

Finite Grand Unified Theories and Inflation

S. Mukaigawa, T. Muta and S. D. Odintsov [†]

Department of Physics, Hiroshima University
Higashi-Hiroshima, Hiroshima 739

Abstract

A class of finite GUTs in curved spacetime is considered in connection with the cosmological inflation scenario. It is confirmed that the use of the running scalar-gravitational coupling constant in these models helps realizing a successful chaotic inflation. The analyses are made for some different sets of the models.

1 Introduction

It is now a common understanding to assume the presence of the inflationary stage in the early universe (see Ref. 1 for a review). Among various models in the inflationary scenario the chaotic inflation model seems to be the most successful and promising.¹ In the chaotic inflation model, however, we need the fine-tuning of some coupling constants such as scalar-gravitational coupling constant² ξ . The scalar-gravitational term associated with this coupling constant is required in any quantum field theory in curved space-time in order to guarantee the multiplicative renormalizability of the theory³ (see Ref. 4 for a general review). Applying the renormalization group argument we find that the coupling constant ξ starts running.^{3,4,5} The behavior of the running coupling constant at strong gravitational field has been investigated for various models in Refs. 3 and 5 (see Ref. 4 for a review).

In a recent paper⁶ an interesting observation has been made on an implication of the running coupling constant in realizing the inflation scenario.

[†]On leave of absence from Department of Physics, Tomsk Pedagogical Institute, 634041 Tomsk, Russia

The authors of Ref. 6 report that the use of the running scalar-gravitational coupling constant in a specific field theory⁷ helps constructing a successful model of the chaotic inflation. The behaviour of the running ξ in their model is typical of $\lambda\varphi^4$ -theory³ so that

$$\begin{aligned}\xi(t) &= \frac{1}{6} + \left(\xi - \frac{1}{6}\right)(1 - a^2\lambda t)^\alpha, \\ \lambda(t) &= \frac{\lambda}{1 - a^2\lambda t},\end{aligned}\tag{1}$$

where a^2 and α are suitable constants, and t is the RG parameter, $t = \frac{1}{2} \ln \frac{\varphi^2}{\mu^2}$ with μ the renormalization scale. Depending on the sign of the exponent α it is possible to have an asymptotic conformal invariance or not.^{3,4,5} In fact $\xi(t) \rightarrow 1/6$ (for $\alpha < 0$) as $t \rightarrow \infty$ (the infrared limit) which is the case of Ref. 6. For $\alpha > 0$, $|\xi(t)| \rightarrow \infty$ as $t \rightarrow \infty$.^{3,4,5} In Ref. 6 one of the simplest supersymmetric model, i.e. the Wess-Zumino model, was taken into account and the effective potential in the flat direction of the model was considered in order to restrict oneself to the effect of quadratic terms of the model. Thus the model exhibits the typical behavior as mentioned above.

There is an another type of supersymmetric models which are called finite GUTs in which the behaviour of $\xi(t)$ is qualitatively different from the one which we have seen in Eq. (1). The purpose of this note is to examine a possibility of the chaotic inflation in the finite GUTs (including some possible finite non-supersymmetric theories).

2 Finite GUTs

Let us consider the typical (supersymmetric or non-supersymmetric) finite GUTs in curved spacetime. In curved space-time the theory is not completely finite⁸ according to an appearance of the divergence in the vacuum energy (in the external gravitational field sector). Nevertheless, the matter sector remains unaffected.⁸ The behavior of coupling constants in the matter sector is given as follows,⁸

$$\begin{aligned}g^2(t) &= g^2, \quad h(t)^2 = k_1 g^2, \quad f(t) = k_2 g^2, \\ \xi(t) &= \frac{1}{6} + \left(\xi - \frac{1}{6}\right) \exp [cg^2 t],\end{aligned}\tag{2}$$

where $g(t)$, $h(t)$ and $f(t)$ are running coupling constants respectively, $g_2 \ll 1$, and k_1, k_2 and c are certain numerical constants determined by the group

structure of the theory. Note that c may be positive, negative or zero depending on the nature of the theory. Let us consider a concrete example. The $SU(2)$ gauge theory with $SU(N)$ global invariance (the Lagrangian is written in the book previously mentioned;⁴ see Eq. (3.130) therein.) includes gauge fields, Weyl spinors and scalars in the adjoint representation of $SU(2)$. With respect to the global group $SU(N)$ the spinors and scalars belong to the fundamental representation and six-dimensional antisymmetric adjoint representation, respectively. In the case of flat space-time the theory has been introduced in Ref. 9.

The theory has two regimes. In the first regime it is $N = 4$ extended supersymmetric gauge theory which is finite to all orders of perturbation theory in flat spacetime. The direct calculation yields⁸ $c = 3/(2\pi^2)$. In the second regime the theory corresponds to the one-loop finite non-supersymmetric theory in flat spacetime and $c \approx 27/(4\pi)^2$. There are some other finite theories where c could be negative or zero.⁸

Consider the scalar sector of the finite theory taken in the flat direction of the effective potential where interaction terms do not contribute. The renormalization-group-improved effective action in curved spacetime^{4,10} coupled with the classical Einstein gravity is given by

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left[\frac{M_{pl}}{8\pi} R + Z(t) \partial_\mu \varphi(t) \partial^\mu \varphi(t) + m^2(t) \varphi^2(t) - \xi(t) R \varphi^2(t) \right]. \quad (3)$$

where $t = \frac{1}{2} \ln \frac{\varphi^2}{\mu^2}$. The variation of the running mass as a function of t is considered to be small, i.e. $m^2(t) \simeq m^2$. We choose the gauge in which the anomalous dimension for the scalar field vanishes (i.e. the renormalization for the scalar field is finite), then

$$Z(t) = 1, \quad \varphi(t) = \varphi. \quad (4)$$

Under this circumstance the effective action reads

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left[\frac{M_{pl}}{8\pi} R + \partial_\mu \varphi \partial^\mu \varphi + m^2 \varphi^2 - \xi(t) R \varphi^2 \right]. \quad (5)$$

It should be noted here that in our present model it is not necessary to incorporate the effect of the running φ due to the property (4). In the model employed in Ref. 6 Eq. (4) does not hold so that the effect of the running φ plays an important role in deriving the cosmological predictions while in Ref. 6 this effect is not fully taken into account.

Field equations for the theory characterized by Eq. (5) are given by

$$\begin{aligned} \left(\square - m^2 + \xi\varphi R + \frac{1}{2} \frac{d\xi}{d\varphi}(\varphi)R \right) \varphi &= 0, \\ R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{8\pi}{M_{pl}}T_{\mu\nu}. \end{aligned} \quad (6)$$

Rewriting these equations in the Friedmann-Robertson-Walker Universe with scale factor a we obtain¹¹

$$\begin{aligned} \ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi + \left[\xi(\varphi)\varphi + \frac{1}{2} \frac{d\xi}{d\varphi}(\varphi) \right] \times \\ \left[\{6\xi(\varphi) - 1 + 12\frac{d\xi}{d\varphi}(\varphi)\varphi + 6\frac{d^2\xi}{d\varphi^2}(\varphi)\varphi^2\} \dot{\varphi}^2 + 2m_2\varphi^2 \right. \\ \left. + \{6\xi(\varphi)\varphi + 3\frac{d\xi}{d\varphi}(\varphi)\varphi^2\} \{\ddot{\varphi} + 3H\dot{\varphi}\} \right] \left[\frac{M_{pl}}{8\pi} - \xi(\varphi)\varphi^2 \right]^{-1} = 0, \end{aligned} \quad (7)$$

where $H = \dot{a}/a$.

3 Chaotic inflation

As it has been established in Refs. 11 there are two saddle points of Eq. (7) for negative ξ . They are given by

$$\varphi_{\text{cr}} = \pm \frac{M_{pl}}{\sqrt{-8\pi\xi}}, \quad \dot{\varphi} = 0. \quad (8)$$

It is discussed in Ref. 6 (and in the preceding works cited there) that the initial conditions for inflaton φ in order to have a successful chaotic inflation as well as a sufficient period of the inflation are

$$-\frac{M_{pl}}{\sqrt{8\pi|\xi|}} < \varphi < \frac{M_{pl}}{\sqrt{8\pi|\xi|}}, \quad (9)$$

and

$$|\varphi| \geq 5M_{pl}. \quad (10)$$

Actually for negative as well as positive ξ we have two qualitatively different situations summarized by the condition (9) and (10)

Our purpose now is to examine initial conditions (9) and (10) for finite GUTs with running $\xi(t)$ given by Eq. (2). It should be noted that in Eq. (2) $\xi = \xi(\mu)$ and $g^2 = g^2(\mu)$ are initial values of the running coupling constants $\xi(t)$ and $g^2(t)$ respectively at a certain RG scale μ . We start with a very small initial value of $\xi(t)$ and so we may practically set $\xi(\mu) = 0$. Then the total variation of $\xi(t)$ comes purely from the running effect. We start with the minimal theory at scale μ . We wish to plot $\varphi_{\text{cr}} - \varphi$ as a function of φ . The relation between $\varphi_{\text{cr}} - \varphi$ and φ is easily obtained by using Eq. (2) and Eq. (8) with $t = \frac{1}{2} \ln \frac{\varphi^2}{\mu^2}$. In Figs.1, 2, 3 and 4 the behavior of $\varphi_{\text{cr}} - \varphi$ is shown in four typical cases with $(\mu = 50M_{pl}, c < 0)$, $(\mu = 50M_{pl}, c > 0)$, $(\mu = 2M_{pl}, c < 0)$ and $(\mu = 2M_{pl}, c > 0)$ respectively. (Note here that the behavior of $\varphi_{\text{cr}} - \varphi$ as a function of φ is symmetric around $\varphi = 0$ and so we need not to consider the case $\varphi < 0$).

Figs. 1, 2, 3 and 4

Let us examine whether we can have any region where $\varphi_{\text{cr}} - \varphi$ is positive (i. e. $\varphi_{\text{cr}} > \varphi$) for small φ starting with $\varphi \geq 5M_{pl}$. We clearly see that in all four cases $\varphi_{\text{cr}} - \varphi$ becomes positive if $|c|g^2 \sim 10^{-3}$. It is important to note that the chaotic inflation is realized independent of the sign of c . Thus we conclude that for a wide class of the finite GUTs we have successful chaotic inflations.

4 Conclusions

Working within the framework of the finite GUTs we examined the mechanism⁶ that may lead to the successful chaotic inflation. We find that in a wide class of the finite GUTs the chaotic inflation is realized as far as the gauge coupling constant is kept sufficiently small. In this sense the finite GUTs are worth for further investigations in connection with the early universe scenario.

In this regards it is very interesting to note that the finite GUTs in curved spacetime are one of the possible candidates to give a solution to the cosmological constant problem (see the second reference in Ref. 10) due to the exponential running of the effective cosmological constant. These favorable properties of the theories under discussion indicate that the cosmological applications of the finite GUTs (in particular the N=4 super Yang-Mills theory) should be considered more seriously.

References

1. A. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic Publishers, 1990.
2. T. Futamase, T. Rothman and R. Matzner, *Phys. Rev.* D39 (1989) 405; T. Futamase and K. Maeda, *Phys.Rev.* D39 (1989) 405.
3. I. L. Buchbinder and S. D. Odintsov, *Izv. Vuzov. Fizika (Sov. Phys. J.)* N12 (1983) 108; *Yad. Fiz. (Sov. J. Nucl. Phys.)* 40 (1984) 1338; *Lett. Nuovo Cim.* 42 (1985) 379.
4. I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, *Effective Action in Quantum Gravity*, IOP Publishing, Bristol and Philadelphia, 1992.
5. T. Muta and S. D. Odintsov, *Mod. Phys. Lett. A6* (1991) 3641.
6. T. Futamase and M. Tanaka, preprint OCHA-PP-95, 1997; hep-ph/9704303.
7. M. Tanaka, hep-th/9701063.
8. I. L. Buchbinder, S. D. Odintsov and I. M. Lichtzier, *Class. Quant. Grav.* 6 (1989) 605.
9. M. Böhm and A. Denner, *Nucl. Phys.* B282 (1987) 206.
10. I. L. Buchbinder, S. D. Odintsov, *Class. Quant. Grav.* 2 (1985) 721; E. Elizalde and S. D. Odintsov, *Phys. Lett.* B333 (1994) 331.
11. L. Amendola, M. Litterio and F. Occhionero, *Int. J. Mod. Phys.* A5 (1990) 3861; A. Barroso, J. Casasayas, P. Crawford, P. Moniz and A. Nunes, *Phys.Lett.* B275(1992)264.

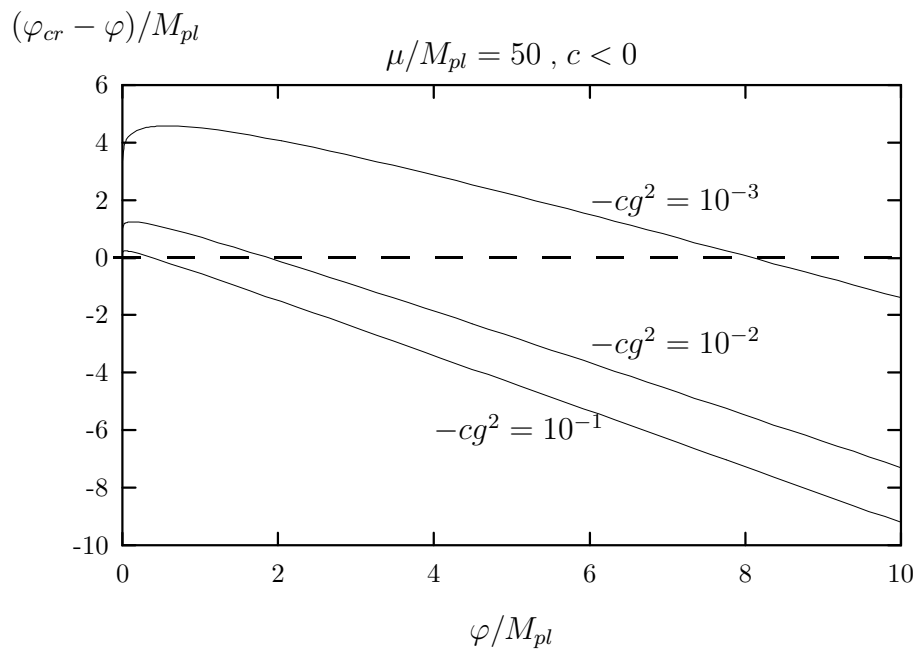


Figure 1: Behavior of $\varphi_{cr} - \varphi$ as a function of φ for $\mu = 50M_{pl}$ and $c < 0$. The three typical cases of the gauge coupling strength are shown: $-cg^2 = 10^{-3}, 10^{-2}, 10^{-1}$.

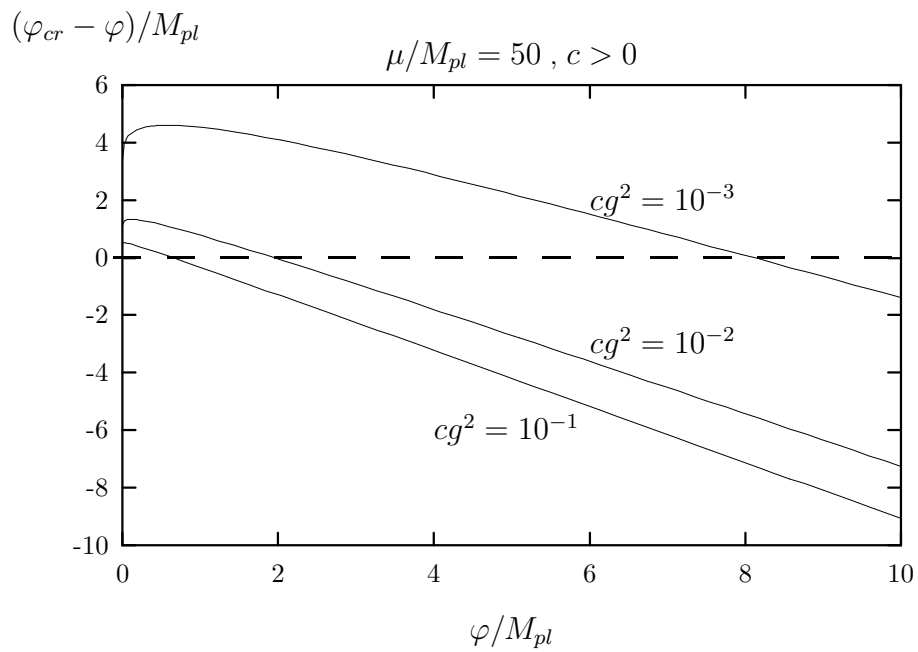


Figure 2: Behavior of $\varphi_{cr} - \varphi$ as a function of φ for $\mu = 50M_{pl}$ and $c > 0$. The three typical cases of the gauge coupling strength are shown: $cg^2 = 10^{-3}, 10^{-2}, 10^{-1}$.

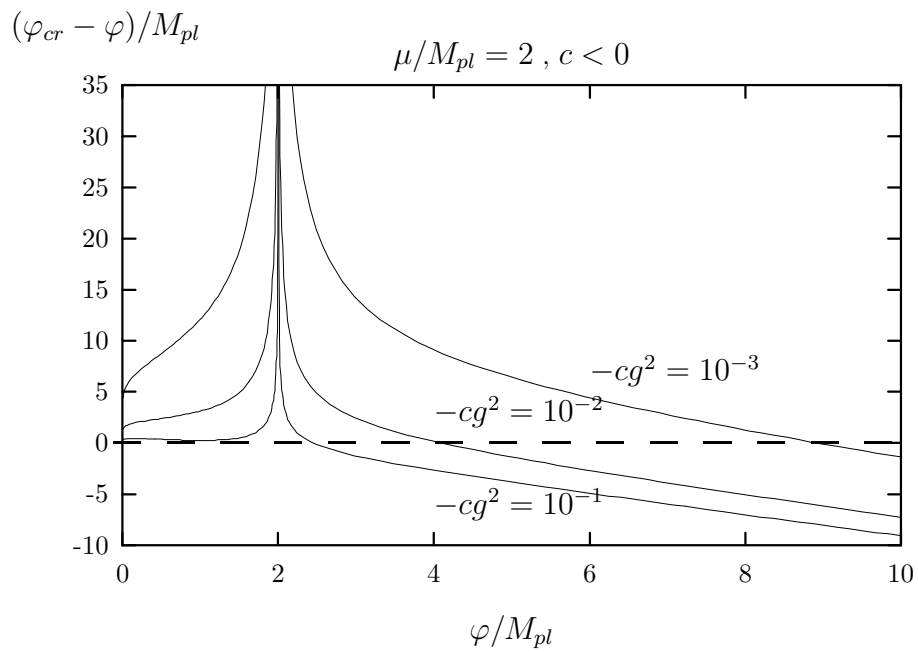


Figure 3: Behavior of $\varphi_{cr} - \varphi$ as a function of φ for $\mu = 2M_{pl}$ and $c < 0$. The three typical cases of the gauge coupling strength are shown: $-cg^2 = 10^{-3}, 10^{-2}, 10^{-1}$.

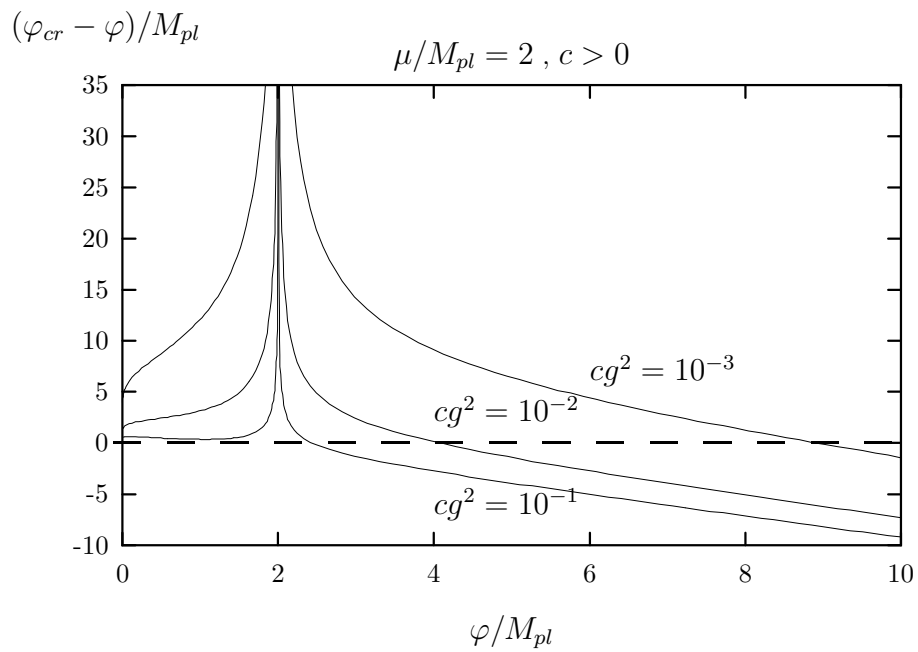


Figure 4: Behavior of $\varphi_{cr} - \varphi$ as a function of φ for $\mu = 2M_{pl}$ and $c > 0$. The three typical cases of the gauge coupling strength are shown: $cg^2 = 10^{-3}, 10^{-2}, 10^{-1}$.