J. Fac. Appl. Biol. Sci.,
Hiroshima Univ. (1980), **19**: 1 – 9

Study on the Flow Behavior in the Low Flow Region of Skim Milk Solutions

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Received: March 11, 1980
(Figs. 1–4, Tables 1–3)

INTRODUCTION

The flow equations of liquid foods are important for the designing of various apparatuses and for the control of them. In former papers[1–6], we have studied the flow equaitons of heated starch solutions under various temperatures.

Although numerous investigatores have studied the viscosity of milk[7–13] and skim milk[14–19], their flow equations have not been reported in these studies excepted two literatures[17, 19].

In this paper, we took up the flow behavior of slim milk solutions under low concentrations at various temperatures, and could determine the flow equation in the low flow region.

For the measuring, we used a capillary tube viscometer operated under various pressures, the same as the one used in the former studies[1–6].

EXPERIMENTAL

1. Sample Preparation

The powdered skim milk used in this study was prepared commercially in Japan (Yukijirushi Nyugyo Co.). The main components as printed on the box are: protein: 35.0 wt%, lipid: 1.0wt%, non-fibrous carbohydrate: 52.0 wt%, ash: 8.0 wt% and water: 4.0 wt%.

The skim milk solutions used in this study were prepared in the desired concentrations from powdered skim milk, under vigorous agitation by stirring for 30 minutes at 30°C.

The solution obtained was degassed with a laboratory vacuum drier for 60 minutes at 110 mmHg and 30°C. The values of the concentration of the prepared samples were determined by measuring the weight of the degassed solutions.

After the skim milk solutions had been kept at the desired temperature for 30 minutes, the flow behavior was measured by a capillary tube viscometer at a set temperature.

2. Tube viscometer

The capillary tube viscometer used in this study was the same as the one descrived formerly[1].

The grass capillary tubes used in our experiments were that Cap.–A: 0.0369 cm radius and 25.9 cm length, Cap.–B: 0.0746 cm and 26.2 cm, respectively. The diameters of the capillary tubes were calculated from the weight of mercury required to fill it throughly.

The capillary tube and the sample feeder were made up in a constant temperature water bath. The pressure difference of the fluid through the capillary tube could be obtained by means of a vacuum pump. The values of the pressure difference were measured with a mercury or a water manometer.

The volumetric flow rates of the fluid were taken from the time required for a determined volume of fluid to pass through the tube. All measurements were made upon 5.00 ~ 25.3 wt% skim milk solutions at constant temperatures from 30 to 60°C.

FLOW EQUATION

1. Flow equation

The flow equation of many non-Newtonian liquid foods can be expressed as follows[7,8]:

$$\gamma = (1/K)(g_c\tau - g_c\tau_y)^n \tag{1}$$

where, γ(s^{-1}) is the shear rate, τ(g_f/cm^2) is the shear stress, K ($g^n/cm^n \cdot sec^{2n-1}$) is the fluid consistency index, n(–) is the flow behavior index, τ_y(g_f/cm^2) is the yield stress and g_c ($g \cdot cm/g_f \cdot sec^2$) is the gravitational conversion factor. The viscometric parameters K, n and τ_y in Eq. (1) can be determined from the experimental results for the flow behavior of skim milk solutions.

For Newtonian fluids which is n=1 and τ_y=0 in Eq.(1), the flow equation becomes:

$$\gamma = (1/\mu)\, g_c\tau \tag{2}$$

where, μ(g/cm·sec) is the viscosity which is one of the well known physical properties.

In a circular tube, the relation between the shear stress and the pressure difference ΔP (g_f/cm^2) acting on the cylinderical flow of radius r (cm) and length L (cm) can be expressed as follows:

$$2\pi r L \cdot \tau = \pi r^2 \cdot \Delta P \tag{3}$$

Combining Eqs. (1) and (3), we obtain [1,20,21]:

$$Q = \{2\pi g_c^n r_w^3 (\tau_w - \tau_y)^{n+1}/K\tau_w^3\}\{(\tau_w-\tau_y)^2/2(n+3)$$

$$+\tau_y(\tau_w-\tau_y)/(n+2)+\tau_y^2/2(n+1) \tag{4}$$

$$\tau_w = r_w \Delta P/2L \tag{5}$$

where, Q (cm^3/sec) is the volumetric flow rate of flowing fluid, r_w (cm) is the radius of the capillary tube and τ_w (g_f/cm^2) is the shear stress at the wall.

2. Calibration of apparatus

The pressure difference ΔP in Eq. (5) must be taken from the following equation:

$$\Delta P = \Delta P_m - m_p \rho \bar{u}^2 / g_c \tag{6}$$

where, ΔP_m (g_f/cm^2) is the observed value of pressure difference in capillary tube viscometer, m_p (–) is the Hagenbach coefficient, ρ(g/cm^3) and $\bar{u}$ (cm/sec) are the density and the mean velosity of flowing fluid.

The second term of the right-hand side in Eq.(6) is usually called the Hagenbach correction [22]. The value of the Hagenbach coefficient which is the corrected factor for the contraction and enlargement effect between the large diameter of the sample feeder and the small diameter of the capillary tube and so on, may be obtained from the experiments with standard solutions of known viscosity. The value of density of the fluids in Eq. (6) may be measured by means of a psycrometer.

3. Calculation method of viscometric parameters

The viscometric parameters n, τ_y and K in Eq.(4) were calculated using a non-linear least square method [23] from the experimental data of the volumetric flow rate and the pressure difference. The digital electric computer program of the calculation of these viscometric parameters has been described in previous papers [1,5]. In this paper, the digital electric computer in The Computation Center of Nagoya Univ., FACOM M-200 was used.

The values of K can be represented by the Andrade or Arrhenius type equation as follows:

$$K = A \exp(B/T) = A \exp(E/R_g T) \tag{7}$$

where, $T(^\circ K) = t(^\circ C) + 273.2$ is the absolute temperature and $R_g = 1.987$ cal/g-mol$\cdot^\circ$K is the gas constant. The values of A and B or E can be obtained from the relationships of $\log(K)$ and $1/T$ by a linear least square method [23].

The value of K has been related with the temperature and the concentration, respectively, by using two log-log type equations by Buckingham [17]. These equations are compricated and can not be combined to one equation. In this study, we nsed Eq.(7).

RESULTS AND DISCUSSION

1. Calibrated results

The relationships between the volumetric flow rate Q (cm^3/sec) and the pressure difference ΔP_m (g_f/cm^2) for the prepared skim milk solutions were determined by a capillary tube viscometer.

The values of the Hagenbach coefficient m_p (–) were calculated from the experimental data with glycerin-water solutions of known viscosity [24, 25] and density [26, 27]. The density of the solutions was measured at the observing temperature by using a pycrometer. The flow behavior of the glycerin-water solutions is Newtonian. The relations of the Hagenbach coefficient m_p (–) and the Reynolds number Re (–) for the Cap.–A and –B which obtained for 17.8 ~ 59.6 wt% glycerin-water solutions at 30 ~ 60°C, are shown in Fig. 1. Nearly smooth curves are obtained in the laminar flow region for each capillary tube. The values of m_p are nearly con stant in the high Reynolds number region, but in the low region the values vary with the Reynolds number.

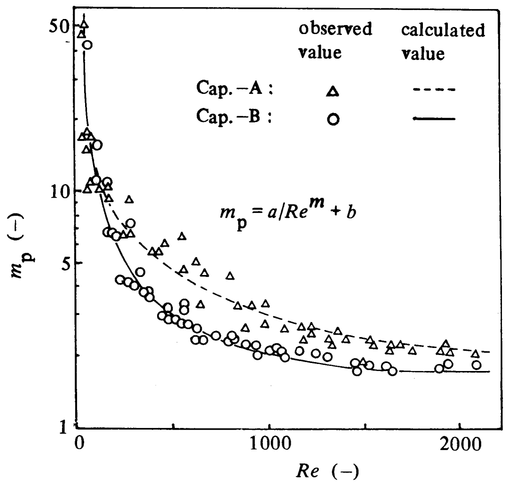

Fig. 1. Hagenbach coefficient m_p vs. Reynolds number Re.

These relationships can be represented by the following equation.

$$m_p = a/Re^m + b \tag{8}$$

$$Re = 2 r_w \bar{u} \rho / \mu \tag{9}$$

where, μ (g/cm·sec) is the viscosity of flowing fluid. The values of a, m and b in Eq.(8) can be obtained by the non-linear least square method [23] using a digital electric computer. The calculated results for the capillary tubes used in this study are shown in Table 1. The calculated values of m_p using the results of Table 1 are illustrated by the solid and broken lines in Fig. 1.

Table 1. Hagenbach coefficient's parameters

Cap.	a	m	b
–A	437	0.748	0.612
–B	4910	1.300	1.505

2. Flow behavior

The data of the volumetric flow rate Q (cm^3/sec) vs. the shear stress at the wall τ_w (g$_f$/cm^2) of the prepared skim milk solutions for the Cap.–A and –B are plotted in Figs. 2 and 3, respectively. The values of τ_w for the given pressure difference ΔP_m can be calculated by Eqs.(5) and (6) using the value of m_p from Eq.(8). In Figs. 2 and 3, it is clear that the value of consistency increases with increasing concentration, and decreases with increasing temperature.

High volumetric flow rate appears at the low concentration and the high temperature regions and appears in the large capillary radius. In these high flow region, the value of consistency increases appreciably, and flow behavior is very complicated. In the low flow

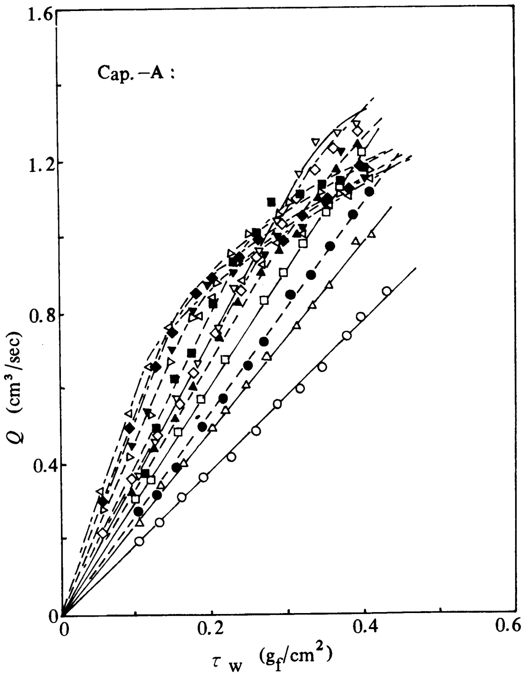

Fig. 2. Volumetric flow rate Q vs. shear stress τ_w.

S(wt%)	t = 30	40	50	60 °C
5.00	◇	▷	◆	◁
10.36	●	▲	■	▼
16.97	○	△	□	▽

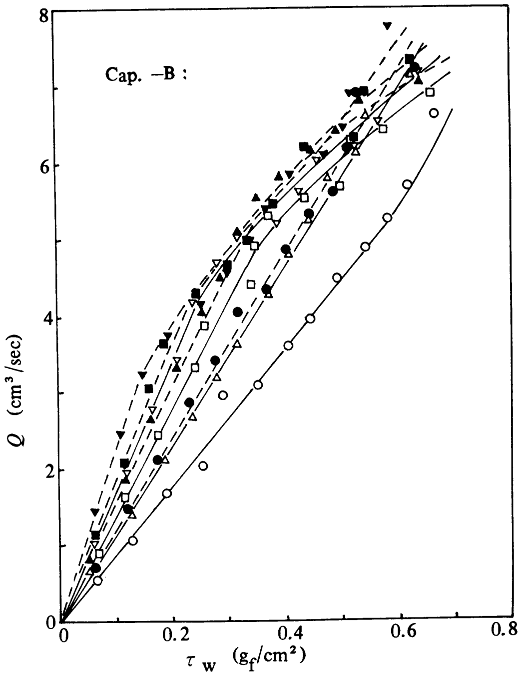

Fig. 3. Volumetric flow rate Q vs. shear stress τ_w.

S(wt%)	t = 30	40	50	60 °C
20.51	●	▲	■	▼
25.28	○	△	□	▽

region, the data follow a straight line as in Figs. 2 and 3. For these straight line regions, Eq. (1) can be expressed with n = 1.0 and τ_y = 0.0. The values of K which fixed n = 1.0 and τ_y = 0.0 in Eq.(1) for these region are listed in Table 2.

Table 2. Viscometric parameters of skim milk for low flow region

Cap	S(wt%)	K* (g/cm·sec)			
		t = 30	40	50	60 °C
–A	5.00	0.01061	0.008555	0.007417	0.006652
–A	10.36	0.01475	0.01133	0.009849	0.008251
–A	16.97	0.02057	0.01556	0.01266	0.01031
–B	20.51	0.02579	0.02042	0.01730	0.01403
–B	25.28	0.03570	0.02832	0.02292	0.01893
water**	0.0	0.008007	0.006560	0.005494	0.004688

where, * : the values fixed n = 1.0 and τ_y = 0.0
** : the values in Handbook

For the values of K in Table 2, the values of E in Eq.(7) are found to be between 3.18 × 10³ and 4.68 × 10³ cal/g-mol. The average value is obtained as being 3.96 × 10³ cal/g-mol. The values of A which fixed E = 3.96 × 10³ cal/g-mol in Eq.(7) are listed in

Table 3.

Table 3. Arrhenius parameters of skim milk for low flow region

Cap.	S(wt%)	A* (g/cm·sec)
–A	5.00	1.510×10^{-5}
–A	10.36	2.033×10^{-5}
–A	16.97	2.934×10^{-5}
–B	20.51	3.571×10^{-5}
–B	25.28	4.654×10^{-5}
water	0.0	1.144×10^{-5}

where, * : the values fixed $E = 3.96 \times 10^3$ cal/g-mol

The values of A in Table 3 increase with the increasing of the concentration. This relation is expressed as follows:

$$A = 10^{\alpha s+\beta} \tag{10}$$

where, S (wt%) is the concentration of solids in the flowing fluid.

For the data of tomato concentrates, this relation has been expressed with the following equation by Harper and Sahrigi[28].

$$A = \alpha' s^{\beta'} \tag{11}$$

The results in Table 2 better agree with Eq. (10) than with Eq. (11). Eq.(10) proposed by us is preferable to Eq.(11), because when the value of S is zero, Eq.(11) can not be calculated.

The values of α and β in Eq.(10) can be calculated by a linear least square method[23]. The fluid consistency index K (g/cm·sec) which fixed $n = 1.0$ and $\tau_y = 0.0$ in Eq.(1) for the straight line region of data, is shown as follows:

$$K = 10^{0.0241s-4.94} \exp(3.96 \times 10^3/R_g T) \tag{12}$$

where, $t = 30 \sim 60\ °\mathrm{C}$, $S = 0 \sim 25.3$ wt%

The relationships between K and t are shown in Fig. 4. The curves in Fig. 4 are the calculated results by means of Eq.(12).

For the high flow rate region of data, the flow equation is very complicated, because the value of consistency increases appreciably with the increasing of the flow rate. The Reynolds numbers of the winding points from the straight lines to the curves are between 1800 and 2500 for $S = 5.00 \sim 25.28$ wt%. We considered that these phenomena are phehaps due to the change of flow pattern from laminar flow to the turburent flow.

For all the data in Figs. 2 and 3, the values of n, τ_y and K in Eq.(1) were calculated. The values of n in Eq.(1) were found to be between 0.8 and 1.2, and the values of τ_y could be overlooked. The values of n decrease with the increasing of the temperature and increase with the increasing of the concentration. However, since the values of m_p

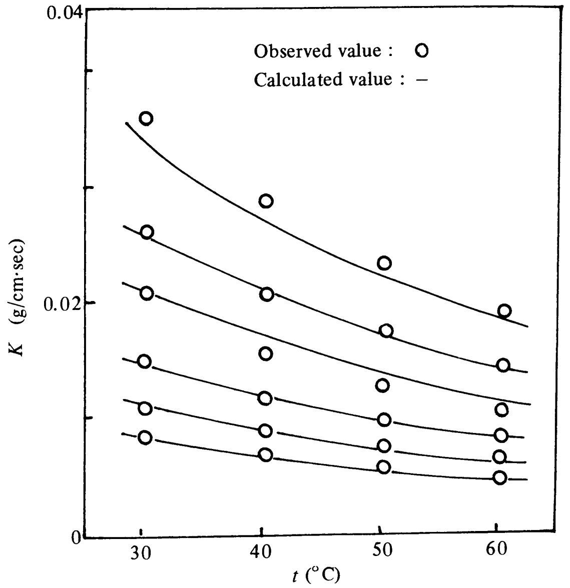

Fig. 4. Viscometric parameter K vs. temperature t.
From under line : S(wt%)
Water : 0.0
Cap.–A : 5.00, 10.36, 16, 97
Cap.–B : 20.51, 25.28

are used one in the laminar flow region, these parameters are not adequate. These results are not shown in this paper, because the flow behavior is too complicated.

This paper was presented on the occasion of the 45th Annual Meeting of the Society of Chemical Engineers, Japan (April 2, 1980 ; Osaka).

SUMMARY

The flow equations of liquid foods are important for the designing and controlling of various apparatuses. In this paper, we studied the flow behavior of skim milk solutions under low concentration at various temperatures. For measuring the flow behavior, we used a capillary tube viscometer operated under various pressures.

For the low flow region, a Newtonian flow equation was applied. The viscometric parameters are shown as follow:

$$\gamma = (1/K)(g_c \tau - g_c \tau_y)^n, \quad n = 1.0, \quad \tau_y = 0.0,$$

$$K = 10^{0.0241\,s-4.94} \exp(3.96 \times 10^3 / R_g T) \quad (\text{g/cm} \cdot \text{sec})$$

$$t = 30 \sim 60\ ^\circ\text{C}, \quad S = 0 \sim 25.3\ \text{wt\%}$$

where, γ(sec $^{-1}$): shear rate, τ (g_f/cm^2): shear stress, g_c = 980.7 g·cm/g_f·sec^2 : gravitational conversion factor and R_g = 1.982 cal/g-mol·°K: gas constant.

NOTATIONS

A, B and E	:	constants in Eq.(7)
a, m and b	:	constants in Eq.(8)
g_c	:	gravitational conversion factor ($g \cdot cm/g_f \cdot sec^2$)
K	:	fluid consistency index ($g^n/cm^n \cdot sec^{2n-1}$)
L	:	length of capillary tubes (cm)
m_p	:	Hagenbach coefficint in Eq.(6) (–)
n	:	flow behavior index (–)
ΔP	:	pressure difference (g_f/cm^2)
Q	:	volumetric flow rate of fluids (cm^3/sec)
Re	:	Reynolds number (–)
R_g	:	gas constant ($cal/g\text{-}mol \cdot {}^0K$)
r and r_w	:	radius and radius of capillary tubes (cm)
S	:	concentration of skim milk solutions (wt%)
T and t	:	temperature (${}^0K$) and (${}^0C$)
u	:	average velocity of fluids (cm/sec)
$\dot{\gamma}$	:	shear rate (sec^{-1})
μ	:	viscosity of fluids (g/cm·sec)
ρ	:	density of fluids (g/cm^3)
τ and τ_w	:	shear stress and shear stress at the wall of capillary tubes (g_f/cm^2)
τ_y	:	yield stress (g_f/cm^2)

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脱脂粉乳水溶液の低流速域における流動特性に関する研究

久保田清・松本俊也・鈴木寛一・保坂秀明

液状食品に関する流動方程式は，各種装置を設計，制御をしていく場合に必要となる。本研究では，低濃度の脱脂粉乳水溶液の流動特性に関する研究を温度を変えて行なった。流動特性の測定には，圧力を変化させて操作ができる毛管形粘度計を使用した。

低流速域において，ニュートン流動方程式が適用できた。得られた粘性パラメータを次に示す。

$\gamma = (1/K)(g_c\tau - g_c\tau_y)^n$,

$n = 1.0,\ \tau_y = 0.0$

$K = 10^{0.0241S-4.94}\exp(3.96\times10^3/R_gT)$ (g/cm・sec)

$t = 30\sim60$ °C, $S = 0\sim25.3$ wt%

ここで，γ (sec^{-1})：せん断速度，τ (g$_f$/cm^2)：せん断応力，$g_c = 980.7$ g・cm/g$_f$・sec^2：重力換算係数，$R_g = 1.987$ cal/g－mol・^{0}K：気体定数である。