Modified data fidelity speed in anisotropic diffusion

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Modified Data Fidelity Speed in Anisotropic Diffusion

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Abstract— **In this paper, we use an anisotropic diffusion in a level set framework for low-level segmentation of necrotic femoral heads. Our segmentation is based on three speed terms. The first one includes an adaptive estimation of the contrast level. We use the entropy for evaluating our diffusion on synthetic 3D data. We notice that using the data fidelity term in the last iterations excessively penalizes the diffusion process. To provide better segmentation results, we propose some modifications in the data fidelity speed: we propose to build its refrence data term from previous iterations results and hence lessening influence of initial noisy data.**

I. Introduction

Working on a project aiming at providing the experts with semi-automatic tools for low-level 3D segmentation of joints, we focus here on developing a method for evaluating the necrotic volume in femoral head. This was the objective of many works such that of Zoroofi *et al.* in their recent paper [1].

Fig. 1. Initial 23rd slice of a necrotic femoral head – MRI data set.

We use 3D MR data sets in the study of different patients. Figure 1 shows an initial slice of a 71x71x41 voxels MR data volume with a 1x1x1.5 mm^3 resolution. The segmentation is performed using an anisotropic diffusion formulated in a level set framework. Anisotropic diffusion was early introduced by Perona and Malik in their famous paper [2]. Their objective was to preserve edges while smoothing. However their model was ill-posed. To solve this problem, Catté *et al.* proposed [3] to evaluate the gradient on a pre-filtered image.

Others have then introduced new terms in the diffusion equation. Alvarez *et al.* used mean curvature in anisotropic diffusion [4] and benefited from the invariance of such a term for the luminance. Some works [5]–[6] have established a strong relationship between anisotropic diffusion and level set method approaches and thus allow the formulation of anisotropic diffusion in a level set framework. Such formulation provides many advantages – such as stable numerical schemes – and enables us to introduce different propagation terms easily.

While segmenting 3D data of necrotic femoral heads, we make use of such level set formulation with three speed terms. In the next section, we develop our diffusion method. Then, in the second section, we discuss an entropy evaluation of our diffusion process. The observation of its dynamics while the iterations are progressing, lead us to modify the third speed term as described in section 3. Section 4 presents some results on both synthetic and real data.

II. Anisotropic Diffusion Based on Three Speed Terms

Within a level set framework, a generic diffusion-based process can be seen as a propagation of multiple fronts at a time. We use a process governed by the non-linear PDE:

$$\begin{cases} I_{t=0} = I_0 \\ \dfrac{\partial I_t}{\partial t} = -\alpha H(I_t)\, g(|\vec{\nabla}\tilde{I}_t|)\,|\vec{\nabla} I_t| + \beta \vec{\nabla} g(|\vec{\nabla}\tilde{I}_t|)\cdot \vec{\nabla} I_t \\ \qquad -\gamma(I_t - I_0)|\vec{\nabla} I_t| \end{cases}, \quad (1)$$

where t denotes the time scale parameter, $\tilde{I}$ is the spatial smoothed data, H is the mean curvature, $\vec{\nabla}$ is the derivative operator, α, β and γ are weighting coefficients between the three propagations speed terms. The function g controls the degree of the diffusion by stopping the diffusion on edges.

The first term in the second member of (1), containing the coefficient α, monitors the propagation by gradient curvature speed – *i.e.*, the non-linear diffusion term. The second term, with a coefficient β, enables to enhance the contrast on edges, and the third term, weighted by the coefficient γ, is a data fidelity term.

Black *et al.* [7] unified anisotropic diffusion and robust statistics where valid edges can be seen as outliers in a robust estimation framework. While getting an instantaneous estimate of standard deviation of the contrast noise through the MED L-estimator, we devise the filtering equation:

$$\begin{cases} I_{t=0} = I_0 \\ \hat{\sigma}_t = 1.4826\ M_{u\in D_I}(|\vec{\nabla} I_{t,u}|) \\ \dfrac{\partial I_t}{\partial t} = -\alpha H(I_t)\, g(|\vec{\nabla}\tilde{I}_t|, \hat{\sigma}_t)\,|\vec{\nabla} I_t| \\ \qquad + \beta \vec{\nabla} g(|\vec{\nabla}\tilde{I}_t|, \hat{\sigma}_t)\cdot \vec{\nabla} I_t - \gamma(I_t - I_0)|\vec{\nabla} I_t| \end{cases}, \quad (2)$$

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where M denotes the median operator and $g(x,\sigma)$ the weighting function $\exp(-x^2/\sigma^2)$ of the Leclerc robust M-estimator[1]. Unlike a usual setting of the optimization of a robust norm, we do not set $\hat{\sigma}_t = \hat{\sigma}_0 \ \forall t$. This brings out a hybrid iterative scheme merging an L-estimator and a PDE. Thus, it becomes difficult to theoretically ensure the convergence of this highly non-linear process. However, our experience shows that such a simulated-annealing-like process gives better results. Moreover, if needed, a time regularization effect could be carried out through some damped smoothing function f while introducing the relationship $\hat{\sigma}^*_n = f(\hat{\sigma}_n, \hat{\sigma}_{n-1}, \ldots)$ in the discrete numerical scheme.

While propagating the fronts, such a versatile scheme enables to change the weighting coefficients of the three terms. And, hence, one can benefit from the action of each term in some iteration and then limit theirs actions in others. In this paper, as our target is to show the influence of the third speed term, and then proposing some modifications on it, we will not use the second speed term and his coefficient will always be set to 0.

III. Quantitative Evaluation of the Segmentation

In addition to visual evaluation of the results, to evaluate the diffusion we use a quantitative evaluation based on the entropy of the results. The entropy E of the data can be calculated through (3) as the sum of the entropy in different regions. Let R denote a region of the data. Let N denote the number of regions and V the volume of the region. The volume being approximated by the number of voxels in the region, the entropy is

$$E(I) = \sum_{j=1}^{N} \frac{V_j}{V_I} E(R_j),$$
$$E(R_j) = -\sum_{m \in V_j^v} \frac{S_j(m)}{V_j} \log \frac{S_j(m)}{V_j}, \quad (3)$$

where $S_j(m)$ is the number of voxels in the region j with a gray-level value m [8].

The lower entropy we have, the more homogenous the regions are and the best results we obtain. We evaluate the results entropy while the diffusion is advancing. So, the entropy of the results decreases while the diffusion process is progressing (Figure 2). This evaluation provides us with a method to measure the influence of each term of propagation in (2) and hence enable us to optimally configure their weighting coefficients. Figure 2 shows the evolution of the entropy of the results with two different configurations. The first one uses only the first term for diffusion and the second introduces the data fidelity speed term.

The data used in this test is a synthetic 71x71x41 voxels data of necrotic femoral head of with a 1x1x1.5mm^3 resolution that represents four different regions obtained from manual segmentation of a real data. A gaussian white noise with a standard deviation $\sigma = 10$ is added to the segmented result data.

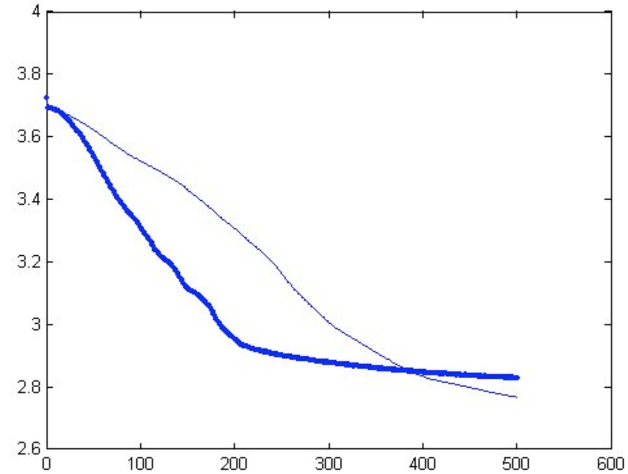


Fig. 2. Entropy evaluation of diffusion with two different configurations (thin line: $\alpha = 1, \beta = 0, \gamma = 0$, thick line: $\alpha = 1, \beta = 0, \gamma = 0.1$).

Figure 2 shows the advantages of activating of the third speed term. The thick curve passes quickly under the blue one and shows that using the third speed term leads to lower entropy results than using only the first speed term.

However, from a given rank of iteration, the data fidelity speed limits the diffusion process. The classical approach using (2) reduces the coefficient γ while iterations are progressing. So, we should take advantage of the third term in the first iterations without limiting the diffusion in the last ones.

IV. Modified Data Fidelity Speed

It is clear that the third speed term contributes to a better diffusion in the first iterations. However, in the last ones, this term penalizes the diffusion because the influence of the noise in the initial data is still important. Therefore, motivated by the results in the first iterations where the diffusion is more important using the data fidelity speed as showed in figure 2, we propose to modify the third term speed and to use it along all the diffusion process.

The main idea is to replace I_0 in the data fidelity speed by another data estimation. So, we propose to construct the reference data from results in the previous iterations instead of taking one iteration result as a reference in the third speed term. While diffusion is progressing, the noise in the data is decreasing and using these results in the data fidelity speed will limit the effect of noise from the action of the third speed term in the last diffusion iterations.

Using the same notation as above and introducing the dynamic data reference, equation 2 becomes:

$$\begin{cases} I_{t=0} = I_0 \\ \hat{\sigma}_t = 1.4826 \ M_{u \in D_t}(|\vec{\nabla} I_{t,u}|) \\ \dfrac{\partial I_t}{\partial t} = -\alpha H(I_t) g(|\vec{\nabla}\tilde{I}_t|, \hat{\sigma}_t) |\vec{\nabla} I_t| \\ \qquad + \beta \vec{\nabla} g(|\vec{\nabla}\tilde{I}_t|, \hat{\sigma}_t) \cdot \vec{\nabla} I_t - \gamma (I_t - I^*_{t-\varphi}) |\vec{\nabla} I_t| \end{cases}, \quad (4)$$

[1] Indeed, the relationship (2) involves two implicit assumptions: (i) the mean components of the signed contrast noise vector are null $\forall t$; (ii) it is more relevant to get a standard noise estimate $\hat{\sigma}$ from I rather than $\tilde{I}$.

where $I^*_{t-\varphi}$ is the new reference data and φ is the time delay. The notation I^* denotes a time scale smoothed data.

The influence of each past iteration results will not be the same while building the new evolving ground-data reference. The introduction of new results in the construction will be progressive. We propose to use a weighting function to govern the influence of the previous results. From certain iteration level, the old iteration results will be omitted in the construction. With such a modification, we attempt to limit the influence of the noise in the last iterations while supporting the diffusion process. Within the numerical scheme, the reference data is constructed according to a function *h* as specified in the discrete convolution:

$$I^*_{n-L} = \frac{\sum_{i=1}^{N} h(i)\, I_{n-i}}{\sum_{i=1}^{N} h(i)} \quad , \qquad (5)$$

where *L* is the discrete time delay and the range [*n*-1,*n*-*N*] is a time segment centered on *n*-*L*.

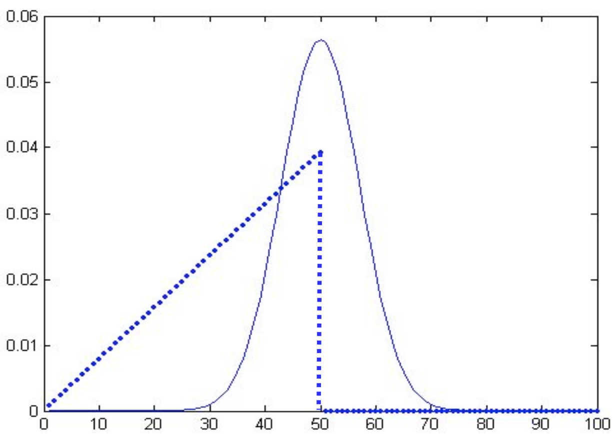


Fig. 3. Functions used in reference data construction (dotted: the crawl function; solid: the gaussian function).

For example, we can use a crawl function for the function *h* in equation 5 (Figure 3). Alternatively, the older results can also be truncated more softly while using other smoothing functions such as a gaussian. For comparison, in both functions, we choose the results that have the higher influence in the construction of the reference data as the 50th last results.

We compare the results of diffusion using these two functions in the construction of reference data to the results using the diffusion without the data fidelity speed term and to results using this term without modification.

V. Results

The diffusion process acts in three dimensions, however we present in this section results on some slices of the 3D results. The functions used for reference data reconstruction are the crawl function of figure 3 and the gaussian one with standard deviation $\sigma = 10$.

We present here the curves of the quantitative evaluation of the diffusion using the modified data fidelity speed while iterations are progressing (Figure 4).

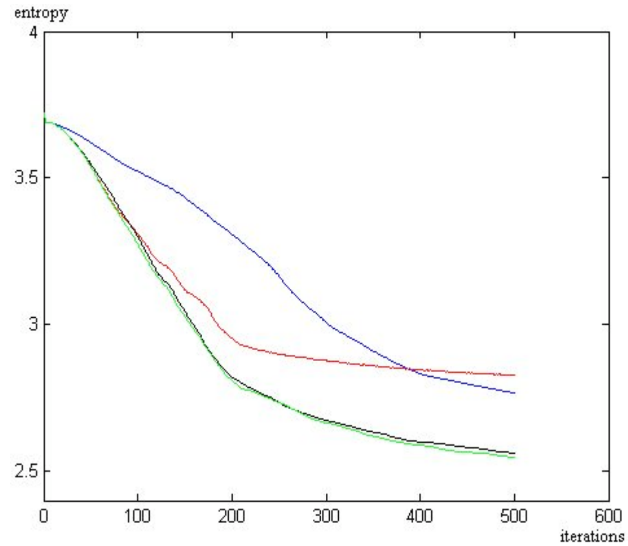
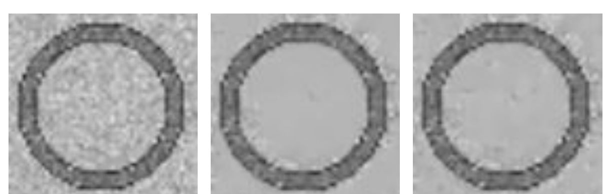


Fig. 4. Evaluation of modified data fidelity term action (blue: only first speed term; red: $\alpha = 1$, $\beta = 0$, $\gamma = 0.1$, initial 3rd term; green: gaussian modification in 3rd term with $\alpha = 1$, $\beta = 0$, $\gamma = 0.1$; black: crawl modification in 3rd term with $\alpha = 1$, $\beta = 0$, $\gamma = 0.1$).

With both modifications, the crawl and the gaussian functions, the entropies of results are lower than using only the first term speed or the first and third term without modification (Figure 4).

In figure 5, we present our segmentation results on a synthetic data of 50x50x50 voxels isotropic volume containing two concentric spheres with different rays: 18 and 22 voxels. We also present the results of the modified segmentations on the 19th slice of the synthetic femoral head data volume discussed in section 3.

For tests on synthetic data we use the parameters $\alpha = 1, \beta = 0, \gamma = 0.1$ and 500 iterations.



Initial 22th slice (left); building reference data with crawl function (midle) and gaussian function (right).

Initial 19th slice (left); building reference data with crawl function (midle) and gaussian function (right).

Fig. 5. Visual evaluation of synthetic data segmentations.

We also present on figure 6 some results using both the gaussian and the crawl constructions of the reference data on the real volume data presented in the introduction. For tests on real data, we use the same configuration as previously and 300 iterations.

Fig. 6. Results with past reference data built from a crawl function (top); scaled differences between initial data and result without using the modification (middle); differences between initial data and result using the modification – same scale as middle image (bottom).

The differences between the result and the initial data in figure 6 show that the edges are well preserved. We obtain similar results using gaussian or crawl construction of the reference data. Both lead to a good segmentation. Figure 7 shows that using a larger inertia time-delay in construction of this data fidelity term yields a better segmentation – but this introduces certainly a higher memory cost.

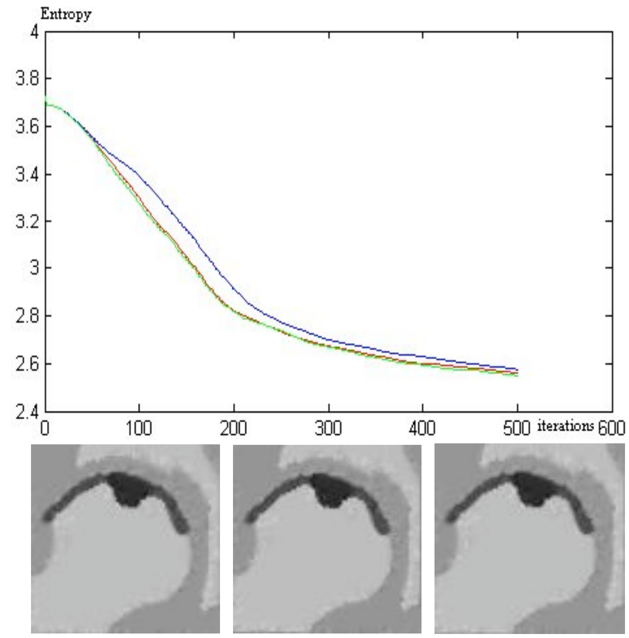


Fig. 7. Results evaluation using the crawl function with different time delay: $\varphi = 30$ (blue, left), $\varphi = 50$ (red, midle), $\varphi = 70$ (green, right).

VI. Conclusion and Future Work

In this paper, we modify the data fidelity speed term in the diffusion process. Such a modification allows the use of this term along all iterations and hence performs the diffusion process without deteriorating edges.

The modification of the data fidelity speed term is based on its construction as a combination from the last results. We used a crawl function and a gaussian one to show the benefit of such modification.

In a future work, we will focus on the study of the best recursive filter to estimate reference data more efficiently.

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