

# QCD Anomaly Coupling for the $\eta' - g - g$ Vertex in Inclusive Decay $B \rightarrow \eta' X_s$

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## Abstract

The QCD anomaly coupling of  $\eta' - g - g$  is treated as the axial vector current triangle anomaly. By assuming the divergent axial vector coupling of the  $\eta'$  meson with the quark line in the triangle diagram, we calculate the QCD anomaly of  $\eta' - g - g$  as well as the QED anomaly of  $\eta' - \gamma - \gamma$  with only one common parameter  $\kappa_{\eta'}$ . We obtain consistent results of the branching ratios for  $B \rightarrow \eta' X_s$ ,  $J/\psi \rightarrow \eta' \gamma$  and  $\eta' \rightarrow \gamma \gamma$  when comparing with experimental data.

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The first observation of  $B \rightarrow \eta' X_s$  decays [1, 2] has stimulated much theoretical interests. A number of interpretations have been proposed to explain the observed large decay branching fraction. One of the interpretations is based on  $b \rightarrow c\bar{c}s$  decay with the assumption of the intrinsic  $c\bar{c}$  content in the  $\eta'$  wave function[3], while the others are based on the  $b \rightarrow sg^*$  penguin transition followed by  $g^* \rightarrow \eta'g$  through QCD anomaly [4, 5, 6, 7, 8, 9]. The observed recoil mass spectrum favors the second interpretation[2]. Thus the QCD anomaly coupling of the  $\eta'$  meson with gluon fields is important in the  $B \rightarrow \eta' X_s$  decay process. However there are still no consensus among the authors on the second interpretation. Some of them conclude that the calculation within the standard model (SM) is sufficient to account for the experimental data[4, 7, 9] while the others[5, 6] conclude not, stressing that new physics is needed for the proper interpretation. The key point of the confusing situation comes from the QCD anomaly itself. Refs.[4, 5, 7] stress that the QCD anomaly coupling for the  $\eta' - g - g$  vertex is highly nonperturbative so that it is unpredictable. Because the  $\eta' - g - g$  vertex is treated differently in Refs.[4, 5, 6, 7], the different results of the branching ratio of  $B \rightarrow \eta' X_s$  are drawn.

In the present paper, we try to make clear the above confusing situation on the  $\eta' - g - g$  coupling. Given the unique properties of the axial current triangle anomaly which has first been discovered in QED and successfully solved the problem of the  $\pi^0 \rightarrow \gamma\gamma$  decay[10] the  $\eta' - g - g$  coupling is also treated as the axial current anomaly here (Fig.1). The vertex of the  $\eta'$  meson coupling with the quark line in the triangle diagram is assumed to be given by  $i\kappa_{\eta'}\not{p}\gamma_5$  which comes from the PCAC hypothesis[11]. Here  $\kappa_{\eta'}$  is introduced as a coupling parameter. It is reasonable to treat  $\kappa_{\eta'}$  as a constant, because it only involves the inner properties of the  $\eta'$  meson.

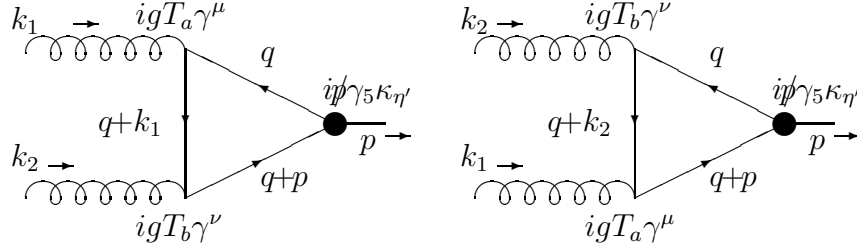


Figure 1: The lowest-order diagrams which contribute to QCD anomaly coupling of  $\eta' - g - g$ . Higher order diagrams do not contribute to the anomaly term according to the Adler-Bardeen theorem[12]

The routing of the loop momentum  $q$  is shown in Fig.1. We write the amplitude  $T^{\mu\nu\lambda}$  directly by following the Feynman rule,

$$T^{\mu\nu\lambda} = \int \frac{d^4q}{(2\pi)^4} (-1) \left\{ \text{Tr} \left[ \frac{i}{\not{q} - m} i\gamma^\lambda \gamma_5 \kappa_{\eta'} \frac{i}{\not{q} + \not{p} - m} ig\gamma^\nu T_b \frac{i}{\not{q} + \not{k}_1 - m} igT_a \gamma^\mu \right] + \left( \begin{array}{c} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{array} \right) \right\} \quad (1)$$

Maintaining the vector current Ward identities  $k_{1\mu} T^{\mu\nu\lambda} = 0$  and  $k_{2\nu} T^{\mu\nu\lambda} = 0$  we obtain,

$$p_\lambda T^{\mu\nu\lambda} = 2mT^{\mu\nu} - \frac{1}{2\pi^2} \left( \frac{1}{2} \delta_{ab} g^2 \kappa_{\eta'} \right) \varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}, \quad (2)$$

where

$$T^{\mu\nu} = \left( \frac{1}{2} \delta_{ab} g^2 \kappa_{\eta'} \right) \int \frac{d^4q}{(2\pi)^4} (-1) \left\{ \text{Tr} \left[ \frac{1}{\not{q} - m} \gamma_5 \frac{1}{\not{q} + \not{p} - m} \gamma^\nu \frac{1}{\not{q} + \not{k}_1 - m} \gamma^\mu \right] + \left( \begin{array}{c} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{array} \right) \right\} \quad (3)$$

For the details of the derivation of eq.(2) and (3), the readers may refer to Ref.[13].

After a few steps of algebraic calculation eq.(3) can be finally converted into the following Feynman integration.

$$T^{\mu\nu} = \left( \frac{1}{2} \delta_{ab} g^2 \kappa_{\eta'} \right) 8m I(k_1^2, k_2^2, p^2) \varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}, \quad (4)$$

where

$$I(k_1^2, k_2^2, p^2) = \frac{-1}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{1}{(x^2 - x)k_1^2 + (y^2 - y)k_2^2 - 2xyk_1 \cdot k_2 + m^2}. \quad (5)$$

Now some properties of eq.(2) should be pointed out. The second term in the right hand side of eq.(2) violates the axial current Ward identity  $p_\lambda T^{\mu\nu\lambda} = 2mT^{\mu\nu}$ , it is called anomaly term. The factor  $\frac{1}{2\pi^2} \left( \frac{1}{2} \delta_{ab} g^2 \kappa_{\eta'} \right)$  is momentum independent. Substituting eq.(4) into eq.(2) we get

$$p_\lambda T^{\mu\nu\lambda} = [16m^2 I(k_1^2, k_2^2, p^2) - \frac{1}{2\pi^2}] \left( \frac{1}{2} \delta_{ab} g^2 \kappa_{\eta'} \right) \varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}. \quad (6)$$

After practising the numerical calculation through the use of the computer we find for the whole range of the momentum squared  $k_1^2$  and  $k_2^2$ , which is favored by the decay  $B \rightarrow \eta' X_s$ , the first term in the square brackets of eq.(6) is almost two orders of magnitude smaller than the second one. Hence we can safely drop the first term. Now the QCD anomaly coupling for the  $\eta' - g - g$  vertex is finally given by

$$A^{\mu\nu} = -\frac{\alpha_s(\mu) \kappa_{\eta'}}{\pi} D \delta_{ab} \varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}, \quad (7)$$

where  $D = \sqrt{3} \cos\theta$  which is introduced by taking into account the contributions of three quarks contained in  $\eta'$  meson. The angle  $\theta$  is the pseudoscalar mixing angle defined by  $\eta' = \eta_0 \cos\theta + \eta_8 \sin\theta$  and our choice of the angle is  $\theta = -19.5^\circ$ .

We use eq.(7) to calculate the branching ratio for  $B \rightarrow \eta' X_s$ . We use the branching ratio of  $b \rightarrow sg\eta'$  to estimate the inclusive process. The strong penguin induced  $b \rightarrow s$  current is [14]

$$\frac{G_f}{\sqrt{2}} \frac{g_s}{4\pi^2} V_t \bar{s} T_a \{ \Delta F_1 (q^2 \gamma_\mu - q_\mu) L - F_2 i \sigma_{\mu\nu} q^\nu m_b R \} b, \quad (8)$$

where  $\Delta F_1 \simeq -5$ ,  $F_2 \simeq 0.286$  and  $V_t = v_{ts}^* v_{tb}$ . For the convenience of expressing the result for the branching ratio of  $b \rightarrow sg\eta'$  we first define some variables:  $x \equiv m_x^2/m_b^2$  with  $m_x$  the invariant mass of the hadronic system in recoil from the  $\eta'$  meson in the final state,  $y \equiv k_1^2/m_b^2$  with  $k_1$  the momentum of the virtual gluon  $g^*$  which connects the penguin diagram with the  $\eta'$  meson,  $x' \equiv m_{\eta'}^2/m_b^2$  and  $x'_s \equiv m_s^2/m_b^2$ . The branching ratio can be expressed as

$$\begin{aligned} Br(b \rightarrow sg\eta') &= \frac{1}{\Gamma_B} \frac{G_f^2 |V_t|^2 m_b^5 g_s^2 (mb) m_b^2}{192\pi^3 16\pi^4 4} \left( \frac{\alpha_s(\mu) \kappa_{\eta'}}{\pi} D \right)^2 \\ &\times \int dx \int dy \left\{ \frac{1}{2} |\Delta F_1|^2 [-2(x - x'_x)^2 y + (1 - y - x'_s)(y - x')(2x + y - x' - 2x'_s)] \right. \\ &\quad \left. - Re(\Delta F_1 F_2^*) (1 - y - x'_s)(y - x')^2 / y \right. \\ &\quad \left. + \frac{1}{2} |F_2|^2 [2(x - x'_s)^2 y^2 - (1 - y - x'_s)(y - x')(2(x - x'_s)y - (y - x')(1 - x'_s))] / y^2 \right\}. \quad (9) \end{aligned}$$

It should be stressed that our result of the anomaly coupling for the  $\eta' - g - g$  vertex is obtained by assuming the divergent axial vector coupling between  $\eta'$  and the quark line in the triangle diagram. If we only use it to explain one experiment, the validity of our method cannot be tested. So we should use the same method and the same parameter  $\kappa_{\eta'}$  to other processes. Now we will use it to calculate  $\eta' \rightarrow \gamma\gamma$  decay which is an electromagnetic decay process, so it is completely different from

$\eta' - g - g$  coupling in which strong interaction is involved. Calculating the similar triangle diagrams as shown in Fig.1, where the coupling  $i\kappa_\eta \not{p} \gamma_5$  is the same, only the strong coupling  $igT_a \gamma^\mu$ ,  $igT_b \gamma^\nu$  are changed to be the QED coupling  $iQe\gamma^\mu$  and  $iQe\gamma^\nu$ . We find the amplitude of  $\eta' \rightarrow \gamma\gamma$  is

$$A_{\gamma\gamma} = -\frac{2\alpha\kappa_{\eta'}}{\pi} D' \varepsilon^{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \varepsilon_\mu^* \varepsilon_\nu^*. \quad (10)$$

where  $\alpha$  is the fine-structure constant, i.e.,  $\alpha = 1/137$ ,

$$D' = \left[ \frac{1}{\sqrt{3}}(Q_u^2 + Q_d^2 + Q_s^2) \cos\theta + \frac{1}{\sqrt{6}}(Q_u^2 + Q_d^2 - 2Q_s^2) \sin\theta \right] \cdot N_c \simeq \frac{7}{3\sqrt{6}},$$

where  $N_c = 3$  is the color number. Finally we get

$$\Gamma_{\eta' \rightarrow \gamma\gamma} = \frac{1}{64\pi} \left( \frac{14}{3\sqrt{6}} \frac{\alpha\kappa_{\eta'}}{\pi} \right)^2 m_{\eta'}^3. \quad (11)$$

Comparing eq.(11) with experimental data,  $\Gamma_{\eta' \rightarrow \gamma\gamma}^{exp.} = (4.28 \pm 0.43) \times 10^{-6} \text{GeV}$  [15], we find

$$\kappa_{\eta'} = 7.06 \pm 0.35 \text{GeV}^{-1}. \quad (12)$$

Substitute the value of  $\kappa_{\eta'}$  into eq.(9), and take  $m_b = 4.8 \text{GeV}$ ,  $m_s = 0.15 \text{GeV}$ ,  $\frac{1}{\Gamma_B} \frac{G_f^2 |V_t|^2 m_b^5}{192\pi^3} \approx 0.2$ , we can get,

$$Br(b \rightarrow s g \eta') = (4.9 \pm 0.5) \times 10^{-4}, \quad (13)$$

here the error bar  $\pm 0.5$  comes from  $\pm 0.35$  in eq.(12), and the experimental cut has been taken into account in eq.(13). Comparing eq.(13) with the experimental data  $Br(B \rightarrow \eta' X_s) = (6.2 \pm 2.0) \times 10^{-4}$  [2] we find that eq.(13) is fairly consistent with the data.

Some remarks should be given. First, to get eq.(13), we have used the running strong coupling constant  $\alpha_s(\mu)$  in eq.(7)[16]. Although it is smaller than the result which is obtained without taking into account the running of  $\alpha_s(\mu)$ , it is well

within the experiment error bar. In contrast, it will be too large to account for the experimental data if we do not take into account the running of  $\alpha_s(\mu)$ . Second, at  $\mu = m_{\eta'}$  we find the anomaly coupling in eq.(7) is  $\frac{\alpha_s(m_{\eta'})\kappa_{\eta'}}{\pi}D \doteq 1.8\text{GeV}^{-1}$ . It is also consistent with the anomaly coupling for the  $\eta' - g - g$  vertex which AS[4] extracted from the experimental data of  $Br(J/\psi \rightarrow \eta'\gamma)$ . Hence the QCD anomaly coupling for the  $\eta' - g - g$  vertex derived by calculating the triangle diagram can also give the correct result of  $Br(J/\psi \rightarrow \eta'\gamma)$ . Third, in general, it is believed that the QCD anomaly coupling of  $\eta' - g - g$  is nonperturbative and hence it is unpredictable. But now our calculation shows that the nonperturbative part can be successfully separated, which is absorbed into the coupling parameter  $k_{\eta'}$ . With the parameter  $k_{\eta'}$  the strong coupling of  $\eta' - g - g$  can be predicted from the knowledge of  $\eta' \rightarrow \gamma\gamma$ , which is an electromagnetic decay process. It is very interesting to note that the QCD anomaly and QED anomaly can be treated in an uniform way.

In summary, by calculating AVV (Axial vector-Vector-Vector current) triangle diagram, we find that QCD anomaly coupling for the  $\eta' - g - g$  vertex and QED anomaly coupling  $\eta' - \gamma - \gamma$  can be treated with one parameter  $\kappa_{\eta'}$ . The calculation of these three decay process  $B \rightarrow \eta'X_s$ ,  $J/\psi \rightarrow \eta'\gamma$ , and  $\eta' \rightarrow \gamma\gamma$  can be consistent with experiment at the same time. SM is sufficient to account for  $B \rightarrow \eta'X_s$  data within the present experimental error bars.

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$$\alpha_s^{(f)}(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_f^2)} \left[ 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln \ln(\mu^2/\Lambda_f^2)}{\ln(\mu^2/\Lambda_f^2)} \right]$$

with  $\beta_0 = 11 - \frac{2}{3}f$  and  $\beta_1 = 102 - \frac{38}{3}f$ , where  $f$  is the number of the quarks which are active in the scale of  $\mu$ .  $\Lambda_4 = 300MeV$  is used in this paper, and to keep the running of  $\alpha_s^{(f)}(\mu)$  continuous at  $\mu = m_c = 1.5GeV$ ,  $\Lambda_3 = 353MeV$  is taken here.