# Economically Optimal Marine Reserves Without Spatial Heterogeneity in a Simple Two-Patch Model 

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#### Abstract

Bioeconomic analyses of spatial fishery models have established that marine reserves can 2 be economically optimal (i.e., maximize sustainable profit) when there is some type of spatial heterogeneity in the system. Analyses of spatially continuous models and models with more 4 than two discrete patches have also demonstrated that marine reserves can be economically optimal even when the system is spatially homogeneous. In this note we analyze a spatially 6 homogeneous two-patch model and show that marine reserves can be economically optimal in this case as well. The model we study includes the possibility that fishing can damage 8 habitat. In this model, marine reserves are necessary to maximize sustainable profit when dispersal between the patches is sufficiently high and habitat is especially vulnerable to damage.


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## Introduction

Marine reserves are zones where extractive fishing is prohibited (Lubchenco et al., 2003). While there is growing scientific consensus that marine reserves are useful for biological conservation, their economic costs and benefits are debated (Hart, 2006; Hart and Sissenwine, 2008), with consequences for their political feasibility as a fisheries management tool. To help understand the biological and economic circumstances under which marine reserves may be economically beneficial, theoreticians have turned to studying bioeconomic models that include a spatial dimension (Herrera and Lenhart, 2010). The analysis and interpretation of these models can be difficult, so to keep matters simple models often divide the habitat into two patches, one of which is designated as an unfished reserve. Analyses of two-patch models have found that reserves can be economically beneficial (i.e., increase yield or profit) when there is an asymmetry in either the biological or economic characteristics between the reserve and fished patches. In particular, closures may support elevated catch levels if they export larvae or adult fish into fished areas (Pezzey et al., 2000; Costello and Polasky, 2008), if they are sited in areas that are more expensive to fish than open areas (Sanchirico and Wilen, 2001; Sanchirico et al., 2006), or if they stabilize catch levels in the face of environmental stochasticity (Hannesson, 2002).

What is more surprising is that reserves can be economically optimal even when space is completely homogeneous. Using a partial differential equation model that treats space as a one-dimensional continuum, we have shown that marine reserves can be profit maximizing even when the biological and economic parameters are the same at every location (Moeller and Neubert, 2013). Similarly, White et al. (2008) found economically optimal reserves in a model that approximates a continuous habitat with a system of many identical patches arranged in a ring; however, they did not find any case where reserves were required to maximize profit when there were only two patches. Their study followed on from a paper by White and Kendall (2007) who found that fishing yield could be improved when one patch was closed to fishing and the other was completely harvested. They did not investigate the
effect of closed areas on profit.
These results leave open the possibility that reserves might only be profit maximizing in a spatially homogeneous model if there are more than two patches. The purpose of our present paper is to present a counterexample to that idea. We show here that marine reserves may emerge as part of the economically optimal (i.e., profit maximizing) management strategy even in completely homogeneous two-patch settings.

What distinguishes our model from previous two-patch models (aside from spatial homogeneity) is that we account for the possibility that fishing may negatively impact habitat quality. Destructive fishing practices like bottom trawling reduce the complexity of benthic habitats, affecting survivorship of fish populations (Collie et al., 1997; Watling and Norse, 1998; Hiddink et al., 2006). Here, we assume that fishing directly reduces the habitat's carrying capacity, with the severity of the habitat damage scaling with fishing effort intensity (Auster, 1998; Fogarty, 2005). This habitat-quality feedback, coupled with the dispersal of fish between patches, drives an asymmetrical distribution of fishing effort, in which habitat quality is preserved in one half of the habitat to sustain elevated fish stocks and profits. Although habitat damage as a result of fishing is often neglected in the bioeconomic literature, some theoretical work has attempted to incorporate these feedbacks into marine reserve models (see Armstrong and Falk-Petersen (2008) for a review). These treatments generally take a retroactive perspective, focusing on the recovery of the habitat fraction in reserve (Rodwell et al., 2003; Armstrong, 2007), or are limited in their economic analysis of reserve viability (Lindholm et al., 2001; Upton and Sutinen, 2005) (but see Moeller and Neubert (2013) for an exception).

We begin with a brief examination of the optimal harvest of a single population. This will allow us to address the effects of harvest-related habitat impacts in isolation from spatial considerations. It will also provide context for the results of our analysis of the two-patch model. As it turns out, dispersal changes the relationship between optimal fishing effort and the degree to which fishing impacts habitat quality.

## Analysis and Results

## One-Patch Model

Imagine a stock whose density $N(t)$, in the absence of harvesting, is governed by the logistic differential equation:

$$
\begin{equation*}
\frac{d N}{d t}=r N\left(1-\frac{N}{K}\right) \tag{1}
\end{equation*}
$$

In most bioeconomic models, the effect of harvesting is included in model (1) by subtracting a fishing mortality term from the right hand side. Thus

$$
\begin{equation*}
\frac{d N}{d t}=r N\left(1-\frac{N}{K}\right)-q E N \tag{2}
\end{equation*}
$$

where $E$ is a measure of the fishing effort (Clark, 1990). The positive proportionality constant $q$ is the so-called "catchability coefficient;" it depends upon characteristics of the habitat and on the harvesting technology. Variables, parameters, and their typical units are summarized in Table 1.

In our model, we follow Fogarty (2005) and imagine that in addition to increasing fish mortality directly, fishing also damages habitat such that it reduces the environmental carrying capacity $K$. In particular, we will replace the constant $K$ in (2) with

$$
\begin{equation*}
K(E)=\frac{K_{0}}{1+g E} \tag{3}
\end{equation*}
$$

The larger the value of the parameter $g$, the more carrying capacity is suppressed by effort. In this sense, $g$ represents the sensitivity of the habitat to fishing. Substituting our effortdependent carrying capacity (3) into the stock equation (2) gives

$$
\begin{equation*}
\frac{d N}{d t}=r N\left[1-\frac{(1+g E) N}{K_{0}}\right]-q E N \tag{4}
\end{equation*}
$$

For a fixed level of effort, the equilibrium stock density is found by setting $d N / d t=0$

$$
\begin{equation*}
\frac{d n}{d \tau}=n[1-(1+\gamma h) n]-h n \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
\pi=h(n-w) \tag{8}
\end{equation*}
$$

These two equations depend on only two parameters: the dimensionless habitat sensitivity

$$
\begin{equation*}
\gamma=\binom{r}{q} g \tag{9}
\end{equation*}
$$

and the dimensionless cost

$$
\begin{equation*}
w=\left(\frac{1}{q p K_{0}}\right) c . \tag{10}
\end{equation*}
$$

In this rescaled version of our model the sole owner's problem is to maximize the dimensionless profit $\pi$ by choosing the dimensionless effort $h$.

For model (7) the stable equilibrium stock size, found by setting $d n / d \tau=0$, is given by

$$
n= \begin{cases}(1-h) /(1+\gamma h), & \text { if } 0 \leq h \leq 1  \tag{11}\\ 0, & \text { if } h>1\end{cases}
$$

As we see in Fig. 1, the equilibrium stock declines with effort and declines more quickly if habitat is vulnerable to fishing damage (i. e., when $\gamma>0$ ). By substituting the equilibrium size (11) into (8) and differentiating with respect to $h$, one can find the profit maximizing effort level

$$
h^{*}= \begin{cases}(1-w) / 2, & \text { if } \gamma=0 \text { and } w \leq 1  \tag{12}\\ \gamma^{-1}[-1+\sqrt{(1+\gamma) /(1+\gamma w)}], & \text { if } \gamma>0 \text { and } w \leq 1 \\ 0, & \text { if } w>1,\end{cases}
$$

as well as the resulting optimal stock size

$$
n^{*}= \begin{cases}(1+w) / 2, & \text { if } \gamma=0 \text { and } w \leq 1,  \tag{13}\\ \gamma^{-1}[-1+\sqrt{(1+\gamma)(1+\gamma w)}], & \text { if } \gamma>0 \text { and } w \leq 1 \\ 1 & \text { if } w>1\end{cases}
$$

The maximum sustainable profit, $\pi^{*}$, is then given (using equation (8)) by $h^{*}\left(n^{*}-w\right)$.
By plotting $h^{*}, n^{*}$ and $\pi^{*}$ as functions of $\gamma$ (Fig. 2), we see that as habitat sensitivity increases, equilibrium profit decreases, and less effort is required to maximize profit. Despite this decrease in effort, the equilibrium stock size also decreases as habitat sensitivity increases. These results are consistent with those of Fogarty (2005) who studied yield (as
opposed to profit) using a similar model.

## Two-Patch Model

Let us now expand our view slightly and consider two stocks, $N_{1}$ and $N_{2}$, coupled by individuals who disperse between them with constant per capita emigration rate $D$. Assume that these two stocks live in completely identical habitats. That is, each habitat "patch" has exactly the same population growth rate and responds to harvesting in exactly the same way. The population dynamic portion of our model then becomes

$$
\begin{equation*}
\frac{d N_{i}}{d t}=r N_{i}\left(1-\frac{N_{i}}{K\left(E_{i}\right)}\right)-q E_{i} N_{i}+D\left(N_{j}-N_{i}\right), \quad \text { for } i=1,2, \quad j \neq i, \tag{14}
\end{equation*}
$$

where $K(E)$ is again given by formula (3). If we also assume that the costs, prices, and catchability coefficients are identical in both patches then the sole owner's objective is to maximize

$$
\begin{equation*}
\tilde{\Pi}=\sum_{i=1}^{2} p q E_{i} N_{i}-c E_{i} \tag{15}
\end{equation*}
$$

over nonnegative values of $E_{1}$ and $E_{2}$.
Before analyzing model (14)-(15), we again rescale the variables and parameters (listed in Table 1) via

$$
\begin{equation*}
n_{i}=\left(\frac{1}{K_{0}}\right) N_{i}, \quad \tau=r t, \quad h_{i}=\left(\frac{q}{r}\right) E_{i}, \quad \delta=\frac{D}{r}, \quad \text { and }, \quad \tilde{\pi}=\left(\frac{1}{r p K_{0}}\right) \tilde{\Pi} \tag{16}
\end{equation*}
$$

to arrive at the dimensionless version of the two-patch population dynamic model

$$
\begin{equation*}
\frac{d n_{i}}{d \tau}=n_{i}\left[1-\left(1+\gamma h_{i}\right) n_{i}\right]-h_{i} n_{i}+\delta\left(n_{j}-n_{i}\right) \tag{17}
\end{equation*}
$$

(for $i$ and $j$ equal to 1 or 2 , and with $j \neq i$ ) and the economic model

$$
\begin{equation*}
\tilde{\pi}=\sum_{i=1}^{2} h_{i}\left(n_{i}-w\right) . \tag{18}
\end{equation*}
$$

This dimensionless form (17)-(18) reemphasizes the fact that the two patches in the model are completely identical: harvesting effort costs the same $(w)$ in each patch, each patch is equally sensitive to harvest $(\gamma)$, and individuals in both patches have the same emigration rate $(\delta)$.

Given all this spatial homogeneity, it would be reasonable for a sole owner to think that applying effort equally, at the rate $h^{*}$, in each patch would maximize profit. It turns out that this is not always true (Fig. 3). In the absence of dispersal $(\delta=0)$ or when habitat is invulnerable $(\gamma=0)$, it is indeed optimal to apply the same fishing effort in each patch. However, when both $\delta$ and $\gamma$ are positive, it can be optimal to focus all of one's effort in one patch and treat the other patch as an unfished reserve. ${ }^{1}$ The more sensitive the habitat, the lower the emigration rate at which reserves become optimal (Fig. 4). In the limit of extreme habitat sensitivity ( $\gamma$ very large), fishing in both patches would generate infinitesimal profit due to habitat degradation. Thus, so long as $\delta>0$, it becomes more profitable to close one of the patches and catch emigrating fish in the other. The threshold values of $\delta$ and $\gamma$ above which reserves become optimal depend upon the cost of fishing $w$. The larger $w$, the more sensitive the habitat or the higher the emigration rate must be for reserve optimality.

In addition to allowing for the possibility of spatially asymmetric harvesting, the twopatch model behaves differently than the single-patch model with regard to the relationship between optimal effort and habitat sensitivity (compare Fig. 2, Fig. 5). Optimal aggregate effort in the two-patch model $\left(h_{1}^{*}+h_{2}^{*}\right)$ does not monotonically decline with habitat sensitivity. Rather, it declines until the point at which it becomes optimal to stop harvesting in one of the patches. Aggregate effort then increases until the habitat becomes extremely sensitive,

[^0]when it decreases again. Aggregate optimal stock size $\left(n_{1}^{*}+n_{2}^{*}\right)$ and aggregate optimal profit $\left(\tilde{\pi}^{*}\right)$ decrease with habitat sensitivity, just as they do in the one-patch model.

## Discussion

Three points emerge from our brief analysis. The first is that we have found a counter example to the idea that marine reserves would not be optimal in a homogeneous two-patch model. In the (admittedly stylized) system we analyzed here the two patches are intrinsically identical, both biologically and economically. Nevertheless, we have shown that it can be most profitable to harvest in one patch and establish a reserve in the other. This result, along with our previous analysis of a continuous-space model (Moeller and Neubert, 2013) and the results of White and Kendall (2007) and White et al. (2008), lead us to a warning: Though it may be tempting to manage spatially homogeneous systems in a homogeneous manner, one should resist this temptation absent a careful analysis.

The second point is that ignoring feedbacks between fishing and habitat quality can lead to qualitative errors in the design of spatial management strategies. In our model, marine reserve creation is only economically optimal when fishing damages habitat. Thus, from a purely economic perspective, reserve benefits may be limited by the context of the affected fishery. However, in the many cases where fishing damages habitat, our results establish that the economic benefits of marine reserves should not be dismissed out of hand. The dependence of these results on habitat sensitivity and dispersal rates serves to highlight the importance of tailoring management strategies to particular fisheries and their biological effects.

The third point is that, if habitat is sensitive to damage from fishing, there may be substantial value to be gained by implementing technologies that reduce the impact of fishing gear. For example, in the one-patch system, altering gear to reduce $\gamma$ from 10 to 0 would result in a quintupling of profit (Figure 2). Optimal effort levels also depend upon habitat
sensitivity, with potential implications for fishery employment (Neubert and Herrera, 2008).
Our results come from intentionally simple models, in which we have assumed environmental homogeneity and analyzed equilibrium outcomes. While these models served our purpose of illustrating the optimality of marine reserves even in homogeneous two-patch systems, we note some important caveats. First, by focusing on model equilibria, we have ignored time-dependent processes such as discounting, or the potentially dynamic response of habitat quality to fishing pressure. Thus, our results do not account for the potential costs of management transitions to the profit-maximizing steady state, which may impact the economic optimality of marine reserves depending on the timescale of analysis. Second, we have not considered the many ways that environmental heterogeneity can influence profitmaximizing fishing distributions. Other authors have considered these effects, and shown that reserves are more likely to be optimal in places where costs (either implicit, as where fishing is more deleterious to fish stocks, or explicit, as where costs per unit of fishing effort are greater) are high relative to other locations (Pezzey et al., 2000; Costello and Polasky, 2008; Sanchirico and Wilen, 2001; Sanchirico et al., 2006).

Over the past decade, increasing attention has been paid to the economic benefits of both terrestrial and marine reserves (e.g., Grafton et al., 2005; Naidoo and Adamowicz, 2005). These benefits include activities that take place within reserves (e.g., ecotourism), and spillover of benefits into adjacent areas (e.g., pollination and pest control services). Reserves can also act as a population refuge for harvested stocks, and have long been implemented for their conservation benefits in this regard. Our model highlights a special intersection of conservation and economic interests by focusing on a case in which the act of harvesting a stock (fishing) reduces the carrying capacity (damages the habitat) of that stock. Thus, the establishment of reserves protects both a source stock population and intact habitat, enhancing the economic value of the fishery.

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Table 1: Variables and parameters used in the models, together with their typical units.

| Symbol | Description | Typical Units | Dimensionless Version |
| :---: | :---: | :---: | :---: |
| Variables: |  |  |  |
| $N, N_{i}$ | stock, in patch $i$ | tonnes | $n, n_{i}$ |
| $E, E_{i}$ | fishing effort, in patch $i$ | vessels | $h, h_{i}$ |
| $\Pi, \Pi$ | profit | dollars $\cdot$ day ${ }^{-1}$ | $\pi, \tilde{\pi}$ |
| $t$ | time | days | $\tau$ |
| Parameters: |  |  |  |
| $r$ | per capita population growth rate | $d a y^{-1}$ |  |
| $K_{0}$ | carrying capacity without harvest | tonnes |  |
| $g$ | habitat sensitivity | vessel $^{-1}$ | $\gamma$ |
| q | catchability coefficient | vessel $^{-1} \cdot d a y^{-1}$ |  |
| D | per capita emigration rate | $d a y^{-1}$ | $\delta$ |
| $p$ | market price of stock | dollars $\cdot$ tonne ${ }^{-1}$ |  |
| c | cost of fishing effort | dollars $\cdot$ vessel ${ }^{-1} \cdot$ day $^{-1}$ | $w$ |



Figure 1: Equilibrium of the dimensionless "one-patch" model (7)-(8). Stock size ( $n$ ) and profit $(\pi)$ are shown as functions of effort $(h)$. For the solid curves $\gamma=0$; for the dashed curves $\gamma=5$. For this figure the dimensionless cost of effort $w$ was set to 0.1.


Figure 2: Optimal equilibrium management of the one-patch model (7)-(8). Profitmaximizing effort $\left(h^{*}\right)$, stock size $\left(n^{*}\right)$ and maximum profit $\left(\pi^{*}\right)$ are shown as functions of the dimensionless habitat sensitivity $(\gamma)$. For this figure the dimensionless cost of effort $w$ was set to 0.01 .


Figure 3: Total equilibrium profit for the two-patch model (17). Contours of total equilibrium profit ( $\tilde{\pi}$, equation (18)) are shown as functions of effort in each patch. Profit maximizing combinations of effort are marked with an red star. For this figure the dimensionless cost of effort $w$ was set to 0.01 .


Figure 4: Reserve implementation thresholds for the two-patch model (17). Contours show where management strategies transition from fishing both patches (below the contour), to where one patch is fished and the other is treated as an unfished reserve (above the contour), as a function of habitat sensitivity $\gamma$ and emigration rate $\delta$. Each contour represents a value of $w$.


Figure 5: Optimal equilibrium management of the two-patch model (17)-(18). In each plot, the top curve represents the total of the quantity (either effort, stock size, or profit) in both patches as functions of habitat sensitivity $\gamma$. The distance between the top and bottom curves (shaded) gives the contribution to the plotted quantity from one of the two patches. Because these quantities vary rapidly for small $\gamma$, the top row shows results for $0 \leq \gamma \leq 5$. The bottom row shows the same quantities over a wider range, $0 \leq \gamma \leq 100$. For this figure the dimensionless cost of effort $w$ was set to 0.01 and the dimensionless per capita emigration rate $\delta$ was set to 1 .


[^0]:    ${ }^{1}$ Because the patches are identical, it does not matter which patch is fished and which is placed in reserve.

