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Mathematical Strengths, Difficulties and Misconceptions of Teachers: Analysis of Their Performance in an Achievement Test

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Abstract

Identifying teachers' strengths and weaknesses is of crucial importance for the design of in-service training activities aimed at improving the teaching/learning process. In this connection, this paper presents an examination of teachers' performance in an achievement test. It presents an analysis of the teachers' common strengths, difficulties and misconceptions as reflected in how they went about identifying, analyzing and dealing with mathematical concepts in a given problem. Also, it points out the common areas in mathematics where the teachers significantly made improvement after participating in the training program.

After a careful investigation of the teachers' working papers in the achievement tests, the researcher also found out that the misconceptions learners have surrounding a mathematical concept contribute largely to their difficulty in dealing with that concept. This confirms Rosnicks (1981) finding in his study that focused on the students' ability to translate English sentence into algebraic expressions, and vice versa.

Introduction

In the field of education, it is an accepted notion that a teacher must possess a very clear understanding of a concept to be able to significantly impart information on such concept to the pupils. For this reason, it was believed that studies dealing with the areas of difficulty in mathematics assume that only teachers are knowledgeable and students are the only party having difficulties (Levenberg and Ophir, 2001). However, in a study conducted by Lawrenz (1986) about in-service elementary school teachers' understanding of some physical science concepts, it was found out that while some of the errors the teachers committed were due to lack of content knowledge, others were indicative of serious misconceptions. Thus, teachers, like any other learners, do perpetuate some difficulties and misconceptions.

Hancock (1940) considered misconceptions to arise from faulty reasoning, while Barass (1984) argued that they are misleading ideas. Whichever the case may be, it is believed that misconceptions may come from confusion or lack of knowledge. In the article "Summary of Misconceptions about Decimal Numbers", it was noted that the misconception of the learner is associated with his/her place value difficulties. The difficulty, therefore, that one possesses about a concept may lead to his/her mis-

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conception of other related concepts.

Michael (2001), on the other hand, contended that misconceptions are conceptual or reasoning difficulties that hinder learners' mastery of any discipline. For the learners to be able to confront underlying conceptual difficulties requires overcoming these misconceptions.

Considering the different studies mentioned, in this paper the teacher-participants' answers in the achievement tests given as pretest and posttest in the training program were carefully examined. The pretest was intended to find out the teachers' existing knowledge and skills as well as their strengths, difficulties and misconceptions in mathematics. The posttest was given to determine the improvement in the teachers' knowledge and skills in mathematics and also persistent problems they encountered despite the training. As a whole, this paper presents the teacher-participants mathematical strengths, difficulties and misconceptions in mathematics and also the areas where they made improvements after the training program.

Since part of this paper reveals the effect of the training program on the teachers' mathematical ability, the inclusion of the In-service Education and Training (INSET) Curriculum Context as an independent section in this paper is deemed necessary.

The INSET Curriculum Context

Elementary mathematics education should not only focus on the development of mathematical concepts and skills but the learners' thinking as well. Taking this into account, the in-service training program has two major objectives: to upgrade the teacher-participants' knowledge of mathematical content and to enhance their thinking skills and processes. To carry out these objectives, the teacher-participants were made to engage in two types of sessions: (a) those that dealt directly with the strategies on how to teach a mathematical concept while focusing on the development of thinking skills; and (b) those that were about the curriculum content (UP NISMED Elementary School Mathematics Workgroup Report, 1996). These two areas, however, through the use of manipulative materials were closely intertwined in every session.

The mathematical contents tackled in the training program were based on the Elementary Learning Competencies (ELC) prescribed by the Curriculum Development Division of the Department of Education, Culture and Sports (DECS). These were numbers and operations, fractions, ratio, decimals, linear measurement, area measurement, mass measurement, and geometry. Meanwhile, the strategy-related topics included: thinking skills, assessment, lesson planning, outdoor mathematics, problem solving, and games in mathematics.

On the first day of the training, the teacher-participants were informed that as part of the training program, each of them be required to conduct a demonstration teaching with his/her peers. Thus, towards the end of the program, the session on microteaching was held. In this session, the teacher-participants were tasked to demonstrate a lesson focusing on the "rule formation or concept development" stage only due to time constraint. Its purpose was to provide an opportunity for the teacher-participants to review, practice and enhance the teaching practices and techniques introduced and modeled in many occasions during the training program. The critique that followed each demonstration lesson proved to be very enlightening because it gave the teacher-participants the chance to clarify strategy- and content-related questions. More importantly, they learned from each other's strengths and weaknesses.

The specific objectives of the training program were as follows:

1. To discuss the importance of developing the learners' mathematical concepts;
2. To use practical work effectively in teaching mathematics;
3. To create opportunities for developing higher order thinking ; and
4. To upgrade mastery of the curriculum content.

Method

This study was conducted based on the teacher-participants' performance in the achievement test in the UP NISMED teacher-training program for elementary mathematics. Thirty-five teachers participated in the training program, but two were not able to take both the pretest and the posttest. As a result, only the teacher-participants with complete outputs were considered in the analysis for this study. These teachers teach grades 1 to 4 pupils. They come from different schools in one of the divisions of city schools in Manila, Philippines. The complete enumeration sampling design was used in this study.

Table 1 shows the distribution of teacher-participants by school and grade level, while **Table 2** gives a comprehensive view of the distribution of the subjects by age, educational attainment and number of years in teaching mathematics.

The instrument used was the pretest/posttest (Appendix). The UP NISMED's trainers themselves where the researcher happened to be a member developed the test items based on the result of the needs assessment that was conducted to a considerable number of elementary school teachers in the field. Originally, there was a pool of 55 items, but was trimmed down to 35 items after it was pilot-tested on a group of 46 elementary mathematics teachers from different schools in the Philippines. The 35-item test then consisted of eight (8) multiple-choice and twenty-seven (27) open-ended type questions wherein the teacher-participants were required to draw a figure, partition it in order to conform to a required condition, and to engage with one- or two-step problem solving.

The following steps were performed using the McNemar Symmetry chi-square to assess the significance of the difference between performance of the teacher-participants before and after the training program (Bambico, 2002).

a. A McNemar's test uses the Chi-square distribution. Thus, a square table shown in **Figure 1** was prepared in which the diagonal reflects teacher-participants whose performance in the pretest and posttest had not changed.

Figure 1. A square table used in testing Significance of Change

		Posttest +	Posttest -
Pretest	+	A(+,+)	B(+,-)
	-	C(-,+)	D(-,-)

b. The matched pair samples from the pretest and posttest answer sheets were tallied in the prepared table.
c. The responses in each item in the pretest/ posttest were summarized in the square table as follows:
   A. correct in the pretest, correct in the posttest
   B. correct in the pretest, error in the posttest
   C. error in the pretest, correct in the posttest
   D. error in the pretest, error in the posttest
d. The McNemar test uses the chi-square distribution, based on the formula:

$$\text{Chi-square } (X^2) = \frac{\{/B-C/ - 1\}^2}{B+C}$$

Degree-of-freedom = (rows-1)(columns-1)

e. The computed Chi-square (X^2) values were referred to the table of the distribution of Chi-square with 1 degree of freedom (Bohrnstedt & Knoke, 1994). If the computed Chi-square is less than the critical value found in the table for the desired level of significance, in this case .05, the gains in the responses in the pretest and posttest is not significant. Hence, at the .05 level of significance, the critical value of Chi-square is 3.841.

In addition, the results of the above-mentioned test were used to determine the areas or topics in mathematics where the teacher-participants performed best/ least. Also, it served as a reference when the researcher closely examined the actual answer sheets of the teacher-participants to be able to determine their common strengths, misconceptions and difficulties in mathematics.

Table 1. Distribution of Teacher-participants by School and Grade Level

SCHOOL	Grade 1	Grade 2	Grade 3	Grade 4	TOTAL	Percentage
A	3	2	3	3	11	31.4
B	3	3	3	3	12	34.3
C	3	3	3	3	12	34.3
TOTAL	9	8	9	9	35	100

Table 2. Distribution of the Teacher-participants by Age, Educational Attainment, and No. of Years in Teaching Mathematics.

Age	No response	24-29 years	30-34 years	35-39 years	40-44 years	45-49 years	50 andabove	TOTAL
	4	5	5	5	3	9	4	35
Educational Attainment	No response	BD	MU					TOTAL
	1	27	7					35
No. of Yrs. Teaching Mathematics	No response	No experience	Below 5 years	5-9 years	10-14 years	15-19 years	20 andabove	TOTAL
		3	9	9	4	2	8	35

Legend for educational attainment:
BD Bachelor's Degree
MU With master's units

Results

This study aimed to identify if there were significant changes in the teachers' performance in each item in the achievement test. More specifically, it pointed out the common strengths, misconceptions and difficulties of teachers in mathematics.

Table 3 shows that except for item 5, all items showed positive gains. Of the 34 items with positive gains, 17 or 49% yielded a significant change, through which five items (items 10, 15, 19, 31 and 34) had more than 50% of the teacher-participants who did not give correct answers in the pretest answering them correctly in the posttest.

While item 34 showed the highest significant change with 88% of those who were not able to answer correctly in the pretest giving a correct answer in the posttest, item 10 had the 2nd highest significant change with 61% of those who were not able to answer it correctly in the pretest getting it right in the posttest. Item 19, on the other hand, ranked 3rd in the list of items with significant positive change, although there was 64%, when compared to item 10's 61%, of those who did not answer correctly in the pretest but did it right in the posttest. This was so because in item 19, two teacher-participants who did it correctly in the pretest gave a wrong answer in the posttest.

On the other hand, columns A (+, +) and B (+, -) in table 3 show the teachers' common strengths in mathematics. These columns reveal that more than three-fourths or 75% of the teacher-participants showed a sort of mastery of the topics embraced in 10 items (items 1, 3, 4, 5, 8, 22, 24, 30, 32 and 35) even before the training. Item 5 appeared to be the most mastered item since all but one of the teacher-participants got it right before the training was conducted, and after the training the correct response had reached 100%. For item 3, 91% of the teacher-participants answered it correctly in the pretest, while in item 32, 88% of the teacher-participants demonstrated clear understanding of the concept being measured. For items 4 and 8, 85% of the teacher-participants gave correct answers in the pretest.

Meanwhile, column D (-, -) in table 3 shows the teachers' common difficulties and misconceptions in mathematics. In this column, items 9 and 33 appeared to be the most difficult items, for 70% of the teacher-participants had not mastered the concepts being measured even after the training program. In addition, although items 17 and 29 showed positive gains, the percentage of teachers who were not able to answer these items correctly in the posttest was still very high (48%). Items 6, 7, 31 and 18 on the other hand are additional items which reveal a sort of difficulty and misconception of the teacher-participants in mathematics. For both items 7 and 31, 30% of the teacher-participants did not answer correctly even after the training program.

Analysis and Discussion

<u>Teacher-participants' Profile</u>

All the grades 1 to 4 teachers taken as the subjects were females. This shows that in the teaching profession where a teacher starts building a strong foundation at the early stage of one's education is a world dominated by women (Bambico, 2001). Of the 35 teachers, only seven or 20% had masters units. All the rest were baccalaureate graduates of different courses leading to the teaching profession.

As for the teacher-participants' number of years in teaching mathematics, a little more than half of them had taught mathematics from 1 to 9 years. Six had taught from 10 to 19 years while eight had 20 or more years experience in teaching the subject. There were three teachers who did not have the

Table 3. McNemar's Chi-square test of Change in Knowledge of the Concept Measured per Item of the Pretest/ Posttest (N=33)

ITEM NO.	A(+,+)	B(+,-)	C(-,+)	D(-,-)	Chi sqr
1	26	0	7	0	5.14*
2	19	2	5	7	0.57
3	30	0	2	1	0.5
4	26	2	5	0	0.57
5	32	0	1	0	0
6	11	2	7	13	1.78
7	8	0	15	10	13.07*
8	24	4	4	1	0.12
9	3	1	6	23	2.28
10	4	0	20	9	18.05*
11	18	5	8	2	0.31
12	18	1	12	2	7.69*
13	13	6	10	4	0.56
14	18	3	8	4	1.45
15	9	2	20	2	13.14*
16	19	1	10	3	5.82*
17	4	2	11	16	4.92*
18	8	8	8	9	0.06
19	3	2	21	7	14.09*
20	14	3	12	4	5.82*
21	8	3	13	9	5.06*
22	26	1	6	0	2.28
23	18	0	14	1	12.07*
24	24	1	7	1	3.12
25	9	2	15	7	8.47*
26	19	0	14	0	12.07*
27	18	2	13	0	6.67*
28	12	6	8	7	0.07
29	2	1	14	16	9.60*
30	23	2	6	2	1.12
31	1	5	17	10	5.50*
32	29	0	3	1	1.33
33	2	3	5	23	0.12
34	0	3	29	1	19.53*
35	19	6	8	0	0.07

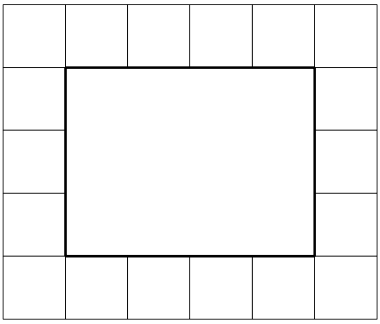
Note: (*) Significant at .05 level, (one-tailed)

chance to teach mathematics at all. The teacher-participants' age, on the other hand, range from 24 to 53 years, with 42 being the average age.

A Glimpse on the Teachers' Performance in the Achievement Test Items as a Result of INSET

Based on the result of the study, the following are the top 3 items with the highest significant change.

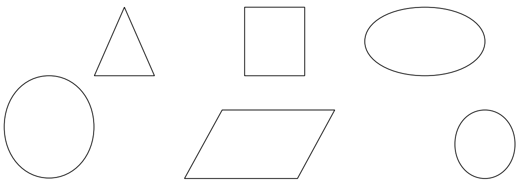
Item 34: *In the grid draw a rectangle having an area of 12 sq units and a perimeter of 14 units*.
Content: Concept of area and perimeter
Answer:



Explanation about the correct answer:
The problem involves application of area and perimeter. To be able to answer this problem the teacher-participants must have a clear understanding of the concept of area and perimeter. Since the formula for getting the area of a rectangle is *l* x *w*, indirectly this problem involves finding the factors of 12. There are three pairs of numbers (they must be whole numbers) whose product is 12. These are: 12 & 1, 6 & 2, and 4 & 3. Then, draw the three rectangles represented by each of the pairs. Remember that each pair represents the sides of the rectangle. Now, applying the idea that perimeter is the sum of all the sides of a rectangle, identify which of the three given rectangles gives a perimeter of 14 units. Thus, the answer is the rectangle whose sides are 4 units and 3 units.

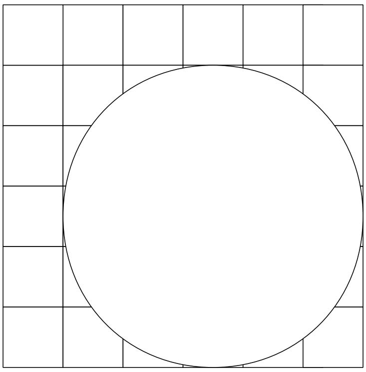
In this item, results in the pretest supported the findings of a study that maintains that area and perimeter are topics known to be points of difficulty in mathematics (Suydam, 1984). The results may reflect that, in particular, confusion of the two persists not only among students but among teachers as well. Perhaps, concrete materials were not used to develop the concept of each so that the distinction between the two is not made clear to them. But more interestingly, after the training program where the use of manipulatives was extensively employed to illustrate these concepts, only 4 or about 12% of the teacher-participants failed to answer it correctly.

Item 10: *What is the ratio of the circles to the total number of shapes?*



Content: Concept of ratio
Answer: 2 is to 6 or 2 : 6
Explanation about the correct answer:
This problem involves part-to-whole ratio. There are six shapes in all of which two are circles. Thus, the ratio of circles to the total number of shapes is 2 is to 6. In ratio notation it is 2 : 6.

Item 19: *What is the radius of the circle?*



Content: Concept of radius of a circle

Answer: 2.5 units

Explanation about the correct answer:

In a grid, one space is considered as one unit and an nxn grid is referred to as a grid with n units on both sides. In this problem, the circle is inscribed in a 6x6 grid through which only one unit on both sides was untouched. Given this condition, the teacher-participant can see that the circle has a diameter of 5 units. The radius of any circle is defined as half of its diameter. Thus, in the problem, the radius of the circle is equal to 2.5 units.

<u>Common Strengths in Mathematics as Reflected in the Pretest</u>

Discussions about some of the items that show teachers' common strengths in mathematics are presented below.

Item 5: *Which is greater 1/3 or 1/5?*

Content: Comparing fractions

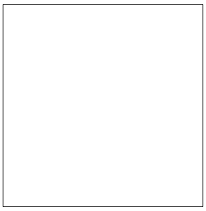
Answer: 1/3

Explanation about the correct answer:

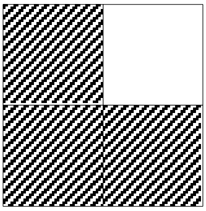
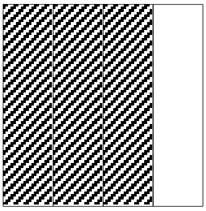
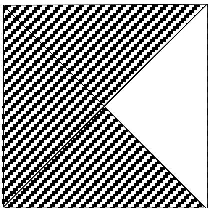
This problem involves comparing unit fractions (fractions having numerators of one). To be able to compare such fractions, it will help to recall the inverse relationship between the number of equal parts a whole is divided into, and the size of each part. Then, relate the denominator of a unit fraction to the size of each part; that is, the more equal parts there are, the greater the denominator, and the smaller each piece. This would lead to the idea that the greater the denominator of a unit fraction, the smaller the value of the fraction.

Clearly, one important thing to point out is that the teacher-participants had enough knowledge on comparing unit fractions. This may be due to their ability to relate the denominator of a fraction to the size of each fractional part. As a result, they seemed to have a thorough understanding of the rule in comparing unit fractions; that is, the unit fraction with the smaller denominator has a higher value than the unit fraction with the larger denominator, and vice versa.

Item 3: *Shade 3/4 of the square below*.



Content: Representation of fraction

Answer:  or  or 

Explanation about the correct answer:

The whole square stands for one whole unit. The whole unit has to be partitioned into four equal parts. One part represents 1/4. Thus, to show 3/4, shade three of the four equal parts.

By examining the working papers of the teacher-participants, it shows that they had acquired an adequate understanding of equal shares or equal parts. Also, the concept that fraction is based on the idea of dividing a unit into equal parts is very clear to them.

Item 32: *What does 4^2 mean? Write it in another way*.

Content: Square of a number

Answer: 4 x 4 or 16

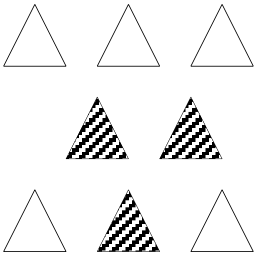
Explanation:

The expression 4^2 may be viewed in several ways. One, it may be interpreted simply as squaring a number; that is, 4 x 4 which is equivalent to 16. But it may also be associated as something similar to the formula for getting the area of a figure, say, a square. In this case, it is given by the formula S x S or S^2.

If S is equal to 4, then S^2 is 4 x 4 which is also equal to 16. Though both approaches are correct and would give the same result, each has its distinct interpretation.

Moreover, in item 32, the teacher-participants demonstrated a clear knowledge of the concept that to get the square of a number is to multiply the number twice. The fact that they have recognized the *base* and the *exponent* in the expression 4^2 is an indication of having a mental grasp on that concept.

Item 4: *What part of the set of triangles is shaded?*



Content: Representation of fractions

Answer: 3/8

Explanation about the correct answer:

In answering the problem, the teacher-participant has to see the set of triangles as the whole unit that is divided into 8 equal parts. The problem asks what part of the set is shaded. The answer is 3 out of 8. In fraction notation it is 3/8.

Item 8: *What is the ratio of leaves to the flowers?*

Content: Concept of ratio

Answer: 2 is to 3 or 2:3

Explanation about the correct answer:

Just like in item 10, item 8 deals with the concept of ratio. But while item 10 is about a part-to-whole ratio, item 8 requires the teacher-participants to deal with part-to-part ratio.

In answering this problem the teacher-participant has to see that there are five objects which consist of two leaves and three flowers. The problem asks for the ratio of leaves to flowers. The answer is 2 is to 3. In ratio notation it is 2:3.

The answer sheets of the teacher-participants revealed that they reasonably have sufficient knowledge on visualizing the ratio of two given objects that constitute a set. Also, they showed a remarkable potential to express the ratio of two numbers using a colon (:).

<u>Common Difficulties and Misconceptions in Mathematics as Reflected in both Pretest and Posttest</u>

After a thorough examination of the teacher-participants answer sheets, the following items were consistent to be the areas of difficulties and misconceptions of the teachers both in the pretest and in the posttest.

Item 9: *There are six pencils and ball pens in a box. If the ratio of pencils to the ball pens is 1:2, how many ball pens are there?*

Content: Concept of ratio

Answer: 4   Frequency of responses: 4 (12%)

Explanation about the correct answer:

Items 8 and 9 are somewhat similar since they both deal with the concept of a part-to-part ratio. But while item 8 is a one-step problem because it simply asks for the ratio of two objects given the set, item 9 is considered a two-step problem. It involves part-to-part ratio with the total number of items given. The ratio 1:2 suggests that for every pencil, there are 2 ball pens, and together there should be 3 items. But there are 6 items in the box, instead of 3. This calls for doubling the number of pencils and ball pens. Therefore, there should be 4 ball pens.

Incorrect Responses:

a) 12   Frequency of responses: 23 (70%)

Analysis for the error:

The teacher-participants must have multiplied 6 by 2, probably thinking that there were 6 pencils, following the clue in the given ratio (1:2) that the number of ball pens is twice this number. The teacher-participants showed an understanding of the meaning of ratio "1:2", but they failed to recognize that 6 refers to the total number of pencils and ball pens. Overall, this result underscores the fact that the teacher-participants do have a great deal of difficulty comprehending the problem situation.

b) Other answers: 2:4, 2, 3, 10, 24   Frequency of responses: 6 (18%)

Analysis for the error:

The teacher-participant who answered 2:4 showed an understanding of a part-to-part ratio given the whole, but he/she failed to identify which part refers to the number of ball pens.

The problem was an open-ended type of test which required the teacher-participants to supply the answer. Probably, some of them just guessed the answer to the problem.

Item 33: *Which of the following would most likely be measured in milligram?*

a. a teaspoonful of water
b. an ordinary paper clip
c. the diameter of a Mongol pencil
d. the thickness of 10 sheets of paper

Content: Mass Measurement

Answer: b   Frequency of responses: 5 (15%)

Explanation about the correct answer:

In answering this problem, the teacher-participant has to know that milligram is one of the standard units used for measuring mass. An ordinary paper clip is a small object that has mass and is preferably measured using milligrams. In this problem, option “a” suggests a unit of measure used for measuring volume, while options “c” and “d” call for a unit of linear measurement.

Incorrect Responses: a   Frequency of responses: 24 (73%)

Analysis for the error:

Clearly, the teacher-participants failed to distinguish milligram from milliliter. A teaspoonful of water implies a unit for measuring volume of a liquid for which a milliliter should be used rather than milligram. This response also shows that a little less than 3/4 of the teacher-participants demonstrate lack of understanding of the different units for measuring mass and volume.

Other incorrect response: d Frequency of responses: 4 (12%)
Analysis for the error:
Apparently, the teacher-participants failed to notice that thickness is a word commonly associated with linear measurement and is to be measured using millimeter. In this option, there seems to be unfamiliarity of mathematical words implicating "length" or "distance".

Item 17: *An embroidered piece of cloth covers an area of one sq. decimeter.*
How many square centimeters does the same piece of cloth cover?
Content: Area Measurement
Answer: 100 sq cm Frequency of responses: 6 (18%)
Explanation about the correct answer:
The embroidered piece of cloth has an area of one square decimeter, thus giving a side measuring 1 dm each. The problem asks for the number of square centimeters covered by the same piece of cloth. Therefore, the decimeter unit has to be converted to a centimeter unit. This will give the piece of cloth a new dimension of 10 cm x 10 cm which covers 100 sq cm in all.

Incorrect responses:
a) 40 Frequency of responses: 7 (21%)
Analysis for the error:
The answer suggests that the teacher-participants succeeded in the first step. That is, changing 1 dm to 10 cm, thus, getting a square with a side of 10-cm each. However, they were unable to make a distinction between area and perimeter. Instead of getting the area, they added the 4 sides together, which is the perimeter of the square, thus getting 40 as the answer. Clearly, they failed to see that the number of square centimeters a figure could cover is the same as its area.
It might also be possible that the teacher-participants could distinguish area from perimeter, but they interchanged the formula for getting the former to that of the latter.

The misconception based on this answer lies on the teachers' lack of conceptual understanding on area and linear measurements. More specifically, there was a misconception that to find the number of units covered by a figure, add the 4 sides.

b) 10 Frequency of responses: 11 (33%)
Analysis for the error:
The working paper of the teacher-participants showed that they succeeded in transforming 1 dm to 10 cm as the problem requires, but made no attempts to continue. They were unable to see that the problem asks for the area covered by the piece of cloth, and not the number of units on its side. The teacher-participants misunderstood the problem.

c) Other responses:
4 Frequency of response: 1 (3%)
1 Frequency of response: 1 (3%)
no answer Frequency of responses: 7 (21%)
Analysis for the error:
The teacher-participants showed lack of conceptual understanding of area measurement. Just like those in response (b), they misunderstood the problem.

Item 29: *List all the factors of 72.*
Content: Comprehension of multiplication
Answer: 1,2,3,4,6,8,9,12,18,24,36,72 Frequency of responses: 3 (9%)
Explanation about the correct answer:
The problem requires the teacher-participants to write all the factors of 72. One approach is to make a factor tree for 72, starting with one of the largest possible factors, say, 36. Then, break 36 down to all its possible factors, this time it will be a lot easier than dealing with 72. Now, list down all distinct factors of 72, and these are: 1,2,3,4,6,8,9,12,18,24,36,72.
Another approach is to do the prime factorization of 72 and then combining some factors to form bigger factor of this number.

Incorrect responses and analysis for the error:
Answers varied tremendously. Though most of them did give 2 or more correct factors, they did not provide the complete list of factors which the problem requires. Those who failed to give all the factors might have had limited knowledge on factorization.

Item 6: *Arrange the following from greatest to smallest:*
5/6, 1/2, 3/3
Content: Ordering Fractions
Answer: 3/3, 5/6, 1/2 Frequency of responses: 13 (39%)
Explanation about the correct answer:
Three-thirds (3/3) is equal to 1. It is the largest fraction of the three because the other two are less than 1 since their numerators are less than their denominators.
When 1/2 is transformed to a fraction similar to 5/6, the result would be 3/6 which is, of course, the smallest of the three given fractions. Thus, the fractions are arranged from greatest to smallest in this order: 3/3, 5/6, 1/2.

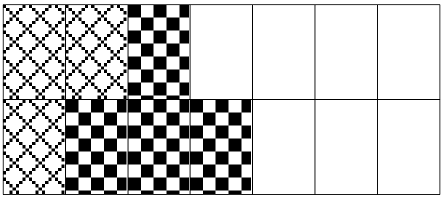
Incorrect Responses:
a. 3/3, 1/2, 5/6 Frequency of responses: 12 (36%)
b. 1/2, 3/3, 5/6 Frequency of responses: 7 (21%)
c. 5/6, 3/3, 1/2 Frequency of response: 1 (3%)
Analysis for the error:
Fifty-seven percent (36% and 21%) thought correctly that 3/3 is greater than 5/6, but the same teacher-participants considered 1/2 to be greater than 5/6. Seven or 21% considered 1/2 to be even greater than 3/3. Apparently, the lone teacher-participant in response (c) arranged the fractions based on

the descending order of both the numerators and the denominators. This misconception reinforces the idea that individuals tend to judge the size of a fraction in terms of the numerical value of their numerator and denominator.

Meanwhile, response (b) is in the opposite direction. If the teacher-participants misunderstood the problem to mean "smallest to greatest" then it demonstrates the same misconception described in response (c).

The error pattern in this item may be ascribed to the teacher-participants difficulty in developing procedures to finding equivalent fractions as an initial step to comparing and ordering fractions. Also, it might be that the teacher-participants have a good mental representation of a fraction, but they have difficulty co-ordinating the size of the numerator and denominator of a fraction. As a result, they wrongfully conclude that 1/2 is greater than 3/3 because they think only about the size of the parts (the halves and the thirds) and cannot simultaneously consider how many parts are being considered with respect to the total number of parts.

Item 7: *A rectangular region is divided into a number of equal parts. Write an addition sentence that shows you are adding the two shaded parts.*



Content: Concept and interpretation of fractions

Answer: 3/14 + 4/14 = 7/14 Frequency of responses: 8 (24%)

Note: When using decimals the number sentence becomes:

0.21 + 0.29 = 0.50

Explanation about the correct answer:

The rectangle which is the whole unit is divided into 14 equal parts. The problem requires the teacher-participants to represent the sum of the two shaded areas in a number sentence. One shaded part is 3/14; the other is 4/14. The sum of the two shaded areas may be expressed as:

3/14 + 4/14 = 7/14.

Incorrect responses:

a) 3+4 = 7 or 4+3 = 7 Frequency of responses: 6 (18%)

Analysis for the error:

The teacher-participants must have confused the concept of fractions with whole numbers. Instead of taking the rectangle as a whole unit and the squares as parts of that unit, they considered each square as one unit; thus representing the two shaded areas as 3 and 4.

b) 3/7 + 4/7 = 7/7 Frequency of responses: 1 (3%)

Analysis for the error:

The whole rectangle has two shaded areas which together cover seven squares. The teacher-participants considered the seven squares as constituting the whole unit rather than just part of the whole unit, which is the rectangle. This response revealed that the teacher-participants showed lack of understanding of the problem, or probably the lack of ability to identify the whole unit within the context of the problem.

c) 1/7 + 1/7 Frequency of responses: 3 (9%)

Analysis for the error:

Just like in response (b), the teacher-participants considered the seven squares as together comprising the whole unit. But, in addition to this error, they also showed difficulty in naming the fractional parts of the whole.

d) 7/14 + 7/14 Frequency of responses:3 (9%)

Analysis for the error:

The teacher-participants must have recognized the combined region as comprising 7/14 of the whole rectangle. But instead of decomposing 7/14 into 3/14 and 4/14, it is not clear why 7/14 was added twice, coming up with a total of 14/14. They failed to see that 14/14 refers to the whole fraction. They should have realized that the shaded regions are just part of the whole rectangle.

The difficulty in this item is similar to the findings in a study conducted by Goldberg and McDermott (1987). The findings noted difficulties with diagrammatic representation. In general, these difficulties may stem from the lack of understanding of the meaning of the concepts in the diagram, or probably not having practiced drawing diagrams of the like.

Item 31: *What number will you write in the box in order to make the statement true?*

$$3 + 4 \times 2 = \square$$

Content: Concept of a number sentence

Answer: 11 Frequency of responses: 6 (18%)

Explanation about the correct answer:

When performing the four fundamental operations in a number sentence, the general rule "MDAS", which stands for multiplication, division, addition and subtraction, in that sequence must be applied. This means that multiplication and division shall be performed prior to addition and subtraction. Of the two pairs, whichever comes first from left to right shall be performed first. In this item, however, only the addition and multiplication operations are present. Hence, following the general rule, 4x2 which is 8 must be done first, then perform 3+8. Thus, the answer is 11.

Incorrect responses:

a) 14 Frequency of responses: 20 (61%)

Analysis for the error:

The teacher-participants must have performed 3+4 and then multiplied 7 which is the sum to 8, thus getting a result of 14.

In this item, the misconception lies on performing operations given a number sentence. This misconception shows that more than half of the total number of participants thought that to perform a series of operations in a number sentence, simply perform the given operation whichever appears first from left to right.

b) 24 Frequency of responses: 7 (21%)

Analysis for the error:

If the teacher-participants overlooked the number sentence to mean 3x4x2 instead of 3+4x2, then the result "24" will be correct. If this was the case, then they would also be correct whichever pair they choose to multiply first over the other.

Item 18: *Which of the following objects will likely measure one square decimeter?*

a. your palm

b. one-half sheet of bond paper

c. the area covered by an ordinary paper clip

d. your ID card

Content: Area Measurement

Answer: d Frequency of responses: 16 (48%)

Explanation about the correct answer:

To answer this problem, one has to make an estimate of the size of a 10 cm x 10 cm figure since one square decimeter gives the area of a figure with 10 cm x 10 cm dimensions. Among the options, "d" suggests the nearest estimate.

Incorrect responses:

a Frequency of responses: 2 (6%)

b Frequency of responses: 6 (18%)

c Frequency of responses: 7 (21%)

no answer Frequency of responses: 2 (6%)

Analysis for the error:

In option "c", apparently the teacher-participants erroneously interpreted one sq dm as one sq cm; thus, coming up with a very small area in the size of a paper clip. While the teacher-participants in option "b" over-estimated the area of 1 sq dm, the ones in option "a" under-estimated it. In general, the teacher-participants, as this error suggests, lack visualization of the area of a given dimension.

Conclusion

In general, it is apparent that teachers performed relatively well on items dealing with basic concepts on rational numbers like comparing fractions, representation of fractions and ratio. However, there was a matter for consideration expressed over teachers' ability to apply relational understanding of concepts in rational numbers involved in word problems.

Also, it is evident that teachers do not possess a clear distinction of mass from area measurement, and also area from perimeter. And lastly, the teachers have insufficient knowledge on factoring a number, and in applying the four basic operations in a number sentence.

Implications for teaching

Simply lecturing teachers on the different topics in mathematics will not help give up their misconceptions. It is because the difficulty and misconceptions that they hold are mostly on understanding mathematical concepts by heart and in comprehending situations in the context of the problem. Taking these into account, teacher education programs should focus on providing teachers opportunities that help develop understanding mathematical concepts. Also, teacher educators should supply them with numerous examples that would give them enough experience and idea on how to effectively deal with problem solving. More importantly, teachers should be taught with the necessary skills that would enable them to, eventually, identify misconceptions held by pupils in their own classrooms.

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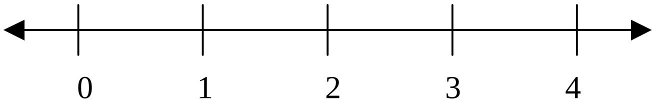
Using GraphPad's McNemar Test Calculator.
[Online] URL: http://www.graphpad.com/calculators/McNemar-Ex.cfm.

Appendix. Pretest/ Posttest of the Training Program

Directions: Supply the required answer to each of the items. In case of multiple choice, encircle the letter of the best answer. You are being encouraged to make drawings or diagrams if it would help you analyze the questions/ problems clearly.

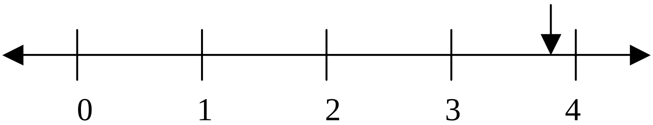
[Note: For items 3, 4, 5, 6, 7, 8, 9, 10, 17, 18,19, 29, 31, 32, 33, and 34, they are presented in the Analysis and Discussion section; thus, are being omitted in the list below.]

1. In the number line below, indicate clearly the location of 2 3/4.



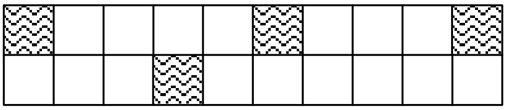
2. There are 10 chocolate cakes of the same size on the table. Represent 2 3/4 by making a drawing below.

11. In the number line what decimal fraction represents the point indicated by the arrow?



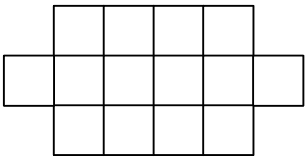
12. What is 4/5 in decimal fraction?

13. Write a decimal fraction to represent the shaded region.



14. Arrange the three numbers from smallest to greatest.
    0.2 , 0.7 , 1.0 ______, ______, ______

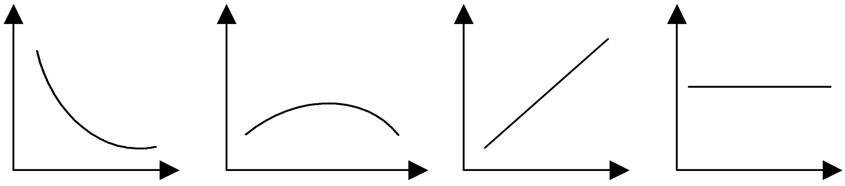
15. Find the perimeter of the region below.



16. The side of a table measures 1.5 metres. This is equivalent to how many centimeters?

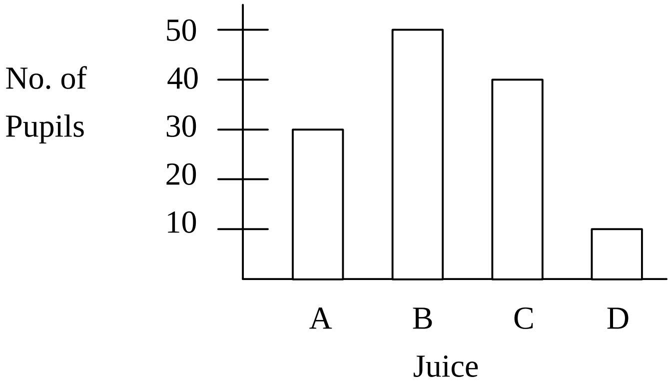
20. Which of the following is likely the temperature of a typical day in January in the Philippines?
    a. 10ºC c. 30ºC
    b. 20ºC d. 40ºC

21. One day last May, Rey recorded, then graphed, the temperature from 7 am to 7 pm. Which of the following graphs would likely show this information?

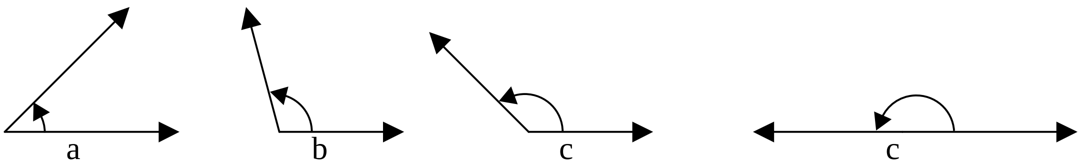


a b c d

22. The following is the result of a survey conducted in order to find out which is the most popular juice in school. What is the difference in votes between the most popular and the least popular juice?



23. Which is the most appropriate graph to use in determining how a variable changes from time to time?
    a. line graph c. bar graph
    b. pictograph d. circle graph
24. Which of the following angles measures most closely to 150?



25. Which of the following measures close to 100 centimetres?
    a. height of the door of this room c. width of the blackboard
    b. waistline of an average person d. length of an unused Mongol pencil
26. How many grams are there in 2 kilograms?
27. Which of the numbers below is divisible both by 6 and 8?
    a. 64 c. 200
    b. 124 d. 264

28. A certain number divided by 24 yields 16, remainder 11. What is the number?
30. Give two odd numbers having a sum of 36.
35. Mang Oscar bought a rectangular lot 12 m long and 5 m wide. He constructed a green house that occupied 1/3 of the lot. What is the area of the green house?