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# AN OFF-AXIS JET MODEL FOR GRB 980425 AND LOW-ENERGY GAMMA-RAY BURSTS

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## ABSTRACT

Using a simple off-axis jet model of gamma-ray bursts (GRBs), we can reproduce the observed unusual properties of the prompt emission of GRB 980425, such as the extremely low isotropic equivalent $\gamma$-ray energy, the low peak energy, the high fluence ratio, and the long spectral lag when the jet with the standard energy of $\sim 10^{51}$ ergs and the opening half-angle of $10° \lesssim \Delta\theta \lesssim 30°$ is seen from the off-axis viewing angle $\theta_v \sim \Delta\theta + 10\gamma^{-1}$, where $\gamma$ is a Lorentz factor of the jet. For our adopted fiducial parameters, if the jet that caused GRB 980425 is viewed from the on-axis direction, the intrinsic peak energy $E_p(1+z)$ is ∼2.0–4.0 MeV, which corresponds to those of GRB 990123 and GRB 021004. We also discuss the connection of GRB 980425 in our model with the X-ray flash, and the origin of a class of GRBs with small $E_\gamma$.

*Subject headings:* gamma rays: bursts — gamma rays: theory

## 1. INTRODUCTION

Recently, a very luminous gamma-ray burst (GRB), GRB 030329 at the distance of 0.8 Gpc ($z = 0.1685$), was confirmed to be associated with supernova SN 2003dh (Stanek et al. 2003; Uemura et al. 2003; Price et al. 2003; Hjorth et al. 2003). The geometrically corrected $\gamma$-ray energy $E_\gamma$ of this event, $\sim 5 \times 10^{49}$ ergs, is a factor of 20 smaller than the standard value, if the jet break time of ∼0.48 days is assumed (Vanderspek et al. 2003; Price et al. 2003). GRB 980425 was the first GRB associated with a supernova event, SN 1998bw at $z = 0.0085$ (36 Mpc; Galama et al 1998; Kulkarni et al. 1998; Woosley, Eastman, & Schmidt 1998; Pian et al. 2000, 2003). There are some other events that might be associated with supernovae (Della Valle et al. 2003; Wang & Wheeler 1998; Germany et al. 2000; Rigon et al. 2003). Therefore, the association of the long-duration GRBs with supernovae is strongly suggested and at least some GRBs arise from the collapse of a massive star.

In this context, it is important to investigate whether GRB 980425/SN 1998bw is similar to more or less typical long-duration GRBs like GRB 030329/SN 2003dh. However, GRB 980425 showed unusual observational properties. The isotropic equivalent $\gamma$-ray energy is $E_{\rm iso} \sim 6 \times 10^{47}$ ergs, and the geometrically corrected energy is $E_\gamma \sim 3 \times 10^{46}$ ergs $(\Delta\theta/0.3)^2$, where $\Delta\theta$ is the unknown jet opening half-angle. These energies are much smaller than the typical values of GRBs, $E_\gamma \sim 1 \times 10^{51}$ ergs (Bloom, Frail, & Kulkarni 2003; Frail et al. 2001). Bloom et al. (2003) claim that there should be some events with small $E_\gamma$ such as GRB 980519 and GRB 980326 and that GRB 980425 might be a member of this class. The other properties of GRB 980425 are also unusual: the large low-energy flux, the long spectral lag, the low variability, and the slowly decaying X-ray luminosity of its counterpart detected and monitored by *BeppoSAX* and by *XMM-Newton* (Frontera et al. 2000a; Norris, Marani, & Bonnell 2000; Fenimore & Ramirez-Ruiz 2000; Pian et al. 2000, 2003).

Previous works suggest that the above peculiar observed properties of GRB 980425 might be explained if the standard jet is seen from the off-axis viewing angle (Ioka & Nakamura 2001; Nakamura 1999; Nakamura 2001; see also Maeda et al. 2002, Iwamoto 1999, Dado, Dar, & De Rújula 2003, and Dar & De Rújula 2000). Following this scenario, the relativistic beaming effect reduces $E_{\rm iso}$ and hence $E_\gamma$. The quantity $E_{\rm iso}$ is roughly proportional to $\delta^2$–$\delta^3$ for the typical observed spectrum, where $\delta = \{\gamma\,[1 - \beta\cos(\theta_v - \Delta\theta)]\}^{-1}$ is the Doppler factor and $\theta_v$ is the viewing angle (Yamazaki, Ioka, & Nakamura 2002). Since $E_{\rm iso}$ is $\sim 10^4$–$10^5$ times smaller than the standard value, $\delta$ should be 20–$10^2$ times smaller than the usual value. Then the peak energy $E_p$ ($\propto\delta$) becomes 20–$10^2$ times smaller than the on-axis $E_p$, which is measured when the jet is seen from the on-axis viewing angle. However, the observed $E_p$ of GRB 980425 (∼50 keV) is only a factor 4 or 5 smaller than the typical value of ∼250 keV. Therefore, one might consider that GRB 980425 belongs to a different class of GRBs.

It is well known that the distribution of $E_p$ is lognormal with a mean of $\langle E_p\rangle \sim 250$ keV (Preece et al. 2000). Ioka & Nakamura (2002) showed that if the distribution of the intrinsic $E_p$ [i.e., $E_p(1+z)$] is lognormal, the redshifted one is also lognormal, under the assumption that the redshifts of the observed GRBs are random. Therefore, $\langle E_p(1+z)\rangle \sim 570$ keV since the mean value of the measured redshifts is ∼1.3 (Bloom et al. 2003). There are some GRBs with even higher intrinsic peak energy; for example, $E_p(1+z) \sim 2.0$ MeV for GRB 990123 (Amati et al. 2002), and $E_p(1+z) \sim 3.6$ MeV for GRB 021004 (Barraud et al. 2003). Furthermore, Figure 3 of Schaefer (2003) shows that the highest value of $E_p(1+z)$ detected by BATSE is about 4 MeV. Since GRB 980425 is the nearest GRB, the redshift factor is not important. In this sense, the peak energy of GRB 980425 is at least a factor of ∼10 smaller than the usual one of ∼570 keV. Suppose that the intrinsic $E_p$ of GRB 980425 is similar to that of GRB 990123 and GRB 021004 when the jet of GRB 980425 is seen from the on-axis viewing angle. Then the observed $E_p$ of GRB 980425 is $\sim 10^2$ times smaller than the intrinsic $E_p$ of GRB 990123 and GRB 021004. This is the reason why we are inclined to reconsider the off-axis jet model for GRB 980425.

In this Letter, assuming a rather large on-axis $E_p$, we reconsider the prompt emission of GRB 980425 using the simple model in Yamazaki et al. (2002) and Yamazaki, Ioka, & Nakamura (2003b) to reproduce its unusual observed quantities. In § 2, in order to extract the observational properties that should be compared with our theoretical model for prompt emission of the GRB, we analyze the BATSE data of GRB 980425. In § 3, we describe a simple jet model, including the cosmological

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effect. We assume a uniform jet with a sharp edge. Numerical results are shown in § 4. Section 5 is devoted to discussions. Throughout this Letter, we adopt a flat universe with $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.7$.

## 2. SPECTRAL ANALYSIS FOR PROMPT EMISSION OF GRB 980425 USING BATSE DATA

In our simple jet model of GRBs, the time dependence of spectral indices is not treated, although it is known that the spectral parameters of GRB 980425 changed over time (Galama et al. 1998; Frontera et al. 2000a). Hence, we should discuss the time-averaged observed spectral properties of GRB 980425 before we apply our model to them.

Using the BATSE data of GRB 980425, we analyze the spectrum within the time of the FWHM of the peak flux in the light curve of BATSE channel 2 (50–110 keV). This time interval approximately corresponds to portions "B" and "C" in Frontera et al. (2000a), when most of photons arrived at the detector and the spectral shape was approximately constant with time. We fitted the observed spectrum with a smoothly broken power-law function given by Band et al. (1993) that is characterized by the energy at the spectral break $E_0$ and the low- and high-energy photon indices $\alpha$ and $\beta$, respectively. For the case of $\beta < -2$, the peak energy is derived as $E_p = (2 + \alpha)E_0$. The best-fit spectral parameters are

$$\alpha = -1.0 \pm 0.3,$$

$$\beta = -2.1 \pm 0.1,$$

$$E_p = 54.6 \pm 20.9 \text{ keV}.$$

The reduced $\chi^2$ is 1.10 for 31 degrees of freedom. These results are consistent with those derived by the previous works (Frontera et al. 2000a; Galama et al. 1998). Although the photon indices are the typical values of GRBs, $E_p$ is lower than the typical values of GRBs (Preece et al. 2000). This spectral property is similar to one of the recently identified class of the X-ray flash (Kippen et al. 2003; Heise et al. 2001).

The observed fluence of the entire emission between 20 and 2000 keV is $S(20\text{–}2000 \text{ keV}) = (4.0 \pm 0.74) \times 10^{-6}$ ergs cm$^{-2}$, so the isotropic equivalent $\gamma$-ray energy becomes $E_{\rm iso} = (6.4 \pm 1.2) \times 10^{47}$ ergs. The fluence ratio is $R_s = S(20\text{–}50 \text{ keV})/S(50\text{–}320 \text{ keV}) = 0.34 \pm 0.036$. In the following sections, we reproduce the above results using our prompt emission model.

## 3. MODEL OF PROMPT EMISSION OF GRBs

We use a simple jet model of prompt emission of GRBs adopted in Yamazaki et al. (2003b), in which the cosmological effect is included (see also Yamazaki et al. 2002, Yamazaki, Ioka, & Nakamura 2003a, and Ioka & Nakamura 2001). We adopt an instantaneous emission of an infinitesimally thin shell at $t = t_0$ and $r = r_0$. Then the observed flux of a single pulse is given by

$$F_\nu(T) = \frac{2(1+z)r_0 c A_0}{d_L^2} \frac{\Delta\phi(T) f[\nu_z\gamma([1 - \beta\cos\theta(T)])]}{[\gamma(1 - \beta\cos\theta(T))]^2}, \quad (1)$$

where $1 - \beta\cos\theta(T) = (1+z)^{-1}(c\beta/r_0)(T - T_0)$ and $A_0$ determines the normalization of the emissivity. A detailed derivation of equation (1) and the definition of $\Delta\phi(T)$ are found in Yamazaki et al. (2003b). In order to have a spectral shape similar to that derived by the previous section, we adopt the following form of the spectrum in the comoving frame:

$$f(\nu') = \begin{cases} (\nu'/\nu'_0)^{1+\alpha_B} \exp(-\nu'/\nu'_0) \\ \quad \text{for } \nu'/\nu'_0 \le \alpha_B - \beta_B, \\ (\nu'/\nu'_0)^{1+\beta_B}(\alpha_B - \beta_B)^{\alpha_B - \beta_B} \exp(\beta_B - \alpha_B) \\ \quad \text{for } \nu'/\nu'_0 \ge \alpha_B - \beta_B, \end{cases} \quad (2)$$

with $\alpha_B = -1$ and $\beta_B = -2.1$. Equations (1) and (2) are the basic equations for calculating the flux of a single pulse, which depends on the following parameters; $\gamma$, $\gamma\nu'_0$, $\theta_v$, $\Delta\theta$, $r_0/c\beta\gamma^2$, $z$, and $A_0$. In the next section, the viewing angle $\theta_v$ and the jet opening half-angle $\Delta\theta$ are mainly varied. The other parameters are fixed as follows. The quantity $\gamma$ is fixed as $\gamma = 100$. The isotropic $\gamma$-ray energy is calculated as $E_{\rm iso} = 4\pi(1 + z)^{-1} d_L^2 S(20\text{–}2000 \text{ keV})$, where $S(\nu_1 - \nu_2)$ is the observed fluence in the energy range $h\nu_1$–$h\nu_2$ keV. We fix the amplitude $A_0$ so that the geometrically corrected $\gamma$-ray energy $E_\gamma = (\Delta\theta)^2 E_{\rm iso}/2$ is the observationally preferred value when we see the jet from the on-axis viewing angle $\theta_v = 0$. It is shown that $E_\gamma$ is tightly clustering about a standard energy $\mathcal{E}_\gamma$ of $\sim 10^{51}$ ergs (Bloom et al. 2003; see also Frail et al. 2001 and Panaitescu & Kumar 2002). Bloom et al. (2003) derived this energy as

$$\log \mathcal{E}_\gamma = \log [1.15 \times 10^{51} (h/0.7)^{-2} \text{ ergs}] \pm 0.07, \quad (3)$$

so that $\mathcal{E}_\gamma = (0.98\text{–}1.35) \times 10^{51}$ ergs at the 1 $\sigma$ level and $\mathcal{E}_\gamma = (0.51\text{–}2.57) \times 10^{51}$ ergs at 5 $\sigma$ level. Note that the smaller jet opening half-angle $\Delta\theta$ corresponds to the larger $A_0$ (Yamazaki et al. 2003b).

Practical calculations show that when the jet with $\alpha_B = -1$ and $\beta_B = -2.1$ is seen from the on-axis viewing angle $\theta_v = 0$, the observed peak energy becomes $E_p^{(\theta_v=0)} \sim 1.54\gamma\nu'_0(1 + z)^{-1}$, which is independent of $\Delta\theta$ being larger than $\sim\gamma^{-1}$. In order to reproduce the observed quantities of GRB 980425, we adopt the value $\gamma\nu'_0 = 2600$ keV, which yields $E_p^{(\theta_v=0)}(1+z) \sim 4.0$ MeV. For comparison, we consider another case of $\gamma\nu'_0 = 1300$ keV, which reads $E_p^{(\theta_v=0)}(1+z) \sim 2.0$ MeV. These values correspond to the intrinsic $E_p$ of GRB 021004 and GRB 990123, respectively. Note here that in our jet model, the quantities that will be calculated in the next section do not depend on $r_0/c\beta\gamma^2$; for example, $E_{\rm iso} \propto A_0(r_0/c\beta\gamma^2)^2 \propto (r_0/c\beta\gamma^2)^0$ since $A_0 \propto (r_0/c\beta\gamma^2)^{-2}$. The value of $r_0/c\beta\gamma^2$ will be determined when we discuss the spectral lag in § 5.

## 4. ISOTROPIC ENERGY, PEAK ENERGY, AND FLUENCE RATIO

We now calculate the isotropic equivalent $\gamma$-ray energy $E_{\rm iso}$ as a function of $\theta_v$ and $\Delta\theta$. Then the peak energy $E_p$ and the fluence ratio $R_s = S(20\text{–}50 \text{ keV})/S(50\text{–}320 \text{ keV})$ are computed for the set of $\Delta\theta$ and $\theta_v$ that reproduces the observed $E_{\rm iso}$ of GRB 980425.

For fixed $\Delta\theta$ and $\mathcal{E}_\gamma$, $E_{\rm iso}$ is calculated as a function of the viewing angle $\theta_v$. The result is shown in Figure 1. When $\theta_v \lesssim \Delta\theta$, $E_{\rm iso}$ is essentially constant, and when $\theta_v \gtrsim \Delta\theta$, $E_{\rm iso}$ is considerably smaller than the typical value of $\sim 10^{51}$–$10^{53}$ ergs because of the relativistic beaming effect. In order to explain the observation, $\theta_v$ should be $\sim 21°$ in the case of $\Delta\theta = 15°$ and $\theta_v \sim 25°$ in the case of $\Delta\theta = 20°$. This result does not depend on $\gamma\nu'_0$ so much.

The upper panels of Figures 2 and 3 show $\theta_v^*$, for which $E_{\rm iso}$ becomes equal to the observed values, as a function of $\Delta\theta$ in the case of $\gamma\nu'_0 = 2600$ keV and 1300 keV, respectively. Since the emissivity ($\propto A_0$) of the jet is small for large $\Delta\theta$, the rela-

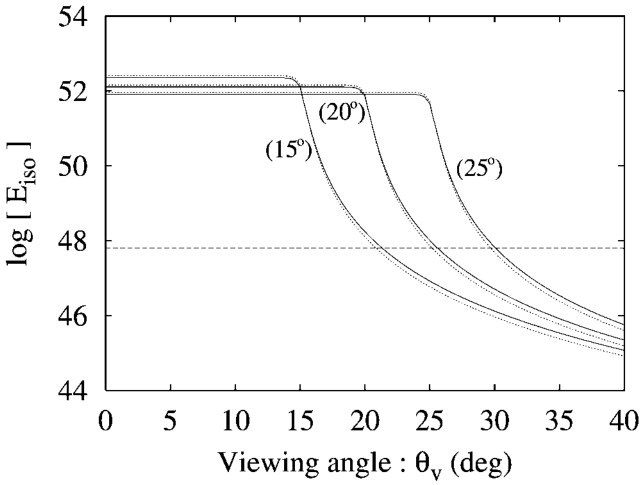


FIG. 1.—Isotropic equivalent $\gamma$-ray energy $E_{\rm iso}$ shown as a function of the viewing angle $\theta_v$ for a fixed jet opening half-angle $\Delta\theta$. The source is located at $z = 0.0085$. The values of $\Delta\theta$ are shown in parentheses. The solid lines correspond to the case of $\gamma\nu_0' = 2600$ keV, and the dotted lines to $\gamma\nu_0' = 1300$ keV. Other parameters are fixed as $\alpha_B = -1$, $\beta_B = -2.1$, $\gamma = 100$, and $\mathcal{E}_\gamma = 1.15 \times 10^{51}$ ergs. The horizontal dashed line represents the observed value of GRB 980425, $E_{\rm iso} = 6.4 \times 10^{47}$ ergs. The value of $E_{\rm iso}$ in the on-axis case, $\theta_v < \Delta\theta$, is slightly smaller for $\gamma\nu_0' = 2600$ keV than for $\gamma\nu_0' = 1300$ keV. This is because the amplitude $A_0$ is fixed so that we should observe a constant $E_\gamma$ from the source at $z = 1$, and the $K$-correction is larger for $\gamma\nu_0' = 1300$ keV than for $\gamma\nu_0' = 2600$ keV.

tivistic beaming effect should be weak for large $\Delta\theta$. Therefore, the value of $\theta_v^* - \Delta\theta$ is a decreasing function of $\Delta\theta$. For such $\theta_v^*$, we calculate the fluence ratio $R_s^* = R_s^{(\theta_v = \theta_v^*)}$ and the peak energy $E_p^* = E_p^{(\theta_v = \theta_v^*)}$. The middle and the lower panels of Figures 2 and 3 show the results. The quantity $E_p^*$ is proportional to the Doppler factor $\delta \sim \{\gamma[1 - \beta \cos(\theta_v^* - \Delta\theta)]\}^{-1}$. Therefore, when $\Delta\theta$ increases, $\theta_v^* - \Delta\theta$ decreases so that $E_p^*$ increases. Since we fix spectral indices $\alpha_B$ and $\beta_B$, $R_s^*$ depends only on $E_p^*$. Hence, if $E_p^*$ is large, the spectrum is hard, and $R_s^*$ is small. For the fiducial parameters of $\gamma\nu_0' = 2600$ keV, $\mathcal{E}_\gamma = 1.15 \times 10^{51}$ ergs, and $E_{\rm iso} = 6.4 \times 10^{47}$ ergs, $\Delta\theta$ should be between $\sim 18°$ and $\sim 31°$, and then $\theta_v^*$ ranges between $\sim 24°$ and $\sim 35°$ in order to reproduce the observed values of $R_s$ and $E_p$. When $\mathcal{E}_\gamma$ is varied from $0.51 \times 10^{51}$ to $2.6 \times 10^{51}$ ergs (at the 5 $\sigma$ level), the allowed region with $20° \lesssim \Delta\theta \lesssim 30°$ can exist even in the case of $\gamma\nu_0' = 1300$ keV.

Note that $\gamma$ does not affect our results for observed $R_s^*$ and $E_p^*$. When $\gamma$ is large, $\theta_v^*$ becomes small because the observed flux for fixed $\theta_v$ becomes small because of the stronger relativistic beaming effect. However, we can see that $\gamma(\theta_v^* - \Delta\theta)$ remains almost unchanged even if $\gamma$ is varied. Then, for fixed $\gamma\nu_0'$, $E_p^*$ remains constant since $E_p^* \propto \nu_0'\delta \sim 2\gamma\nu_0'\{1 + [\gamma(\theta_v^* - \Delta\theta)]^2\}^{-1}$. The quantity $R_s^*$ depends only on $E_p^*$, so that $\gamma$ does not affect the estimate of $R_s^*$.

## 5. DISCUSSION

We considered the time-averaged emissions, which means that successive emissions from multiple subjets (or shells) are approximated by one spontaneous emission caused by a single jet (Yamazaki et al. 2002). We choose $\alpha_B = -1$, $\beta_B = -2.1$, $\gamma = 100$, and $\gamma\nu_0' = 2600$ keV for the canonical set of parameters. As a result, when the jet with an opening half-angle of $\Delta\theta \sim 10°$–$30°$ is seen from the off-axis viewing angle of $\theta_v \sim \Delta\theta + 6°$, observed quantities can be well explained. Derived $\theta_v$ and $\Delta\theta$ are consistent with those suggested in Nakamura (1999, 2001) and Maeda et al. (2002). We may also be able

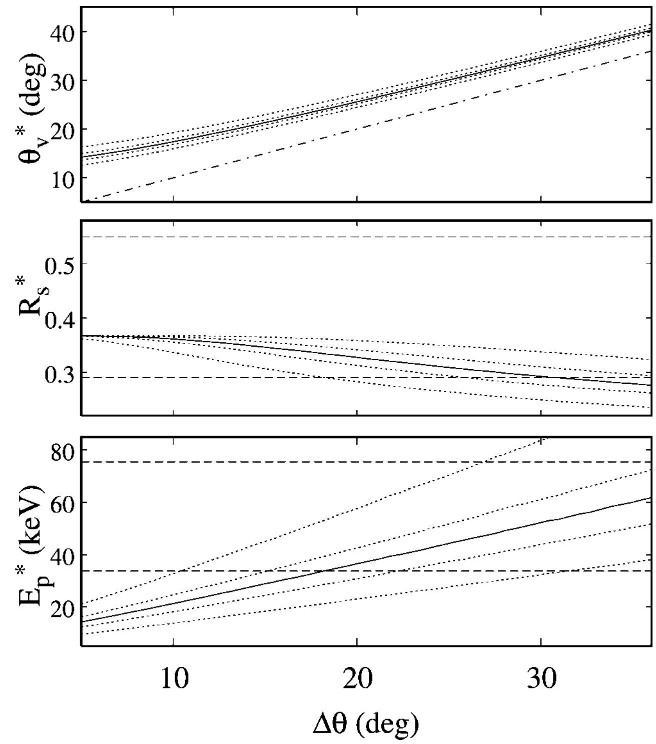


FIG. 2.—*Upper panel*: $\theta_v^*$ for which $E_{\rm iso}$ is the observed value of GRB 980425. *Middle and lower panels*: The fluence ratio $R_s^* = R_s^{(\theta_v = \theta_v^*)}$ and the peak energy $E_p^* = E_p^{(\theta_v = \theta_v^*)}$, respectively. The solid lines correspond to the fiducial case of $E_{\rm iso} = 6.4 \times 10^{47}$ ergs and $\mathcal{E}_\gamma = 1.15 \times 10^{51}$ ergs. The dotted lines represent regions where $E_{\rm iso}$ becomes $(6.4 \pm 1.2) \times 10^{47}$ ergs when $\mathcal{E}_\gamma$ is in the 1 $\sigma$ and 5 $\sigma$ level around the fiducial value, respectively. Other parameters are fixed as $\alpha_B = -1$, $\beta_B = -2.1$, $\gamma = 100$, and $\gamma\nu_0' = 2600$ keV. The dot-dashed line in the upper panel represents $\theta_v^* = \Delta\theta$. The horizontal dashed lines in the middle and lower panels represent the observational bounds $R_s = 0.42 \pm 0.13$ and $E_p = 54.6 \pm 20.9$ keV, respectively.

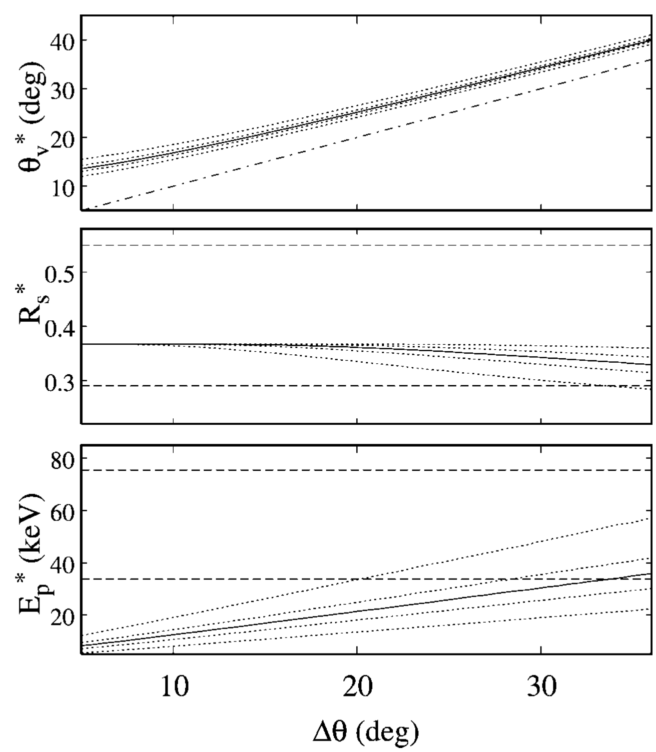


FIG. 3.—Same as Fig. 2, but for $\gamma\nu_0' = 1300$ keV

to explain the observed low variability since only subjets at the edge of the cone contribute to the observed quantities (see the discussion in Yamazaki et al. 2002). If the jet is seen from an on-axis viewing angle (i.e., $\theta_v < \Delta\theta$), the intrinsic peak energy $E_p(1 + z)$ is ∼4.0 MeV, which is almost the same as the highest one (Schaefer 2003; Amati et al. 2002; Barraud et al. 2003).

As we have mentioned in § 3, $E_{\rm iso}$, $E_p^*$, and $R_s^*$ do not depend on the parameter $r_0/\beta c\gamma^2$. In order to estimate the value of $r_0/\beta c\gamma^2$, we discuss the spectral lag of GRB 980425 (Ioka & Nakamura 2001). In our model, we can calculate the spectral lag $\Delta T$, which is defined, for simplicity, as the difference of the peak time between BATSE energy channels 1 and 3. We obtain $\Delta T/(r_0/c\beta\gamma^2) = 0.97$–1.34. Therefore, the observed value of $\Delta T = 3$ s (Norris et al. 2000) can be explained when $r_0/c\beta\gamma^2 = (2.2$–3.1) s, which is in the reasonable parameter range.

The observed quantities of small $E_p$ and large fluence ratio $R_s$ (see also Frontera et al. 2000a) are the typical values of the X-ray flash (Heise et al. 2001; Kippen et al. 2003; see also Barraud et al. 2003 and Arefiev, Priedhorsky, & Borozdin 2003). The operational definition of the X-ray flash detected by *BeppoSAX* is a fast X-ray transient with a duration of less than $\sim 10^3$ s that is detected by Wide Field Cameras and not detected by the Gamma Ray Burst Monitor (GRBM; Heise et al. 2001). If the distance to the source of GRB 980425 that has an opening half-angle of $\Delta\theta = 20°$ were larger than ∼86 Mpc, the observed flux in the γ-ray band would have been less than the limiting sensitivity of the GRBM, $\sim 5 \times 10^{-7}$ ergs cm$^{-2}$ in 40–700 keV band (Band 2003), so that the event would have been detected as an X-ray flash.

We might be able to explain the origin of a class with low $E_\gamma$, pointed out by Bloom et al. (2003). Let us consider the jet seen from a viewing angle $\theta_v \sim \Delta\theta + \gamma_i^{-1}$, where $\gamma_i$ is the Lorentz factor of a prompt γ-ray–emitting shell. Due to the relativistic beaming effect, observed $E_\gamma$ of such a jet becomes an order of magnitude smaller than the standard energy (see Fig. 1). At the same time, the observed peak energy $E_p$ is small because of the relativistic Doppler effect. In fact, the observed $E_p$ of GRB 980326 and GRB 981226 are ∼35 and ∼60 keV, respectively (Amati et al. 2002; Frontera et al. 2000b). In our model, the fraction of GRBs with low $E_\gamma$ becomes $2/(\gamma_i\Delta\theta) \sim 0.1$ since the mean value of $\Delta\theta \sim 0.2$, while a few GRBs with low $E_\gamma$ are observed in ∼30 samples (Bloom et al. 2003). In later phase, the Lorentz factor of the afterglow-emitting shock $\gamma_f$ is smaller than $\gamma_i$, so that $\theta_v < \Delta\theta + \gamma_f^{-1}$. Then the observed properties of the afterglow may be similar to the on-axis case $\theta_v \ll \Delta\theta$; hence, the observational estimation of the jet break time and the jet opening angle remains the same.

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## REFERENCES


Amati, L., et al. 2002, A&A, 390, 81
Arefiev, V. A., Priedhorsky, W. C., & Borozdin, K. N. 2003, ApJ, 586, 1238
Band, D. L. 2003, ApJ, 588, 945
Band, D., et al. 1993, ApJ, 413, 281
Barraud, C., et al. 2003, A&A, 400, 1021
Bloom, J. S., Frail, D. A., & Kulkarni, S. R. 2003, ApJ, 594, 674
Dado, S., Dar, A., & De Rújula, A. 2003, A&A, 401, 243
Dar, A., & De Rújula, A. 2000, preprint (astro-ph/0008474)
Della Valle, M., et al. 2003, A&A, 406, L33
Fenimore, E. E., & Ramirez-Ruiz, E. 2000, preprint (astro-ph/0004176)
Frail, D. A., et al. 2001, ApJ, 562, L55
Frontera, F., et al. 2000a, ApJS, 127, 59
———. 2000b, ApJ, 540, 697
Galama, T. J., et al. 1998, Nature, 395, 670
Germany, L. M., Reiss, D. J., Sadler, E. M., Schmidt, B. P., & Stubbs, C. W. 2000, ApJ, 533, 320
Heise, J., in 't Zand, J., Kippen, R. M., & Woods, P. M. 2001, in Proc. Second Rome Workshop: Gamma-Ray Bursts in the Afterglow Era, ed. E. Costa, F. Frontera, & J. Hjorth (Berlin: Springer), 16
Hjorth, J., et al. 2003, Nature, 423, 847
Ioka, K., & Nakamura, T. 2001, ApJ, 554, L163
———. 2002, ApJ, 570, L21
Iwamoto, K. 1999, ApJ, 512, L47
Kippen, R. M., Woods, P. M., Heise, J., in 't Zand, J. J. M., Briggs, M. S., & Preece, R. D. 2003, in AIP Conf. Proc. 662, Gamma-Ray Burst and Afterglow Astronomy 2001: A Workshop Celebrating the First Year of the *HETE* Mission, ed. G. R. Ricker & R. K. Vanderspek (Melville: AIP), 244
Kulkarni, S. R., et al. 1998, Nature, 395, 663
Maeda, K., Nakamura, T., Nomoto, K., Mazzali, P. A., Patat, F., & Hachisu, I. 2002, ApJ, 565, 405
Nakamura, T. 1999, ApJ, 522, L101
———. 2001, Prog. Theor. Phys. Suppl., 143, 50
Norris, J. P., Marani, G. F., & Bonnell, J. T. 2000, ApJ, 534, 248
Panaitescu, A., & Kumar, P. 2002, ApJ, 571, 779
Pian, E., et al. 2000, ApJ, 536, 778
———. 2003, Adv. Space Res., in press (astro-ph/0304521)
Preece, R. D., Briggs, M. S., Mallozzi, R. S., Pendleton, G. N., Paciesas, W. S., & Band, D. L. 2000, ApJS, 126, 19
Price, P. A., et al. 2003, Nature, 423, 844
Rigon, L., et al. 2003, MNRAS, 340, 191
Schaefer, B. E. 2003, ApJ, 583, L71
Stanek, K. Z., et al. 2003, ApJ, 591, L17
Uemura, M., et al. 2003, Nature, 423, 843
Vanderspek, R., et al. 2003, GCN Circ. 1997 (http://gcn.gsfc.nasa.gov/gcn/gcn3/1997.gcn3)
Wang, L., & Wheeler, J. C. 1998, ApJ, 504, L87
Woosley, S. E., Eastman, R. G., & Schmidt, B. P. 1999, ApJ, 516, 788
Yamazaki, R., Ioka, K., & Nakamura, T. 2002, ApJ, 571, L31
———. 2003a, ApJ, 591, 283
———. 2003b, ApJ, 593, 941