

Dynamical CP violation in composite Higgs models

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(Received 5 January 1993; revised manuscript received 24 March 1993)

The dynamical origin of CP violation in electroweak theory is investigated in composite Higgs models. The mechanism of spontaneous CP violation proposed in another context by Dashen is adopted to construct simple models of dynamical CP violation. Within the models the size of the neutron electric dipole moment is estimated and the constraint on the ϵ parameter in K -meson decays is discussed.

PACS number(s): 11.30.Er, 11.30.Qc, 12.15.Cc

I. INTRODUCTION

CP violation is described by phases appearing in the Kobayashi-Maskawa matrix [1] in the standard theory of quarks and leptons. The CP -violating phases are introduced only when the number of the quark-lepton generations is equal to or greater than three. In other words the reason why we have CP violation in nature is that we have three generations of quarks and leptons. The CP violating phases are partially determined by experimental data in the neutral K -meson system. The prediction for the neutron electric dipole moment [2] based on the Kobayashi-Maskawa CP violating phases (KM phases) is extremely small and is well below the experimental bound [3]. Thus the standard theory with Kobayashi-Maskawa CP violation is consistent with the present experimental situation.

The KM phases are introduced as free parameters in the standard theory. From the point of view of the fundamental theory of quarks and leptons this situation is not satisfactory and we would like to see where the theoretical origin of the KM phases describing the CP violation is.

One possibility of explaining the KM phases through a more fundamental origin is to introduce the complex vacuum expectation values for the Higgs field as discussed by Weinberg more than a decade ago [4]. In this approach it is required to have at least three Higgs doublets in order to interpret the full KM phases. This mechanism suggests that the spontaneous electroweak symmetry breaking has something to do with the origin of CP violation.

Pushing forward this idea we are naturally led to the composite Higgs models where the Higgs field is replaced by a composite system of fundamental fermions. There are a variety of composite Higgs models including the technicolor model [5], top-quark condensation model [6,7], fourth-generation model [8], and color-sextet quark model [9]. In the composite Higgs models CP violation may occur if the complex vacuum expectation value would result for the composite field $\bar{\psi}\psi$ with a fundamental fermion ψ . The realization of such a circumstance was suggested a long time ago by Dashen in another context [10].

The idea of Dashen will be recapitulated in the next

section and will be applied straightforwardly to the composite Higgs models. Eichten, Lane, and Preskill [11] have adopted Dashen's idea in the technicolor model to elucidate the mechanism of the dynamical CP violation. In their paper the general framework of generating dynamical CP violation was presented and some physical consequences were pointed out. Later Goldstein [12] has reconsidered the problem and constructed a model of the dynamical CP violation with two quark and techniquark doublets. This model, however, fails to give rise to the CP -violating phase unless one introduces extra leptons or one assumes an existence of the strong CP violation in the technicolor sector.

In this paper we would like to construct some simple examples of dynamical CP violation in the composite Higgs models. In our models we assume the presence of two flavors of up- (down-)type extra fundamental quarks and three flavors of up-(down-)type ordinary quarks. We start with the Lagrangian with flavor symmetry (i.e., all fermions massless) in which a nonvanishing vacuum expectation value develops for the composite field $\bar{\psi}\psi$ with ψ the fundamental fermion. To this Lagrangian we add flavor-symmetry-breaking terms to realize the quark mass hierarchy. We consider transformations which mixes the flavors of quarks. We find a special solution for the transformations which gives the true vacuum with the proper direction. According to this special solution the CP violating terms are generated in the flavor-symmetry-breaking part of the Lagrangian.

The main purpose of our argument is to show the usefulness of the Dashen mechanism for the dynamical CP violation in a transparent way. Our model is too simple to explain the KM phases practically and should be elaborated to reproduce the standard theory as a low-energy effective theory. If our model has something to do with nature, it has to be consistent with the existing experimental observations. Thus we calculate the contribution in our model to the electric dipole moment of the neutron and the ϵ parameter in K decays. Both quantities are found to be consistent with the experimental data if the cutoff Λ existing in the model is larger than 800 TeV which is consistent with the cutoff set by the flavor-changing neutral current (FCNC) restriction [13].

It should be remarked that any model of the spontaneous CP violation suffers from the cosmological domain-

wall problem. In this paper we are interested in constructing simple examples of dynamical CP violation and we tentatively circumvent the problem by assuming that dynamical CP violation takes place before the inflation period.

II. DASHEN MECHANISM IN COMPOSITE HIGGS MODELS

In this section we briefly review the Dashen mechanism of spontaneous CP violation with the application to the composite Higgs models.

We start with the Lagrangian \mathcal{L}_0 symmetric under the flavor group

$$G_F = \prod_{\rho} U_V(n_{\rho}) \otimes U_A(n_{\rho}), \quad (2.1)$$

where n_{ρ} is the number of quark flavors belonging to the irreducible representation ρ in the underlying gauge group and U_V (U_A) is the unitary group associated with the vector (axial-vector) currents. Here by the term ‘‘quark’’ we mean the ordinary quarks as well as the fermions required for generating the composite Higgs field. The quark fields included in the Lagrangian \mathcal{L}_0 are all massless to guarantee the underlying gauge symmetry and the flavor symmetry.

We assume that the flavor symmetry G_F is broken dynamically by the presence of the nonvanishing vacuum expectation value for the composite field $\bar{\psi}\psi$ made of fermion fields ψ :

$$\langle \bar{\psi}\psi \rangle \neq 0. \quad (2.2)$$

Here we have chosen the vacuum for which

$$\langle \bar{\psi}\gamma_5\psi \rangle = 0. \quad (2.3)$$

The condensation (2.2) is responsible for generating the mass of quarks according to the dynamical breaking of the flavor symmetry G_F . The remaining flavor symmetry if any will be denoted by S_F . As is well known the vacuum satisfying Eqs. (2.2) and (2.3) is not unique and thus we have degenerate vacua in G_F/S_F . These degenerate vacua point to arbitrary direction in G_F/S_F .

We add to \mathcal{L}_0 the term \mathcal{L}' which explicitly breaks the flavor symmetry G_F . We assume that \mathcal{L}' is CP invariant. The degeneracy of the vacua mentioned above is now resolved in the system described by the total Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}'. \quad (2.4)$$

The direction of the vacuum thus determined, however, does not necessarily guarantee the conditions (2.2) and (2.3). Hence we need to make a transformation on the field to recover the conditions

$$\psi' = U\psi, \quad (2.5)$$

with U the transformation belonging to G_F . By this transformation the form of the symmetry-breaking term \mathcal{L}' will be modified so that CP -violating terms, in general, show up in \mathcal{L}' . We will call this mechanism of spontaneous CP violation [10] the Dashen mechanism. In the following we would like to apply the Dashen mechanism to

the case of the composite Higgs models.

In electroweak theory the Higgs fields are introduced as elementary scalar fields. Accordingly the Higgs boson mass, Higgs self-coupling constant, and Higgs-fermion Yukawa-coupling constants are all arbitrary parameters. In the composite Higgs model the Higgs particle appears as a composite system of some fundamental fermions and some of the parameters in the standard electroweak theory are predictable in principle. The Lagrangian corresponding to this model may be given by

$$\mathcal{L}_0 = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{EW}} + \mathcal{L}_{\text{dyn}}, \quad (2.6)$$

where \mathcal{L}_{QCD} is the ordinary QCD Lagrangian for quarks, \mathcal{L}_{EW} is the electroweak Lagrangian without Higgs fields, and \mathcal{L}_{dyn} is the dynamical term which is assumed to be responsible for generating the fermion-antifermion condensation as well as the composite Higgs system as a bound state (this term may be thought of as a low-energy effective Lagrangian stemming from the more fundamental Lagrangian).

The Higgs particle appears as a bound state of the fundamental fermions ψ and the bound state is assumed to generate a condensation:

$$\langle \bar{\psi}\psi \rangle \neq 0. \quad (2.7)$$

The fundamental fermions as well as the ordinary quarks acquire a mass according to the condensation. The mass of the fundamental fermions should be of the order of the weak scale in order to guarantee that the resulting effective theory be the standard electroweak theory.

In the technicolor model [5] the fundamental fermion is the techniquark, in the top-quark-condensation model [6] it is the top quark with a mass close to the weak scale, in the fourth-generation model [8] it is the heavy quark in the assumed fourth generation, and in the color-sextet model [9] it is the quark in the sextet representation of the color $SU(3)$.

For the flavor-symmetry-breaking term \mathcal{L}' we choose the four-fermion interaction made of fundamental fermions and ordinary quarks.

In the following we would like to present simple models of the dynamical CP violation in the composite Higgs models.

III. SIMPLE MODELS OF DYNAMICAL CP VIOLATION

A. General formalism

Here we first present a general argument in constructing simple models of the dynamical CP violation in the composite Higgs model. We consider n_{ρ} flavors of fundamental quarks in the representation ρ of the color $SU(3)$ or other symmetry group (we call this symmetry governing the fundamental quarks the symmetry S) and n_3 flavors of ordinary quarks in the triplet representation of the color $SU(3)$. The fundamental quarks may or may not have a color degree of freedom.

We will discuss transformations which mix the flavors of the fundamental and ordinary quarks among themselves. Since this transformation has to conserve charges,

the mixing occurs only among the up-type (or down-type) fundamental and ordinary quarks. For simplicity we consider only up-type fundamental and ordinary quarks.

According to Goldstein's analysis [12] one finds that only two flavors of the fundamental and ordinary quarks are not sufficient to realize the Dashen mechanism. Hence we try a model with two flavors of the up-type fundamental quarks Q and three flavors of the up-type ordinary quarks q :

$$Q=(U,C), \quad q=(u,c,t). \quad (3.1)$$

We assume that Q belongs to the N -plet of the fundamental symmetry S and q belongs to the color triplet. It is understood that our model equally applies to the system of the down-type quarks

$$Q=(D,S), \quad q=(d,s,b). \quad (3.2)$$

In the following by the term "quark" we generically mean both fundamental and ordinary quarks.

As a G_F breaking Hamiltonian density \mathcal{H}' we take the four-fermion terms

$$\begin{aligned} \mathcal{H}' = -\mathcal{L}' = & G_{rr'ss'}^Q \bar{Q}_{Lr} Q_{Rr'} \bar{Q}_{Rs} Q_{Ls'} \\ & + G_{rr'ss'}^{Qq} \bar{Q}_{Lr} Q_{Rr'} \bar{q}_{Rs} q_{Ls'} + \text{H.c.} \\ & + G_{rr'ss'}^q \bar{q}_{Lr} q_{Rr'} \bar{Q}_{Rs} Q_{Ls'}, \end{aligned} \quad (3.3)$$

where $G_{rr'ss'}^Q$, $G_{rr'ss'}^{Qq}$, and $G_{rr'ss'}^q$ are coupling parameters among fundamental quarks Q and ordinary quarks q which depend on the flavor $U(2)$ indices r, r' and the flavor $U(3)$ indices s, s' , respectively. In Eq. (3.3) the fundamental symmetry indices and color indices are suppressed and are understood to be contracted between adjoining quarks. There would be other possibilities of contracting these indices. We, however, confine ourselves to the case of Eq. (3.3).

$$\begin{aligned} E(W) = & G_{rr'ss'}^Q \langle 0 | \bar{Q}_{Lr} U_{Lr'}^{\dagger} U_{Rr'}^Q Q_{Rr'} \bar{Q}_{Ru} U_{Rus}^{\dagger} U_{Ls'u}^Q Q_{Lu'} | 0 \rangle \\ & + G_{rr'ss'}^{Qq} \langle 0 | \bar{Q}_{Lr} U_{Lr'}^{\dagger} U_{Rr'}^Q Q_{Rr'} \bar{q}_{Ru} U_{Rus}^{\dagger} U_{Ls'u}^q q_{Lu'} + \text{H.c.} | 0 \rangle \\ & + G_{rr'ss'}^q \langle 0 | \bar{q}_{Lr} U_{Lr'}^{\dagger} U_{Rr'}^q q_{Rr'} \bar{Q}_{Ru} U_{Rus}^{\dagger} U_{Ls'u}^Q q_{Lu'} | 0 \rangle. \end{aligned} \quad (3.9)$$

Since the state $|0\rangle$ is invariant under S_F , we may express the amplitudes

$$\begin{aligned} \langle 0 | \bar{Q}_{Lr} Q_{Rr'} | 0 \rangle &= \Delta_0^Q \delta_{rr'}, \quad \langle 0 | \bar{q}_{Lr} q_{Rr'} | 0 \rangle = \Delta_0^q \delta_{rr'}, \\ \langle 0 | \bar{Q}_{Lr} Q_{Rr'} \bar{Q}_{Rs} Q_{Ls'} | 0 \rangle &= \Delta^Q \delta_{rr'} \delta_{ss'} + \Delta'^Q \delta_{rs} \delta_{r's'}, \\ \langle 0 | \bar{Q}_{Lr} Q_{Rr'} \bar{q}_{Rs} q_{Ls'} | 0 \rangle &= \Delta^{Qq} \delta_{rr'} \delta_{ss'}, \\ \langle 0 | \bar{q}_{Lr} q_{Rr'} \bar{Q}_{Rs} Q_{Ls'} | 0 \rangle &= \Delta^{qQ} \delta_{rr'} \delta_{ss'} + \Delta'^{qQ} \delta_{rs} \delta_{r's'}, \end{aligned} \quad (3.10)$$

where parameters Δ are chosen to be real. After some algebra we obtain

$$\begin{aligned} E(W) = & g_{rr'ss'}^Q U_{r'r}^Q U_{s's}^{\dagger} + r g_{rr'ss'}^{Qq} U_{r'r}^Q U_{s's}^{\dagger} + \text{H.c.} \\ & + r^2 g_{rr'ss'}^q U_{r'r}^q U_{s's}^{\dagger} + \text{const.}, \end{aligned} \quad (3.11)$$

where matrices U^Q and U^q and parameters $g_{rr'ss'}^Q$, $g_{rr'ss'}^{Qq}$,

We require the CP invariance and Hermiticity of the Lagrangian (3.3). We then have

$$\begin{aligned} G_{rr'ss'}^Q &= G_{s'sr'r}^Q = (G_{s'sr'r}^Q)^*, \\ G_{rr'ss'}^{Qq} &= (G_{rr'ss'}^{Qq})^*, \\ G_{rr'ss'}^q &= G_{s'sr'r}^q = (G_{s'sr'r}^q)^*, \end{aligned} \quad (3.4)$$

where indices r, r', s, s' represent flavors of Q and q , i.e., U, C, u, c, t .

Our first task is to find the correct vacuum under the Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}', \quad (3.5)$$

where \mathcal{L}_0 is the Lagrangian given by Eq. (2.6) and \mathcal{L}' is given by Eq. (3.3). Let us denote by $|\bar{0}\rangle$ the ground state (vacuum) for a system governed by the Lagrangian (3.5) and by $|0\rangle$ the ground state for \mathcal{L}_0 which is invariant under S_F . To find the ground state $|\bar{0}\rangle$ we try to minimize the energy

$$E(W) = \langle \bar{0} | \mathcal{H}' | \bar{0} \rangle = \langle 0 | W^{\dagger} \mathcal{H}' W | 0 \rangle, \quad (3.6)$$

by suitably choosing the transformation W in G_F . The transformation W is induced by the transformation U of fermion fields Q and q :

$$Q'_{L,R} = U_{L,R}^Q Q_{L,R}, \quad q'_{L,R} = U_{L,R}^q q_{L,R}, \quad (3.7)$$

where $U_{L,R}^Q$ is the transformation belonging to the left-handed (right-handed) flavor $U(2)$ for fundamental quarks Q and $U_{L,R}^q$ belonging to the $U(3)$ for ordinary quarks q . The transformation W is a function of these fermion transformations:

$$W = W(U), \quad (3.8)$$

where we represent generically by U the transformations $U_{L,R}^Q$ and $U_{L,R}^q$. We find

$g_{rr'ss'}^q$, and r are given by the relations

$$U^Q = U_R^Q U_L^{Q\dagger}, \quad U^q = U_R^q U_L^{q\dagger}, \quad (3.12)$$

$$g_{rr'ss'}^Q = G_{rr'ss'}^Q \Delta^Q, \quad r g_{rr'ss'}^{Qq} = G_{rr'ss'}^{Qq} \Delta^{Qq}, \quad (3.13)$$

$$\begin{aligned} r^2 g_{rr'ss'}^q &= G_{rr'ss'}^q \Delta^q, \\ r &= \langle \bar{q}q \rangle / \langle \bar{Q}Q \rangle. \end{aligned} \quad (3.14)$$

Here we introduced the parameter r in order to show explicitly the relative size of the three kinds of parameters $G_{rr'ss'}^Q$, $G_{rr'ss'}^{Qq}$ and $G_{rr'ss'}^q$. The parameter r is the ratio of the ordinary and fundamental mass scale [5] and its size is assumed to be

$$r \sim (1 \text{ GeV} / 1 \text{ TeV})^3 = 10^{-9}. \quad (3.15)$$

Our task is to minimize $E(W)$ given in Eq. (3.11) by changing U and find the solution for U . With U determined in this procedure we rewrite \mathcal{L}' to see whether CP -violating terms are generated in \mathcal{L}' .

B. Special solutions

A comprehensive model of the dynamical CP violation may be obtained within our framework if we find a general solution for U which minimizes $E(W)$ of Eq. (3.11) and generates CP -violating terms in \mathcal{L}' . However, it is not a simple task to determine the full matrix elements of U including off-diagonal elements. For our present purpose it is enough to confine ourselves to the case of diagonal matrices U and hence we specialize our model by setting

$$\begin{aligned} g_{rr'ss'}^Q &= g_{rs}^Q \delta_{rr'} \delta_{ss'} , \\ g_{rr'ss'}^{Qq} &= g_{rs}^{Qq} \delta_{rr'} \delta_{ss'} , \\ g_{11}^Q &< 0, \quad g_{22}^Q < 0, \quad g_{12}^Q = 0 . \end{aligned} \quad (3.16)$$

In this case Eq. (3.11) takes the simple form

$$\begin{aligned} E(W) &= g_{11}^Q |U_{11}^Q|^2 + g_{22}^Q |U_{22}^Q|^2 + r(g_{rs}^{Qq} U_{rr}^Q U_{ss}^{q\dagger} + \text{H.c.}) \\ &\quad + O(r^2) . \end{aligned} \quad (3.17)$$

Here and in the following we neglect terms of $O(r^2)$ in $E(W)$. If we parametrize the diagonal elements of U by

$$\begin{aligned} U_{rr}^Q &= u_r^Q \exp(i\theta_r^Q) \\ U_{rr}^q &= u_r^q \exp(i\theta_r^q) , \end{aligned} \quad (3.18)$$

the above equation (3.17) is rewritten as

$$\begin{aligned} E(W) &= g_{11}^Q (u_1^Q)^2 + g_{22}^Q (u_2^Q)^2 + 2rg_{rs}^{Qq} u_r^Q u_s^q \cos(\theta_r^Q - \theta_s^q) \\ &\quad + O(r^2) . \end{aligned} \quad (3.19)$$

We would like to find a set of parameters u and θ which minimizes $E(W)$ given by Eq. (3.19).

Since we are interested only in the solution which generates the CP violation, we take into account the condition of either

$$\begin{aligned} g_{11}^{Qq} g_{21}^{Qq} g_{12}^{Qq} g_{22}^{Qq} &< 0 , \\ g_{11}^{Qq} g_{21}^{Qq} g_{13}^{Qq} g_{23}^{Qq} &< 0 , \end{aligned} \quad (3.20)$$

or

$$g_{12}^{Qq} g_{22}^{Qq} g_{13}^{Qq} g_{23}^{Qq} < 0 .$$

Moreover the resulting quark mass matrix M has to reflect the size of the realistic quark mass m_u , m_c , and m_t and hence we require

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} = g^{Qq} \begin{pmatrix} \varepsilon & & \\ & \varepsilon & \\ & & 1 \end{pmatrix} \quad (\varepsilon \ll 1) , \quad (3.21)$$

where ε is a small number of order

$$\frac{m_{u,c}}{m_t} \sim \frac{1}{100} .$$

A simple choice of the set of coupling constants g which respects the above condition is, for example,

$$\begin{aligned} g_{11}^{Qq} &= g_{12}^{Qq} < 0, \quad g_{21}^{Qq} = g_{22}^{Qq} < 0 , \\ \alpha_1 g_{13}^{Qq} &= g_{11}^{Qq} < 0, \quad \alpha_2 g_{23}^{Qq} = -g_{22}^{Qq} > 0 , \end{aligned} \quad (3.22)$$

where α_1 and α_2 are a small number of order $\frac{1}{10}$.

We expand parameters u 's and θ 's in powers of r and look for the minimum of $E(W)$ to the first order of r :

$$\begin{aligned} E &= E^0 + E^1 r + O(r^2) , \\ u &= u^0 + u^1 r + O(r^2) , \\ \theta &= \theta^0 + \theta^1 r + O(r^2) , \end{aligned} \quad (3.23)$$

where we have omitted the indices i and j and the superscript Q or q in the parameters u and θ . After some algebra we find

$$E^0 = g_{11}^Q (u_1^0)^2 + g_{22}^Q (u_2^0)^2 , \quad (3.24)$$

$$\begin{aligned} E^1 &= 2g_{11}^Q u_1^0 u_1^1 + 2g_{22}^Q u_2^0 u_2^1 \\ &\quad + 2g_{11}^{Qq} [u_1^0 u_1^0 \cos(\theta_1^0 - \theta_1^0) + u_1^0 u_2^0 \cos(\theta_1^0 - \theta_2^0) + \alpha_1^{-1} u_1^0 u_3^0 \cos(\theta_1^0 - \theta_3^0)] \\ &\quad + 2g_{22}^{Qq} [u_2^0 u_1^0 \cos(\theta_2^0 - \theta_1^0) + u_2^0 u_2^0 \cos(\theta_2^0 - \theta_2^0) + \alpha_2^{-1} u_2^0 u_3^0 \cos(\theta_2^0 - \theta_3^0)] . \end{aligned} \quad (3.25)$$

With the choice of coupling constants (3.22) we find by looking at Eq. (3.24) that the minimum of $E(W)$ is attained when $u_1^Q = u_2^Q = 1$ and $\theta_1^Q = \theta_2^Q$ (u_1^Q and u_2^Q are less than 1 by unitarity of the matrix U). We differentiate Eq. (3.25) by θ to obtain the condition of the minimum:

$$\begin{aligned} g_{11}^{Qq} \sin(\theta_1^0 - \theta_1^0) + g_{12}^{Qq} \sin(\theta_2^0 - \theta_1^0) &= 0 , \\ g_{13}^{Qq} \sin(\theta_1^0 - \theta_3^0) + g_{23}^{Qq} \sin(\theta_2^0 - \theta_3^0) &= 0 , \\ 2g_{11}^{Qq} \sin(\theta_1^0 - \theta_1^0) + g_{13}^{Qq} \sin(\theta_1^0 - \theta_3^0) &= 0 , \\ 2g_{22}^{Qq} \sin(\theta_2^0 - \theta_1^0) + g_{23}^{Qq} \sin(\theta_2^0 - \theta_3^0) &= 0 . \end{aligned} \quad (3.26)$$

We look for a nontrivial solution with $u_1^Q = u_2^Q = 1$ for Eq. (3.26). For this purpose we rewrite Eq. (3.26) in the form

$$\begin{aligned}\alpha_1 &= -2 \frac{\sin(\theta_1^{Q0} - \theta_3^{Q0})}{\sin(\theta_1^{Q0} - \theta_1^{Q0})}, \\ \alpha_1 &= 2 \frac{\sin(\theta_2^{Q0} - \theta_3^{Q0})}{\sin(\theta_2^{Q0} - \theta_1^{Q0})}, \\ \alpha &= - \frac{\sin(\theta_2^{Q0} - \theta_1^{Q0})}{\sin(\theta_1^{Q0} - \theta_1^{Q0})},\end{aligned}\quad (3.27)$$

where α is given by

$$\alpha \equiv g_{11}^{Qq} / g_{22}^{Qq}. \quad (3.28)$$

We require that the strong CP violation in the sector of ordinary quarks is absent and therefore

$$\theta_1^{Q0} + \theta_2^{Q0} + \theta_3^{Q0} = 0. \quad (3.29)$$

Neglecting the difference between u and c we set

$$\theta_1^{Q0} = \theta_2^{Q0}, \quad (3.30)$$

and we have

$$\theta_1^{Q0} = \theta_2^{Q0} = -\frac{1}{2}\theta_3^{Q0} \equiv \theta^{Q0}. \quad (3.31)$$

Since we wish to keep the parameters α_1 and α_2 smaller than α by order of $\frac{1}{10}$ we set

$$\begin{aligned}\theta_1^{Q0} + 2\theta^{Q0} &= -\varepsilon_1, \\ \theta_2^{Q0} + 2\theta^{Q0} &= \pi + \varepsilon_2,\end{aligned}\quad (3.32)$$

where ε_1 and ε_2 are a small number of order $\frac{1}{10}$.

Inserting Eq. (3.32) into (3.27) and neglecting terms of $O(\varepsilon_1^2)$ and $O(\varepsilon_2^2)$ we obtain

$$\begin{aligned}\alpha_1 &= \frac{\varepsilon_1}{2 \sin \theta}, \\ \alpha_2 &= \frac{\varepsilon_2}{2 \sin \theta}, \\ \alpha &= 1 + (\varepsilon_1 + \varepsilon_2) \frac{\cos \theta}{\sin \theta},\end{aligned}\quad (3.33)$$

where we have set $\theta = -3\theta^{Q0}$. Equation (3.33) is the general solution for our present problem. It will provide us with a model in which the CP violation is dynamically generated and the realistic quark mass matrix is obtained.

We use the above solution to rewrite $\mathcal{L}' (= -\mathcal{H}')$ in the new quark fields Q' and q' obtained by transforming Q and q by U . We find

$$\begin{aligned}\mathcal{H}' &= G_{1111}^Q \bar{U}'_L U'_R \bar{U}'_R U'_L + G_{2222}^Q \bar{C}'_L C'_R \bar{C}'_R C'_L \\ &+ G_{1111}^{Qq} [(e^{i(\theta - \varepsilon_1)} \bar{U}'_L U'_R + \alpha^{-1} e^{i(\pi + \theta + \varepsilon_2)} \bar{C}'_L C'_R)(\bar{u}'_R u'_L + \bar{c}'_R c'_L) \\ &+ (\alpha_1^{-1} e^{-i\varepsilon_1} \bar{U}'_L U'_R - \alpha_2^{-1} \alpha^{-1} e^{i(\pi + \varepsilon_2)} \bar{C}'_L C'_R) \bar{t}'_R t'_L] + \text{H.c.} \\ &+ G_{1111}^q \bar{u}'_L u'_R \bar{u}'_R u'_L + \dots + G_{1313}^q e^{i\theta} \bar{u}'_L t'_R \bar{u}'_R t'_L + \dots,\end{aligned}\quad (3.34)$$

where we chose $U_L^Q = U_L^q = 1$. Obviously we see that the CP-violating terms are generated in Eq. (3.34).

For $u_1^Q = u_2^Q = 1$ the mass matrix reads

$$M \sim g_{11}^{Qq} \begin{pmatrix} \alpha \cos(\theta_1^{Q0} - \theta^q) + \cos(\theta_2^{Q0} - \theta^q) & 0 & 0 \\ 0 & \alpha \cos(\theta_1^{Q0} - \theta^q) + \cos(\theta_2^{Q0} - \theta^q) & 0 \\ 0 & 0 & \alpha_1^{-1} \cos(\theta_1^{Q0} + 2\theta^q) - \alpha_2^{-1} \cos(\theta_2^{Q0} + 2\theta^q) \end{pmatrix}. \quad (3.35)$$

If we take into account Eq. (3.33) we find

$$M \sim g_{11}^{Qq} \begin{pmatrix} \frac{\varepsilon_1 + \varepsilon_2}{\sin \theta} & 0 & 0 \\ 0 & \frac{\varepsilon_1 + \varepsilon_2}{\sin \theta} & 0 \\ 0 & 0 & 2 \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 \varepsilon_2} \sin \theta \end{pmatrix}. \quad (3.36)$$

Hence the ratio of the quark masses is given in the present model by

$$\frac{m_{u,c}}{m_t} = \frac{\varepsilon_1 \varepsilon_2}{2 \sin^2 \theta}. \quad (3.37)$$

By keeping $\varepsilon_1 \sim \varepsilon_2 \sim \frac{1}{10}$ and $\sin^2 \theta \sim 1$ we obtain

$$\frac{m_{u,c}}{m_t} \sim \frac{1}{100}. \quad (3.38)$$

Thus our model well reflects the real situation. We may elaborate our model by tuning parameters ε_1 , ε_2 , and θ so as to reproduce the actual quark masses. In this paper, however, we will not go into the detail of such analysis.

It should be noted that the absence of the strong CP violation has been required only in the sector of ordinary quarks q [See Eq. (3.29)] and no such requirement has been set in the sector of fundamental quarks Q . Our point of view is that the strong CP problem in the sector of the fundamental quarks should be resolved when the

underlying theory of the fundamental quarks is disclosed.

Of course we can construct a model without the strong CP violation in the sector of the fundamental quarks, i.e., $\theta_1^Q + \theta_2^Q = 0$, in our present approach although we consider such a model to be unnecessary for the present purpose. A typical example which respects the condition $\det U^Q = \det U^q = 1$ is given by

$$U^Q = \begin{pmatrix} e^{\pm i(\pi/4)} & 0 \\ 0 & e^{\mp i(\pi/4)} \end{pmatrix}, \quad (3.39)$$

$$U^q = \begin{pmatrix} e^{\mp i(\pi/2)} & 0 & 0 \\ 0 & e^{\mp i(\pi/2)} & 0 \\ 0 & 0 & e^{\pm i\pi} \end{pmatrix},$$

where we required

$$g_{11}^{Qq} = g_{12}^{Qq} = -g_{21}^{Qq} = -g_{22}^{Qq} > 0, \quad (3.40)$$

$$g_{13}^{Qq} = 2g_{11}^{Qq} > 0,$$

$$g_{23}^{Qq} = -2g_{22}^{Qq} > 0.$$

The quark mass matrix corresponding to the solution (3.39) is given by

$$M \propto g_{11}^{Qq} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 2 \end{pmatrix}, \quad (3.41)$$

C. Models

We found the CP -violating interaction Lagrangian as a result of special solutions of the minimum $E(W)$ condition. Thus we succeeded in constructing the simple model of the dynamical CP violation. In deriving the model we made some simplifying assumptions. This simplification made the model far from explaining the real situation in standard theory. For example, our model Hamiltonian does not reproduce the KM matrix correctly. In order to get the full KM matrix we have to relax our assumptions and minimize $E(W)$ with the full expression of the transformation matrix U . (We have to abolish the assumption that U be a diagonal matrix.) This attempt will be made in a separate work. We are, however, interested in estimating physical effects in low-energy phenomena which are predicted by the Hamiltonian. Such estimation may help examine whether our model serves as a prototype of the real theory of the dynamical CP violation for standard theory.

The system of quarks we assumed consists of the up-type two-flavor fundamental quarks Q and three-flavor ordinary quarks q as shown in Eq. (3.1). We have not yet specified the symmetry group S to which the fundamental quarks Q belong.

A natural possibility is to identify the symmetry group S to the technicolor $SU(N)$. In this case the fundamental quark Q is the techniquark [5] belonging to the N -dimensional fundamental representation of the technicolor $SU(N)$. Another possibility is to identify the symmetry group S to the color $SU(3)$. In this case the fundamental quark Q is the color-sextet quark [9] belonging to the six-dimensional representation of the color $SU(3)$.

These two possibilities fit the previous argument quite well and constitute two practical models of the dynamical CP violation.

It is also possible to identify the fundamental quarks Q to the top quark in the top-quark condensation model [6] (or in the top-color model [7]). In this case, however, we are not able to get the nontrivial CP -violating phase within our framework.

Yet another possibility is to identify the fundamental fermion Q to the quark in the assumed fourth generation [8]. In this case, it is again impossible to obtain the nontrivial CP -violating phase in our approach.

In the following application we keep in mind the technicolor model as well as the color-sextet quark model.

IV. LOW-ENERGY EFFECTS

In our simple model introduced in the last section the KM matrix is real and diagonal. This is because we have taken a particular choice for a G_F breaking Lagrangian \mathcal{L}' and have neglected the higher-order terms in r . Starting with the more general assumption we could have obtained the KM matrix with off-diagonal elements and complex phases.

In this section we consider possible low-energy effects originating from the model Lagrangian (3.34). By this analysis we will be able to compare the low-energy CP -violating effects of dynamical origin with the one in the standard origin of the CP violation (i.e., through the KM phase).

Our Hamiltonian reads

$$H = H_0 + H'_{\text{cons}} + H'_{\text{viol}}, \quad (4.1)$$

where H_0 is the Hamiltonian derived from Lagrangian \mathcal{L}_0 , H'_{cons} is the CP conserving part of the Hamiltonian defined by integrating Eq. (3.34) over the space variables, and H'_{viol} is the CP -violating part. In the following we consider two typical low-energy effects derived from the Hamiltonian (4.1).

A. Neutron electric dipole moment

Since the Lagrangian \mathcal{L}' includes the energy scale at which the four-fermion interactions are induced from the more fundamental gauge theory, it is expected that our estimate of the neutron electric dipole moment depend on this energy scale. This means that this fundamental energy scale, i.e., the cutoff parameter Λ , may be constrained by the experimental information on the neutron electric dipole moment.

We estimate the size of the contribution to the neutron electric dipole moment coming from our CP -violating Lagrangian \mathcal{L}' given in Eq. (3.34).

The neutron electric dipole moment d_n is given in terms of the quark dipole moments d_u and d_d in the naive quark model such that

$$d_n = (4d_d - d_u)/3. \quad (4.2)$$

The electric dipole moment of quarks is calculated through the following term in the quark electromagnetic form factor at zero-momentum transfer:

$$-d_q \bar{u} \sigma_{\mu\nu} \gamma_5 q^\nu u, \quad (4.3)$$

where the index q represents the u or d quark, q^ν is the momentum transfer for quarks (momentum carried by the virtual photon), and u is the Dirac spinor for quark q .

We start with the Lagrangian \mathcal{L}' given in Eq. (3.34). For the later calculational convenience we introduce auxiliary fields B and use the following effective Lagrangian instead of the four-fermion type Lagrangian (3.34):

$$\mathcal{L}' = -\bar{\psi} \left[B^\dagger \frac{1-\gamma_5}{2} + B \frac{1+\gamma_5}{2} \right] \psi + G^{-1} B^\dagger B. \quad (4.4)$$

The use of the above auxiliary-field Lagrangian makes it easier to classify the relevant Feynman diagrams contributing to the quark electric dipole moment and to perform the higher-order loop calculations.

At the one-loop level, the diagrams shown in Fig. 1 contribute to the quark electromagnetic form factor. As is easily seen the diagram in Fig. 1(a) has no tensor structure corresponding to the electric dipole moment. The contribution of Fig. 1(b) to the electric dipole moment is found to vanish. Thus there is no one-loop contribution to the quark electric dipole moment.

We next examine the two-loop contribution to the quark electric dipole moment. The relevant diagrams are shown in Fig. 2. The Feynman amplitudes corresponding to these diagrams are, in general, quartically divergent. The quartically divergent part of the amplitudes, however, has no tensor structure of the electric dipole moment

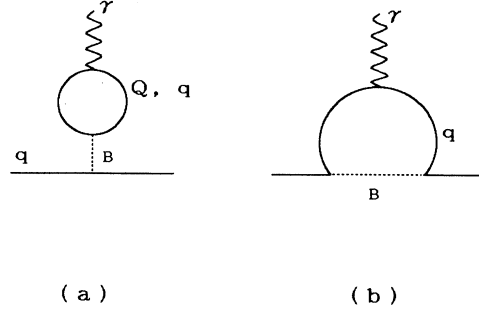


FIG. 1. One-loop diagrams for the electromagnetic vertex function of quarks represented by the use of auxiliary field B .

and hence the leading contribution of these diagrams to the quark electric dipole moment is quadratically divergent. As is seen by direct calculations, the diagrams in Figs. 2(a) and 2(b) have no quadratically divergent contribution to the quark electric dipole moment. The reason for this is that the helicity of the quark flips three times in these diagrams. Accordingly the leading quadratic divergence exists only in the diagram in Fig. 2(c). In the following we will calculate the quadratically divergent part of the Feynman amplitude corresponding to the diagram in Fig. 2(c).

The Feynman amplitude F corresponding to the diagram in Fig. 2(c) reads

$$\begin{aligned}
F = \sum_{i,j,k} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} & \left[G_{jiuk}^q G_{kjiu}^q \left(\frac{1+\gamma_5}{2} \frac{1}{m_i - \not{p}_1 + \not{p}'} \frac{1-\gamma_5}{2} \frac{1}{m_j - \not{p}_1 + \not{p} + \not{p}'} \right. \right. \\
& \times \frac{1-\gamma_5}{2} \frac{1}{m_k - \not{p}_1 + \not{p}} (Q_k e \gamma_\mu) \frac{1}{m_k - \not{p}_2 + \not{p}} \frac{1+\gamma_5}{2} \\
& + \frac{1+\gamma_5}{2} \frac{1}{m_i - \not{p}_1 + \not{p}'} \frac{1-\gamma_5}{2} \frac{1}{m_j - \not{p}_1 + \not{p} + \not{p}'} (Q_j e \gamma_\mu) \\
& \times \frac{1}{m_j - \not{p}_2 + \not{p} + \not{p}'} \frac{1-\gamma_5}{2} \frac{1}{m_k - \not{p}_2 + \not{p}} \frac{1+\gamma_5}{2} \\
& + \frac{1+\gamma_5}{2} \frac{1}{m_i - \not{p}_1 + \not{p}'} (Q_i e \gamma_\mu) \frac{1}{m_j - \not{p}_2 + \not{p}'} \frac{1-\gamma_5}{2} \\
& \left. \times \frac{1}{m_j - \not{p}_2 + \not{p} + \not{p}'} \frac{1-\gamma_5}{2} \frac{1}{m_k - \not{p}_2 + \not{p}} \frac{1+\gamma_5}{2} \right) \\
& + G_{ukji}^q G_{iukj}^q (\gamma_5 \rightarrow -\gamma_5) \Big], \quad (4.5)
\end{aligned}$$

where $p_1(p_2)$ is the momentum of the incoming (outgoing) quark. Here in Eq. (4.5) the charge Q_j is equal to $\frac{2}{3}$ for up-type quarks, i.e., $j=u,c,t$, and is equal to $-\frac{1}{3}$ for down-type quarks, i.e., $j=d,s,b$. By extracting the quadratically divergent part F^{div} of Eq. (4.5) we obtain

$$F^{\text{div}} = \frac{2\Lambda^2}{(4\pi)^4} \sum_{i,j,k} Q_e \text{Im}(G_{jiuk}^q G_{kjiu}^q) m_j (i A_j k_\mu \gamma_5 - B_j \sigma_{\mu\nu} q^\nu \gamma_5), \quad (4.6)$$

where A_j and B_j are given by

$$\begin{aligned}
A_j &= 2 \left[\ln \frac{\Lambda^2}{m_j^2} - 1 \right] \\
&+ \int_0^1 dx \int_0^1 dy \int_0^{1-y} dz \left\{ (3-5y) \ln \left| \frac{x(1-x)+1-y-z}{1-y-z} \right| - \frac{3x(1-x)(1-2y)}{x(1-x)+1-y-z} + \frac{1}{2} \frac{x(1-x)}{[x(1-x)+1-y-z]^2} \right. \\
&\quad \left. \times \left[x(1-x) \left[3(3-7y) - \frac{4}{x}(2-3y) \right] + (1-y-z) \left[2(3-7y) - \frac{2}{x}(2-3y) \right] \right] \right\}, \\
B_j &= \int_0^1 dx \int_0^1 dy \int_0^{1-y} dz \frac{1}{x} \left[\frac{-x(1-x)[2x(1-x)+(1-y-z)]}{[x(1-x)+1-y-z]^2} \right].
\end{aligned} \tag{4.7}$$

After some algebra we derive the following formula for the quadratically divergent part of the electric dipole moment of the up quark d_u :

$$d_u = \frac{2}{3} e \frac{\Lambda^2}{(4\pi)^4} \sum_{i,j,k} \text{Im}(G_{jiuk}^q G_{kjiu}^q) m_j (A_j + B_j). \tag{4.8}$$

Performing the integration in Eq. (4.7) we finally find the explicit expression for the quadratically divergent part of the up-quark electric dipole moment:

$$d_u = \frac{2}{3} e \frac{2\Lambda^2}{(4\pi)^4} \sum_{i,j,k} \text{Im}(G_{jiuk}^q G_{kjiu}^q) m_j \left[2 \ln \frac{\Lambda^2}{m_j^2} - 2.01 \right]. \tag{4.9}$$

Apparently the dominant contribution in the above formula to the up-quark dipole moment comes from the top-quark intermediate state. Keeping only the top-quark contribution to Eq. (4.9) and taking into account that

$$\text{Im}(G_{jiuk}^q G_{kjiu}^q) \sim \frac{g^4}{4\Lambda^4} \sim \frac{(2\pi)^2}{\Lambda^4}, \tag{4.10}$$

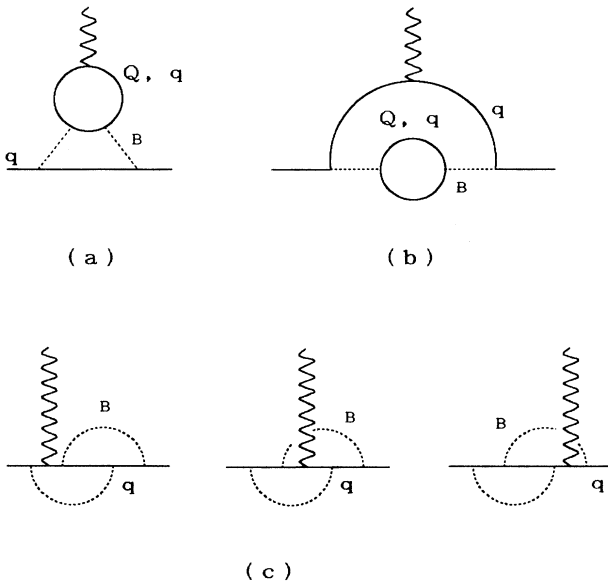


FIG. 2. Two-loop diagrams for the electromagnetic vertex function of quarks.

we find

$$d_u = e \frac{m_t}{48\pi^2 \Lambda^2} \left[4 \ln \frac{\Lambda}{m_t} - 2.01 \right]. \tag{4.11}$$

Since $d_u \gg d_d$, we find that $d_n = d_u/3$. Assuming that $m_t = 140$ GeV and taking into account the experimental upper bound of the neutron electric dipole moment [3] we realize that the effective cutoff of the loop integral should satisfy

$$\Lambda > 800 \text{ TeV}. \tag{4.12}$$

The above lower bound for the cutoff Λ is of the same order as the one set by the FCNC restriction [13]. If we use the value of Λ set by the FCNC restriction which will be described in Eq. (4.22) and calculate d_n through Eq. (4.11), we find $d_n \sim 5 \times 10^{-27} e$ cm. This prediction is surely much smaller than the present experimental bound. In the standard model with the KM phase the neutron electric dipole moment is calculated and is found to be extremely small [2]. Our results (4.11) and (4.12) guarantee this property of the standard model.

B. K -meson system

The only known experimental information on the CP violation exists in the K -meson decays. In this subsection we discuss the ε parameter which is determined by measuring the charge asymmetry in the semileptonic decay of the K^0 meson and the 2π decay of the K_L^0 meson.

K^0 -meson states $|K_L^0\rangle$ and $|K_S^0\rangle$ are defined as

$$\begin{aligned}
|K_L^0\rangle &= \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} [(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle], \\
|K_S^0\rangle &= \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} [(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle] \\
&(|K^0\rangle = -CP|\bar{K}^0\rangle). \tag{4.13}
\end{aligned}$$

With the nonvanishing ε the K -meson mass eigenstates are different from the eigenstates of CP . We have

$$\begin{aligned}
\varepsilon &= \frac{\langle K^0 | H | \bar{K}^0 \rangle^{1/2} - \langle \bar{K}^0 | H | K^0 \rangle^{1/2}}{\langle K^0 | H | \bar{K}^0 \rangle^{1/2} + \langle \bar{K}^0 | H | K^0 \rangle^{1/2}} \\
&\simeq \frac{\langle K^0 | H'_{\text{viol}} | \bar{K}^0 \rangle}{\langle K^0 | (H_0 + H'_{\text{cons}}) | \bar{K}^0 \rangle}. \tag{4.14}
\end{aligned}$$

Here we require that

$$|\langle K^0 | (H_0 + H'_{\text{cons}}) | \bar{K}^0 \rangle| \gg |\langle K^0 | H'_{\text{viol}} | \bar{K}^0 \rangle|. \quad (4.15)$$

The Hamiltonian H'_{viol} contains the term

$$i \text{Im}(G) \int d^3x \bar{s}_L d_R \bar{s}_R d_L + \text{H.c.}, \quad (4.16)$$

where G is the corresponding four-fermion coupling constant and s and d represent the s - and d -quark fields. Although in our model $\text{Im}(G)$ vanishes, we consider here the more general case in which $\text{Im}(G) \neq 0$. Using the PCAC (partial conservation of axial-vector current) relation [it should be remembered that a specific choice of the contraction of color indices is made in Eq. (3.3)], we find

$$\langle K^0 | \bar{s}_L d_R \bar{s}_R d_L | \bar{K}^0 \rangle = -\frac{B_K(\mu) f_K^2 m_K^4}{2(m_s + m_d)^2}, \quad (4.17)$$

where $B_K(\mu)$ is the so-called B parameter, f_K is the K -meson decay constant and m_K , m_d , and m_s are the mass of the K meson, d quark, and s quark, respectively. After some calculation we obtain

$$\langle K^0 | H'_{\text{viol}} | \bar{K}^0 \rangle \simeq -i \frac{\text{Im}(G) B_K(\mu) f_K^2 m_K^3}{4(m_d + m_s)^2} \langle K^0 | K^0 \rangle. \quad (4.18)$$

We see by definition

$$\langle K^0 | (H_0 + H'_{\text{cons}}) | \bar{K}^0 \rangle \simeq \frac{1}{2} \left[\Delta M - \frac{i}{2} \Delta \Gamma \right] \langle K^0 | K^0 \rangle, \quad (4.19)$$

where ΔM and $\Delta \Gamma$ are the K_L - K_S difference of the mass and decay width, respectively. Accordingly we obtain

$$\varepsilon \simeq \frac{-i \text{Im}(G) B_K(\mu) f_K^2 m_K^3}{2[\Delta M - (i/2)\Delta \Gamma](m_d + m_s)^2}. \quad (4.20)$$

By inserting experimental data in Eq. (4.20) we find

$$\text{Im}(G) \sim 10^{-9} \text{ TeV}^{-2}. \quad (4.21)$$

The above result (4.21) is about 10^2 times smaller than that obtained by the FCNC restriction [13]:

$$\text{Re}(G) < 10^{-7} \text{ TeV}^{-2}. \quad (4.22)$$

V. CONCLUSION

Applying Dashen's mechanism to the composite Higgs models we succeeded in finding simple models of the dynamical CP violation. Although our models have to be further elaborated to explain the actual KM phase, they represent an essential ingredient of the dynamical CP violation in the standard model and may be thought of as prototype models which accommodate the CP violation in the standard model.

In order to see whether our model could be in conformity with experimental situations we examined low-energy consequences of our model. By estimating the ε parameter in the neutral K -meson decays and the neutron electric dipole moment we derived the lower bound on the cutoff parameter using the available experimental informations. The cutoff parameter signals, at the scale determined by the low-energy data, the existence of the deeper theory for which our model is an effective theory. The lower bound we obtained is consistent with the one required by the constraint on the flavor-changing neutral current.

Although our model is a simple toy model for the dynamical CP violation, it may be elaborated to fully account for the CP violation in the standard model. The investigation in this direction is in progress.

ACKNOWLEDGMENTS

The authors would like to thank T. Onogi for discussions and suggestions and K. Yamawaki for useful informations. S.H. was supported in part by Monbusho Grant-in-Aid under the contract No. 040011. T.M. was supported in part by Monbusho Grant-in-Aid for Scientific Research (C) under the contract No. 04640301.

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