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CONTROL OF SOIL HETEROGENEITY AND USE OF THE PROBABLE ERROR CONCEPT IN PLANT BREEDING STUDIES

By H. K. Hayes

The importance of plant breeding has become more generally recognized in recent years. This to some extent, probably is a result of a growing appreciation of the need of obtaining varieties resistant to fungus and insect parasites. The problem of a determination of the comparative value of numerous new productions has become greatly complicated, however, by the increasing number of new varieties which must be compared.

The Mendelian mode of attack in the synthesis of new varieties necessitates the study of large numbers of new strains. The realization that most, if not all, important economic characters are the result of the interaction of many factors plus environment is, perhaps, the main reason from a genetic standpoint why large numbers should be used.

For these reasons the question of better methods for trying out new varieties has received considerable study altho at present investigators differ as to methods. In breeding self-fertilized crops, such as most small grains, a uniformity of procedure for comparative trial of large numbers of new strains has been evolved. The plan developed in the United States for the preliminary trial of numerous strains of small grains is known as the rod-row method. There is some variation in details such as the use of single- or three-row plots; the rate of seeding for varieties which differ in grain size; the number of replications; and the method of sowing the grain. Further study will make it possible eventually to standardize these procedures; it is not feasible, however, to carry on the studies in all localities in the same manner as environmental conditions, the number of strains under comparison, and other factors greatly influence the accuracy and value of the methods.

The subject uppermost in the minds of many field plot technicians at the present time may be stated in the form of a question,—what is the best means of comparing and eliminating varieties after all possible care has been taken to conduct the study in a desirable manner with the facilities available? This has led to a critical study of the use of probable errors in the elimination of new varieties.

It seems unnecessary in the present paper to review the literature in this field as this has been discussed in various papers.[1] The different

[1] See Journal American Society of Agronomy, Vol. 15, and various reports of the Varietal Standardization Committee in the same journal.

uses of the probable error concept being studied can be grouped under three general heads: (1) the calculation of probable errors for each separate strain in the test; (2) the use of "Student's method" of comparing varieties; and (3) the calculation of a probable error of the experiment by the use of check plots or some method whereby the results from all varieties can be used for the calculation of a single probable error for the experiment.

A detailed comparison of the three methods would be of interest. At present, however, it seems desirable to point out briefly some of the difficulties and desirable points in each altho (3), the calculation of a single probable error of the experiment will be discussed in some detail.

With a single variety harvested in one-fortieth acre plots and under rather carefully conducted experimental conditions it was learned that when the yields of four systematically replicated plots were used for each calculation that with a total of 50 comparisons there were two probable error calculations which differed from each other to such an extent that one was over 900 per cent larger than the other [Kiesselbach (7), Hayes (5)]. This result, which on purely mathematical bases, might be expected somewhat frequently seems a sufficient reason to doubt the wisdom of using in the comparison of two varieties probable error calculations based on as few as 3 or 4 plots. This opinion is at some variance with a recent one as stated by Love (10) who certainly deserves much credit for stimulating a more general interest in the subject. Love says, "In answer to the criticism of some that the probable error concept cannot be applied to a small number of observations, it is well to add that if this is so we should place little dependence on a mean or other constant obtained from a few observations" and also "yet when necessary it is better to calculate it," that is, a probable error, "from two or three plots rather than not use it at all." Perhaps there is little difference of opinion after all between Love and the writer for it is hardly ever necessary to calculate a probable error from few comparisons. If the probable error itself is no more accurate than the mean it is scarcely conceivable that it is of great value as an aid in comparing two means.

The method developed by "Student" (15, 16) for comparing two series of results was first brought to the attention of American investigators by Love (8). In its application to varietal trials this method involves pairing like plots and the use of student tables for small numbers of replications. In this discussion the term "Students Method" refers to the pairing of results. The same philosophy would apply to the standard deviation of successive differences used with the ordinary tables of probabilities. In varietal or strain comparisons "Student's Method" is applicable for those cases where paired comparisons are available and while under certain conditions its use greatly facilitates the study, it is perhaps not as valuable for the plant breeder as for other investigators. It may be desirable to point out some limitations in its use for the plant breeder.

In a comparison of two wheat varieties, one of which is resistant to stem rust and the other susceptible, let us suppose that both have approximately the same yielding ability when rust is not a factor and in addition let us suppose that one is resistant and the other susceptible under rust epidemic conditions. In a five-year test we may further suppose that the yield of the susceptible variety is reduced 50 per cent by rust one year when rust was a major factor. All data are hypothetical in this problem but there is no reason for believing that such a case might not occur.

The hypothetical comparison is outlined here:

Year	Variety 1	Variety 2	D	D'	D'^2
1	40	42	—2	6	36
2	42	40	2	2	4
3	40	42	—2	6	36
4	42	40	2	2	4
5	40	20	20	16	256
			5|20		5|336
			4		67.20

$Z=\frac{4}{8.2}=4.9$ S.D.$=\sqrt{67.20}=8.2$

The chances are 4.24:1 that the difference is significant (9). A separate probable error calculated each year would have led without doubt to the conclusion that the varieties were significantly different in yielding ability one year out of five.

To be strictly accurate in field plot trials where there is correlation in the yielding ability of nearby plots Student's method should be used only for paired comparisons. In variety trials conducted for several years the yearly averages represent seasonal pairs but do not adequately take into account soil heterogeneity. Two writers have pointed out in the field of agronomy [Kemp (6), Richey (11)] that the probable error of a difference should consider the correlation between the two series of comparable variables. If there is a correlation in the yielding ability of nearby plots this leads to a reduction in the probable error of a difference and a perfect correlation or 1 reduces the probable error of a difference to approximately 0. In plant breeding studies, however, where small plots are used the use of the correlation coefficient is perhaps not as necessary as in some other kind of field studies. This is to some extent due to the fact that over a three-year period two varieties will not be grown in the same relationship to each other.

Some plant breeders, likewise, change the order of the series thus partially overcoming the error which arises from neglecting the correlation coefficient in computing the probable error of a difference.

The computation of a single probable error in percentage, representing the error which arises from soil heterogeneity and other uncontrollable errors, is a convenient method for the plant breeder. The present paper is largely a consideration of the method of computing probable errors by what is called the "Deviation from the Mean Method" used for several years by the writer. It is similar in nature to the pairing method outlined by Wood and Stratton (17) in that it allows all plots of all varieties in the test to be used in computing the probable error of the experiment, i.e., the probable error which represents soil heterogeneity and other errors of a non-systematic nature. After presenting the method of calculation and the use of the probable error obtained, results will be given which indicate the extent of error in rod-row trials resulting from the failure to use the correlation coefficient in obtaining the probable error of a difference.

THE DEVIATION FROM THE MEAN METHOD OF CALCULATING A PROBABLE ERROR

A plant breeder, in the majority of cases, centers his attention chiefly upon a few characters at a time and discards rather freely. For this reason the strains which are under trial are perhaps not widely different in general adaptability. For several years in the Minnesota plant breeding studies a comparison of probable errors as calculated by different methods has been made.

Check plots of standard varieties—Marquis wheat, Manchuria (Minn. No. 184) barley, and Victory oats—have been planted every fifth to tenth plot throughout the rod-row trials and probable errors calculated from these plots[2] have been used as standards for comparison of other methods of obtaining probable errors. In a previous paper [Hayes, (5)] probable errors obtained by the deviation from the mean method and Wood and Stratton's pairing method were compared with the probable errors as obtained from the use of check plots. It was found that the deviation from the mean method gave results which were closer to those obtained from check plots than were the probable errors calculated from the pairing method. Garber and others (2), at West Virginia, have compared probable errors computed from check plots with those obtained from the deviation from the mean method for a three-year period for buckwheat and for two years for oats and wheat. Agreements were good except for buckwheat, in 1921, when the deviation from the mean method gave a probable error twice as

[2] The formula used was $P.E. = \pm .6745 \sqrt{\frac{\Sigma(d^2)}{n}}$

great as by the use of check plots. The only other wide variation was in 1923 for wheat studies, the percentage probable error by the check plot method being 12.9 and by the deviation from the mean method, 18.9.

The computation method where the yields obtained from different varieties are used to calculate a single probable error in percentage may be given as follows:

COMPUTING OF PROBABLE ERRORS BY THE DEVIATION FROM THE MEAN METHOD

1. Express in per cent the deviation of each plot of each variety from its variety mean.
2. Square these percentage deviations and sum by adding all squared deviations.
3. Divide the sum of the squared deviations by the total number of deviations and extract the square root.
4. After extracting the square root multiply the value obtained by $\pm.6745$. The value obtained is the probable error of a single plot test in percentage.
5. To compute the probable error of any number of systematically distributed plots, (n) divide the P.E. of a single plot by the $\sqrt{n}$.
6. Multiply the yield by the percentage probable error to obtain a probable error in bushels.

The method of computation may be illustrated altho much larger numbers are desirable than are used in the illustration. One of the advantages of the plan is that so many deviations are obtained that an accurate estimate of the extent of variability is reached. The plan is illustrated for two varieties each grown in four systematically distributed plots. Commonly, at least 100 deviations are available in plant breeding studies.

ILLUSTRATION OF METHOD

Variety	Yield each plot in bu.	Mean	Dev. in bu.	Dev. in per cent	Dev. in per cent squared
Marquis x Iumillo	35.4		4.6	11.50	132.25
II-15-44	48.0	40.0	8.0	20.00	400.00
	37.3		2.7	6.75	45.56
	39.2		0.8	2.00	4.00
Marquis x Kanred	35.5		2.7	7.07	49.98
II-15-58	44.4	38.2	6.2	16.23	263.41
	37.4		0.8	2.09	4.37
	35.3		2.9	7.59	57.61

$\sqrt{\dfrac{958.18}{8}} = 10.9 =$ S.D. single plot in per cent.

$\dfrac{10.9}{\sqrt{4}} = 5.45 =$ P.E. in per cent for 4 systematically distributed plots by the deviation from the mean of the variety method.

It is apparent in the illustration given that Plot 2 in each variety was in a favorable position and that there is some correlation in yielding ability of neighboring plots.

The criterion of the accuracy of the method is a comparison with probable errors obtained from the yields of check plots of a standard variety where the probable error $= \pm .6745 \sqrt{\frac{\Sigma(d^2)}{n}}$.

Results obtained from a six-year comparison of the two methods are presented in Table I.

TABLE I

PROBABLE ERRORS OF A SINGLE PLOT TEST AS OBTAINED FROM CHECK PLOTS OF A STANDARD VARIETY AND FROM A COMBINATION OF OTHER VARIETIES IN THE TRIAL*

Crop	Year	Check plot method (Single standard variety)			Deviation of mean method (All varieties)		
		Av. yield	No. plots	P.E.	Av. yield	No. deviations	P.E.
		Bu.		Per cent	Bu.		Per cent
S. wheat	1919	20.3	42	11.8	20.0	108	15.2
	1920	25.8	60	10.1	25.0	252	9.6
	1921	18.6	44	10.2	17.8	140	10.2
	1922	35.4	52	9.2	33.8	292	8.6
	1923	30.9	26	5.2	32.2	360	5.7
	1924	26.9	64	9.0	25.7	328	9.9
Average		26.3		9.3	25.8		9.9
Oats	1919	56.4	48	7.6	62.3	132	6.1
	1920	80.7	42	9.2	77.1	788	9.4
	1921	50.7	43	7.5	51.8	388	7.9
	1922	115.6	55	4.7	100.6	160	5.1
	1923	61.6	55	7.0	61.0	579	6.1
	1924	59.0	60	10.4	58.3	292	6.7
Average		70.7		7.7	68.5		6.9
Barley	1919	41.9	52	9.5	29.7	80	9.2
	1920	49.1	32	10.0	41.5	188	10.1
	1921	30.9	62	10.0	28.1	232	9.6
	1922	60.7	48	11.6	60.4	520	10.1
	1923	56.1	34	7.1	54.1	324	7.5
	1924	42.5	24	7.9	40.8	76	8.6
Average		46.9		9.4	42.4		9.2
W. wheat	1924	45.2	24	5.3	45.6	88	5.1

* Miss Alma Schweppe, Fred Griffee, and H. E. Brewbaker assisted in the computations.

The two methods of computation gave very similar results. The average for the six-year period where the probable errors for the check plot method are given first and for the deviation of the mean method next are as follows: for wheat 9.3%, and 9.9; for oats 7.7 and 6.9; and for barley 9.4 and 9.2. The only differences of any magnitude were in 1919 in which case the probable errors by the two methods are 11.8 and 15.2, respectively, for wheat; and in 1924, 10.4 and 6.7, respectively, for oats. Only a single year's comparison is given for winter wheat as in other years winterkilling, occurring in spots in the nursery, prevented obtaining probable errors of any significance. Probable errors in percentage obtained by the two methods in the season in which winterkilling was not a factor are 5.3 and 5.1, respectively.

THE USE OF A PROBABLE ERROR IN ELIMINATING VARIETIES

As pointed out in an earlier paper (5) there is good agreement in general between mathematical expectation and actual reduction in the size of the probable error which is obtained by replication. To determine the probable error for several replicated plots the probable error in percentage calculated for a single plot is divided by $\sqrt{n}$ where n=the number of systematically distributed plots. This formula assumes a lack of correlation of yields of plots used in the computations. Probably such an assumption has no serious error in these studies altho there is, as will be shown later, correlation between the yielding ability of plots which are at some distance from each other. To determine the probable error for a series of years percentages are used and an average probable error is obtained by the formula $\frac{1}{N}\sqrt{a^2+b^2 \ldots\ldots n^2}$ where N=number of means averaged and a, b, n=separate probable errors in percentage for the respective years.[3]

The use which is made of such a probable error for the rod-row trials for any particular season may be illustrated by the probable error application the first year in which a large number of new rust-resistant hybrid oat lines were grown in the trials.

Two varieties of mid-season oats, Minota (Minn. No. 512) and Victory (Minn. Accession No. 514), have consistently proved the most desirable for central Minnesota. Both are good yielders, stand up better than the average, and, in general, are otherwise desirable except that they are highly susceptible to black stem rust, *Puccinia graminis avenae*. Erikss. and Henn. White Russian, which has received the variety name White Tartar (Etheridge, 1) is highly stem rust resistant in Minnesota but is late in maturity and on the average yields less than either Victory or Minota. From crosses between White Russian with either Victory or Minota, homozygous resistant lines were selected which were apparently homozygous in other characters. Approximately 1000 lines were grown in F_5 and a considerable number were

[3] See Mellor, J. W. "Higher mathematics for the students of chemistry or physics," pp. 498-566. Longmans, Green and Co.: New York and London. 1916.

discarded on the basis of undesirable appearance. The better homozygous lines were placed in rod-row trials in the sixth generation after the cross.

At University Farm, St. Paul, each variety in the rod-row trial is, as a rule, grown in four systematically distributed plots of three rows each. A foot is cut off from the end of each row and the center row only of each three-row plot is used for the yield test.

In 1923, there were 176 lines in rod-row trials which were obtained from either White Russian x Minota or White Russian x Victory crosses. The highest yielding line produced 73.4 bushels, while the lowest produced 48.1 bushels. Some varieties could be discarded on the basis of undesirable plant or grain characters but the greater part could be discarded only on a yield basis.

The calculated probable error for three systematically distributed plots was $\frac{6.1}{\sqrt{3}} = 3.5$ per cent (see Table I). Only three plots were used in 1923 because of the large number of new strains in the test.

On the basis of their probable errors any two varieties could be compared. The yields of Minota and of a hybrid which yielded somewhat less might be compared in the following manner:

The yield of Minota was 60.7 ± 2.1. Multiplying 2.1 by $3 \times \sqrt{2}$ gives 8.8. By subtracting 8.8 from the yield of Minota a yield of 52.9 results. The chances are 22:1 that any variety yielding less than 52.9 is a lower yielder than Minota, there being only six such varieties. Adding 8.8 to 60.7 gives 69.5 bushels and there were only eight varieties which yielded more than this figure. The probable error in yield for these studies for a three-plot trial was 3.5 per cent which is much lower than is usually reported for such studies. However, out of 176 varieties the chances are 22:1 that eight were better yielders than Minota and that seven were inferior. On this basis it appears very apparent that very few, if any, varieties for this year could be discarded legitimately on a probable error basis. This is exactly what further studies with this material have shown. While varieties have been discarded from time to time this was done largely on the basis of plant and grain characters and not on a probable error basis, for very few varieties were consistently better or inferior to Minota and Victory in yielding ability. The question arises, did the probable error prove valuable in this study? Stem rust was not a factor at University Farm and the conclusion appears legitimate that many varieties have as high yielding ability as Minota or Victory and are known from other trials to be highly rust resistant. Somewhat greater confidence can be placed in the results than if the extent of error from soil heterogeneity was not known.

While in some cases a few varieties can be discarded on a probable error basis for a single year, without danger of eliminating the more desirable ones, most new strains must be tested for several years before discarding on a yield basis alone. Often new strains can be eliminated on the basis that they are much inferior in some important character such as strength of straw or susceptibility to disease. After two or more years some strains can, as a rule, be discarded because of low yielding ability, however, after a three year's trial a somewhat better basis of comparison is available.

Since 1920 the chief studies in barley breeding at University Farm, St. Paul, have had as their main purpose the obtaining of a smooth-awned variety of high-yielding ability which also excelled in other characters. Seventeen smooth-awned strains grown in comparison with Manchuria (Minn. No. 184) for from 2 to 5 years remain in the trials and many others have been discarded. The three varieties that have been tested for 5 years yield as much as Manchuria 184 and one variety, named Velvet, somewhat exceeds Manchuria.

Probable errors for four systematically distributed plots for different years have been computed by the deviation from the mean method.

Year	P.E. in per cent
1920	5.1
1921	4.8
1922	5.1
1923	3.8
1924	4.3
1920-24	2.1
1921-24	2.3
1922-24	2.6
1923-24	2.9

To determine the probable error for Velvet, which yielded an average for the 5 years of 52.6 bushels, multiply by the average probable error or 2.1 per cent. This gives a probable error of 1.1 bushels. Comparing Velvet and Manchuria on a 5-year basis the results are as follows:

Varieties compared	Average yield	Diff./P.E.
Velvet, Minn. 447	52.6±1.1	
Manchuria, Minn. 184	47.9±1.0	
Difference	4.7±1.5	3.1

The chances are about 26:1 that Velvet is superior to Manchuria, Minn. 184.

The same varieties may be compared by Student's method:

Manchuria	Velvet	D	D'	D'^2
42.5	37.5	—5.0	11.7	136.89
56.1	61.8	+5.7	1.0	1.00
61.2	73.2	12.0	5.3	28.09
30.6	32.4	1.8	4.9	24.01
49.0	58.2	9.2	2.5	6.25
		5|33.7		5|196.24
		6.7		39.25

$Z = \frac{6.7}{6.3} = 1.06$ S.D. $= \sqrt{39.25} = 6.3$

On this basis the chances are ~~18.7~~ 9.4 :1 that Velvet is a better yielder than Manchuria. However these varieties were not grown in paired plots and the only kind of paired comparisons that are available are seasonal. To the writer it appears that a probable error which measures soil heterogeneity is more desirable than Student's comparison for results of this nature.

Six smooth-awned strains have been compared with Manchuria for 5 years (1921-24, inclusive). All yield as well as Manchuria. The higher yielder is compared here with Manchuria.

Variety	Av. yields 1921-24	Diff./P.E.
Smooth Awn x Manchuria, II-21-14	54.4±1.4	
Manchuria, Minn. No. 184	47.6±1.2	
Difference	6.8±1.8	3.8

The chances are 95:1 that II-21-14 is a better yielder than Manchuria.

These two varieties may be compared by Student's method.

Year	Manchuria Minn. 184	Smooth-Awn x Manch. II-21-14	D	D'	D'^2
1921	42.5	47.1	4.6	2.2	4.84
1922	56.1	55.9	—0.2	7.0	49.00
1923	61.2	64.5	3.3	3.5	12.25
1924	30.6	49.9	19.3	12.5	156.25
			4|27.0		4|222.34
			6.8		55.59

$Z = \frac{6.8}{7.5} = 0.9$ S.D. $= 7.5$

The chances are 8.22:1 that the hybrid is the better yielder. If, on the other hand, the hybrid had yielded in 1924, 33.8 bushels instead of 49.9 the chances would have been 32.2:1 that the hybrid was the better yielder. Results of this nature have been emphasized strongly by Salmon (13) and it seems evident to the writer that when a probable error for the experiment can be computed that a more logical comparison can be made between varieties growing in rod-row trials than by the use of Student's comparison. Both comparisons given here are taken at random without attempting to find cases where one method indicated the probability of a difference and the other failed to indicate such a probability.

In using a probable error computed by the deviation from the mean method it should be emphasized that such computations are not warranted when dealing with varieties widely different in adaptability, for in such cases it is evident from experimental data [Salmon (12), Stadler (14)] that one variety may be signfiicantly more variable than the other.

In both comparisons of the deviation from the mean and Student's methods the chances were somewhat greater by the deviation from the mean comparison that the varieties were significantly different than when made on the basis of Student's method. On the basis of a strict mathematical requirement the probable error would be decreased from that given by the deviation from the mean method for in the comparisons the correlation coefficient was not used in the determination of the probable error of a difference. The extent of error on this basis may well be pointed out.

THE CORRELATION IN YIELDING ABILITY OF NEARBY PLOTS IN ROD-ROW TRIALS

Harris (3, 4) has emphasized the fact that soil heterogeneity is often very large even on fields appearing highly uniform and has suggested a method of determining the degree of soil heterogeneity by the correlation in yielding ability of contiguously grouped plots. The method is of interest as a means of emphasizing the importance of the factor of soil heterogeneity but as the field must be cropped to a single variety it prevents its use for experimental purposes during the years in which the extent of soil heterogeneity is to be determined. The method furnishes the experimenter with a convenient tool to measure the relative heterogeneity of two fields, thus enabling the selection of the more uniform soil for plot trials.

It is desirable in some cases to measure heterogeneity and still carry on plot or varietal trials at the same time. The following study indicates one way in which this can be done.

As has been previously mentioned four systematically distributed plots are used, as a rule, for the rod-row trials conducted at University Farm. These are commonly grown in the same order in each replicated series. There was considerable evidence in 1924, by an examination of the relative vigor of plant growth, that some sections of the field on which the wheat and oat rod-row trials were being conducted were inferior to others. The yield of each plot of each variety was placed on a percentage basis, the average yield of each four plots being taken as 100. Correlation coefficients were then computed to express the degree of relationship for yielding ability of adjacent plots and for plots separated, respectively, by 1, 2, 3, 4, and 10 intervening plots.

The rows were sown lengthwise of the field and each series of rows was separated from the next series by a three-foot alley. By cutting off one foot from the end of each row, the border effect was minimized, as has been explained, and the central row of each three-row plot was harvested. The yielding ability of each plot was expressed in percentage and only plots lying in the same series were used for the correlations. In the various computations the 16-foot row plots were separated from each other by 3, 6, 9, 12, 15, and 30 feet, respectively.

The correlation coefficients as computed are presented in Table II together with regression equations.

The respective coefficients for the extent of soil heterogeneity in adjacent plots in spring wheat, oats, and winter wheat were .618±.023. .572±.025, and .552±.068, respectively. In spring wheat the extent of heterogeneity for plots separated by 6, 9, 12, 15, or 30 feet was practically the same or slightly more than .4.

TABLE II

CORRELATION OF PERCENTAGE YIELDING ABILITY IN NEARBY PLOTS OF OATS, WINTER WHEAT AND SPRING WHEAT, 1924

Crop	Correlation of	Correlation coefficient	Means		Standard deviations		Regression equation
			X	Y	X	Y	
Oat rod rows	Adjacent plots	.572±.025	100.14	99.79	10.18	10.04	Y=43.30+.5641X
	Separated by 1	.490±.029	100.07	99.37	10.00	9.81	Y=51.27+.4807X
	Separated by 2	.407±.034	100.25	99.38	9.94	9.72	Y=59.48+.3980X
	Separated by 3	.412±.035	100.51	99.46	10.09	9.55	Y=60.26+.3900X
	Separated by 4	.264±.041	100.43	99.30	10.36	9.61	Y=74.70+.2449X
	Separated by 10	.275±.057			10.65	8.18	
Winter wheat rod rows	Adjacent plots	.552±.068					
	Separated by 1	.293±.093					
	Separated by 4	—.114±.118					
Spring wheat rod rows	Adjacent plots	.618±.023	99.74	99.74	13.90	13.95	Y=37.85±.6205X
	Separated by 1	.518±.028	99.76	99.77	14.02	14.05	Y=47.99±.5191X
	Separated by 2	.454±.030	99.74	99.90	14.16	14.18	Y=54.56+.4546X
	Separated by 3	.383±.034	99.75	99.93	14.32	14.48	Y=61.30+.3873X
	Separated by 4	.449±.034	99.84	100.00	14.40	13.95	Y=56.72+.4350X
	Separated by 10	.429					

NOTE.—All computations were checked. H. E. Brewbaker, W. C. Broadfoot, C. H. Goulden, K. S. Quisenberry, S. C. Salmon, George Stewart, D. W. Robertson, and Miss Alma Schweppe assisted in the computations.

With oats, plots separated by 4 or 10 intervening plots had a soil heterogeneity relationship which was expressed by a coefficient of approximately .27 while the relationship between plots separated by 1, 2, or 3 intervening plots was not greatly different than in the spring wheat trials. For winter wheat, adjacent plots showed rather high heterogeneity while plots separated by several feet did not exhibit any soil heterogeneity relationship. By duplicate planting of experimental rod-row, varietal, or strain trials on two separate fields it would be possible to compare soil heterogeneity and determine the most desirable field and in the same season to conduct a yield comparison.

Richey (11) has given the reduction in variability which may be expected from using the regression equation to correct yields on the basis of adjusting to a moving average. He states that correlations of less than .6 will reduce variability so little that adjustment may hardly be worth while altho in some cases it is emphasized that adjustment may materially change the relative standing of a variety, all four plots of which may happen to be placed on relatively poor or good soil.

Table III is taken from Richey's publication.

TABLE III

EXPECTED REDUCTION IN VARIABILITY FOR VARIOUS CORRELATION COEFFICIENTS (RICHEY)

r=	Reduction in variability= $100 \times 1-\sqrt{1-r^2}$	r=	Reduction in variability= $100 \times 1-\sqrt{1-r^2}$
	Per cent		Per cent
0.4	08.4	0.8	40.0
0.5	13.4	0.866	50.0
0.6	20.0	0.9	56.4
0.707	29.3	1.0	100.0

In the spring wheat and oats studies check plots of standard varieties were distributed through the test every 6 plots. The yields of each plot of nearby varieties were adjusted by means of the regression values. The method used is illustrated for several plots in the wheat series.

Plot No.	Actual yield	Percentage yield
Check a	25.1	93
8	27.5	
9	20.3	
10	28.3	
11	24.2	
12	25.7	
Check b	32.2	120

The yielding ability of each check plot was expressed in percentage by dividing its actual yield by the average of all check plots. By this method the yielding ability of Check a was 93 and of b, 120.

If y=nearby plot and x= the check, then the corrected percentage yielding values for Plot 8 would be

$$1.\quad y = 37.85 + .6205\ x = 95.56$$

$$2.\quad y = 56.72 + .4350\ x = 108.92$$

A value of 102 was obtained by adding 1 and 2 and averaging. The corrected yield for Plot 8 in bushels would then be obtained by dividing the actual yield, 27.5 bushels, by 102. In this way the yield of each plot of each variety was adjusted by the yield of the two nearest check plots on the basis of the average relationship as expressed by the calculated regression equations.

After obtaining a corrected yield for each plot of all varieties, except the check plots, a probable error in percentage was calculated by the deviation from the mean method. These probable errors were then compared with probable errors obtained before adjustment. The results are presented in Table IV.

TABLE IV

PROBABLE ERROR FOR A SINGLE PLOT TEST BEFORE ADJUSTMENT OF YIELDS AND AFTER CORRECTING YIELDS ON THE BASIS OF THE REGRESSION VALUES AND IN RELATION TO THE TWO NEAREST CHECK PLOTS

Crop	Probable error in percentage	
	Actual yield	After correction
Spring wheat	9.9	8.0
Oats	6.7	5.8

The actual percentage probable errors are reduced by adjustment to a moving average by approximately 19 and 13 per cent, respectively, which is slightly more than expected on the basis of the standard error of estimate for correlation coefficient of the values obtained. It seems very doubtful whether the reduction in probable error is worth the trouble of making the calculations.

The actual and corrected yields and the relative standing of varieties before and after correction are given in Tables V and VI.

TABLE V

COMPARISON OF YIELDS AND RELATIVE STANDING OF STRAINS BEFORE AND AFTER CORRECTION ON THE BASIS OF THE RELATION EXPRESSED BY THE REGRESSION EQUATION SPRING WHEAT ROD ROWS, 1924

Variety	Yield range after correction	Yield range before correction	Difference + or −	Range after correction	Range actual
II-18-20	32.7	32.7	0.0	1	1
B2-5	30.7	31.5	−0.8	2	2
II-18-44	30.6	31.2	−0.6	3	3
II-17-40	30.0	30.9	−0.9	4	4
II-15-41	29.9	29.7	+0.2	5	10
II-18-21	29.8	29.5	+0.3	6	12
II-17-45	29.7	30.5	−0.8	7	5
B8-11	29.6	29.9	−0.3	8	7
II-18-8	29.6	30.3	−0.7	9	6
II-15-13	29.5	29.6	−0.1	10	11
Ruby	29.3	29.8	−0.5	11	9
II-18-33	29.2	28.6	+0.6	12	18
II-15-24	29.0	29.0	0.0	13	15
II-18-15	28.8	29.8	−1.0	14	8
II-17-37	28.7	29.4	−0.7	15	13
N.D. 149.124	28.7	28.6	+0.1	16	21

TABLE V—*Continued*

Variety	Yield range after correction	Yield range before correction	Difference + or −	Range after correction	Range actual
II-17-23	28.4	28.7	−0.3	17	17
II-17-36	28.2	29.0	−0.8	18	14
II-15-16	28.1	28 1	0.0	19	23
II-17-43	28.0	28.6	−0.6	20	19
II-15-57	28.0	28.6	−0.6	21	20
II-17-22	27.9	28.4	−0.5	22	22
II-15-8	27.9	28.7	−0.8	23	16
II-17-4	27.7	27.0	+0.7	24	27
II-18-17	27.6	27.8	−0.2	25	24
II-17-3	27.2	26.4	+0.8	26	30
N.D. 149.43	27.2	27.0	+0.2	27	26
II-19-11	26.6	27.5	−0.9	28	25
II-18-28	26.5	25.7	+0.8	29	40
II-15-39	26.4	26.1	+0.3	30	35
N.D. 149.178	26.4	26.2	+0.2	31	33
II-18-19	26.2	26.3	−0.1	32	31
II-17-33	26.2	27.0	−0.8	33	28
Kitchener	26.2	26.2	0.0	34	34
II-19-2	26.1	26.6	−0.5	35	29
Red Bobs	26.0	26.1	−0.1	36	36
II-18-27	25.9	25.6	+0.3	37	41
II-17-2	25.8	26.2	−0.4	38	32
II-19-9	25.7	25.4	+0.3	39	44
II-18-36	25.4	25.9	−0.5	40	37
II-17-28	25.4	25.9	−0.5	41	38
II-17-25	25.1	25.4	−0.3	42	45
II-17-35	25.1	25.6	−0.5	43	42
II-17-14	24.9	24.0	+0.9	44	54
II-18-12	24.9	25.5	−0.6	45	43
Kota Bulk	24.7	24.2	+0.5	46	49
II-19-19	24.7	25.3	−0.6	47	46
II-19-23	24.6	24.1	+0.5	48	52
II-18-16	24.6	25.8	−1.2	49	39
II-17-13	24.5	23.6	+0.9	50	59
II-18-34	24.4	24.0	+0.4	51	53
II-18-35	24.3	23.9	+0.4	52	55
II-18-38	24.3	24.6	−0.3	53	48
II-18-39	24.3	24.7	−0.4	54	47
II-19-26	24.0	23.7	+0.3	55	56
N.D. 149.48	24.0	23.7	+0.3	56	57
II-17-15	23.8	23.1	+0.7	57	60
II-17-8	23.7	23.0	+0.7	58	61
II-19-12	23.6	24.2	−0.6	59	50
II-19-18	23.4	24.2	−0.8	60	51
II-17-20	23.4	22.6	+0.8	61	63
II-18-37	23.3	23.6	−0.3	62	58
II-17-10	23.2	22.5	+0.7	63	64
II-17-7	23.0	22.4	+0.6	64	65
N.D. 149.1	22.9	22.9	0.0	65	62
II-17-16	22.0	21.4	+0.6	66	68
II-17-19	21.8	21.1	+0.7	67	69
II-19-29	21.6	21.4	+0.2	68	67
II-17-29	21.5	22.3	−0.8	69	66
II-15-55	20.9	20.7	+0.2	70	70
II-15-59	20.9	20.6	+0.3	71	71

TABLE VI

Comparison of Yields and Relative Standing of Strains before and after Correction on the Basis of the Relation Expressed by the Regression Equation

Oat Rod Rows, 1924

Variety	Yield range after correction	Yield range before correction	Difference + or −	Range after correction	Range actual
II-18-149	72.0	73.3	−1.3	1	1
II-18-114	67.4	68.4	−1.0	2	3
II-18-224	66.6	67.7	−1.1	3	5
II-19-7	66.3	68.1	−1.8	4	4
II-18-150	66.0	68.9	−2.9	5	2
II-18-222	65.7	66.2	−0.5	6	10
II-18-238	65.0	66.5	−1.5	7	9
II-19-4	64.9	66.5	−1.6	8	8
II-18-157	64.8	66.9	−2.1	9	6
I-06-12	64.5	65.9	−1.4	10	11
II-18-171	64.2	65.5	−1.3	11	12
II-19-5	63.4	66.6	−3.2	12	7
II-18-227	63.4	65.1	−1.7	13	13
II-18-221	63.4	62.6	+0.8	14	22
II-18-153	62.2	64.4	−2.2	15	14
II-18-100	62.0	63.0	−1.0	16	19
II-18-108	62.0	62.9	−0.9	17	20
II-19-2	61.8	63.2	−1.4	18	16
II-19-8	61.8	63.7	−1.9	19	15
II-19-6	61.8	63.3	−1.5	20	17
II-18-225	60.8	62.2	−1.4	21	23
II-18-169	60.5	61.5	−1.0	22	25
II-18-152	60.5	63.0	−2.5	23	18
II-19-9	60.3	61.8	−1.5	24	24
II-18-163	60.3	60.9	−0.6	25	27
II-18-151	60.3	62.9	−2.6	26	21
II-18-147	60.1	60.7	−0.6	27	28
II-18-185	59.8	55.2	+4.6	28	46
II-18-148	59.3	61.0	−0.7	29	26
II-18-23	59.2	58.0	+1.2	30	35
II-18-178	58.9	59.7	−0.8	31	30
II-18-228	58.9	59.6	−0.7	32	31
II-18-226	58.5	60.1	−1.6	33	29
II-18-8	58.5	56.8	+1.7	34	38
II-18-161	58.1	58.9	−0.8	35	33
II-18-20	58.1	56.4	+1.7	36	41
II-18-194	57.8	54.6	+3.2	37	48
II-18-18	57.6	55.9	+1.7	38	43
II-19-10	57.1	58.6	−1.5	39	34
II-18-15	57.0	54.9	+2.1	40	47
II-18-155	56.9	59.1	−2.2	41	32
II-18-187	56.7	52.4	+4.3	42	56
II-18-180	56.7	56.5	+0.2	43	39
II-18-193	56.5	53.3	+3.2	44	52
II-18-37	56.5	55.4	+1.1	45	45
II-18-181	56.3	55.7	+0.6	46	44
II-18-145	56.3	57.2	−0.9	47	36
II-18-179	56.1	56.0	+0.1	48	42
II-18-12	55.9	53.9	+2.0	49	49
II-18-146	55.8	56.4	−0.6	50	40
II-18-14	55.6	53.6	+2.0	51	51
II-18-158	55.4	56.9	−1.5	52	37
II-18-19	55.4	53.6	+1.8	53	50
II-18-188	55.3	51.3	+4.0	54	62
II-18-189	55.3	51.7	+3.6	55	59
II-18-184	54.9	51.3	+3.6	56	61
II-18-200	54.8	52.8	+2.0	57	53

TABLE VI—*Continued*

Variety	Yield range after correction	Yield range before correction	Difference + or —	Range after correction	Range actual
II-18-183	54.8	51.5	+3.3	58	60
II-18-195	54.7	52.3	+2.4	59	57
II-18-186	54.6	50.8	+3.8	60	64
II-18-196	54.4	52.6	+1.8	61	55
II-18-13	53.9	52.0	+1.9	62	58
II-18-190	53.8	50.5	+3.3	63	65
II-18-201	53.1	52.6	+0.5	64	54
II-18-6	52.8	51.1	+1.7	65	63
II-18-197	52.4	50.4	+2.0	66	66
II-18-199	51.9	49.9	+2.0	67	68
II-18-198	51.8	49.9	+1.9	68	67
II-18-3	49.6	48.6	+1.0	69	70
II-18-74	49.2	48.8	+0.4	70	69
II-18-60	44.5	43.7	+0.8	71	71

Relative standing of the varieties is changed very little by correction of yields. In the spring wheat the four highest yielders remain in the same relative order after correction as before. Varieties which after correction stood 5 and 6 were tenth and twelfth, respectively, on the basis of actual yield. The greatest change in yielding value was only 1.2 bushels, whereas the probable error for a four-plot trial is 5.0 per cent and for a yield of 25 bushels the probable error in bushels is 1.25. The greatest change in yielding value for spring wheat was about the same as the probable error.

With oats there were several cases where changes in yielding value were relatively somewhat larger than with wheat. The greatest change as a result of correction was 4.6 bushels. This is slightly more than two times the calculated probable error.

In no case is the relative change in yielding ability of particular significance and the correction of yields in such studies, which must be continued for three years at least, and where the better strains will be compared for a six-year period in several localities, appears to be of little value. Correction of yields on the basis of regression values and in relation to a moving average will certainly, where soil heterogeneity is a factor, reduce to some extent the size of the probable error.

SUMMARY

1. A probable error for the experiment measures on the average the extent of error of a non-systematic nature. It measures chiefly errors which arise from soil heterogeneity or other uncontrolled errors of like nature. The method of calculation rests on the assumption that all varieties are equally variable. This is not true for widely different varieties, however, in plant breeding experiments most varieties are rather similar in general adaptability. This is perhaps particularly true

when the Mendelian mode of attack for the synthesis of new varieties is used.

2. In plant breeding studies such as rod-row trials in which several systematically distributed plots are used and where all varieties or strains are similar in general adaptability the combined probable error as obtained from all varieties, by some such method as the "deviation from the mean," was found to be very similar to that obtained from check plots.

3. By expressing probable errors for different seasons in percentage an average for several seasons can be computed. Varieties can be compared for various yearly tests and for combined tests. Better yielding varieties can then be determined and the probabilities that they are significantly better than others can be measured accurately. Such methods, if correctly understood, should lead to more accurate and scientific comparisons.

4. The use of Student's method for varietal trials conducted for several years is not entirely satisfactory as there is no sure control or measure of soil heterogeneity unless the comparisons are made on the basis of adjacent plots. Student's method is valuable for trials where it may be expected that if there is a difference in yielding ability the difference will be consistent from year to year. Such consistence is not commonly obtained in varietal tests. Differences which are a result of varietal resistance or susceptibility and which are only apparent under disease infection conditions may be present only once in every three or four seasons. Student's method used for seasonal averages tends to cover up important differences.

5. By the deviation from the mean method calculated on the basis of numerous strains the odds that two varieties were different in yielding ability for a four- and five-year trial, respectively, were much greater than where Student's method was used and seasonal averages were compared.

6. Adjustment of yields by the regression equation on the basis of a moving average leads to a reduction in the probable error of the experiment providing soil heterogeneity is a factor. If the extent of soil heterogeneity as measured by the correlation coefficient is .6 or lower and if the probable error of a single determination is 10 per cent or less, adjustment of yields will, as a rule, not markedly change the relative standing of varieties. Such changes as occur will be of little importance on the average.

7. An estimate of the extent of soil heterogeneity may be accomplished by placing the yields of each separate set of replications of individual strains on a percentage basis with 100 as an average. By the correlation of yielding ability of plots which are separated by any

particular number of intervening plots the extent of soil heterogeneity may be measured. By placing a similar series of varieties on each of two fields the one with the least soil heterogeneity can be selected for permanent nursery experiments.

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