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River dolphins can act as population trend indicators in degraded freshwater systems: comment

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The record of historical sightings of a species provides a basis for inference about its population status. In many cases, however, it is not possible to reconstruct a reliable sighting record. In an interesting paper, Turvey et al. (2012) used the recollections of the most recent sightings by a number of individuals in local fishing communities to compare population declines among four species in the Yangtze River, the idea being that the most recent sighting is more memorable than earlier ones. Briefly, Turvey et al. (2012) found that the empirical distributions of the most recent sightings of three of these species had similar declining upper tails, suggesting a common pattern of population decline, but found no such decline for the remaining species even though its population was known to be declining. This raises a general question about the relationship between the distribution of the most recent sightings and the overall distribution of sightings. The purpose of this comment is to address some aspects of this question and, in particular, to show that, even in simple situations, this relationship is somewhat complicated.

Let the random variables $T_1, T_2, \ldots, T_n$ be the sighting times for a single individual over the observation period $(0,T)$. These sightings are assumed to arise from a Poisson process with unknown rate function $\beta(t) > 0$ that is directly related to population size. It is a standard statistical result that, conditional on their number n, these sightings represent independent observations from a distribution with probability density function (pdf)

$$f(t) = \frac{\beta(t)}{\int_0^T \beta(u)du} \qquad 0 \leq t \leq T \tag{1}$$

(e.g., Cox and Lewis 1966) with cumulative distribution function (cdf) $F(t)$. Let

$$T_{(n)} = \max\{T_j,\ j = 1,\ 2,\ \ldots, n\} \tag{2}$$

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be the most recent sighting time for this individual. It is another standard statistical result that the pdf of $T_{(n)}$ is

$$g\left(t_{(n)}\right) = n \times F^{n-1}\left(t_{(n)}\right) f\left(t_{(n)}\right) \tag{3}$$

(e.g., David and Nagaraja 2003).

Consider now a number m of independent sighting records arising from this model, all of which contain n sightings. The collection of the most recent sightings extracted from these records is precisely a random sample of size m from the distribution with pdf given in Eq. 3. The question is how the behavior of the pdf g of the most recent sighting times is related to the pdf f of overall sighting times, which, by assumption, is directly related to population size. To be more specific, I will focus here on the relationship between the signs of the derivative of g and f both evaluated at T. From Eq. 3

$$g'(T) = n \times f'(T) + n(n-1)f^2(T). \tag{4}$$

Two general points arise. First, the only term on the right-hand side of Eq. 4 that can be negative is $f'(T)$. It follows that, if $g'(T)$ is negative, then $f'(T)$ must also be negative and, by assumption, the population is declining at the end of the observation period. Second, the converse is not true: it is possible for $f'(T)$ to be negative so that the population is declining at the end of the observation period, but $g'(T)$ to be positive so that the pdf of the most recent sightings is increasing. Moreover, provided $f'(T)$ is finite and $f(T)$ is positive, this is bound to occur for large enough n.

It is instructive to consider some examples. Suppose that the size of a population is constant over the observation period so that, conditional on n, the sightings by each individual are uniformly distributed over $(0, T)$. For convenience, here and following, I will take $T = 1$ so that, in the uniform case, $f(t) = 1$, $0 \leq t \leq 1$. The pdf of the most recent sighting is

$$g\left(t_{(n)}\right) = n \times t_{(n)}^{n-1} \tag{5}$$

which, provided $n > 1$, increases with $t_{(n)}$ with $g'(T) = n(n-1)$. That is, for a constant population size, the distribution of the most recent sighting actually increases with time.

Suppose next that the sighting rate declines linearly over the observation period at rate β. The pdf of sighting time is

$$f(t) = \left(1 + \frac{\beta}{2}\right) - \beta \times t \tag{6}$$

with $0 \leq \beta \leq 2$ where the upper bound ensures that the sighting rate is positive over the observation period. In this case

$$g'(T) = -n\beta + n(n-1)\left(1 - \frac{\beta}{2}\right)^2 \tag{7}$$

which can be shown to be positive if

$$\beta < \frac{2(n - \sqrt{2n-1})}{n-1}. \tag{8}$$

So, for example, if $n = 5$, then $g'(T) > 0$ if $\beta < 1$. To put this into context, if $\beta = 1$, the sighting rate declines by two-thirds over the observation period. For large n, the right-hand side of Eq. 8 approaches 2 so that $g'(T) > 0$ for all values of β.

As a final example, suppose the sighting rate declines exponentially at rate β. In this case, conditional on n, the sighting times follow a truncated exponential distribution with pdf

$$f(t) = \frac{\beta \exp(-\beta t)}{1 - \exp(-\beta)} \qquad 0 \le t \le 1. \tag{9}$$

It is straightforward to show for this model that is positive if

$$\exp(-\beta) > n^{-1}. \tag{10}$$

The quantity on the left-hand side of Eq. 10 is the ratio of the sighting rate at the end of the observation period to the sighting rate at the beginning. So, for example, if $n = 5$, the pdf of the most recent sighting time increases as long as this ratio is greater than 0.2. For large n, the right-hand side of Eq. 10 approaches 0 so that $g'(T)$ is again positive for all values of β.

In the much more realistic case where the numbers $n_1, n_2, \ldots, n_m$ of sightings in the different records are different, the pdf g of the most recent sightings is a mixture of pdf's each of the form in Eq. 3

$$g\left(t_{(n)}\right) = \frac{f\left(t_{(n)}\right)}{m} \sum_{j=1}^{m} n_j F^{n_j - 1}\left(t_{(n)}\right) \tag{11}$$

and it is straightforward to show that

$$g'(T) = \bar{n} f'(T) + \overline{n(n-1)} f^2(T) \tag{12}$$

where the over bar indicates the average. As before, if $g'(T) < 0$, then $f'(T) < 0$, but not the converse. Specific results such as those above about the sign of $g'(T)$ are more complicated and depend on both the average and spread of the sighting numbers. Briefly taking a broader view, if each of the most recent sightings is paired with the overall number of sightings, then it would be possible to fit a parametric model of f and to test, for example, the null hypothesis of a common f among a collection of populations.

The main result of this comment has been that the behavior of the record of most recent sightings of a population depends on both the underlying population trend and the numbers of overall sightings by different observers. Returning to the paper of Turvey et al. (2012), this suggests that, without further assumptions about these overall sighting numbers, the similar rates of decline in most recent sightings among three of the Yangtze species need not imply similar rates of population decline. By the same token, the absence of a decline in most recent sighting rate for the remaining species need not imply a different rate of population decline.

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