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Information losses in continuous-variable quantum teleportation

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It is shown that the information losses due to the limited fidelity of continuous variable quantum teleportation are equivalent to the losses induced by a beam splitter of appropriate reflectivity.

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Quantum teleportation allows the transmission of an unknown quantum state by combining the nonlocal quantum coherence of entangled states with the transmission of classical information obtained in a joint measurement of the unknown input state and one part of the entangled pair [1]. Ideally, the classical information transmitted contains no information about the input state, and the output state is exactly identical to the input state. However, this ideal form of quantum teleportation requires maximal entanglement. In continuous variable quantum teleportation [2–4], only nonmaximal entanglement is available. As a consequence, the output state is not perfectly identical to the input state, while the statistics of the joint measurement depend on the properties of the input state [5]. The classical information channel then carries information on the input state that may be extracted, e.g., in order to eavesdrop on a quantum communication channel [6]. The relationship between the measurement information extracted and the change of the quantum state can be described in terms of a measurement-dependent transfer operator [5]. In the following, the transfer operator describing the continuous variable teleportation process is derived and the equivalence with a feedback compensated beam splitter is established. The information obtained in quantum teleportation can then be identified with the reflected amplitude at the beam splitter, while the output state of the teleportation corresponds to the transmitted beam, displaced by the feedback. The loss of quantum information due to the limited fidelity of the teleportation process is thus shown to be equivalent to the loss of quanta at a beam splitter.

As illustrated in Fig. 1(a), quantum teleportation transfers an unknown quantum state in mode A using an entangled state of a reference mode R and an output mode B. For continuous variables, this is achieved by measuring the difference $\hat{x}_-=\hat{x}_A-\hat{x}_R$ and the sum $\hat{y}_+=\hat{y}_A+\hat{y}_R$ of the orthogonal quadrature components of the input mode A and the reference mode R. The quantum state $|\psi_B(\beta)\rangle$ of the output mode B is then conditioned by the input state $|\psi_A\rangle$ in mode A and the measurement result $\beta=x_-+iy_+$, which ideally defines an eigenstate $|\beta(A,R)\rangle$ of modes A and R. Realistically, the finite resolution due to limited detector efficiencies and other technical problems of the measurement will result in some additional noise, which can be simulated by a classical Gaussian error. In the following, however, we will focus on the ideal quantum limit of the information transfer in order to determine the quantum state distortions originating from nonmaximal entanglement. Using $|q(R,B)\rangle$ to denote the initial entangled state of modes R and B, this conditional state in B can be written as

$$|\psi_B(\beta)\rangle=\langle\beta(A,R)|\psi_A\rangle|q(R,B)\rangle. \tag{1}$$

Note that the output state $|\psi_B(\beta)\rangle$ is not normalized, since the probability $P(\beta)$ of the measurement outcome β is given by

$$P(\beta)=\langle\psi_B(\beta)|\psi_B(\beta)\rangle. \tag{2}$$

Making use of the displacement operator $\hat{D}(\beta)$ and the photon-number expansion of entanglement, the eigenstates $|\beta(A,R)\rangle$ and the entangled state $|q(R,B)\rangle$ can be expressed as

$$|\beta(A,R)\rangle=\frac{1}{\sqrt{\pi}}\sum_{n=0}^{\infty}\hat{D}_A(\beta)|n;n\rangle_{A,R},$$

$$|q(R,B)\rangle=\sqrt{1-q^2}\sum_{n=0}^{\infty}q^n|n;n\rangle_{R,B}, \tag{3}$$

where the entanglement coefficient q provides a quantitative measure characterizing the degree of entanglement obtained by parametric amplification. The quantum state $|\psi_B(\beta)\rangle$ of the output mode B conditioned by the measurement of modes A and R is then given by

$$|\psi_B(\beta)\rangle=\sqrt{\frac{1-q^2}{\pi}}\sum_{n=0}^{\infty}q^n|n\rangle\langle n|\hat{D}_A(-\beta)|\psi_A\rangle. \tag{4}$$

For $q=1$, this state is a copy of the input state displaced by a field difference of $-\beta$. Therefore, the final step of quantum teleportation is the reversal of this displacement by the addition of a coherent field amplitude $g\beta$ to obtain the final out-

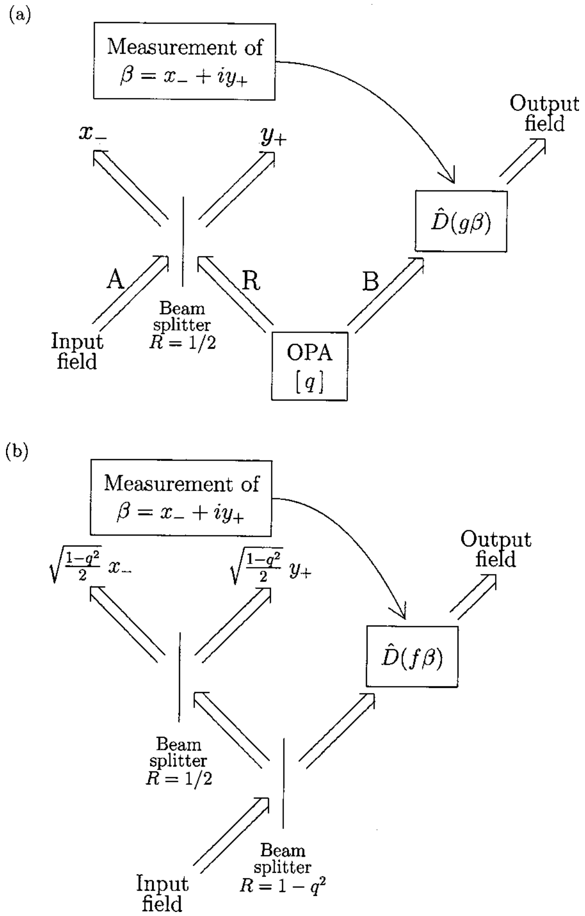

FIG. 1. Comparison of the setups for continuous variable quantum teleportation (a) and the feedback compensated beam splitter (b). R denotes beam splitter reflectivities. All other parameters are as given in the text.

put state, $|\psi_{\text{out}}(\beta)\rangle = \hat{D}(g\beta)|\psi_B(\beta)\rangle$. The gain factor g allows an adjustment of the teleported amplitude [4]. $g=1$ reproduces the average input amplitude in the output and thus optimizes the fidelity for the teleportation of high amplitude coherent states. As discussed in a previous paper [5], the conditional output of quantum teleportation can be described using the transfer operator $\hat{T}_g(\beta)$,

$$|\psi_{\text{out}}(\beta)\rangle = \hat{T}_g(\beta)|\psi_A\rangle,$$

$$P(\beta) = \langle\psi_A|\hat{T}_g^\dagger(\beta)\hat{T}_g(\beta)|\psi_A\rangle,$$

$$\text{with} \quad \hat{T}_g(\beta) = \sqrt{\frac{1-q^2}{\pi}} \sum_{n=0}^{\infty} q^n \hat{D}(g\beta)|n\rangle\langle n|\hat{D}(-\beta). \tag{5}$$

This transfer operator characterizes the teleportation process of an arbitrary quantum state by correlating the extracted information β with the quantum information in the output state $|\psi_{\text{out}}(\beta)\rangle$. When applied to a coherent state $|\alpha\rangle$, the result reads

$$\hat{T}_g(\beta)|\alpha\rangle = \sqrt{\frac{1-q^2}{\pi}} \exp\left(-(1-q^2)\frac{|\alpha-\beta|^2}{2}\right) \times \exp\left((1-gq)\frac{\alpha\beta^*-\beta\alpha^*}{2}\right)|q\alpha+(g-q)\beta\rangle. \tag{6}$$

For coherent states, the output is also a coherent state with an amplitude give by the sum of the attenuated input amplitude $q\alpha$ and a measurement dependent displacement of $(g-q)\beta$. Since the quantum state in the output depends on the randomly varying measurement result β, the teleportation output is generally a statistical mixture of different coherent states. Only in the special case of $g=q$, the amplitude $q\alpha$ of the coherent output state does not depend on β and the output is a well defined pure state even if the output is averaged over all measurement results β.

The attenuation of the signal amplitude described by Eq. (6) corresponds to the losses induced by a beam splitter with a reflectivity of $1-q^2$. For the special case of $g=q$, this property of teleportation has been pointed out previously by Polkinghorne and Ralph [7,8], based on an analysis of the quantum fluctuations in the teleportation. In the following, we will generalize this analogy by deriving the proper transformation of quantum states in a beam splitter measurement and considering the possibility of compensating the beam splitter losses by feedback. This formalism includes all the details necessary for an evaluation of the information obtained on the system and the minimal back action on the signal field. If the reflected amplitude of $\sqrt{1-q^2}\,\alpha$ is measured by eight-port homodyne detection, the corresponding positive operator valued measure is given by projections onto the nonnormalized, nonorthogonal coherent states

$$|P(\beta)\rangle = \sqrt{\frac{1-q^2}{\pi}}|\sqrt{1-q^2}\beta\rangle$$

$$\text{with} \quad \int d^2\beta |P(\beta)\rangle\langle P(\beta)| = \hat{1}, \tag{7}$$

and the transmitted state reads

$$|\psi_{\text{trans}}(\beta)\rangle = \sqrt{\frac{1-q^2}{\pi}}\langle\sqrt{1-q^2}\beta|\sqrt{1-q^2}\alpha\rangle|q\alpha\rangle = \sqrt{\frac{1-q^2}{\pi}}\exp\left(-(1-q^2)\frac{|\alpha-\beta|^2}{2}\right) \times \exp\left((1-q^2)\frac{\alpha\beta^*-\beta\alpha^*}{2}\right)|q\alpha\rangle, \tag{8}$$

where only the probability amplitude of $|\psi_{\text{trans}}(\beta)\rangle$ depends on the outcome β. This result corresponds to Eq. (6) if the gain g is equal to the entanglement coefficient q. The effects of quantum teleportation at a gain of $g=q$ are therefore identical to the effects of a field measurement by eight-port homodyne detection performed on the reflected part of the signal field using a beam splitter of reflectivity $R=1-q^2$. At other gain coefficients, quantum teleportation is equivalent to a feedback compensated beam-splitter measurement in which the losses induced in the transmitted beam are compensated using a linear feedback based on the measurement result β obtained from the reflected light [9]. At a feedback amplitude of $f\beta$, the output state of the feedback compensated beam splitter reads

$$\hat{D}(f\beta)|\psi_{\text{trans}}(\beta)\rangle = \hat{T}_{g=f+q}(\beta)|\alpha\rangle. \quad (9)$$

In particular, a gain of $g=1$ corresponds to a beam-splitter feedback amplitude of $(1-q)\beta$. In this case, the measurement operator of the beam-splitter setup is Hermitian, minimizing the back action of the measurement to the minimal noise required by the Heisenberg principle [10].

Since all quantum states may be expanded in terms of coherent states, Eq. (9) proves the equivalence of continuous variable quantum teleportation and feedback compensated beam splitting with respect to both the changes in the transmitted state and the information obtained in the measurement of β. In the special case of $g=q$, no additional photons are created in the teleportation process, indicating that all photons emitted into the output field B by the parametric amplifier are reabsorbed in the displacement transformation. This effect allows a teleportation of the vacuum with a fidelity of one, making a more reliable distinction of signal pulses from a vacuum background possible. The transmission probability for photons teleported at $g=q$ is equal to q^2. The loss of quantum information in continuous variable teleportation can thus be expressed in terms of photon losses. In an experimental realization of quantum teleportation, the case of $g=q$ can be used to characterize the performance of the setup. Specifically, the point at which $g=q$ can be found by minimizing the output intensity at a vacuum input. The remaining intensity at that point arises from the finite resolution of the measurement, imperfect phase matching, and similar technical problems in the optical setup. It is then possible to separate quantum noise effects from the classical noise contributions in the teleportation setup.

At $g>q$, the loss of quantum information is compensated by the classical information obtained from the measurement of β. However, the original quantum state cannot be restored by this purely classical manipulation, limiting the achievable fidelity to a value well below one. At $g<q$, the displacement actually reduces the quantum information in the output further, until at $g=0$, there is no correlation between the input state and the output density matrix formed by integrating over all measurement results β. In general, the effect of quantum teleportation on the one photon state $|1\rangle$ is given by

$$\hat{T}_g(\beta)|1\rangle = \sqrt{\frac{1-q^2}{\pi}}\exp\left(-(1-q^2)\frac{|\beta|^2}{2}\right)\hat{D}((g-q)\beta) \times((1-q^2)\beta^*|0\rangle + q|1\rangle). \quad (10)$$

This representation of the teleported state has only two components, corresponding to a displaced vacuum and a displaced photon-number state, respectively. At $g=q$, the displacement is zero and the two components correspond to the actual loss or transmission of the photon. At other gain factors, the coherent displacement can generate photon numbers $n>1$ in the output. Figure 1 shows a schematic comparison of the experimental setups for the quantum teleportation setup and for the compensated beam splitter. Both methods employ linear transformations on the input field mode and two vacuum modes, extracting information on the unknown input field from the homodyne detection measurements on two of the output modes. However, in quantum teleportation, the only physical connection between the input field and the output field is given by the measurement dependent displacement. While the beam splitter transmits the attenuated input field by a direct physical interaction, quantum teleportation achieves the same result by combining the entanglement with the classical information β. The entanglement coefficient q is the measure of the nonmaximal entanglement that corresponds to the attenuation of the transmitted amplitude at an equivalent beam splitter [11].

In conclusion, the analysis of the transfer operator $\hat{T}_g(\beta)$ shows that the information transfer in quantum teleportation is essentially equivalent to a feedback compensated beam splitter. This result clarifies the nature of information losses in quantum teleportation and allows an assessment of the information extracted with respect to applications such as continuous variable eavesdropping [6]. Moreover, the analogy provides a quantification of the information transfer properties of nonmaximal entanglement in terms of the photon transmission probability q^2 and simplifies the derivation of quantum coherent transfer properties for few photon inputs, e.g., for the entanglement swapping scheme discussed in [7].

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