac{C_T^I - C_T^{II}}{2S}, \tag{5.36}$$

$$\Delta\lambda_l^R = -\frac{S_T^I + S_T^{II}}{2GS}. \tag{5.37}$$

For the other wrong sign decay parameters, one can obtain three constraints,

$$\frac{R_T^I - R_T^{II}}{2} = -\hat{\lambda}_{\text{wst}}^R + G\hat{\lambda}_l^R, \tag{5.38}$$

$$\frac{B_T^I - B_T^{II}}{2} = S(S\hat{\lambda}_{\text{wst}}^R - G\Delta\lambda_{\text{wst}}^I), \tag{5.39}$$

$$D_T^I = -S(\Delta\lambda_{\text{wst}}^R + S\hat{\lambda}_l^I). \tag{5.40}$$

5.3 Extracting Parameters of Interest: the absence of wrong sign decays

In this section, constraints on parameters in the case with no wrong sign decays are considered. In this limit, the wrong sign decay amplitudes are eliminated so that the relations in Eqs. (5.29-5.31) are kept while r.h.s. in Eq. (5.28) vanishes. One can fix CP and CPT violation of the mixing parameters in B meson system through the observables,

$$z^I = \frac{R_T^I - R_T^{II}}{2S}, \quad z^R = \frac{S_T^I + S_T^{II}}{2GS}. \tag{5.41}$$

For K meson system, CP and CPT violation in mixing is also determined as,

$$z_K^I = \frac{2D_T^I + S_T^I + S_T^{II}}{2S}, \quad z_K^R = \frac{B_T^I - B_T^{II} - (C_T^I - C_T^{II})}{2SG}. \tag{5.42}$$

Chapter 6

Conditions for Authentic Time Reversal

We have learned from the previous chapter that the coefficients of the asymmetry do not vanish in the T symmetric limit; although the main goal in the original suggestion in Ref. [43] is to obtain genuine T violation, it turned out that T-even contribution is allowed. In this chapter, we clarify why the T-conserving parts are included in the coefficients. One can show that, when the following conditions are simultaneously satisfied, the coefficients in Eqs. (5.9-5.13) become a T-violating object:

(1) Equivalence of B meson states.

(2) $\Delta N_R^{\mathrm{e}} = 0$.

In the above conditions, we defined,

$$\Delta N_R = \Delta N_R^{\mathrm{o}} + \Delta N_R^{\mathrm{e}} \tag{6.1}$$

where ΔN_R^{e} (ΔN_R^{o}) stands for the T-even (odd) part. The idea of the condition (1) is originally addressed in Ref. [52].

In the following sections, the interpretation of the conditions (1) and (2) is addressed.

6.1 Condition of State Orthogonality of B mesons

In Chap. 5, we analyzed the time dependent asymmetry that indicates event number difference of the processes which are naively related with T transformation. In Fig. 6.1.1, the diagram with $(f_1, f_2) = (\psi K_L, l^- X)$, which is associated with $B_- \to \bar{B}^0$, is exhibited while its naively time reversed process given as $\bar{B}^0 \to B_-$ is shown in Fig. 6.1.2. As remarked previously, the difference of the rates for these two processes are considered to obtain T violation. However, rather than Fig. 6.1.2, an inverse decay process which is shown in Fig. 6.1.3 is a genuine time reversed process for Fig. 6.1.1. It is straightforward to verify that Fig. 6.1.1 and Fig. 6.1.3 are related with the reverse of time direction. In the processes considered in this thesis, we substituted Fig. 6.1.2 for Fig. 6.1.3 since signal sides of Fig. 6.1.1 and Fig. 6.1.2 are apparently a time reversed process to each other. Since Fig. 6.1.2 is not a genuine time reversed process, the asymmetry is slightly deviated from a T-violating quantity. The equivalence conditions indicate that the initial (final) B meson states of the signal side in Figs. 6.1.2-6.1.3 are the same as each other up to an overall phase factor.

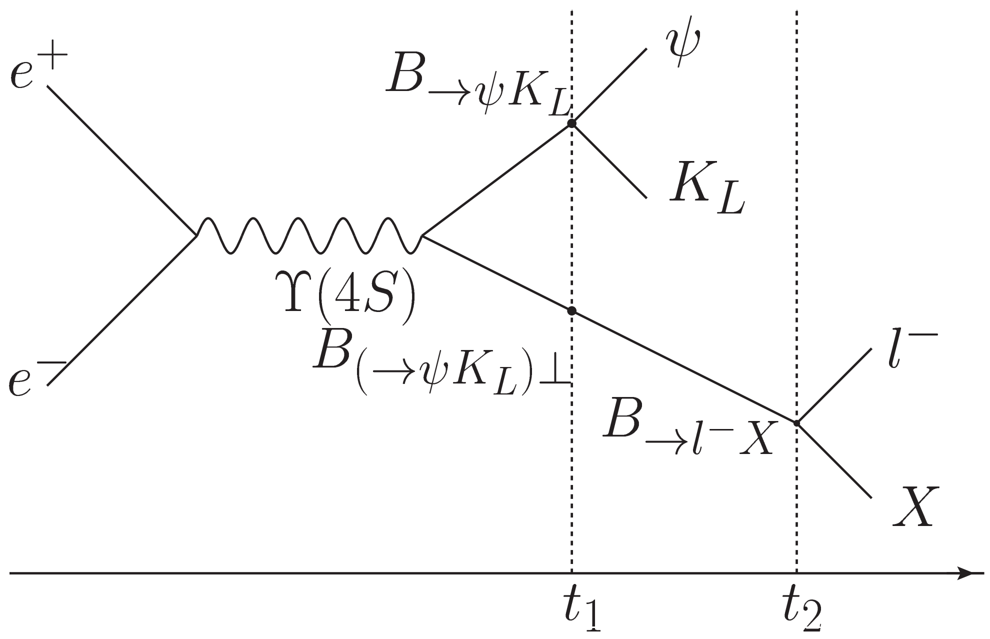

Figure 6.1.1: Process with $(f_1, f_2) = (\psi K_L, l^- X)$. This is reproduced from Ref. [61].

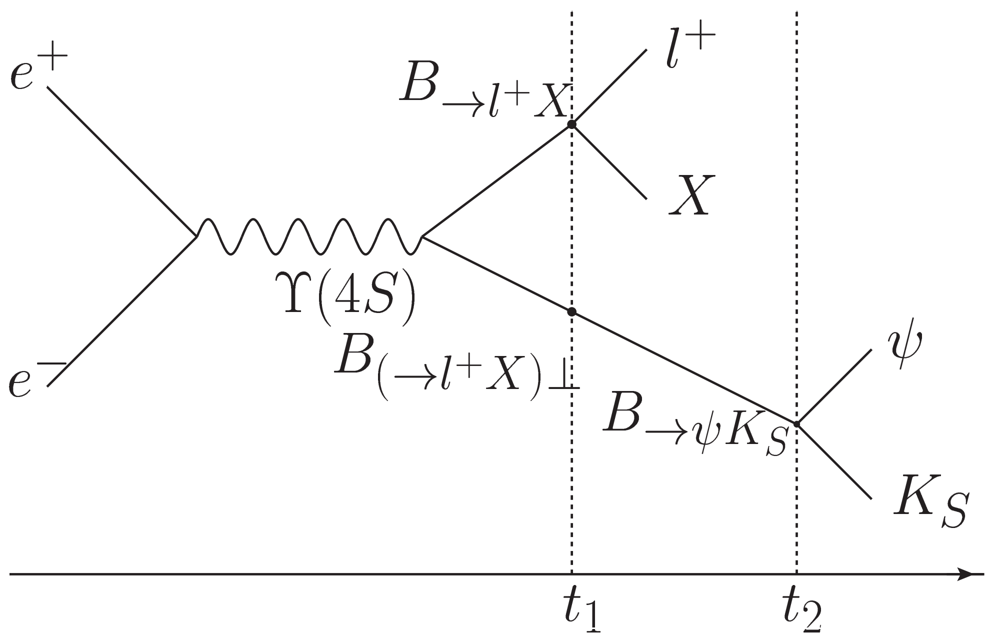

Figure 6.1.2: Process with $(f_3, f_4) = (l^+ X, \psi K_S)$. This is reproduced from Ref. [61].

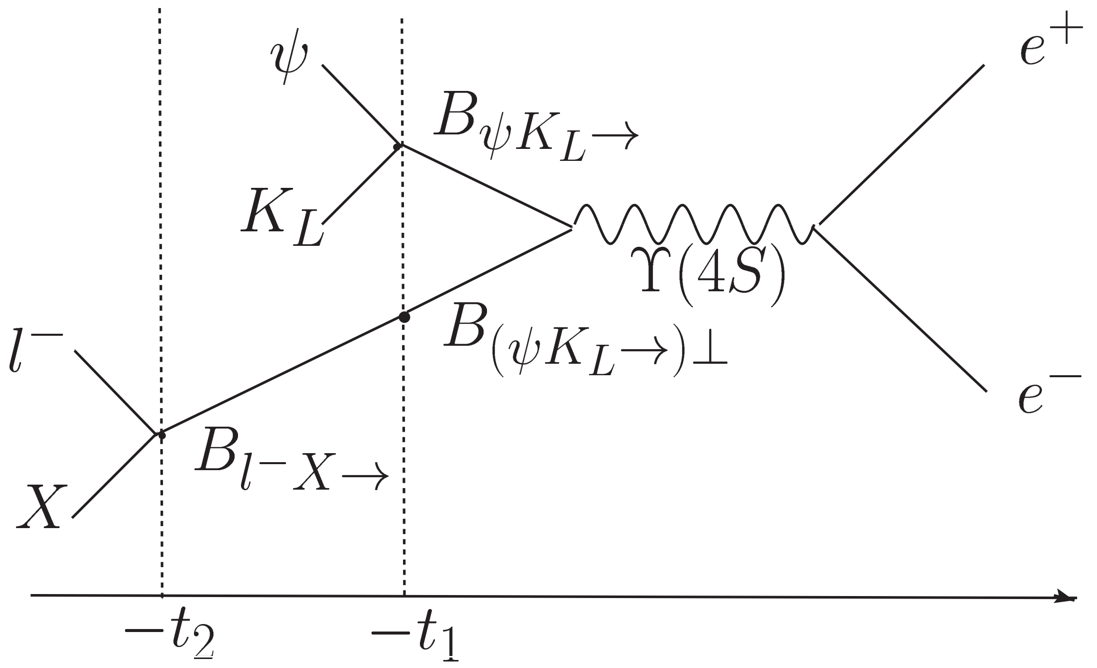

Figure 6.1.3: Inverse process of Fig. 6.1.1. This is reproduced from Ref. [61].

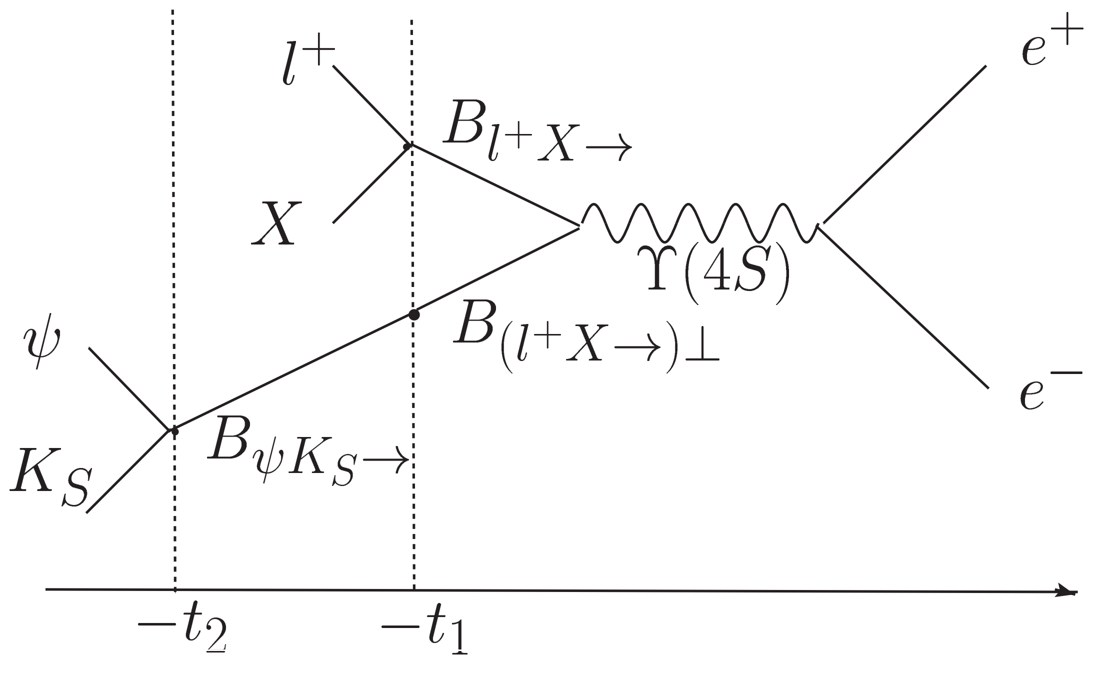

Figure 6.1.4: Inverse process of Fig. 6.1.2. This is reproduced from Ref. [61].

The condition is given as,

$$\begin{cases} |B_{(\to l^+ X)\perp}\rangle \propto |B_{l^- X\to}\rangle \\ |B_{\to\psi K_S}\rangle \propto |B_{(\psi K_L\to)\perp}\rangle \end{cases} . \tag{6.2}$$

The above relations show that B meson states in Fig. 6.1.2-6.1.3 are equivalent. Likewise, Fig. 6.1.4 is a genuine time reversed process of one given in Fig. 6.1.2. We apply state conditions to B meson states in Figs. 6.1.1-6.1.4,

$$\begin{cases} |B_{(\to l^- X)\perp}\rangle \propto |B_{(l^+ X\to)}\rangle \\ |B_{(\to\psi K_L)}\rangle \propto |B_{(\psi K_S\to)\perp}\rangle \end{cases} . \tag{6.3}$$

In the following, we discuss how the conditions are violated, as originally analyzed in Ref. [52]. Up to the normalization factors, violation of the state equivalences in Eqs. (6.2-6.3) is written as,

$$\langle B_{(l^- X\to)\perp}|B_{(\to l^+ X)\perp}\rangle \neq 0, \tag{6.4}$$

$$\langle B_{(l^+ X\to)\perp}|B_{(\to l^- X)\perp}\rangle \neq 0, \tag{6.5}$$

$$\langle B_{(\to\psi K_S)\perp}|B_{(\psi K_L\to)\perp}\rangle \neq 0, \tag{6.6}$$

$$\langle B_{(\psi K_S\to)\perp}|B_{(\to\psi K_L)\perp}\rangle \neq 0. \tag{6.7}$$

The conditions in Eqs. (6.4, 6.5) indicate that one cannot exactly conduct the flavor tagging under the presence of the wrong sign semi-leptonic decays. Similarly, the relations in Eqs. (6.6, 6.7) imply that the CP tagging is contaminated by the presence of CPT violation in $B^0 \to \psi K^0$ decays and wrong strangeness decays. Including overall factors and using our notation, violation of the conditions in Eqs. (6.4-6.7) is shown as,

$$\begin{cases} \langle B_{(\psi K_L\to)\perp}|B_{(\to\psi K_S)\perp}\rangle = N_{(\to\psi K_S)\perp}N_{(\psi K_L\to)\perp}(A_{\psi K^0}A^{\text{ID}}_{\psi K^0} + \bar{A}_{\psi\bar{K}^0}\bar{A}^{\text{ID}}_{\psi\bar{K}^0})\dfrac{\theta_K + \Delta\lambda_{\text{wst}}}{2}, \\ \langle B_{(l^- X\to)\perp}|B_{(\to l^+ X)\perp}\rangle = 2N_{(l^-\to)\perp}N_{(\to l^+)\perp}A_{l^+}\bar{A}^{\text{ID}}_{l^-}\dfrac{p}{q}\lambda_{l^+}, \end{cases} \tag{6.8}$$

$$\begin{cases} \langle B_{(\psi K_S\to)\perp}|B_{(\to\psi K_L)\perp}\rangle = N_{(\to\psi K_L)\perp}N_{(\psi K_S\to)\perp}(A_{\psi K^0}A^{\text{ID}}_{\psi K^0} + \bar{A}_{\psi\bar{K}^0}\bar{A}^{\text{ID}}_{\psi\bar{K}^0})\dfrac{\theta_K - \Delta\lambda_{\text{wst}}}{2}, \\ \langle B_{(l^+ X\to)\perp}|B_{(\to l^- X)\perp}\rangle = 2N_{(l^+\to)\perp}N_{(\to l^-)\perp}\bar{A}_{l^-}A^{\text{ID}}_{l^+}\dfrac{q}{p}\lambda^{-1}_{l^-}, \end{cases} \tag{6.9}$$

where we used the expression for the states defined in App. F. In Eqs. (6.8, 6.9), the effect of mixing-induced CP violation in K meson system is included in terms of our notation of $\Delta\lambda_{\text{wst}}$ in Eq. (4.14). Therefore, Eqs. (6.8, 6.9) show that the wrong sign semi-leptonic decays, the wrong strangeness decays and CPT violation in strangeness changing decays cause tagging ambiguities, which are formulated in terms of the state non-orthogonality.

6.2 Condition of Ratio for Overall Normalization Factor

In this section, we address why the condition (2) is needed to obtain T violating observables. For simplicity, consider the case that the condition (1) is satisfied to demonstrate that violation of $\Delta N_R^{\mathrm{e}} = 0$ gives rise to T-even contribution to the asymmetry.

The following quantities are introduced for convenience,

$$X^{\mathrm{o}} = \frac{X_{(\psi K_L)\perp, l^+X}}{\kappa_{(\psi K_L)\perp, l^+X}} - \frac{X_{(l^-X)\perp, \psi K_S}}{\kappa_{(l^-X)\perp, \psi K_S}}, \quad X^{\mathrm{e}} = \frac{X_{(\psi K_L)\perp, l^+X}}{\kappa_{(\psi K_L)\perp, l^+X}} + \frac{X_{(l^-X)\perp, \psi K_S}}{\kappa_{(l^-X)\perp, \psi K_S}}, \tag{6.10}$$

where $X = \sigma, \mathcal{C}$ and $\mathcal{S}$. Note that $X^{\mathrm{o}}(X^{\mathrm{e}})$ defined in Eq. (6.10) is T-odd (even) due to the explicit forms,

$$S^{\mathrm{o}} = -2S(1 - Gz^R), \quad C^{\mathrm{o}} = 2[C - Sz^I], \quad (\sigma^{\mathrm{o}})^l = 0, \tag{6.11}$$

$$S^{\mathrm{e}} = 2[Gz_K^I + (S^2 - 1)z^I], \quad C^{\mathrm{e}} = 2[z_K^R - Gz^R], \quad (\sigma^{\mathrm{e}})^l = 2G, \tag{6.12}$$

where for σ^o and σ^e, only the leading part is written since small parts of $\Delta\sigma$ and $\hat{\sigma}$ are neglected when multiplied by y in the time dependent asymmetry in Eq. (3.13). In this notation, the observables defined in Eqs. (3.9-3.10) are,

$$\Delta X \simeq X^{\mathrm{o}} - \frac{\Delta N_R}{2} X^{\mathrm{e}} = \left(X^{\mathrm{o}} - \frac{\Delta N_R^{\mathrm{o}}}{2} X^{\mathrm{e}} \right) - \frac{\Delta N_R^{\mathrm{e}}}{2} X^{\mathrm{e}}, \tag{6.13}$$

$$\hat{X} \simeq X^{\mathrm{e}} - \frac{\Delta N_R}{2} X^{\mathrm{o}} = \left(X^{\mathrm{e}} - \frac{\Delta N_R^{\mathrm{o}}}{2} X^{\mathrm{o}} \right) - \frac{\Delta N_R^{\mathrm{e}}}{2} X^{\mathrm{o}}. \tag{6.14}$$

For the case of $\Delta N_R = 0$, it is straightforward to find that $\Delta X(\hat{X})$ is a T-odd (even) object. As shown in Eq. (6.13), one finds that the T-even part of ΔN_R leads to T-even contribution to the observable. The same applies to $\hat{X}$ so that ΔX ($\hat{X}$) deviates from T-odd (even) when ΔN_R^{e} has a non-zero value. Therefore, we can demonstrate that T-even part of ΔN_R yields T-even contribution to the coefficients of time dependent asymmetry in Eqs. (5.4-5.7).

Chapter 7

Summary

In this thesis, we have investigated the precise meaning of the observables in B factory experiments. The processes of $B\bar{B}$ mixing give the methodology to get the information about non-invariance of T violation, as originally suggested in Ref. [43]. The EPR correlation of flavors in $\Upsilon(4S) \to B\bar{B}$ decays enables us to extract the time dependent processes of B meson. For the purpose to obtain a probe of T violation discriminated from CP violation, we utilized the time dependent decay rates for $B_- \to \bar{B}^0$ and $\bar{B}^0 \to B_-$, motivated by the BaBar measurement [50]. Associated with these two processes, the time dependent asymmetry is constructed to investigate violation of the discrete symmetries.

As description of the systems for B and K meson, neutral meson mixing is briefly introduced to obtain the time evolution of wave function. The Weisskopf-Wigner approximation [60] provides drastically simplified formalism to deal with the time dependence of neutral meson states. Incoming and outgoing mass eigenstates in the system with non-Hermitian Hamiltonian are constructed through the orthogonality of the states. To examine the experimental constraints, the CPT violating formulation is adopted for the mixing system.

We attempted to write the time dependent asymmetry in terms of the parameters in the flavor based states. In our notation, the transformation properties for the theoretical parameters are exact so that one can disentangle the transformation laws for the constructed asymmetry. It is well-known that the mass eigenstate of $K_{S,L}$ is deviated from the CP eigenstates. We properly treated these mass eigenstates in the analysis to calculate the contribution from indirect CP violation. CP and CPT violation in K meson mixing is also accounted in the analysis to examine the contribution. Moreover, it is shown that the introduced parameters are invariant under rephasing of quarks.

In the analysis of BaBar [50] and the theoretical study [52], the difference of the overall constants for the rates is eliminated. In general, the ratio of the overall constants for the two decay rates is deviated from unity. Such slight deviation is taken into account in our analysis.

It is shown that the proper function form of the asymmetry is obtained as an extension of one utilized in the experiment [50] if one accounts CPT violation, wrong sign decays, non-zero width difference between B_H and B_L and CP violation in B meson mixing and right sign semi-leptonic decay. We also found that the asymmetry is expressed in terms of the phase convention independent quantities. The effect of mixing-induced CP violation in K meson system is extracted and it gives rise to $\mathcal{O}(10^{-3})$ contribution to the observable, which is comparable to direct CP violation in the $B^0 \to \psi K^0$ decay. Assuming that no cancellation between the parameters occurs, the coefficient of the cosine term in the asymmetry is $\mathcal{O}(10^{-3})$, if the wrong sign semi-leptonic decays and CPT violation are negligible.

The theoretical parametrization for the coefficients of the time dependent trigonometric functions in the asymmetries is explicitly formulated. With our notation, it turned out that the coefficients are slightly deviated from T-odd quantities; there remain contributions which do not vanish in the T symmetric limit. In this sense, we have redemonstrated that the observation of T violation requires the measurement of genuine time reversed processes which include an inverse decay process of B meson.

We have obtained the coefficients of the asymmetry for the processes I-IV, which are apparently considered as T-odd quantities. The proper combinations of the coefficients enable us to extract the theoretical parameters of interest. Provided that the width difference between B_H and B_L is known, the three cases to constrain the parameters are discussed. For the most general case, we can extract the parameters which are associated with sine and cosine of 2β in the SM. We also found that non-zero value of some combination of the coefficient signals CPT violation and/or the presence of the wrong sign decays. For the case of CPT-conserving limit, the observables constrain the parameters for wrong sign semi-leptonic decays and wrong sign strangeness decays of B meson, both of which are extremely suppressed in the SM. As for the case of the absence of wrong sign decays, CPT violation for B and K meson mixing is constrained.

Furthermore, we discussed why the T-conserving contributions appear in the coefficients. It is shown that T-even terms in the asymmetry vanish when several conditions are satisfied. These derived conditions are categorized as two types. The first one is referred to as equivalence conditions which are associated with B meson states for a time reversal-like process and a genuine time reversed process. As suggested in Ref. [52], B mesons for the two processes are not equivalent to each other. We showed violation of the equivalence conditions in our notation including the effects of mixing in K meson system. Since the difference of the overall factors of rates is accounted in this study, one should consider an additional condition to obtain T-odd observables. In particular, it is shown that T-even part of the ratio of the overall factors can be the origin of T-even contribution. One can clarify that if these two conditions are simultaneously satisfied, the coefficients become T-odd quantities.

Through this study, we have learned that the subtle points are involved in the observation of microscopic T violation whereas CP violation is firmly measured in the flavor factory experiments. Obviously, the further check of the discrete symmetries gives us an unique tool to investigate the validity of the theory. Since the ingredients in this study are applicable to future B factory experiments, it is expected that the discrete symmetries will be well-understood by the precise measurement with the principal aim for obtaining a signal of physics beyond the standard model.

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Finally, I would like to express my sincere appreciation for my parents who support me throughout my life.

Appendix A

Coefficients of Master Formula

In this appendix, the coefficients of the master formula for the time dependent decay rate are shown. As originally given in Ref. [52], the coefficients are,

$$
\begin{aligned}
N_{(i)\perp,j} &= \frac{1}{4}\mathcal{N}_i\mathcal{N}_j\{1+(C_i+C_j)(R_M-z^R)\}, \quad \mathcal{N}_i=|A_i|^2+|\bar{A}_i|^2, && \text{(A.1)}\\
\kappa_{(i)\perp,j} &= (1-G_iG_j)\\
&\quad +[(C_i+C_j)(1-G_iG_j)+C_jG_i+C_iG_j]z^R-(S_i+S_j)z^I\\
&\quad +G_iG_j(C_i\theta_i^R+C_j\theta_j^R)-G_iS_j\theta_j^I-G_jS_i\theta_i^I, && \text{(A.2)}\\
\sigma_{(i)\perp,j} &= G_j-G_i\\
&\quad +[C_i(1+G_j-G_i)-C_j(1-G_j+G_i)]z^R+(G_iS_j-G_jS_i)z^I\\
&\quad -C_jG_j\theta_j^R+S_j\theta_j^I+C_iG_i\theta_i^R-S_i\theta_i^I, && \text{(A.3)}\\
\mathcal{C}_{(i)\perp,j} &= -C_iC_j-S_iS_j\\
&\quad -[(C_i+C_j)(C_iC_j+S_iS_j)+C_iG_j+C_jG_i]z^R+(S_i+S_j)z^I\\
&\quad +G_jS_i\theta_j^I-[C_i(1-C_j^2)-C_jS_iS_j]\theta_j^R\\
&\quad +G_iS_j\theta_i^I-[C_j(1-C_i^2)-C_iS_iS_j]\theta_i^R, && \text{(A.4)}\\
\mathcal{S}_{(i)\perp,j} &= C_iS_j-C_jS_i\\
&\quad +[C_iC_j(S_j-S_i)-(C_j^2+G_j)S_i+(C_i^2+G_i)S_j]z^R+(C_j-C_i)z^I\\
&\quad -C_iG_j\theta_j^I+[(C_j^2-1)S_i-C_iC_jS_j]\theta_j^R\\
&\quad +C_jG_i\theta_i^I-[(C_i^2-1)S_j-C_iC_jS_i]\theta_i^R, && \text{(A.5)}
\end{aligned}
$$

where A_i and $\bar{A}_i$ in Eq. (A.1) are the decay amplitude defined in Eq. (4.3). The indices i and j represent the final state of tagging side (f_i) and signal side(f_j), respectively.

Appendix B

Strangeness Changing Decay Amplitudes

In this appendix, B meson decay amplitudes are explicitly given. Here, the amplitudes with the mass eigenstates of K meson are decomposed into definite strangeness amplitudes. For strangeness changing processes with an initial B meson, the decay amplitudes are,

$$\begin{aligned}
A_{\psi K_S} &= \langle \psi K_S^{\rm out} | B_0^{\rm in} \rangle = \frac{1}{2p_K q_K}(q_K\sqrt{1-z_K}A_{\psi K^0} + p_K\sqrt{1+z_K}A_{\psi \bar{K}^0}), && \text{(B.1)}\\
A_{\psi K_L} &= \langle \psi K_L^{\rm out} | B_0^{\rm in} \rangle = \frac{1}{2p_K q_K}(q_K\sqrt{1+z_K}A_{\psi K^0} - p_K\sqrt{1-z_K}A_{\psi \bar{K}^0}), && \text{(B.2)}\\
\bar{A}_{\psi K_S} &= \langle \psi K_S^{\rm out} | \bar{B}_0^{\rm in} \rangle = \frac{1}{2p_K q_K}(q_K\sqrt{1-z_K}\bar{A}_{\psi K^0} + p_K\sqrt{1+z_K}\bar{A}_{\psi \bar{K}^0}), && \text{(B.3)}\\
\bar{A}_{\psi K_L} &= \langle \psi K_L^{\rm out} | \bar{B}_0^{\rm in} \rangle = \frac{1}{2p_K q_K}(q_K\sqrt{1+z_K}\bar{A}_{\psi K^0} - p_K\sqrt{1-z_K}\bar{A}_{\psi \bar{K}^0}), && \text{(B.4)}
\end{aligned}$$

while the inverse decay amplitudes are,

$$\begin{aligned}
A^{\rm ID}_{\psi K_S} &= \langle B_0^{\rm out} | \psi K_S^{\rm in} \rangle = (p_K\sqrt{1-z_K}A^{\rm ID}_{\psi K^0} + q_K\sqrt{1+z_K}A^{\rm ID}_{\psi \bar{K}^0}), && \text{(B.5)}\\
A^{\rm ID}_{\psi K_L} &= \langle B_0^{\rm out} | \psi K_L^{\rm in} \rangle = (p_K\sqrt{1+z_K}A^{\rm ID}_{\psi K^0} - q_K\sqrt{1-z_K}A^{\rm ID}_{\psi \bar{K}^0}), && \text{(B.6)}\\
\bar{A}^{\rm ID}_{\psi K_S} &= \langle \bar{B}_0^{\rm out} | \psi K_S^{\rm in} \rangle = (p_K\sqrt{1-z_K}\bar{A}^{\rm ID}_{\psi K^0} + q_K\sqrt{1+z_K}\bar{A}^{\rm ID}_{\psi \bar{K}^0}), && \text{(B.7)}\\
\bar{A}^{\rm ID}_{\psi K_L} &= \langle \bar{B}_0^{\rm out} | \psi K_L^{\rm in} \rangle = (p_K\sqrt{1+z_K}\bar{A}^{\rm ID}_{\psi K^0} - q_K\sqrt{1-z_K}\bar{A}^{\rm ID}_{\psi \bar{K}^0}). && \text{(B.8)}
\end{aligned}$$

Plugging Eqs. (B.1-B.8) into Eq.(4.4), we can obtain Eqs. (4.13, 4.28-4.30).

Appendix C

List of Coefficients for Time Dependent Decay Rate

In this appendix, we give expressions for the coefficients of the time dependent trigonometric functions in Eq. (3.4). For the processes associated with the final states in Eq.(5.1), the coefficients are,

$$S_{(\psi K_L)\perp,l^-X} = S_{\psi K_L} - S_{\psi K_L} z^R - z^I - G_{\psi K_L}\theta^I_{\psi K_L}, \tag{C.1}$$

$$S_{(l^+X)\perp,\psi K_S} = S_{\psi K_S} + S_{\psi K_S} z^R - z^I - G_{\psi K_S}\theta^I_{\psi K_S}, \tag{C.2}$$

$$C_{(\psi K_L)\perp,l^-X} = C_{\psi K_L} - S_{\psi K_L} S_{l^-} + G_{\psi K_L} z^R + S_{\psi K_L} z^I + \theta^R_{\psi K_L}, \tag{C.3}$$

$$C_{(l^+X)\perp,\psi K_S} = -C_{\psi K_S} - S_{\psi K_S} S_{l^+} - G_{\psi K_S} z^R + S_{\psi K_S} z^I - \theta^R_{\psi K_S}, \tag{C.4}$$

$$\kappa_{(\psi K_L)\perp,l^-X} = 1 - G_{\psi K_L} G_{l^-} - (G_{\psi K_L} + 1) z^R - S_{\psi K_L} z^I, \tag{C.5}$$

$$\kappa_{(l^+X)\perp,\psi K_S} = 1 - G_{\psi K_S} G_{l^+} + (G_{\psi K_S} + 1) z^R - S_{\psi K_S} z^I, \tag{C.6}$$

$$\sigma_{(\psi K_L)\perp,l^-X} = G_{l^-} - G_{\psi K_L} + (1 + G_{\psi K_L}) z^R - S_{\psi K_L}\theta^I_{\psi K_L}, \tag{C.7}$$

$$\sigma_{(l^+X)\perp\psi K_S} = G_{\psi K_S} - G_{l^+} + (1 + G_{\psi K_S}) z^R + S_{\psi K_S}\theta^I_{\psi K_S}, \tag{C.8}$$

$$\frac{S_{(\psi K_L)\perp,l^-X}}{\kappa_{(\psi K_L)\perp,l^-X}} = S_{\psi K_L} + S_{\psi K_L} G_{\psi K_L} G_{l^-} + S_{\psi K_L} G_{\psi K_L} z^R + (S^2_{\psi K_L} - 1) z^I - G_{\psi K_L}\theta^I_{\psi K_L}, \tag{C.9}$$

$$\frac{S_{(l^+X)\perp,\psi K_S}}{\kappa_{(l^+X)\perp,\psi K_S}} = S_{\psi K_S} + S_{\psi K_S} G_{\psi K_S} G_{l^+} - S_{\psi K_S} G_{\psi K_S} z^R + (S^2_{\psi K_S} - 1) z^I - G_{\psi K_S}\theta^I_{\psi K_S}, \tag{C.10}$$

$$\frac{C_{(\psi K_L)\perp,l^-X}}{\kappa_{(\psi K_L)\perp,l^-X}} \simeq C_{(\psi K_L)\perp,l^-X}, \tag{C.11}$$

$$\frac{C_{(l^+X)\perp,\psi K_S}}{\kappa_{(l^+X)\perp,\psi K_S}} \simeq C_{(l^+X)\perp,\psi K_S}, \tag{C.12}$$

$$\frac{\sigma_{(\psi K_L)\perp,l^-X}}{\kappa_{(\psi K_L)\perp,l^-X}} \simeq -G_{\psi K_L}, \tag{C.13}$$

$$\frac{\sigma_{(l^+X)\perp,\psi K_S}}{\kappa_{(l^+X)\perp,\psi K_S}} \simeq G_{\psi K_S}, \tag{C.14}$$

where we keep only the leading term in Eqs. (C.13-C.14) since it is multiplied by y in the formulae of the decay rate in Eq. (3.4). The relation between the coefficients in I and ones in II-IV is addressed in App. E.

Appendix D

Coefficient for Process I

In this appendix, formulae are given for the quantities which appear in Eqs. (5.4-5.7). For process I, $\Delta\mathcal{S}, \Delta\mathcal{C}, \Delta\sigma, \hat{\sigma}, \hat{\mathcal{S}}$ and $\hat{\mathcal{C}}$ are explicitly written. In the derivation of these formulae, Eqs. (C.1-C.14) are utilized. We denote,

$$\kappa_{(1)\perp,2} = \kappa^l_{(1)\perp,2} + \Delta\kappa_{(1)\perp,2}, \quad (D.1)$$

$$\kappa_{(3)\perp,4} = \kappa^l_{(3)\perp,4} + \Delta\kappa_{(3)\perp,4}, \quad (D.2)$$

where the superscript l implies the leading part and Δ denotes the small part which is treated as perturbation.

The ratio of normalizations for rates is,

$$N_R \simeq \frac{\mathcal{N}_3\mathcal{N}_4}{\mathcal{N}_1\mathcal{N}_2}\frac{\kappa^l_{(3)\perp,4}}{\kappa^l_{(1)\perp,2}}\left[1 + (C_3 + C_4 - C_1 - C_2)(R_M - z^R) + \frac{\Delta\kappa_{(3)\perp,4}}{\kappa^l_{(3)\perp,4}} - \frac{\Delta\kappa_{(1)\perp,2}}{\kappa^l_{(1)\perp,2}}\right]. \quad (D.3)$$

Note that for the processes given in Eq. (5.1),

$$\kappa^l_{(1)\perp,2} = \kappa^l_{(3)\perp,4} = 1, \quad (D.4)$$

is satisfied. For the process I, the ratio of overall normalization is given as,

$$N^I_R \simeq \frac{\mathcal{N}_{\psi K_S}\mathcal{N}_{l+X}}{\mathcal{N}_{\psi K_L}\mathcal{N}_{l-X}}[1 + 2(-Sz^I + R_M - G\hat{\lambda}^R_l)]. \quad (D.5)$$

The ratios of $\mathcal{N}_{l+X}/\mathcal{N}_{l-X}$ and $\mathcal{N}_{\psi K_S}/\mathcal{N}_{\psi K_L}$ are slightly deviated from one,

$$\frac{\mathcal{N}_{l+X}}{\mathcal{N}_{l-X}} = 1 - 2(C^l_\xi + \xi^R_l), \quad \frac{\mathcal{N}_{\psi K_S}}{\mathcal{N}_{\psi K_L}} = 1 + 2\hat{\lambda}^R_{\rm wst}. \quad (D.6)$$

Thus, one can obtain,

$$N_R^I = 1 + \Delta N_R^I, \quad (D.7)$$

$$\Delta N_R^I = 2[-Sz^I + R_M + \hat{\lambda}_{\rm wst}^R - G\hat{\lambda}_l^R - C_\xi^l - \xi_l^R], \quad (D.8)$$

where ΔN_R^I is a small number so that we treat it as perturbation. The expressions of ΔS^I and ΔC^I in Eqs. (5.5-5.7) are,

$$\begin{aligned}\Delta \mathcal{S}^I &= \left(\frac{\mathcal{S}_{(\psi K_L)\perp, l^- X}}{\kappa_{(\psi K_L)\perp, l^- X}} - \frac{\mathcal{S}_{(l^+ X)\perp, \psi K_S}}{\kappa_{(l^+ X)\perp, \psi K_S}}\right) - \frac{\Delta N_R^I}{2}\left(\frac{\mathcal{S}_{(\psi K_L)\perp, l^- X}}{\kappa_{(\psi K_L)\perp, l^- X}} + \frac{\mathcal{S}_{(l^+ X)\perp, \psi K_S}}{\kappa_{(l^+ X)\perp, \psi K_S}}\right) \\ &\simeq -2[S(1 - Gz^R) - G\theta_K^I + GS\Delta\lambda_l^R], \quad (D.9)\\ \Delta \mathcal{C}^I &\simeq \frac{\mathcal{C}_{(\psi K_L)\perp, l^- X}}{\kappa_{(\psi K_L)\perp, l^- X}} - \frac{\mathcal{C}_{(l^+ X)\perp, \psi K_S}}{\kappa_{(l^+ X)\perp, \psi K_S}} = 2[C - Sz^I + \theta_K^R + S\Delta\lambda_l^I]. \quad (D.10)\end{aligned}$$

Note that the sub-leading parts of $\Delta\sigma^I$ and $\hat{\sigma}^I$ are suppressed when multiplied with $y\Gamma t$ in Eq. (5.3). We give the expression of the leading parts for $\Delta\sigma^I$ and $\hat{\sigma}^I$,

$$(\Delta\sigma^I)^l = 0, \quad (\hat{\sigma}^I)^l = 2G. \quad (D.11)$$

The expressions for $\hat{\mathcal{S}}^I$ and $\hat{\mathcal{C}}^I$ in Eq. (5.6) are written as follows,

$$\begin{aligned}\hat{\mathcal{S}}^I &= \left(\frac{\mathcal{S}_{(\psi K_L)\perp, l^- X}}{\kappa_{(\psi K_L)\perp, l^- X}} + \frac{\mathcal{S}_{(l^+ X)\perp, \psi K_S}}{\kappa_{(l^+ X)\perp, \psi K_S}}\right) - \frac{\Delta N_R^I}{2}\left(\frac{\mathcal{S}_{(\psi K_L)\perp, l^- X}}{\kappa_{(\psi K_L)\perp, l^- X}} - \frac{\mathcal{S}_{(l^+ X)\perp, \psi K_S}}{\kappa_{(l^+ X)\perp, \psi K_S}}\right) \\ &\simeq 2[G(z_K^I - \Delta\lambda_{\rm wst}^I) - z^I + SR_M + S\hat{\lambda}_{\rm wst}^R - SC_\xi^l - S\xi_l^R], \quad (D.12)\\ \hat{\mathcal{C}}^I &\simeq \frac{\mathcal{C}_{(\psi K_L)\perp, l^- X}}{\kappa_{(\psi K_L)\perp, l^- X}} + \frac{\mathcal{C}_{(l^+ X)\perp, \psi K_S}}{\kappa_{(l^+ X)\perp, \psi K_S}} = 2[z_K^R - \Delta\lambda_{\rm wst}^R - Gz^R - S\hat{\lambda}_l^I]. \quad (D.13)\end{aligned}$$

Appendix E

Relation among Coefficients for Processes I-IV

In this appendix, the relation among the coefficients for the processes I-IV in Eq. (5.1) is shown.

First, the method to obtain the coefficient in process II (IV) from ones in I (III) is addressed. Process I and II (III and IV) are related with interchange of $\psi K_L \leftrightarrow \psi K_S$. Hence, the coefficients of the process II (IV) are obtained by flipping the sign of the mixing parameter q_K and z_K in the process I (III). The replacement of $q_K \to -q_K$ leads to the change of the sign of S, G and $\lambda_{\rm wst}$ so that one can get the relation between II (IV) and I (III).

In the following, we present a simple rule which enables one to obtain the coefficients in process IV from ones in II. The coefficients of the process IV are explicitly given as,

$$
\begin{aligned}
R_T^{\rm IV} &= -Sz^I + \frac{1}{2}(C_{l^+} - C_{l^-})R_M - \xi_l^R - C_\xi^l + \hat{\lambda}_{\rm wst}^R - G\hat{\lambda}_l^R, && \text{(E.1)}\\
C_T^{\rm IV} &= \frac{1}{2}(C_{l^-} - C_{l^+})C + Sz^I + \frac{1}{2}(C_{l^-}\theta_{K_L}^R - C_{l^+}\theta_{K_S}^R) - S\Delta\lambda_l^I, && \text{(E.2)}\\
S_T^{\rm IV} &= \frac{1}{2}(C_{l^-} - C_{l^+})S + SGz^R + \frac{G}{2}(C_{l^+}\theta_{K_S}^I - C_{l^-}\theta_{K_L}^I)\\
&\quad -GS(C_{l^+}{\rm Re}[\lambda_{l^+}] + C_{l^-}{\rm Re}[\lambda_{l^-}^{-1}]), && \text{(E.3)}\\
B_T^{\rm IV} &= S[-Gz_K^I + z^I + \frac{C_{l^-} - C_{l^+}}{2}SR_M + S\xi_l^R] + S^2C_\xi^l - S^2\hat{\lambda}_{\rm wst}^R + SG\Delta\lambda_{\rm wst}^I, && \text{(E.4)}\\
D_T^{\rm IV} &= S[z_K^R - Gz^R] - S\Delta\lambda_{\rm wst}^R + \frac{C_{l^-} - C_{l^+}}{2}S^2\hat{\lambda}_l^I, && \text{(E.5)}\\
E_T^{\rm IV} &= \frac{C_{l^-} - C_{l^+}}{2}GS. && \text{(E.6)}
\end{aligned}
$$

The processes II and IV are related with $l^+X \leftrightarrow l^-X$ in Eq.(5.1). If one interchanges l^+ and l^- in Eq. (D.6), the sign of C_ξ^l and ξ_l^R is reversed. Owing to Eqs. (4.49-4.50), the sign of $\hat{\lambda}_l^I$ and $\Delta\lambda_l^R$ is also flipped. Moreover, one needs to interchange C_{l^+} and C_{l^-} in Eqs. (E.1-E.6) to get the coefficients of the asymmetry for the process II.

Appendix F

Deviation from Orthogonality of B Meson States

In this appendix, the derivation of Eqs. (6.8, 6.9) for orthogonality of B meson states is given. Due to EPR correlation, if a B meson decays into ψK_S at time t_1, another B meson on the opposite side does not decay into ψK_S at the same time. The signal side at time t_1 in Fig. 6.1.1 indicates the B meson state which is orthogonal to the tagging side, *i.e.*,

$$|B_{(\to\psi K_L)\perp}\rangle = N_{(\to\psi K_L)\perp}(\bar{A}_{\psi K_L}|B^0\rangle - A_{\psi K_L}|\bar{B}^0\rangle). \tag{F.1}$$

It is straightforward to verify that the state in Eq. (F.1) is orthogonal to $\langle\psi K_L|$. Likewise, we define B meson states which are orthogonal to the state on the opposite side,

$$\langle B_{(\psi K_L\to)\perp}| = N_{(\psi K_L\to)\perp}(\bar{A}^{\rm ID}_{\psi K_L}\langle B^0| - A^{\rm ID}_{\psi K_L}\langle\bar{B}^0|), \tag{F.2}$$

$$\langle B_{(\psi K_S\to)\perp}| = N_{(\psi K_S\to)\perp}(\bar{A}^{\rm ID}_{\psi K_S}\langle B^0| - A^{\rm ID}_{\psi K_S}\langle\bar{B}^0|), \tag{F.3}$$

$$|B_{(\to\psi K_S)\perp}\rangle = N_{(\to\psi K_S)\perp}(\bar{A}_{\psi K_S}|B^0\rangle - A_{\psi K_S}|\bar{B}^0\rangle). \tag{F.4}$$

The inner product of Eqs. (F.4, F.2) is calculated with our notation including overall normalization,

$$\langle B_{(\psi K_L\to)\perp}|B_{(\to\psi K_S)\perp}\rangle = \frac{N_{(\to\psi K_S)\perp}N_{(\psi K_L\to)\perp}}{2}(A_{\psi K^0}A^{\rm ID}_{\psi K^0} + \bar{A}_{\psi\bar{K^0}}\bar{A}^{\rm ID}_{\psi\bar{K^0}})[\theta_K + \Delta\lambda_{\rm wst}], \tag{F.5}$$

where Eqs. (B.1-B.8) are used. We treated $z_K, \theta_{\psi K^0}, \bar{\theta}_{\psi\bar{K^0}}, \hat{\lambda}_{\rm wst}$ and $\Delta\lambda_{\rm wst}$ as perturbation and ignored the second order contribution of the small quantities. The deviation from zero in Eq. (F.5) was confirmed previously in Ref. [52]. As for orthogonality of B meson states for semi-leptonic decay, the inner products in the first line of Eq. (6.8) and in the second line of Eq. (6.9) are obtained. The B meson states orthogonal to semi-leptonic (inverse) decaying B meson are,

$$\langle B_{(l^- X\to)\perp}| = N_{(l^-\to)\perp}(\bar{A}^{\rm ID}_{l^-}\langle B^0| - A^{\rm ID}_{l^-}\langle \bar{B}^0|), \tag{F.6}$$

$$|B_{(\to l^+ X)\perp}\rangle = N_{(\to l^+)\perp}(\bar{A}_{l^+}|B^0\rangle - A_{l^+}|\bar{B}^0\rangle), \tag{F.7}$$

$$\langle B_{(l^+ X\to)\perp}| = N_{(l^+\to)\perp}(\bar{A}^{\rm ID}_{l^+}\langle B^0| - A^{\rm ID}_{l^+}\langle \bar{B}^0|), \tag{F.8}$$

$$|B_{(\to l^- X)\perp}\rangle = N_{(\to l^-)\perp}(\bar{A}_{l^-}|B^0\rangle - A_{l^-}|\bar{B}^0\rangle). \tag{F.9}$$

Thus, the inner product of the state in Eqs. (F.6-F.7) is,

$$\langle B_{(l^- X\to)\perp}|B_{(\to l^+ X)\perp}\rangle = 2N_{(l^-\to)\perp}N_{(\to l^+)\perp}A_{l^+}\bar{A}^{\rm ID}_{l^-}\frac{p}{q}\lambda_{l^+}. \tag{F.10}$$

As shown above, orthogonality of the B meson states is violated up to the wrong sign semi-leptonic decay amplitude as addressed in Ref. [52].

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