

# The Gender Division of Labor: A Joint Marriage and Job Search

## Model\*

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### Abstract

This paper develops a model combining marriage and the job search, including marital bargaining and wage posting. It considers two types of jobs, full-time and part-time, and workers, male and female. After job-worker matching, male and female individuals find one another in the marriage market. This model has multiple equilibria in terms of gender divisions of labor, and the equilibrium market tightness is socially inefficient because of externalities arising from the expected gains from marriage.

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\*We especially thank Kentaro Hatsumi, Ryoichi Imai, Masaki Nakabayashi, Hideo Owan, Hikaru Ogawa, Dan Sasaki, Masaru Sasaki, Yasuhiro Sato, Takashi Shimizu, and participants in the 17th Labor Economics Conference, the Osaka Workshop on Economics of Institutions and Organizations, the Ryukyuu Workshop and Seminar at Nagoya University for their helpful comments and suggestions.

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# 1 Introduction

Gender gaps in labor market outcomes remain prevalent worldwide, including in many industrialized countries. On the supply side, one widely accepted theory posits that marital specialization contributes to the gender gap in terms of time use for market activities as opposed to domestic work.<sup>1</sup> Two individuals agree to form a family if both perceive higher utility in being married, and thereby specializing in market or domestic work according to their comparative advantages, than in remaining single (Becker, 1981).<sup>2</sup> The evidence does not, however, fully support the idea of a gender difference in time use arising as a consequence of marriage. Fig. 1 presents a comparison of the share of part-time workers in the young adult labor force across OECD countries. Clearly, the gender gap in terms of the portion working part-time varies across countries; moreover, in some countries the gender division arises already in the early stages of workers' careers. To more precisely consider time use by young workers, we must take into account their marital status. Table 1 shows the portion of part-time workers among employed adults aged 21-27 in the United States, classified by marital status, for the year 2006.<sup>3</sup> As shown, women are more likely than men to choose part-time jobs not only after marriage but even *before* getting married.

In this study, we explore the effects of labor market friction on the gender gap in young workers' labor market outcomes, specifically focusing on the choices made before marriage. Choices regarding labor market engagement are typically made early in life but have long-run effects on marital outcomes.<sup>4</sup> Many empirical studies have confirmed difficulties in changing working hours (Altonji and Paxson, 1986, 1988; Kahn and Lang, 1991; Dickens and Lundberg, 1993; Stewart and Swaffield, 1997; Euwals, 2001; Martinez-Granado, 2005; Blundell et al., 2008). Thus, once a single individual chooses a certain job, he/she cannot easily alter his/her working style after marriage; knowing this, one would choose a position suited to marriage even before this life

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<sup>1</sup>Another explanation is gender discrimination by firms (Becker, 1971; Phelps, 1972; Arrow, 1974). For recent studies of this, see Hellerstein, Neumark, and Troske (2002) and Kawaguchi (2003).

<sup>2</sup>Typically, the wife chooses to supply less time to the labor market because of women's generally lower opportunity cost of domestic production within a marriage.

<sup>3</sup>We focus on the data for white workers from the National Longitudinal Survey of Youth (NLSY) 97. Here, a person is a full-time worker if he/she works 35 or more hours per week in his or her primary job; a part-time job is one that offers less than 35 hours per week. For marital status, an individual is classified as married if he or she is married and living with his/her partner or cohabiting even though not legally married.

<sup>4</sup>Recent studies of the marriage market have explained such gender divisions among young workers based on pre-marital investment, especially that in human capital (Vagstad, 2001; Baker and Jacobson, 2007; Iyigun and Walsh, 2007). Since marriages generally lead each gender to specialize in different tasks, some proactive effort must be made to appear attractive in terms of ability to perform these tasks in the marriage market. @@A higher skill individual@@ has at performing gender-specific tasks, the more attractive he or she is in the marriage market. If singles optimize their strategies by anticipating future marital outcomes that are partially defined by domestic specialization, it is natural to invest in particular types of human capital, such as education or training in housework.

event occurred. Although growing attention is being paid by empirical studies to the connection between labor conditions and marriage, such labor market frictions are typically ignored in the theoretical models within the literature on marriage. Therefore, this paper simultaneously considers this irreversibility of job choices as well as marital outcomes in an integrated model.

For this purpose, we develop a directed job search model of individuals, each of whom seeks to marry one partner of the opposite sex.<sup>5</sup> The model has two main features. First, before marriage, each individual chooses his or her job type, selecting either a part-time or full-time job in the framework of the directed search model of Moen (1997), in which firms post wage offers and workers direct their search to the most attractive alternatives. The second feature is that individuals enjoy a marital surplus from domestic specialization. Here, we assume that the larger the difference in time use between partners, the larger the marital benefit for the couple. As such, job choices can be interpreted as a pre-marital investment device for negative assortative matching: unlike job searching in the traditional model, a young worker has an additional incentive to choose a different type of job from that of his/her potential partner in order to enjoy the larger marital surplus.<sup>6</sup> These features allow our model to compare how single workers behave in the light of marriage theory in a labor market with and without frictions.

Using the model, we first derive two marital patterns, characterized by the job choices of young workers. When the expected marital surplus (defined by the potential partner's job-finding rate and domestic specialization with him or her) is relatively large, the likely equilibrium is one in which individuals choose different jobs before marriage (i.e., the division equilibrium). In the opposite case, the non-division equilibrium arises: both spouses choose full-time work. The model also shows the existence of multiple equilibria of young workers' job choices when certain conditions are satisfied. Moreover, the comparative statics results predict that an increase in search intensity raises the full-time job-finding rate in both equilibria and encourages workers to choose the division equilibrium instead of the non-division equilibrium. Finally, our study also demonstrates that the externalities of job choices for the marriage partner result in an inefficient equilibrium in terms of labor market tightness. The inefficiency result arising in the directed search model cannot be derived in the

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<sup>5</sup>More recently, Bonilla and Kiraly (2013) have also succeeded in explaining marriage premium focusing on frictions in the marriage and labor markets rather than relying on the heterogeneous productivity.

<sup>6</sup>The two matching equilibria, called as either positive or negative assortative matching, are studied by Becker (1973), Lam (1988), and Peters and Siow (2002). This model's situation, in which workers seek a potential partner with a different characteristic, is known as "negative assortative mating."

traditional model without marriage.

The rest of this paper is organized as follows. Section 2 presents the directed search model for young workers. Section 3 characterizes the market equilibrium and carries out the comparative statics. In Section 4, we consider the efficiency of the market equilibrium. Section 5 extends the basic model and Section 6 concludes the study.

## 2 Model set-up

Consider a two-period search model in which decisions are made sequentially: individuals first search for jobs, then undertake a marriage search and distribute their resources within the resulting marriage. The population consists of two types of workers based on gender, type  $i \in \{a, b\}$  workers, with the number of each type normalized to one. We assume that workers differ across the two genders but workers within the same gender group are all identical.

Each worker gains utility from consuming two goods: market goods,  $c$ , and domestic goods,  $l$ , according to the utility function:

$$u = c + \pi(l),$$

where  $\pi' > 0$  and  $\pi'' < 0$ . Market goods are produced as a result of worker-firm matching in the labor market, while domestic goods are produced by home production. Workers purchase market goods outside the household, while domestic goods are only traded within the household.

Workers' time and budget constraints differ according to their employment status. Before marriage, young workers can apply for two types of jobs: full-time and part-time. However, due to search friction, there also exists equilibrium unemployment. Thus, there are three work statuses: working in a full-time job, working in a part-time job, and unemployment. Let  $I(i) \in \{F, P, U\}$  denote the working status as a full-time worker, a part-time worker, and an unemployed. The amount of output and hours worked depend on the job type: an employed worker in a full-time job can produce  $y_F$  by spending  $h_F$  hours, while an employed worker in a part-time job can produce only  $y_P (< y_F)$  and spend  $h_P (< h_F)$  in the market. Finally, an unemployed worker cannot produce any market goods but can spend their entire time on domestic production ( $h_U = 0$ ). It is assumed that all workers are endowed with one unit of time to allocate between market and domestic work;

the output of domestic goods is thus given by  $l_{I(i)} = 1 - h_{I(i)}$ . How workers decide on marriage outcomes is discussed in detail later.

## 2.1 The job search

In this study, we consider young workers' job search behaviors based on a directed search model in which the overall job search market is divided into sub-markets, with each sub-market characterized by wage and job type. The matching function is given by  $M = m(su, v) \in (0, 1)$ , where  $u$  and  $v$  are the number of job seekers and the number of vacancies available, respectively, and  $s$  is search intensity parameter.  $m(\cdot, \cdot)$  is the matching function defined on  $\mathbf{R}_+ \times \mathbf{R}_+$  and assumed to be strictly increasing in both arguments, twice differentiable, strictly concave, and homogeneous of degree one. We also assume that  $m(\cdot, \cdot)$  satisfies  $m(u, 0) = m(0, v) = 0$  and that the Inada condition holds for both arguments.

In each sub-market, worker-job matching occurs at the rate of  $sp = sp(\theta) = sM/su = sm(1, \theta)$  for a job searcher and  $q = q(\theta) = M/v = m(1/\theta, 1)$  for a firm seeking to fill a vacancy.  $\theta$  is the measure of labor market tightness in the sub-market, defined as  $\theta = v/su$ . From the assumptions regarding  $m(\cdot, \cdot)$ , we obtain that  $spu = qv$ ,  $dp/d\theta > 0$ , and  $dq/d\theta < 0$  for any  $\theta \in (0, +\infty)$ . We can also assume that  $\lim_{\theta \rightarrow 0} p = 0$ ,  $\lim_{\theta \rightarrow \infty} p = 1$ ,  $\lim_{\theta \rightarrow 0} q = 1$ , and  $\lim_{\theta \rightarrow \infty} q = 0$ . Additionally, we assume that the elasticity of the firm's contact rate with respect to the market tightness,  $\gamma \equiv -(\theta/q)dq/d\theta = 1 - (\theta p)dp/d\theta$ , is constant.<sup>7</sup>

## 2.2 Marriage matching and marital bargaining

Following Baker and Jacobsen (2007), we assume male and female workers are randomly matched to each other in the marriage market with exogenous probability  $\rho$ . After marriage, they negotiate over the intra-household distribution under a Nash bargaining framework. Therefore, the timing of events in the model is summarized as follows.

1. The labor market
2. The marriage market
3. marital bargaining

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<sup>7</sup>This assumption leads to a set of functions, including the Cobb-Douglas function, which is standard in the literature on theoretical and empirical search models (See Petrongolo and Pissarides [?]).

### 3 Equilibrium

In this section, the subgame perfect equilibrium is characterized using backward induction. To simplify, we focus on the symmetric equilibrium, in which workers within the same gender group choose the same job-search strategy. Moreover, we assume that workers can only make decisions regarding work types before marriage, which is relaxed in the Section 5.

#### 3.1 Marital bargaining

Following the classic arguments on intra-household bargaining, such as Manser and Brown (1980) and McElroy and Horney (1981), we consider that a couple can achieve an efficient joint allocation of resources through intra-household bargaining and that its solution is characterized by the level of utility at the point at which their negotiations break-down (i.e., outside options of bargaining) . Here, we assume that each individual's utility at the outside option is that which would be achieved if he/she remained single, and that the bargaining power of each spouse is  $1/2$ .<sup>8</sup> Thus, we first derive the utility levels when the two individuals remain single, then use these utilities as threat-points to obtain the marital bargaining outcome.

##### 3.1.1 Utility as singles

From the budget constraints ( $c = w$ ) and time constraints ( $l_{I(i)} = 1 - h_{I(i)}$ ), the autarkic utility of a type  $i$  worker working in  $I(i)$  is given by

$$\begin{aligned} u_{iI(i)}^S &= w_{iI(i)} + \pi(l_{I(i)}) \\ &= w_{iI(i)} + \pi(1 - h_{I(i)}), \end{aligned} \tag{1}$$

where  $w_{iI(i)}$  is wages and  $h_{I(i)}$  is working hours. Note that if a single worker is unemployed,  $w_{iU} = 0$  and  $h_U = 0$ , so that he/she gains utility only from domestic goods.

For married workers, the levels of consumption of market and domestic goods are determined by Nash bargaining with the outside option defined by Eq. (1).

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<sup>8</sup>To eliminate inefficiency arising in dynamic bargaining, we assume that a worker's outside option is simply his/her autarkic utility. Konrad and Lommerud (2000) and Baker and Jacobsen (2007) find the strategic incentive related to marital bargaining may be the source of such inefficiencies. It should be also mentioned that an alternative interpretation of the outside option in a marriage negotiation is a non-cooperative outcome. See Lundberg and Pollak (1993).

### 3.1.2 Allocation of domestic goods

Next, we characterize the outcome of marital bargaining. The allocation of market and domestic goods is determined by Nash bargaining by the couple.

The outcome of marital bargaining between workers  $a$  and  $b$  can be defined by  $(c_a, c_b, l_a, l_b)$ , which are determined by solving the following problem:

$$\begin{aligned} (c_a, c_b, l_a, l_b) &\in \arg \max \left( c_a + \pi(l_a) - u_{aI(a)}^S \right)^{1/2} \left( c_b + \pi(l_b) - u_{bI(b)}^S \right)^{1/2}, \\ \text{s.t. } c_a + c_b &= w_{aI(a)} + w_{bI(b)}, \\ l_a + l_b &= 2 - (h_{I(a)} + h_{I(b)}), \end{aligned}$$

where  $w_{iI(i)}$  represents the wages of worker  $i$ . The objective function consists of Nash products defined by  $c_a + \pi(l_a) - u_{aI(a)}^S$  and  $c_b + \pi(l_b) - u_{bI(b)}^S$ , which are the gains from the negotiation for type  $a$  and  $b$  workers, respectively. The first constraint is the household budget constraint and states that the household's total market good consumption is bought using their total labor income. The final equation is the time constraint, stating that the couple's total production of domestic goods is the sum of their time spent on domestic production. The outcome of marital bargaining is then chosen to maximize the Nash product of the men and women's marriage surpluses under these budget and time constraints.

From the first-order conditions of the problem, we can characterize the outcome of marital bargaining by the following lemma.

**Lemma 1** *The outcome of intra-household bargaining can be summarized by*

$$l_a = l_b = \frac{h_{I(a)} + h_{I(b)}}{2}, \quad (2)$$

$$\frac{1}{2}S(I(a), I(b)) = c_a + b(l_a) - u_{aI(a)}^S = c_b + b(l_b) - u_{bI(b)}^S, \quad (3)$$

where

$$S(I(a), I(b)) = 2\pi\left(\frac{h_{I(a)} + h_{I(b)}}{2}\right) - \pi(h_{I(a)}) - \pi(h_{I(b)}). \quad (4)$$

**Proof.** See Appendix. ■

Eq. (2) is the condition applying to domestic production within the marriage and shows that domestic goods should be equally distributed as  $l_a = l_b$  from the concavity of  $b(\bullet)$ . Intuitively, due to the implicit cash transfer between husband and wife under the Nash bargaining framework, the couple can achieve an efficient allocation of domestic goods and both can enjoy these goods, irrespective of their job types. The allocation of market goods is determined by satisfying (3). This condition implies that what each spouse gains from the Nash bargaining solution of their marriage,  $c_a + \pi(l_a) - u_{aI(a)}^S$  and  $c_b + \pi(l_b) - u_{bI(b)}^S$ , is equalized to half of the joint marital surplus of the household  $S(I(a), I(b))$ . The model is then tractable by the assumption of quasi-linear utility because the marriage surplus depends only on the job types of the wife and husband.

Eq. (4) can be interpreted as the joint marriage surplus. From the concavity of  $b(\bullet)$ ,  $S(I(a), I(b)) > 0$  if  $I(a) \neq I(b)$ , and  $S(I(a), I(b)) = 0$  if  $I(a) = I(b)$ . Moreover, the marriage surplus has the properties summarized in the following lemma.

**Lemma 2**  $S(F, U) > S(P, U)$ ,  $S(F, U) > S(F, P)$ .

**Proof.** See Appendix.

Lemma 2 states that the more domestic specialization there is, the larger the marital surplus. The first inequality notes that the marital surplus of the full-time worker and his/her unemployed partner is larger than that of the part-time worker and his/her unemployed spouse. Similarly, the second inequality states that the marital surplus of the full-time worker and his/her unemployed partner is larger than that of the full-time worker and his/her part-time-working spouse. ■

Finally, because each gender type of worker is normalized to one, the probabilities that a given worker meets unemployed, full-time, and part-time workers in the marriage market are equal to the respective numbers of these types of workers. Using the definition of the marriage surplus (4) allows us to obtain the expected utility before entering the marriage market as

$$w_{iI(i)} + \pi(1 - h_{I(i)}) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} S(I(i), I(j)).$$

where  $n_{jF}$ ,  $n_{jP}$ , and  $n_{jU}$  are the number of full-time, part-time, and unemployed workers of type  $j (\neq i)$ , respectively.

The first and second terms are the utilities if a worker is single, and the third term is the expected gain from the marriage search.



### 3.2 The job search

Workers choose a job search sub-market to maximize their own expected utility. We focus on the pure strategy behind applying for a job type. Let variable  $\alpha_i \in [0, 1]$  be the probability that a worker applies for full-time jobs. Following Moen (1997), we assume that the labor market is divided into sub-markets, each of which is characterized by wage  $w$  and the job's type. The labor market equilibrium satisfies three requirements:

1. Workers choose a labor sub-market to maximize their expected utility.
2. Firms choose a labor sub-market for posting jobs to maximize their expected profits.
3. The number of jobs is determined by the free-entry condition.

The expected utility if a worker applies to jobs of type  $I(i)$  can be specified in the following way:

$$EU_{iI(i)} = sp(\theta_{iI(i)}) \left[ w_{iI(i)} + \pi(1 - h_{I(i)}) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} S(I(i), I(j)) \right] \\ + (1 - sp(\theta_{iI(i)})) \left[ \pi(1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{i'I(j)} S(U, I(j), ) \right].$$

The right-hand side of the equation is divided into two parts: the expected utilities of being employed and unemployed in accordance with the job-finding rate,  $sp_i$ . In each part, there are two possibilities for marital status, single or married, in accordance with the marital matching probability,  $\rho$ . Moreover, the expected utility when an individual is married is the sum of the weighted functions, depending on the after-marriage time allocation within the household and the partner's employment state, characterized by  $sp_{jI(j)}$ .

The free-entry condition of vacant jobs can be defined as

$$k = q(\theta_{iI(i)}) (y_{I(i)} - w_{iI(i)}). \quad (5)$$

Following Acemoglu and Shimer (1999), we can summarize the labor market equilibrium conditions. Using  $EU_{iI(i)}$ , the labor market equilibrium is characterized as a solution of the following optimization problem:

$$(\alpha_i, w_i, \theta_{iI(i)}) \in \arg \max \alpha_i EU_{iF}(\theta_{iF}) + (1 - \alpha_i) EU_{iP}(\theta_{iP}) \quad \text{s.t. equations (4), (5)}.$$

From the free-entry condition, firms' profits are always zero. Hence, conditions 2 and 3 of the labor market equilibrium, above, are summarized as (5). This depicts a situation in which, given the free-entry condition, workers search for jobs in the labor market in order to maximize their own utility.<sup>9</sup>

By substituting (5) into the expected utility, above, the optimization problem can be rewritten as

$$(\alpha_i, \theta_{iI(i)}) \in \arg \max \alpha_i EU_{iF}^*(\theta_{iF}) + (1 - \alpha_i) EU_{iP}^*(\theta_{iF}) \quad \text{s.t. equations (4), (5)}$$

where

$$\begin{aligned} EU_{iI(i)} = & sp(\theta_{iI(i)}) \left[ y_{I(i)} + \pi(1 - h_{I(i)}) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} S(I(i), I(j)) \right] \\ & + (1 - sp(\theta_{iI(i)})) \left[ \pi(1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{i'I(j)} S(U, I(j), ) \right] - k\theta_{iI(i)}. \end{aligned}$$

We can then characterize the labor market equilibrium using the standard optimization technique.

The first-order condition of  $\theta_{iI(i)}$  yields

$$0 = \frac{\partial p(\theta_{iI(i)})}{\partial \theta_{iI(i)}} \left[ y_{I(i)} + \pi(1 - h_{I(i)}) - \pi(1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} [S(I(i), I(j)) - S(U, I(j))] \right] - k.$$

Using  $p'/q = 1 - \gamma$  and  $\theta q'/q = \gamma$ , the equilibrium condition for  $\theta_{iI(i)}$  is

$$k = (1 - \gamma) q(\theta_{iI(i)}) \left( y_{I(i)} + \pi(1 - h_{I(i)}) - \pi(1 - h_U) + \rho \sum_{j'=\{0, P, F\}} n_{jI(j)} [S(I(i), I(j)) - S(U, I(j))] \right). \quad (6)$$

Equation (6) shows the properties of market tightness at equilibrium. Equilibrium market tightness is an increasing function of productivity  $y_{I(i)}$  and the difference in utility from domestic goods when single,  $\pi(1 - h_{I(i)}) - \pi(1 - h_U)$ . Moreover, the worker's choice of sub-market depends on the gap in expected marriage surpluses between the employed and the unemployed,  $\rho \sum_{j'=\{0, P, F\}} n_{jI(j)} [S(I(i), I(j)) - S(U, I(j))]$ . When workers choose sub-market  $\theta$ , they compare the difference in the level of utility from being employed and unemployed in each sub-market. Thus, if the gap in expected marital surplus is high, the worker applies for a

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<sup>9</sup>If a sub-market subject to the free-entry condition failed to maximize workers' utility, no worker would enter this sub-market at equilibrium.

job in the sub-market with high  $\theta$ .

Next, the optimal solution for  $\alpha_i$  is

$$\begin{aligned}\alpha_i &= 1 \iff EU_{iF}^* > EU_{iP}^*, \\ \alpha_i &= 0 \iff EU_{iF}^* < EU_{iP}^*, \\ \alpha_i &\in [0, 1] \iff EU_{iF}^* = EU_{iP}^*.\end{aligned}$$

The difference between  $EU_{iF}^* - EU_{iP}^*$  is

$$\begin{aligned}EU_{iF}^* - EU_{iP}^* &= sp(\theta_{iF}) \left[ y_F + \pi(1 - h_F) - \pi(1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} [S(F, I(j)) - S(U, I(j))] \right] - sk\theta_{iF} \\ &\quad - sp(\theta_{iP}) \left[ y_F + \pi(1 - h_P) - \pi(1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} [S(P, I(j)) - S(U, I(j))] \right] + sk\theta_{iP}\end{aligned}$$

Substituting (6) into the equation above, we can express the difference in expected utility as

$$EU_{iF}^* - EU_{iP}^* = \frac{\gamma}{1 - \gamma} sk(\theta_{iF} - \theta_{iP}),$$

indicating that  $EU_{iF}^* > EU_{iP}^* \iff \theta_{iF} > \theta_{iP}$ . Making use of (6), the optimal solution for  $\alpha_i$  can also be rewritten as

$$\begin{aligned}\alpha_i &= 1 \iff y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) > \rho \sum_{I(j) \in \{0, P, F\}} n_{jI(j)} [S(P, I(j)) - S(F, I(j))], \quad (7) \\ \alpha_i &= 0 \iff y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) < \rho \sum_{I(j) \in \{0, P, F\}} n_{jI(j)} [S(P, I(j)) - S(F, I(j))], \\ \alpha_i &\in [0, 1] \iff y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) = \rho \sum_{I(j) \in \{0, P, F\}} n_{jI(j)} [S(P, I(j)) - S(F, I(j))].\end{aligned}$$

From the assumption that  $y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) > 0$ , the left-hand sides of the above conditions are always positive.

Finally, the equilibrium numbers of workers are determined as

$$\begin{aligned} n_{iF} &= \alpha_i p_{iF}, \\ n_{iP} &= (1 - \alpha_i) p_{iP}, \\ n_{i0} &= 1 - \alpha_i p_{iF} - (1 - \alpha_i) p_{iP}. \end{aligned} \tag{8}$$

The market equilibrium, defined by  $(\theta_{iI(i)}, \alpha_i, n_{iI(i)})$ , then satisfies conditions (6) - (8).

### 3.3 Subgame perfect equilibrium

Now, let us characterize the subgame perfect equilibrium. To focus on the plausible cases, we make the following assumption:

$$y_F - y_P > \pi(1 - h_P) - \pi(1 - h_F), \tag{9}$$

which implies that unmarried workers always prefer a full-time job to a part-time job.

Under assumption (9), there are then two types of pure strategy equilibria: division and non-division. In the division equilibrium, one type of worker within a couple applies for full-time jobs while the other type applies for part-time jobs. The different jobs undertaken by the couple allow them to enjoy a high marital surplus arising from domestic division of labor after the marriage. Meanwhile, in the non-division equilibrium, both types of workers apply for full-time jobs: they choose the same job type before marriage even though they cannot later obtain the high marital surplus from domestic specialization. Note that under assumption (9), the equilibrium in which both types of workers apply for part-time jobs does not hold. In the following subsections, we will characterize the conditions under which each equilibrium arises.

#### Non-division equilibrium

The non-division equilibrium is defined as  $\alpha_a = \alpha_b = 1$ . Let  $\theta^N \equiv \theta_{aF} = \theta_{bF}$ ,  $q^N = q(\theta^N)$  and  $p^N = p(\theta^N)$ . The share of working types are then  $n_{aF} = n_{bF} = sp^N$ ,  $n_{aP} = n_{bP} = 0$ , and  $n_{a0} = n_{b0} = 1 - sp^N$ . Finally,

from (6) and (4),  $\theta^N$  is characterized by

$$k = (1 - \gamma) q^N \left( y_F + \pi(1 - h_F) - \pi(1 - h_U) + \frac{\rho}{2} (1 - 2sp^N) S(F, U) \right). \quad (10)$$

where

$$S(F, U) = 2\pi \left( 1 - \frac{h_F + h_U}{2} \right) - \pi(1 - h_F) - \pi(1 - h_U),$$

**Lemma 3** *The equilibrium market tightness under the non-division equilibrium must be uniquely determined.*

**Proof.** (10) can be rewritten as

$$\frac{k}{q^N} = (1 - \gamma) \left( y_F + \pi(1 - h_F) - \pi(1 - h_U) + \frac{\rho}{2} (1 - 2sp^N) S(F, U) \right).$$

Because the left-hand side of the above equation is an increasing function of  $\theta^N$  while the right-hand side is a decreasing function of  $\theta^N$ , the equilibrium market tightness must be uniquely determined. ■

The non-division equilibrium exists if the condition to deviate from full-time jobs to part-time jobs does not hold. From the optimal condition for job-type choice (7), this condition is

$$y_F \geq y_F^N \equiv y_P + \pi(1 - h_P) - \pi(1 - h_F) - \rho \frac{(1 - sp^N) [S(F, U) - S(P, U)] - sp^N S(F, P)}{2}. \quad (11)$$

The condition implies that the larger the gap in market outcomes between full-time and part-time jobs,  $y_F - y_P$ , the smaller the difference in the production of domestic goods by a single part-time worker versus a single full-time worker,  $\pi(1 - h_P) - \pi(1 - h_F)$ , and the larger the gap in expected marital surplus between full-time and part-time workers,  $\frac{\rho}{2} [(1 - sp^N) S(F, U) - sp^N S(F, P) - (1 - sp^N) S(P, U)]$ , the more likely the non-division equilibrium is to emerge. Note that the expected marital surplus, the final property of the condition, depends on the job-finding rate of a given worker's potential partners.

### 3.3.1 Division equilibrium

The division equilibrium is defined as  $\alpha_a = 0$  and  $\alpha_b = 1$  (or  $\alpha_a = 1$  and  $\alpha_b = 0$ ). For the remaining sections of this paper, without loss of generality in the division equilibrium, type  $a$  workers always apply for full-time

jobs while type  $b$  workers may apply for part-time jobs. From (4), (6), and the equilibrium market tightnesses, denoted by  $\theta_F^D$  and  $\theta_P^D$ , are given by

$$k = (1 - \gamma) q_P^D \left( y_P + \pi(1 - h_P) - \pi(1 - h_U) + \rho \frac{(1 - sp_F^D) S(P, U) + sp_F^D S(F, P) - sp_F^D S(F, U)}{2} \right), \quad (12)$$

$$k = (1 - \gamma) q_F^D \left( y_F + \pi(1 - h_F) - \pi(1 - h_U) + \rho \frac{(1 - sp_P^D) S(F, U) + sp_P^D S(F, P) - sp_P^D S(P, U)}{2} \right), \quad (13)$$

where  $q_{I \in \{F, P\}}^D = q(\theta_{I \in \{F, P\}}^D)$ ,  $p_I^D = q(\theta_{I \in \{F, P\}}^D)$ , and

$$\begin{aligned} S(F, P) &= 2\pi \left( 1 - \frac{h_F + h_P}{2} \right) - \pi(1 - h_F) - \pi(1 - h_P), \\ S(F, U) &= 2\pi \left( 1 - \frac{h_F + h_0}{2} \right) - \pi(1 - h_F) - \pi(1 - h_U), \\ S(P, U) &= 2\pi \left( 1 - \frac{h_P + h_0}{2} \right) - \pi(1 - h_P) - \pi(1 - h_U). \end{aligned}$$

**Lemma 4** *The equilibrium market tightnesses are uniquely determined iff*

$$\frac{2\gamma k}{s(1 - \gamma)^2 (\theta_F \theta_P)^{1/2} q_P q_F [S(F, U) - S(F, P) + S(P, U)]} > \rho. \quad (14)$$

**Proof.** See Appendix. ■

Equation (14) means that the uniqueness of the market equilibrium is ensured if the friction in the marriage market is sufficiently high (i.e.,  $\rho$  is low). Note that to ensure the existence of the equilibrium, additional assumptions are required (see Appendix).

From the requirements for uniqueness, we can prove the following lemma.

**Lemma 5**  $p_F^D > p_P^D$

**Proof.** See Appendix. ■

There are two conditions for the pre-marriage division equilibrium. The first condition is that no full-time workers deviate to apply for part-time jobs. From (7), the first condition can be rewritten as

$$y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) + \rho \frac{(1 - sp_P^D) [S(F, U) - S(P, U)] + sp_P^D S(F, P)}{2} \geq 0. \quad (15)$$

This condition always holds because from Lemma 2  $y_F + b(1 - h_F) - y_P - h(1 - h_P)$  and  $S(F, U) - S(P, U)$  are positive.

The second condition is that no part-time workers deviate to apply for full-time jobs. This condition is given by

$$y_F \leq y_F^D \equiv y_P + \pi(1 - h_P) - \pi(1 - h_F) - \rho \frac{sp_F^D [S(F, F) - S(F, P)] + (1 - sp_F^D) [S(F, U) - S(P, U)]}{2}. \quad (16)$$

From this condition, we can see that if the gap in market outcomes between full-time and part-time jobs,  $y_F - y_P$ , is relatively small, the difference in production of domestic goods between the single part-time worker and the single full-time worker,  $\pi(1 - h_P) - \pi(1 - h_F)$ , is relatively large, and the expected marital surplus of a part-time worker is sufficiently larger than that of a full-time worker,

$$\frac{\rho}{2} [sp^N S(F, P) + (1 - sp^N) S(P, U) - (1 - sp^N) S(F, U)],$$

then workers are likely to choose the division equilibrium. Again, the difference in expected marital surpluses between the two job types is affected by the job-finding rate of a worker's potential partners.

Finally, from thresholds (11) and (16), we can easily show the following useful corollary.

**Corollary 1** *If the full-time job-finding rate is high, the domain for the division equilibrium is large while that of the non-division equilibrium is small.*

To understand the intuition behind this, suppose that type  $a$  workers apply for full-time jobs. If the job-finding rate of type  $a$  workers is high, type  $b$  workers are more likely to marry full-time workers. Consequently, type  $b$  workers apply for part-time jobs to obtain a larger marital surplus.

### 3.4 Multi-equilibria domain

To show the existence of the domain in which there are multiple equilibria, we first present the following lemma.

**Lemma 6**  $p_F^D > p^N$

**Proof.** See Appendix. ■

The intuition behind Lemma 6 is as follows. As seen in the worker's expected utility, his/her final outcome is defined by his/her own employment and marital status as well as his/her partner's employment status. Here, in the case that the worker cannot marry or marries an unemployed partner, there is no difference in utility between the two equilibria. However, if the partner is employed, the utility gap between being employed and unemployed is larger in the division equilibrium, in which the partner has chosen a part-time job, than in the non-division equilibrium, in which the partner has chosen a full-time job.

To interpret our results intuitively, let us first consider the non-division equilibrium. If the worker is employed in a full-time job, she/he cannot gain the marital surplus if her/his partner works in a full-time job. If she/he is unemployed, however, she/he can enjoy the maximum marital surplus arising from domestic specialization through the combination of unemployment (him- or herself) and full-time work (the wife or husband). Next, we consider the division equilibrium with the partner working a part-time job. Suppose that the individual in question applies for a full-time job and is employed. In this case, she/he can obtain the marital surplus because she/he has chosen a different use of time than his/her partner. On the other hand, when she/he is unemployed, she/he can still obtain the marital surplus through the combination of unemployment (himself) and part-time work (his or her partner). This surplus, however, is less than that arising in the non-division equilibrium in which she/he is unemployed and her/his partner is a full-time worker. In sum, as being employed is more attractive under the division equilibrium than under the non-division equilibrium, the individual has a greater incentive to find a job under the former equilibrium. As a result, the  $p$  characterizing the sub-market a worker chooses is larger under the division equilibrium.

From equations (11) and (16), we can easily show that  $y_F^N$ ,  $y_F^B$ , and  $y_F^D$  are increasing functions of  $p_F$ . Combining with Lemma 6, we can show the following proposition.

**Proposition 1** *There is a domain with multiple equilibria: if  $y_F \in [y_F^N, y_F^D]$ , the market equilibrium may be either the pre-marriage division equilibrium or non-division equilibrium.*

The condition in Proposition 1,  $y_F^N < y_F^D$ , can be interpreted as follows.  $y_F^D$  and  $y_F^N$  represent the thresholds at which a worker chooses a full-time or part-time job, given that his/her partner has chosen a full-time job. The larger this value, the more attractive the choice of a part-time job becomes. To understand this mechanism, it is useful to consider the position of a given worker.



Suppose that a type  $a$  worker is applying for a full-time job. In this case, with a low unemployment rate for type  $a$  workers, type  $b$ 's choice of the full-time job entails a smaller eventual marital surplus. Moreover, from Corollary 1, it becomes more likely that type  $b$  workers choose the part-time job (i.e., the smaller  $\alpha$ : the division equilibrium). Moreover, the larger difference in utility levels between being employed and unemployed as a type  $a$  full-time worker under the division equilibrium (see Lemma 6), the more likely that type  $a$  workers choose the sub-market with the higher  $p$  and, hence, the lower unemployment rate. On the other hand, if the unemployment rate for type  $a$  workers is relatively high, a type  $b$  worker has the incentive to choose a full-time job, aiming at the larger marital surplus (i.e., the non-division equilibrium holds). Meanwhile, from Lemma 6, type  $a$  workers apply to the sub-market with a lower  $p$  and a higher unemployment rate.

The result that the future secondary workers choose to supply higher level of their labor with higher likelihood their potential partner is unemployed, may be similar to the idea of added worker effect presented by Heckman and MacCurdy (1980). Our model differs, however, in the incentives led by the possibility of the partner's unemployment. While in the idea of added worker effect, the choice of higher labor supply in response to the higher likelihood of primary worker's unemployment plays a role of insurance for the family, our model indicates that the unemployment itself gives higher marital surplus resulting in higher incentive to choose full-time job. In the standoff over the existence of added worker effect (Lundberg, 1985; Cullen and Gruber, 2000), our result may give a new explanation for the observed phenomenon shown in Table 1 and Fig. 1.

### 3.5 Comparative statics

Through total differentiation of (10), (12), and (13) with respect to  $s$ , we can show the following comparative statics results.

**Proposition 2** *When the search intensity is high, (i) the rate of finding full-time jobs is high in both equilibria and (ii) the domain of the non-division equilibrium is small while that of the division equilibrium is large.*

**Proof.** See Appendix. ■

The intuition behind Proposition 2 is as follows. For Part 1 of Proposition 2, a rise in  $s$  has two effects: one is a direct positive effect on the job-finding rate of a type  $a$  worker. The other is an indirect effect arising from the job-finding rate of his potential partner, a type  $b$  worker. An increase in  $s$  also induces a rise in the

partner's job-finding rate, increasing the probability that the partner is employed. Consequently, the worker has a smaller incentive to become employed because there is a smaller marital surplus gap between being employed and unemployed. However, since the former dominates the latter (see Appendix), an increase in  $s$  must lead to a higher job-finding rate for the worker him/herself.

For Part 2 of Proposition 2, an increase in  $s$  accompanied by a rise in the partner's job-finding rate reduces the attractiveness of full-time jobs compared to part-time jobs for type  $b$  workers (i.e., an increase in the threshold classifying the equilibrium). Consequently, it is more likely that the worker chooses the part-time job; hence, the domain of the division equilibrium grows and that of the non-division equilibrium shrinks.

More intuitively, these results arise from the fact that the marital surplus would be reduced were a type  $b$  worker to choose a full-time job when her partner's likelihood of being unemployed (job-finding rate) is relatively low (high). Therefore, the full-time job becomes less attractive to type  $b$  workers under both equilibria.

## 4 The social planner's problem

In this section, we compare outcomes under the market equilibrium with the socially optimal allocation in order to consider the efficiency. For this purpose, we first characterize social welfare as the sum of expected utilities for the two gender groups in each job type weighted by the number of workers holding each type of job:

$$\Omega = \alpha_a EU_{aF} + (1 - \alpha_a) EU_{aP} + \alpha_b EU_{bF} + (1 - \alpha_b) EU_{bP}, \quad (17)$$

where

$$\begin{aligned} EU_{iI(i)} &= s \left( p_{iI(i)} \left( y_{I(i)} + \pi (1 - h_{I(i)}) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} S(I(i), I(j)) \right) - k\theta_{iI(i)} \right) \\ &\quad + (1 - sp_{iI(i)}) \left( \pi (1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} S(U, I(j)) \right), \\ S(I(i), I(j)) &\equiv 2\pi \left( \frac{h_{I(i)} + h_{I(j)}}{2} \right) - \pi (1 - h_{I(i)}) - \pi (1 - h_{I(j)}), \end{aligned}$$

and

$$n_{i0} = 1 - \alpha_i sp_{iF} - (1 - \alpha_i) sp_{iP},$$

$$n_{iP} = (1 - \alpha_i) sp_{iP},$$

$$n_{iF} = \alpha_i sp_{iF}.$$

Note that since the expected profits arising from jobs are zero due to the free-entry condition, social welfare is equivalent to workers' aggregate expected utility. Now that total social welfare is defined, we check the derivative of the social welfare function with respect to the labor market tightness around the market equilibrium to characterize the efficiency properties of equilibrium market tightness.

Marginal effects of market tightness around the market equilibria are:

$$\begin{aligned} \frac{\partial \Omega}{\partial \theta_F} \Big|_{\alpha_a = \alpha_b = 1, \theta_{aF} = \theta_{bF} = \theta^N} &= \frac{\rho}{2} (1 - 2sp^N) S(F, U) \frac{\partial p_F}{\partial \theta_F}, \\ \frac{\partial \Omega}{\partial \theta_F} \Big|_{\alpha_a = 1, \alpha_b = 0, \theta_{aF} = \theta_F^D, \theta_{bP} = \theta_P^D} &= \frac{\rho}{2} [sp_P^D (S(F, P) - S(P, U)) + (1 - sp_P^D) S(F, U)] \frac{\partial p_F}{\partial \theta_F}, \\ \frac{\partial \Omega}{\partial \theta_P} \Big|_{\alpha_a = 1, \alpha_b = 0, \theta_{aF} = \theta_F^D, \theta_{bP} = \theta_P^D} &= \frac{\rho}{2} [sp_F^D [S(F, P) - S(F, U)] + (1 - sp_F^D) S(P, U)] \frac{\partial p_P}{\partial \theta_P}. \end{aligned}$$

In these calculations, if  $\frac{\partial \Omega}{\partial \theta} = 0$ , then the market tightness at the market equilibrium is socially optimal. At all equilibria, however, the equilibrium market tightness is not socially optimal. This result can be summarized in the following proposition.

**Proposition 3** *Market equilibria are generally socially inefficient.*

**Proof.** See Appendix. ■

Intuitively, the equations above represent the effects on an individual worker through a change in marital surplus as a result of his/her partner's actions in choosing a sub-market; such changes are not recognized by the partner. The terms in square brackets refer to the change in the potential partner's expected joint marital surplus arising from the worker's job choice. In both the second and third equations, the first term captures the difference in the expected joint marital surplus between being employed and unemployed when one's partner is employed. In the second equation, this term represents the difference in the marital surplus between the case when a worker finds a full-time job and that when he/she fails to do so, given that the partner is employed as

a full-time worker. Since the sign of the total effect depends on  $S(F, P) - S(P, U)$ , it is not determined. In the third equation, the total effect of the term in the first set of brackets is negative because  $S(F, P)$  is always dominated by  $S(F, U)$  from Lemma 2.

The first equation and the second term in square brackets in the second and third equations refer to the case in which the partner is unemployed. Note that there is no marital surplus when both members of the couple allocate their time in the same way. Also, the second term is always positive because a given worker's employment always has positive effects on the marital surplus as long as the partner is unemployed. Thus, the total effect is the sum of the first and second terms.

When an individual chooses his/her job type, he/she cares about the change in his/her own marital surplus but not that of his/her potential partner. However, as discussed above, the model indicates that his/her sub-market choice also influences his/her partner's expected utility through the change in the marital surplus, since the sum of the first and second terms is not zero. Accordingly, these externalities lead to an inefficient market equilibrium. In conventional studies using the directed job search model, it is known that the market equilibrium always achieves a socially efficient outcome (Moan, 1997; Acemoglu and Shimer, 1999). Our model, however, shows that the market equilibrium may not be efficient because of the externalities for potential marriage partners arising through changes in the marital surplus. Indeed, our result can be reduced to the existing result of social optimality (i.e., the signs on the equations being equal to zero) when workers never marry ( $\rho = 0$ ) or when the first and second terms in brackets of the second and the third equations offset each other.

## 5 Extension

In the sections above, we considered a young worker's job search problem under the strong assumption that workers can only make decisions regarding work types before marriage. However, as Becker's classic household production theory indicates, it is natural that multiple family members reallocate their time after marriage, a result that has been supported by many empirical studies and the data presented in Table 1. Moreover, Pissarides (1994), among others, have argued that such possibilities in job search models include quitting work into unemployment and job-to-job quitting. In this section, we demonstrate that our main results are robust under the setting reflecting this realistic situation. Specifically, we extend and modify the main model of the previous sections as follows:

- We allow the case in which either the male or female partner quits his/her job after marrying.
- Firms can offer the contract  $\{W_{iI(i)}, w_{iI(i)}\}$ .  $W_{iI(i)}$  is a recruitment bonus, which is paid until the worker marries;  $w_{iI(i)}$  is the wage paid if the worker remains in the same job after his/her marriage<sup>10</sup>. Note that  $w_{iI(i)}$  must be lower than  $y_{I(i)}$  due to the IR condition for jobs<sup>11</sup>.
- The timing of the model is
  1. Labor markets are opened.
  2.  $W_{iI(i)}$  is paid to the worker.
  3. Marriage markets are opened.
  4. The worker decides whether to quit his/her job.
  5.  $W_{iI(i)}$  is paid to the worker if he/she continues to work.
  6. Production and consumption occur.

According to the modifications above, the problems facing young workers and firms are also modified: workers choose their job types taking into account the possibility of quitting their jobs, and firms internalize the cost of workers potential quitting in their wage profile. In the following analysis, the young worker's problem is solved through backward induction, as done for the basic model set-up.

## 5.1 Intra-household bargaining

We first characterize the outcomes of the intra-household allocation taking place in the second stage. Since the conditions for job quitting and the division equilibrium cannot simultaneously hold, we focus only on the case in which job quitting may occur within the marriages of two full-time workers. If both partners are full-time workers, intra-household bargaining can be characterized by the following problem:

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<sup>10</sup>To ensure the efficiency of job separation, the existence of a recruitment bonus are typically assumed in the random search (see, Kawata 2015, Kawata and Sato 2012) and the directed search (see, Menzio and Shi 2010, 2011) models.

<sup>11</sup>If  $y_{I(i)} < w_{iI(i)}$ , a employers have an incentive to fire a worker because the cost of hiring worker dominates its benefit.

$$\max_{Q \in \{1,0\}, \{c,l\} \in \mathbb{R}^2} (c_a + \pi(l_a) - u_F^S)^{1/2} (c_b + \pi(l_b) - u_F^S)^{1/2},$$

$$\text{s.t. } c_a + c_b = (1 - Q_{aF})w_{aF} + (1 - Q_{bF})w_{bF}, \quad (18)$$

$$l_a + l_b = 2 - Q_a h_0 - (1 - Q_a)h_F - Q_b h_0 - (1 - Q_b)h_F, \quad (19)$$

where  $Q_i$  is an indicator function:  $Q_i = 1$  if a member of the couple quits his/her job, while  $Q_i = 0$  if he/she does not.

The optimal condition for  $Q_a = 1$  (i.e., quitting a job) is:

$$\begin{aligned} Q_a = 1 &\iff (c_a + \pi(l_a) - u_F^S)^{1/2} (c_b + \pi(l_b) - u_F^S)^{1/2} |_{Q_a=1} \\ &\geq \max \left\{ (c_a + \pi(l_a) - u_F^S)^{1/2} (c_b + \pi(l_b) - u_F^S)^{1/2} |_{Q_a=0, Q_b=1}, (c_a + \pi(l_a) - u_F^S)^{1/2} (c_b + \pi(l_b) - u_F^S)^{1/2} |_{Q_a=Q_b=0} \right\}. \end{aligned} \quad (20)$$

Equation (20) indicates that whether an individual quits working after marrying is determined by comparing utility levels (i.e., bargaining outcomes) related to three cases: the bargaining outcome achieved by the worker quitting his/her job; that in which his/her partner quits his/her job; and that in which both continue their careers as full-time workers. If the worker can enjoy a higher marital surplus by quitting than in either of the other two states, then he/she quits his/her job.

Solving the problem yields the demand functions for market and domestic goods:<sup>12</sup>

$$l_a = l_b = 1 - \frac{h_F + Qh_0 + (1 - Q)h_F}{2}, \quad (21)$$

$$\begin{aligned} c_a + \pi \left( 1 - \frac{h_F + Qh_0 + (1 - Q)h_F}{2} \right) - u_F^S &= c_b + \pi \left( 1 - \frac{h_F + Qh_0 + (1 - Q)h_F}{2} \right) - u_F^S \\ &= \frac{1}{2} \hat{S}(F, F), \end{aligned} \quad (22)$$

where

$$\hat{S}(F, F) = -Q_{aF}w_{aF} - Q_{bF}w_{bF} + 2\pi \left( 1 - \frac{Q_a h_0 + (1 - Q_a)h_F + Q_b h_0 + (1 - Q_b)h_F}{2} \right) - 2\pi(1 - h_F).$$

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<sup>12</sup>See Appendix.

$\hat{S}(F, F)$  is the surplus arising from a marriage between two full-time workers, characterized by the possibility of job quitting. In it, the first two terms represent the expected wages forgone as a result of ceasing to work, while the latter term is the additional benefit from domestic specialization. Note that, unlike in the basic model, the marriage surplus of two full-time workers is positive. This arises because we do not exclude the possibility of benefitting from specialization by either partner quitting his/her job after the marriage. As seen in the previous sections, the demand functions show that the additional consumption of market and domestic goods in the marriage is equalized between partners.

Substituting equations (21) and (22) into (20) yields the condition for a worker of type  $a$  to quit his/her job as a function of wage rates and the domestic production technology:

$$\begin{aligned}
Q &= 1 \iff \frac{1}{2}\hat{S}(F, F)|_{Q_a=1, Q_b=0} \geq \max \left\{ \frac{1}{2}\hat{S}(F, F)|_{Q_a=0, Q_b=1}, \frac{1}{2}\hat{S}(F, F)|_{Q_a=Q_b=0} \right\} & (23) \\
&\iff -w_{aF} + 2\pi \left( 1 - \frac{h_0 + h_F}{2} \right) - 2\pi(1 - h_F) > \max \left\{ -w_{bF} + 2\pi \left( 1 - \frac{h_0 + h_F}{2} \right) - 2\pi(1 - h_F), 0 \right\} \\
&\iff 0 < \min \left\{ w_{bF} - w_{aF}, 2\pi \left( 1 - \frac{h_0 + h_F}{2} \right) - 2\pi(1 - h_F) - w_{aF} \right\}.
\end{aligned}$$

With this condition, it is implied that a type  $a$  worker quits her/his job if (i) his/her wage rate is lower than that of his/her partner (i.e.,  $w_{aF} < w_{bF}$ ) and (ii) the earnings the couple must forgo from either quitting work is sufficiently smaller than the resulting benefit in terms of the gap in utility levels between the two states (both continuing full-time jobs versus some domestic specialization) (i.e.,  $w_{aF} < 2\pi \left( 1 - \frac{h_0 + h_F}{2} \right) - 2\pi(1 - h_F)$ ).

## 5.2 The job search

Now that the marital outcome in the second stage has been characterized under the possibility of job quitting, we consider the young workers' job searching. Here, to pin down the equilibrium, we assume the productivity of type  $a$  workers in full-time jobs is slightly lower than that of type  $b$  workers.

Using  $\hat{S}(F, F)$ , the expected utility can be defined as

$$EU_{iF} = sp(\theta_{iF}) \left[ W_{iF} + w_{iF} + \pi(1 - l_F) + \frac{\rho}{2} \left[ n_{jF}\hat{S}(F, F) + n_{jU}S(F, U) \right] \right] + (1 - sp(\theta_{iF})) \left[ \pi(1 - h_U) + \frac{\rho}{2}n_{jF}S(F, U) \right],$$

and the free-entry condition for vacant jobs can be defined as:<sup>13</sup>

$$k = q(\theta_{iIF}) [(1 - \rho n_{iF} Q_{iF}) (y_F - w_{iF}) - W_{iF}]. \quad (24)$$

Note that the free-entry condition differs from that of the basic model; the post-marriage job termination probability is  $\rho n_{jF} Q_a$ , as an employed worker may quit her/his job if she/he meets a full-time worker with the probability of  $\rho n_{jF}$ . Thus, under this free-entry condition, the search cost for one worker,  $k$ , is equal to the expected benefit of employing him/her (i.e., the expected profit she/he brings to the firm,  $q(\theta_{iIF}) (y_F - w_{iF})$ ), minus the possible cost of her/him quitting,  $q(\theta_{iIF}) \rho n_{iF} Q_{iF} (y_F - w_{iF})$ , and the recruitment bonus,  $(W_{iF})$ .

Finally, using (7), we obtain the following proposition:

**Definition 1** *There exist three types of equilibria: (i) "Breadwinner" equilibrium ( $\alpha_a = \alpha_b = 1$ ,  $Q_a = 1$ , and  $Q_b = 0$ ), (ii) Non-division equilibrium ( $\alpha_a = \alpha_b = 1$  and  $Q_a = Q_b = 0$ ), (iii) Division equilibrium ( $\alpha_a = 0$ ,  $\alpha = 1$  and  $Q_a = Q_b = 0$ ).*

Each equilibria holds under the following conditions:

**Proposition 4** *The breadwinner equilibrium holds if*

$$\begin{aligned} y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) &> \frac{\rho}{2} \left[ n_F^B [S(P, F) - \hat{S}(F, F)] + n_U^B [S(P, U) - S(F, U)] \right], \\ 0 &< 2\pi \left( 1 - \frac{h_0 + h_F}{2} \right) - 2\pi(1 - h_F) - y_F, \end{aligned}$$

where

$$\begin{aligned} \hat{S}(F, F) &= -y_F + 2\pi \left( 1 - \frac{h_0 + h_F}{2} \right) - 2\pi(1 - h_F), \\ 0 &= p' \left( \theta_F^B \right) \left[ y_F + \pi(1 - l_F) - \pi(1 - h_U) + \frac{\rho}{2} \left[ s p_F^B \hat{S}(F, F) + (1 - 2p_F^B) S(F, U) \right] \right] - k. \end{aligned}$$

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<sup>13</sup>The Appendix confirms that  $w_i = y_i$  holds in the second-best equilibrium in which firms cannot make a contract for  $Q_i$  when they employ a new worker.



The non-division equilibrium holds if

$$\begin{aligned} y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) &> \frac{\rho}{2} [sp_F^N S(P, F) + (1 - sp_F^N) [S(P, U) - S(F, U)]], \\ 0 &> 2\pi \left(1 - \frac{h_0 + h_F}{2}\right) - 2\pi(1 - h_F) - y_F, \end{aligned}$$

where

$$0 = p' \left(\theta_F^N\right) \left[ y_F + \pi(1 - l_F) - \pi(1 - h_U) + \frac{\rho}{2} (1 - 2sp_F^N) S(F, U) \right] - k.$$

The division equilibrium holds if

$$y_F + \pi(1 - h_F) - y_P - \pi(1 - h_P) < \frac{\rho}{2} \left\{ sp_F^D [S(P, F) - \hat{S}(F, F)] + (1 - sp_F^D) [S(P, U) - S(F, U)] \right\},$$

where

$$\begin{aligned} \hat{S}(F, F) &= \max \left\{ -y_F + 2\pi \left(1 - \frac{h_0 + h_F}{2}\right) - 2\pi(1 - h_F), 0 \right\}, \\ k &= (1 - \gamma) q_P^D \left( y_P + \pi(1 - h_P) - \pi(1 - h_U) + \rho \frac{(1 - sp_F^D) S(P, U) + sp_F^D S(F, P) - sp_F^D S(F, U)}{2} \right), \\ k &= (1 - \gamma) q_F^D \left( y_F + \pi(1 - h_F) - \pi(1 - h_U) + \rho \frac{(1 - sp_P^D) S(F, U) + sp_P^D S(F, P) - sp_P^D S(P, U)}{2} \right). \end{aligned}$$

The argument above can be summarized in Fig. 2, which compares the results with those of the basic model presented in the previous sections. Fig. 2 classifies marriage patterns depending on the relationship between the productivities of full-time and part-time workers. The horizontal line represents the productivity of full-time workers while the vertical line represents that of part-time workers. As can be seen in Fig. 2, the division equilibrium is likely to emerge, as the slope is positive, when the productivity gap between full-time and part-time workers is small (i.e., the full-time workers' (the part-time workers') productivity is relatively small (large)). This result can be interpreted as follows. When there is a small productivity gap between full-time and part-time workers, the benefit from choosing different types of jobs within the couple (i.e., greater consumption of domestic production goods due to specialization) is constant, while its cost (given by the pay gap between full-time and part-time jobs) becomes small. Consequently, with higher net benefits, the workers choose different jobs followed by the division equilibrium.

On the other hand, the domain of the non-division equilibrium in the analysis of the basic model can be broken down into two domains if we permit the possibility of quitting a job. The condition distinguishing the non-division equilibrium from the breadwinner equilibrium is determined by the level of full-time workers' productivity. When this is relatively small, the breadwinner equilibrium is likely to emerge. Intuitively, this result can also be interpreted through a comparison of the costs and benefits of job-quitting: the reduction in full-time workers' productivity entails that the cost of job-quitting (i.e., the earnings forgone) decreases while the benefit from domestic specialization that results from job quitting is constant.

In sum, therefore, the results of the basic model essentially hold even when we incorporate the possibility of workers quitting their jobs.

## 6 Conclusion

In this paper, we formulated a directed search model in which individuals may marry a partner and then enjoy a marital surplus resulting from domestic specialization. The paper has made two key contributions to the literature by combining the theories of marriage and the job search. First, we introduced labor market friction into the theory of marriage. Previous studies on marriage with a frictionless labor market assume that gender division in tasks takes place after the marriage in accordance with each spouse's comparative advantage. However, taking account of labor market friction, we succeeded in showing that gender divisions occur before marriage, as noted in Table 1. Moreover, this finding leads to the multiple equilibria result in which single workers have two strategies: the two members of a couple choose different job types or they choose the same job type. This result partially replicates the variance among industrial countries in the degree of gender divisions within the young labor force, as shown in Fig. 1.

The other contribution of this study is that incorporating the benefit of domestic specialization within marriage enables us to provide one possible explanation for why individuals, especially female workers as indicated in Table 1, dare to choose part-time jobs even before marriage. In our model, workers do not choose a part-time job if there is no possibility of marriage, which corresponds to the existing job search model of single workers. Moreover, the possibility of reaping the marital surplus allows for the final result of inefficiency arising from a worker making his/her job choice without knowing that the decision also affects the welfare of his/her potential partner. This result is in contrast to the result of the traditional directed search model in

which the market equilibrium is always efficient (Moen, 1997; Acemoglu and Shimer, 1999).

Before closing, we must mention some assumptions and limitations of our study. The model assumes that individuals meet their partners with an exogenous probability. However, in the real world, individuals partially control this probability by making themselves attractive partners through pre-marriage investments. One extension would thus be to endogenize the probability of meeting a potential partner, making it depend on a worker's employment status. Better understanding of marital matching could bring our model a step closer to facilitating a more comprehensive analysis of the connection between marriage and the labor market.

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# Appendix

## Proof of Lemma 1

As is well known, the results of intra-household bargaining can be characterized by the solution to the following problem:

$$\begin{aligned} (c_a, c_b, l_a, l_b) &\in \arg \max \left( c_a + \pi(l_a) - u_{aI(a)}^S \right)^{1/2} \left( c_b + \pi(l_b) - u_{bI(b)}^S \right)^{1/2}, \\ \text{s.t. } c_a + c_b &= w_{aI(a)} + w_{bI(b)}, \\ l_a + l_b &= 2 - (h_{I(a)} + h_{I(b)}). \end{aligned}$$

First-order conditions are

$$c_a : \frac{1}{2} \left( \frac{c_b + \pi(l_b) - u_{bI(b)}^S}{c_a + \pi(l_a) - u_{aI(a)}^S} \right)^{1/2} = \lambda_c \quad (\text{A-1})$$

$$c_b : \frac{1}{2} \left( \frac{c_a + \pi(l_a) - u_{aI(a)}^S}{c_b + \pi(l_b) - u_{bI(b)}^S} \right)^{1/2} = \lambda_c \quad (\text{A-2})$$

$$l_a : \frac{1}{2} \pi'(l_a) \left( \frac{c_b + \pi(l_b) - u_{bI(b)}^S}{c_a + \pi(l_a) - u_{aI(a)}^S} \right)^{1/2} = \lambda_l \quad (\text{A-3})$$

$$l_b : \frac{1}{2} \pi'(l_b) \left( \frac{c_a + \pi(l_a) - u_{aI(a)}^S}{c_b + \pi(l_b) - u_{bI(b)}^S} \right)^{1/2} = \lambda_l \quad (\text{A-4})$$

where  $\lambda_c$  and  $\lambda_l$  are the Lagrangian multipliers associated with the household budget and time constraints, respectively. First, substituting equations (A-1) and (A-2) into equations (A-3) and (A-4) yields

$$\begin{aligned} \pi'(l_a) &= \frac{\lambda_l}{\lambda_c}, \\ \pi'(l_b) &= \frac{\lambda_l}{\lambda_c}. \end{aligned}$$

Combining the above equations shows that  $l_a = l_b$ . From the constraints  $l_a + l_b = h_{I(a)} + h_{I(b)}$ , the allocation of domestic goods is then  $l_a = l_b = 1 - (h_{I(a)} + h_{I(b)})/2$ .

Next, combining (A – 1) and (A – 2) yields

$$c_a + \pi(l_a) - u_{aI(a)}^S = c_b + \pi(l_b) - u_{bI(b)}^S. \quad (25)$$

Now, let us define the joint surplus as

$$S = c_a + \pi(l_a) - u_{aI(a)}^S + c_b + \pi(l_b) - u_{bI(b)}^S$$

From the budget constraint  $c_a + c_b = w_{aI(a)} + w_{bI(b)}$ , the definition of  $u_{iI(i)}^S$ , (1), and the equilibrium condition for domestic goods, (2), the joint surplus can be rewritten as

$$S(I(a), I(b)) = 2\pi\left(1 - \frac{h_{I(a)} + h_{I(b)}}{2}\right) - \pi(1 - h_{I(a)}) - \pi(1 - h_{I(b)}).$$

Combining with equation (25) then yields the equilibrium allocation of market goods as

$$c_a + \pi(l_a) - u_{aI(a)}^S = c_b + \pi(l_b) - u_{bI(b)}^S = \frac{1}{2}S(I(a), I(b)) = \frac{1}{2}\left[2\pi\left(1 - \frac{h_{I(a)} + h_{I(b)}}{2}\right) - \pi(1 - h_{I(a)}) - \pi(1 - h_{I(b)})\right].$$

## Proof of Lemma 2

The differentiation of (4) yields

$$\frac{\partial S(I(i), I(j))}{\partial h_{I(i)}} = -\pi'\left(1 - \frac{h_{I(i)} + h_{I(j)}}{2}\right) + \pi'(1 - h_{I(i)}).$$

From the concavity of  $\pi(\bullet)$ ,

$$\frac{\partial S(I(i), I(j))}{\partial h_{I(i)}} > 0 \iff h_{I(i)} > h_{I(j)}.$$

Because  $h_F > h_P > h_U$ , we can show that

$$S(F, U) > S(P, U),$$



and

$$S(F, U) > S(F, P).$$

### Proof of Lemma 4

Using (13) and (12),  $\theta_{F1}^D(\theta_P^D)$  and  $\theta_{F2}^D(\theta_P^D)$  are defined as

$$k = (1 - \gamma) q_P^D (y_P + \pi(1 - h_P) - \pi(1 - h_0)) + \rho \frac{\left(1 - sp_F^D(\theta_{F1}^D(\theta_P^D))\right) S(P, U) + sp_F^D(\theta_{F1}^D(\theta_P^D)) S(F, P) - sp_F^D(\theta_{F1}^D(\theta_P^D)) S(F, U)}{2}, \quad (\text{A-6})$$

$$k = (1 - \gamma) q_F^D(\theta_{F2}^D(\theta_P^D)) \left( y_F + \pi(1 - h_F) - \pi(1 - h_0) + \rho \frac{(1 - sp_P^D) S(F, U) + sp_P^D S(F, P) - sp_P^D S(P, U)}{2} \right), \quad (\text{A-7})$$

and  $\Gamma(\theta_P^D) = \theta_{F1}^D(\theta_P^D) - \theta_{F2}^D(\theta_P^D)$ . The equilibrium market tightness can be defined as  $\Gamma(\theta_P^D) = 0$ .

Total differentiation of (A-6) and (A-7) with respect to  $\theta_P^D$  yields

$$\begin{aligned} \frac{\partial \theta_{F1}^D}{\partial \theta_P^D} &= - \frac{2\gamma k}{sp_P^D q_F^D \rho [S(P, U) - S(F, P) + S(F, U)] (1 - \gamma)^2} < 0, \\ \frac{\partial \theta_{F2}^D}{\partial \theta_P^D} &= - \frac{sp_F^D q_P^D \rho [S(F, U) - S(F, P) + S(P, U)] (1 - \gamma)^2}{2\gamma k} < 0. \end{aligned}$$

There is then a maximum value of  $\bar{\theta}_P^D$  as  $\theta_{F1}^D(\bar{\theta}_P^D) = 0$ . More formally, using (A-6),  $\bar{\theta}_P^D$  can be defined as

$$k = (1 - \gamma) q_P^D(\bar{\theta}_P^D) \left( y_P + \pi(1 - h_P) - \pi(1 - h_0) + \rho \frac{S(P, U)}{2} \right).$$

From (A-7),  $\theta_{F2}^D(\bar{\theta}_P^D)$  can be characterized as

$$k = (1 - \gamma) q_F^D(\theta_{F2}^D(\bar{\theta}_P^D)) \times \left( y_F + \pi(1 - h_F) - \pi(1 - h_0) + \rho \frac{\left(1 - sp_P^D(\bar{\theta}_P^D)\right) S(F, U) + sp_P^D(\bar{\theta}_P^D) S(F, P) - sp_P^D(\bar{\theta}_P^D) S(P, U)}{2} \right).$$

Assume that  $y_F + \pi(1 - h_F) - \pi(1 - h_0) + \rho \frac{S(F, P) - S(P, U)}{2} > 0$ , under which  $\Gamma(\bar{\theta}_P^D)$  must be negative because

$\theta_{F2}^D$  ( $\bar{\theta}_P^D$ ) must be positive.

To ensure the existence of an equilibrium, we additionally assume that  $\theta_{F1}^D(0) > \theta_{F2}^D(0)$ , which implies that  $\Gamma(0) > 0$ . Combining  $\Gamma(0) > 0$  and  $\Gamma(\bar{\theta}_P^D) < 0$  shows the existence of an equilibrium as  $\Gamma(\theta_P^D) = 0$ .

Finally, the differentiation of  $\Gamma(\theta_P^D)$  is

$$\begin{aligned} \frac{\partial \Gamma(\theta_P^D)}{\partial \theta_P} &= -\frac{2\gamma k}{sp_P^D q_F^D \rho [S(P, U) - S(F, P) + S(F, U)] (1-\gamma)^2} + \frac{sp_F^D q_P^D \rho [S(F, U) - S(F, P) + S(P, U)] (1-\gamma)^2}{2\gamma k} \\ &= -\frac{2\gamma k - s\sqrt{\theta_F^D \theta_P^D} q_F^D q_P^D \rho [S(F, U) - S(F, P) + S(P, U)] (1-\gamma)^2}{2\gamma k sp_P^D q_F^D \rho [S(P, U) - S(F, P) + S(F, U)] (1-\gamma)^2} \\ &\quad \times \frac{2\gamma k + s\sqrt{\theta_F^D \theta_P^D} q_F^D q_P^D \rho [S(F, U) - S(F, P) + S(P, U)] (1-\gamma)^2}{2\gamma k sp_P^D q_F^D \rho [S(P, U) - S(F, P) + S(F, U)] (1-\gamma)^2} \end{aligned}$$

Thus, the equilibrium market tightness is uniquely determined iff

$$2\gamma k - s\sqrt{\theta_F^D \theta_P^D} q_F^D q_P^D \rho [S(F, U) - S(F, P) + S(P, U)] (1-\gamma)^2 > 0 \iff \frac{\partial \Gamma(\theta_P^D)}{\partial \theta_P} < 0. \quad (\text{A-8})$$

## Proof of Lemma 5

To prove Lemma 5, we use proof by contradiction. To do so, we first show an important property of  $\Gamma(\cdot)$ .

Define  $\hat{\theta}_{F1}^D(\hat{\theta}_{P1}^D) = \hat{\theta}_{P1}^D$  and  $\hat{\theta}_{F2}^D(\hat{\theta}_{P2}^D) = \hat{\theta}_{P2}^D$ . Further, note that if  $\theta_P^D > \theta_F^D$  in equilibrium, then  $\theta_P^D$  must be greater than  $\hat{\theta}_{P1}^D$  and  $\hat{\theta}_{P2}^D$  because  $\theta_{F1}^D$  and  $\theta_{F2}^D$  are decreasing functions of  $\theta_P^D$ .

From (A-6) and (A-7),  $\hat{\theta}_{F1}^D$  and  $\hat{\theta}_{F2}^D$  can be characterized as

$$\begin{aligned} \frac{k}{(1-\gamma) q_F^D(\hat{\theta}_{F1}^D)} + \rho sp_F^D(\hat{\theta}_{F1}^D) \frac{S(P, U) + S(F, U) - S(F, P)}{2} &= y_P + \pi(1-h_P) - \pi(1-h_U) + \rho \frac{S(P, U)}{2}, \\ \frac{k}{(1-\gamma) q_F^D(\hat{\theta}_{F2}^D)} + \rho sp_F^D(\hat{\theta}_{F2}^D) \frac{S(P, U) + S(F, U) - S(F, P)}{2} &= y_F + \pi(1-h_F) - \pi(1-h_U) + \rho \frac{S(F, U)}{2}, \end{aligned}$$

The left-hand sides of the above equations are increasing functions of  $\hat{\theta}_{F1}^D$  and  $\hat{\theta}_{F2}^D$  because  $q' < 0$  and  $p' > 0$ .

Consequently,  $\hat{\theta}_{F1}^D < \hat{\theta}_{F2}^D$  and then  $\hat{\theta}_{P1}^D < \hat{\theta}_{P2}^D$  because  $y_F + b(1-h_F) - b(1-h_U) > y_P + b(1-h_P) - b(1-h_U)$

and  $S(F, U) > S(P, U)$ . Moreover, because  $\theta_{F1}^D$  is a decreasing function of  $\theta_P^D$ ,  $\hat{\theta}_{F2}^D > \theta_{F1}^D(\hat{\theta}_{P2}^D)$  and then

$$\Gamma(\hat{\theta}_{P2}^D) < 0.$$

Consequently, if we suppose the equilibrium market tightness is  $p(\theta_F^D) > p(\theta_P^D)$ , then the differentiation of  $\Gamma(\theta_P^D)$  near the market equilibrium,  $\frac{\partial \Gamma(\theta_P^D)}{\partial \theta_P} \Big|_{\Gamma(\theta_P^D)=0}$ , must be positive because  $\Gamma(\hat{\theta}_{P2}^D) < 0$ . This is contradicted by the condition of uniqueness (A-8).

## Proof of Lemma 6

Here we also use proof by contradiction. Suppose that  $p_F^D < p^B$ . Equations (13) and (10) show that  $p_F^D < p^B \iff$

$$\begin{aligned} & y_F + \pi(1 - h_F) - \pi(1 - h_U) + \rho \frac{(1 - sp_P^D)S(F, U) + sp_P^D S(F, P) - sp_P^D S(P, U)}{2} \\ < & y_F + \pi(1 - h_F) - \pi(1 - h_U) + \frac{\rho}{2}(1 - 2sp^N)S(F, U) \end{aligned}$$

The above inequality can be rewritten as

$$0 > \frac{s\rho}{2} [(2p^N - p_P^D)S(F, U) + p_P^D S(F, P) - p_P^D S(P, U)].$$

$(2p^N - p_P^D)S(F, 0) + p_P^D S(F, P) - p_P^D S(0, P)$  is an increasing function of  $p^N$ . Moreover, combining inequalities  $p_F^D < p^B$  and  $p_P^D < p_F^D$  (Lemma 5) yields  $p^B > p_P^D$ , and then

$$\begin{aligned} & \frac{s\rho}{2} [(2p^N - p_P^D)S(F, U) + p_P^D S(F, P) - p_P^D S(P, U)] \\ > & \frac{s\rho}{2} [(2p_P^D - p_P^D)S(F, U) + p_P^D S(F, P) - p_P^D S(P, U)] \\ = & \frac{s\rho}{2} p_P^D [S(F, U) + S(F, P) - S(P, U)]. \end{aligned}$$

From  $S(F, U) > S(P, U)$ ,  $S(F, U) + S(F, P) - S(P, U)$  must be positive; this is then a contradiction.

## Proof of Proposition 2

Total differentiation of equations (10), (12), and (13) with respect to  $s$  yields

$$\frac{d\theta^N}{ds} = -q^N p^N \rho S(F, U) \left[ - \left( y_F + \pi(1 - h_F) - \pi(1 - h_U) + \frac{\rho}{2}(1 - 2sp^N)S(F, U) \right) \frac{\partial q^N}{\partial \theta^N} + q^N \rho s S(F, U) \frac{\partial p^N}{\partial \theta^N} \right]^{-1},$$

and

$$\begin{aligned}
0 &= \left( y_P + \pi(1 - h_P) - \pi(1 - h_U) + \rho \frac{(1 - sp_F^D)S(P, U) + sp_F^D S(F, P) - sp_F^D S(F, U)}{2} \right) \frac{\partial q_P^D}{\partial \theta_P^D} \frac{d\theta_P^D}{ds} \\
&\quad - sq_P^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} \frac{\partial p_F^D}{\partial \theta_F^D} \frac{d\theta_F^D}{ds} \\
&\quad - q_P^D p_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2}, \\
0 &= -sq_F^D \rho \frac{S(F, U) - S(F, P) + S(P, U)}{2} \frac{\partial p_P^D}{\partial \theta_P^D} \frac{d\theta_P^D}{ds} \\
&\quad + \left( y_F + \pi(1 - h_F) - \pi(1 - h_U) + \rho \frac{(1 - sp_P^D)S(F, U) + sp_P^D S(F, P) - sp_P^D S(P, U)}{2} \right) \frac{\partial q_F^D}{\partial \theta_F^D} \frac{d\theta_F^D}{ds} \\
&\quad - q_F^D p_P^D \rho \frac{S(F, U) - S(F, P) + S(P, U)}{2}.
\end{aligned}$$

Using equations (10), (12), and (13), and  $\theta q'/q = -\gamma$  and  $p'/q = (1 - \gamma)$ , the above equations can be rewritten

as

$$\begin{aligned}
\frac{d\theta^N}{ds} &= -(p^N)^2 \rho S(F, U) \left[ \frac{\gamma}{1 - \gamma} k + (1 - \gamma) p^N q^N \rho s S(F, U) \right]^{-1} < 0, \\
\frac{d\theta_P^D}{ds} &= -(1 - \gamma) p_P^D p_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2A} \left[ \gamma k - s \theta_P^D q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right], \\
\frac{d\theta_F^D}{ds} &= -(1 - \gamma) p_P^D p_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2A} \left[ \gamma k - s \theta_F^D q_F^D q_P^D \rho \frac{S(F, U) - S(F, P) + S(P, U)}{2} (1 - \gamma)^2 \right],
\end{aligned}$$

where

$$\begin{aligned}
A &= \left( \gamma k + s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right) \\
&\quad \times \left( \gamma k - s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right). \tag{A-9}
\end{aligned}$$

Note that from (14),  $A > 0$ .

The impact on the rate of finding a job in the non-division equilibrium is

$$\begin{aligned}
\frac{\partial sp^N}{\partial s} &= p^N + s \frac{\partial p^N}{\partial \theta^N} \frac{\partial \theta^N}{\partial s} \\
&= p^N \frac{\gamma}{1 - \gamma} k \left[ \frac{\gamma}{1 - \gamma} k + (1 - \gamma) p^N q^N \rho s S(F, U) \right]^{-1} > 0.
\end{aligned}$$

The impact on the rate of finding a full-time job in the division equilibrium is

$$\begin{aligned}
\frac{dsp_F^D}{ds} &= p_F^D + s \frac{\partial p_F^D}{\partial \theta_F^D} \frac{\partial \theta_F^D}{\partial s} \\
&= p_F^D [1 \\
&\quad - s \frac{\partial p_F^D}{\partial \theta_F^D} (1-\gamma) p_P^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2A} \left( \gamma k - s \theta_F^D q_F^D q_P^D \rho \frac{S(F,U) - S(F,P) + S(P,U)}{2} (1-\gamma)^2 \right)] \\
&= p_F^D [1 \\
&\quad - s (1-\gamma)^2 q_F^D p_P^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2A} \left( \gamma k - s \theta_F^D q_F^D q_P^D \rho \frac{S(F,U) - S(F,P) + S(P,U)}{2} (1-\gamma)^2 \right)] \\
&= \frac{p_F^D}{A} [A \\
&\quad - s (1-\gamma)^2 q_F^D p_P^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2} \left( \gamma k - s \theta_F^D q_F^D q_P^D \rho \frac{S(F,U) - S(F,P) + S(P,U)}{2} (1-\gamma)^2 \right)]
\end{aligned}$$

Using  $\theta_F^D > \theta_P^D$ ,  $\theta_F > (\theta_F^D \theta_P^D)^{1/2}$ , and then

$$\begin{aligned}
\frac{dsp_F^D}{ds} &= \frac{p_F^D}{A} [A \\
&\quad - s (1-\gamma)^2 q_F^D p_P^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2} \left( \gamma k - s \theta_F^D q_F^D q_P^D \rho \frac{S(F,U) - S(F,P) + S(P,U)}{2} (1-\gamma)^2 \right)] \\
&> \frac{p_F^D}{A} \left[ A - s (1-\gamma)^2 q_F^D p_P^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2} \right. \\
&\quad \left. \times \left( \gamma k - s (\theta_F^D \theta_P^D)^{1/2} q_F^D q_P^D \rho \frac{S(F,U) - S(F,P) + S(P,U)}{2} (1-\gamma)^2 \right) \right]
\end{aligned}$$

Substituting (A-9) into the above inequality yields

$$\begin{aligned}
\frac{dsp_F^D}{ds} &> \frac{p_F^D}{A} \left[ \gamma k + s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2} (1-\gamma)^2 \right. \\
&\quad \left. - s (1-\gamma)^2 q_F^D p_P^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2} \right] \\
&\quad \times \left[ \gamma k - s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P,U) - S(F,P) + S(F,U)}{2} (1-\gamma)^2 \right].
\end{aligned}$$

Using  $\theta_F^D > \theta_P^D$  again,  $\theta_P < \left(\theta_F^D \theta_P^D\right)^{1/2}$ , and the above inequality can then be written as

$$\begin{aligned}
\frac{dsp_F^D}{ds} &> \frac{p_F^D}{A} \left[ \gamma k + s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right. \\
&\quad \left. - s (1 - \gamma)^2 q_F^D p_P^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} \right] \\
&\quad \times \left[ \gamma k - s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right] \\
&> \frac{p_F^D}{A} \left[ \gamma k + s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right. \\
&\quad \left. - s (1 - \gamma)^2 (\theta_P \theta_F)^{1/2} q_F^D q_P^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} \right] \\
&\quad \times \left[ \gamma k - s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right] \\
&= \frac{p_F^D}{A} \gamma k \left( \gamma k - s (\theta_P \theta_F)^{1/2} q_P^D q_F^D \rho \frac{S(P, U) - S(F, P) + S(F, U)}{2} (1 - \gamma)^2 \right)
\end{aligned}$$

From (14),  $dsp_F^D/ds$  must be positive.

Finally, the impacts on thresholds  $y_F^N$  and  $y_F^D$  are

$$\begin{aligned}
\frac{\partial y_F^N}{\partial s} &= \frac{\partial y_F^N}{\partial sp_F^N} \frac{\partial sp_F^N}{\partial s} = \rho \frac{S(F, P) + S(F, U) - S(P, U)}{2} \frac{\partial sp_F^N}{\partial s}, \\
\frac{\partial y_F^D}{\partial s} &= \frac{\partial y_F^D}{\partial sp_F^D} \frac{\partial sp_F^D}{\partial s} = \rho \frac{S(F, P) + S(F, U) - S(P, U)}{2} \frac{\partial sp_F^D}{\partial s}
\end{aligned}$$

Because  $S(F, P) + S(F, U) - S(P, U) > 0$ , both  $\partial y_F^N/\partial s$  and  $\partial y_F^D/\partial s$  must be negative.

### Proof of Proposition 3

First differentiation of (17) with respect to market tightness yields

$$\begin{aligned}
\frac{\partial \Omega}{\partial \theta_{iF}} &= \left( y_F + \pi (1 - h_F) - \pi (1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} (S(F, I(j)) - S(U, I(j))) \right) \frac{\partial p_{iF}}{\partial \theta_{iF}} - k \\
&\quad + \alpha_j \frac{\rho}{2} [sp_{jF} (S(F, F) - S(F, U)) + (1 - sp_{jF}) S(F, U)] \frac{\partial p_{iF}}{\partial \theta_{iF}} \\
&\quad + (1 - \alpha_j) \frac{\rho}{2} [sp_{jP} (S(F, P) - S(P, U)) + (1 - sp_{jP}) S(F, U)] \frac{\partial p_{iF}}{\partial \theta_{iF}},
\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \Omega}{\partial \theta_{iP}} &= \left( y_P + b(1 - h_P) - b(1 - h_U) + \frac{\rho}{2} \sum_{I(j) \in \{F, P, U\}} n_{jI(j)} [S(P, I(j)) - S(U, I(j))] \right) \frac{\partial p_{iP}}{\partial \theta_{iP}} - k \\ &+ \alpha_j \frac{\rho}{2} [sp_{jF} (S(F, P) - S(F, U)) + (1 - sp_{jF}) S(P, U)] \frac{\partial p_{iP}}{\partial \theta_{iP}} \\ &+ (1 - \alpha_j) \frac{\rho}{2} [-sp_{jP} S(P, U) + (1 - sp_{jP}) S(P, U)] \frac{\partial p_{iP}}{\partial \theta_{iP}}.\end{aligned}$$

Next, we evaluate the above results of first differentiation near the market equilibrium. By substituting the equilibrium market tightness at each equilibrium, (10), (13), and (12), the first terms of the above equations are eliminated, and then

$$\begin{aligned}\frac{\partial \Omega}{\partial \theta_F} \Big|_{\alpha_a = \alpha_b = 1, \theta_{aF} = \theta_{bF} = \theta^N} &= \frac{\rho}{2} (1 - 2sp^N) S(F, U) \frac{\partial p_F}{\partial \theta_F}, \\ \frac{\partial \Omega}{\partial \theta_F} \Big|_{\alpha_a = 1, \alpha_b = 0, \theta_{aF} = \theta_F^D, \theta_{bP} = \theta_P^D} &= \frac{\rho}{2} [sp_F^D (S(F, P) - S(P, U)) + (1 - sp_F^D) S(F, U)] \frac{\partial p_F}{\partial \theta_F}, \\ \frac{\partial \Omega}{\partial \theta_P} \Big|_{\alpha_a = 1, \alpha_b = 0, \theta_{aF} = \theta_F^D, \theta_{bP} = \theta_P^D} &= \frac{\rho}{2} [sp_F^D [S(F, P) - S(F, U)] + (1 - sp_F^D) S(P, U)] \frac{\partial p_P}{\partial \theta_P}.\end{aligned}$$

## 6.1 Derivation of (21) and (22)

The Lagrangian function is defined as

$$\begin{aligned}L &= (c_a + \pi(l_a) - u_F^S)^{1/2} (c_b + \pi(l_b) - u_F^S)^{1/2} + \lambda_c [c_a + c_b - (1 - Q_{aF}) w_{aF} - (1 - Q_{bF}) w_{bF}] \\ &+ \lambda_l [l_a + l_b - 2 + Q_a h_0 + (1 - Q_a) h_F + Q_b h_0 + (1 - Q_b) h_F].\end{aligned}$$

The first-order conditions for consumption are

$$\frac{\partial L}{\partial c_i} = 0 = \frac{1}{2} \left( \frac{c_j + \pi(l_j) - u_F^S}{c_i + \pi(l_i) - u_F^S} \right)^{1/2} + \lambda_c, \quad (26)$$

$$\frac{\partial L}{\partial l_i} = 0 = \frac{\pi'(l_i)}{2} \left( \frac{c_j + \pi(l_j) - u_F^S}{c_i + \pi(l_i) - u_F^S} \right)^{1/2} + \lambda_l, \quad (27)$$

Solving the problem subject to the conditions (26) and (27) and the constraints (18) and (19) gives the demand functions.

## 6.2 Proof of $w_i = y_i$ in the second-best equilibrium

### 6.2.1 First-best problem

To characterize the labor market equilibrium, we define the first-best problem as:

$$\max_{W_{iF}, w_{iF}, \theta_{iF}, Q_{iF}} EU_{iI(i)} \quad \text{s.t. (24), } w_{iF} \leq y_F.$$

Under the constraint  $w_{iF} \leq y_F$ , the offered wage cannot exceed the productivity of the worker; this comes from the IR condition of jobs: if  $w_{iF} > y_F$ , the firm has an incentive to fire its workers. Again, we can reduce the problem by making use of the free-entry condition and the labor market tightness condition to characterize the equilibrium.

Substituting (24) into the objective function yields:

$$\begin{aligned} \max_{w_{iF}, \theta_{iF}, Q_{iF}} EU_{iF} &= sp(\theta_{iI(i)}) \left[ (1 - \rho n_{jF} Q_{iF}) y_F + \rho n_{jF} Q_{iF} w_{iF} + \pi (1 - l_F) + \frac{\rho}{2} \left[ n_{jF} \hat{S}(F, F) + n_{jU} S(F, U) \right] \right] \\ &\quad + (1 - sp(\theta_{iF})) \left[ \pi (1 - h_U) + \frac{\rho}{2} n_{jF} S(U, F) \right] - sk\theta_{iF}, \\ \text{s.t. } w_{iF} &\leq y_F. \end{aligned}$$

The first-order differentiation with respect to  $w$  is

$$\frac{\partial EU_{iI(i)}}{\partial w_i} = sp(\theta_{iF}) \rho n_{jF} Q_{iF} > 0,$$

which implies that the higher wage rate, the more attractive the job. Because the upper bound of  $w_i$  is  $y_F$ , the first-best condition is

$$w_i = y_F. \tag{28}$$

The optimal condition of  $Q_{iF} = 1$  is

$$Q_{iF} = 1 \iff EU_{iF}|_{Q_{iF}=1} \geq EU_{iF}|_{Q_{iF}=0},$$

meaning that the expected utility in the case of job quitting is larger than that without job quitting.



Using the definition of  $EU_{iF}$ , the condition above can be rewritten as

$$\begin{aligned} Q_{iF} = 1 &\iff (1 - \rho n_{jF}) y_F + \rho n_{jF} w_{iF} + \pi (1 - l_F) + \frac{\rho}{2} \left[ n_{jF} \hat{S}(F, F) |_{Q_{iF}=1} + n_{jU} S(F, U) \right] \\ &\geq y_F + \pi (1 - l_F) + \frac{\rho}{2} \left[ n_{jF} \hat{S}(F, F) |_{Q_{iF}=0} + n_{jU} S(F, U) \right]. \end{aligned}$$

Substituting the definition of  $\hat{S}(F, F)$ ,

$$Q_{iF} = 1 \iff -y_F + \frac{1}{2} w_{iF} + \pi \left( 1 - \frac{h_0 + Q_b h_0 + (1 - Q_b) h_F}{2} \right) - \pi \left( 1 - \frac{h_F + Q_b h_0 + (1 - Q_b) h_F}{2} \right) \geq 0.$$

Substituting  $w_{iF} = y_F$ ,

$$Q_{iF} = 1 \iff -\frac{1}{2} y_F + \pi \left( 1 - \frac{h_0 + Q_b h_0 + (1 - Q_b) h_F}{2} \right) - \pi \left( 1 - \frac{h_F + Q_b h_0 + (1 - Q_b) h_F}{2} \right) \geq 0. \quad (29)$$

Finally, the first-order condition for  $\theta$  is

$$0 = p'(\theta_{iI(i)}) \left[ (1 - \rho n_{jF} Q_{iF}) y_F + \rho n_{jF} Q_{iF} w_{iF} + \pi (1 - l_F) - \pi (1 - h_U) + \frac{\rho}{2} \left[ n_{jF} \hat{S}(F, F) + (n_{jU} - n_{jF}) S(F, U) \right] \right] - sk$$

Substituting  $w_i = y_F$  yields

$$0 = p'(\theta_{iI(i)}) \left[ y_F + \pi (1 - l_F) - \pi (1 - h_U) + \frac{\rho}{2} \left[ n_{jF} \hat{S}(F, F) + (n_{jU} - n_{jF}) S(F, U) \right] \right] - sk. \quad (30)$$

### 6.2.2 Second-best problem

We define the second best problem as

$$\max_{W_i, w_i, \theta_{iI(i)}, Q_i} EU_{iI(i)} \text{ s.t. } (24), w_i \leq y_F, Q_i = 1 \iff 2\pi \left( 1 - \frac{h_U + h_F}{2} \right) - w_i \geq 2\pi (1 - h_F)$$

In this problem, there are additional constraints, as  $Q_i$  is determined by the intra-household decision.

**Lemma 7** *The labor market equilibrium can be characterized by*

$$\begin{aligned}
Q_{bF} &= 0 \\
Q_{aF} &= 1 \iff -\frac{1}{2}y_F + 2\pi \left(1 - \frac{h_0 + h_F}{2}\right) - 2\pi(1 - h_F) > 0, \\
w_i &= y_i, \\
0 &= p'(\theta_F) \left[ y_F + \pi(1 - l_F) - \pi(1 - h_U) + \frac{\rho}{2} \left[ n_F \hat{S}(F, F) + (n_U - n_{jF}) S(F, U) \right] \right] - sk,
\end{aligned}$$

where

$$\hat{S}(F, F) = -Q_{aF}w_{aF} + 2\pi \left(1 - \frac{Q_a h_0 + (1 - Q_a)h_F + h_F}{2}\right) - 2\pi(1 - h_F).$$

**Proof.** We can easily show that the conditions for the first-best problem are still feasible. Let us now denote  $y_{iF}$  as the productivity of type  $i$  workers. If the wage is the same as in the first-best case,  $w_{iF} = y_{iF}$ , the quitting condition can be rewritten as

$$Q_a = 1 \iff 0 < \min \left\{ y_{bF} - y_{aF}, 2\pi \left(1 - \frac{h_0 + h_F}{2}\right) - 2\pi(1 - h_F) - y_{aF} \right\}.$$

Because we have assumed that the productivity of type  $a$  workers is lower than that of type  $b$  workers ( $y_{aF} < y_{bF}$ ), the first condition of  $y_{bF} - y_{aF} > 0$  must hold. Then the quitting condition can be reduced to:

$$0 < 2\pi \left(1 - \frac{h_0 + h_F}{2}\right) - 2\pi(1 - h_F) - y_{aF},$$

which is the same as in the first-best condition (29) under  $Q_j = 0$ . Finally, the only constraint on  $\theta_{iI(i)}$  is (24), as in the first-best problem. ■

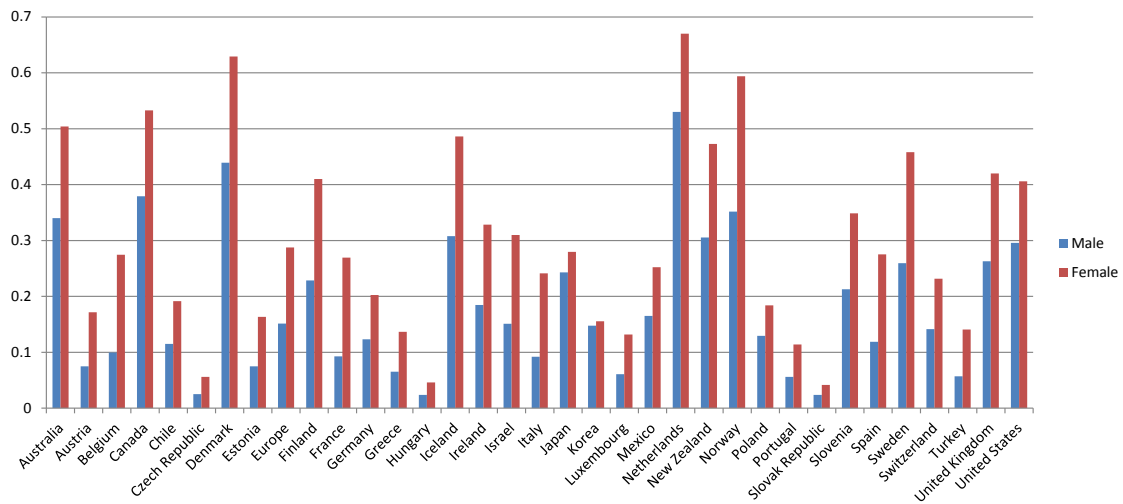


Figure1. Average part-time share (aged 15-24, year 2001-2012)

	Single Male	Single Female	Marriage Male	Marriage Female
# of Full-time employment	829	442	610	685
# of Part-time employment	185	165	92	271
# of Unemployment	64	37	36	46
Part-time/employment	18.24%	27.18%	13.11%	28.35%
Part-time-time/Labor force	17.16%	25.62%	12.47%	27.05%

Table1. The ratio of part-time workers in young workers

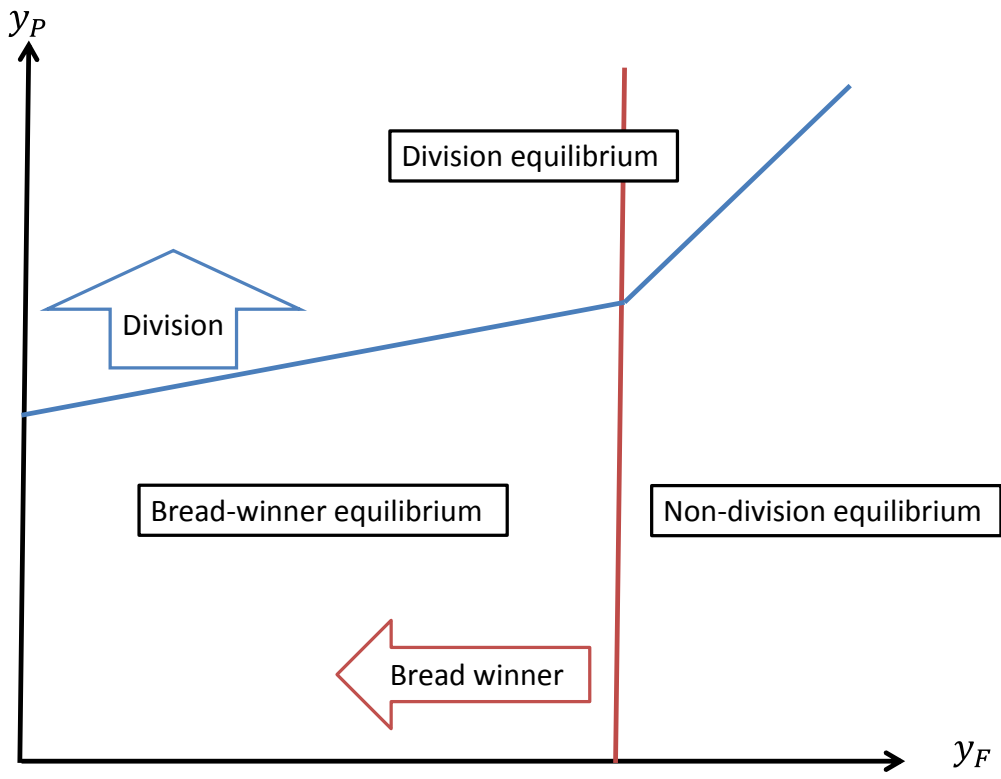


Figure 2. Equilibria in extended model