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Vehicle routing problem for information collection in wireless networks

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Abstract: Advances in computer network architecture add continuously new features to vehicle routing problems. In this work, the Wireless Transmission Vehicle Routing Problem (WT-VRP) is studied. It looks for a route to the vehicle responsible for collecting information from stations as well as an efficient information collection planning. The new feature added here is the possibility of picking up information via wireless transmission, without visiting physically the stations of the network. The WT-VRP has applications in underwater surveillance and environmental monitoring. We discuss three criteria for measuring the efficiency of a solution and propose a mixed integer linear programming formulation to solve the problem. Computational experiments were done to access the numerical complexity of the problem and to compare solutions under the three criteria proposed.

1 INTRODUCTION

The vehicle routing problem (VRP) is one of the most studied problems in operations research. The intensive research on the topic is due not only to its computational complexity but also to the numerous applications in fields such as logistic, maritime transportation, telecommunications, production, among many others. Different variants of the VRP have been treated in the past 50 years (see (Laporte, 2009)). Among the best-known variants we can cite the VRP with Multiple Depots (Montoya-Torres et al., 2015), the Pickup-and-delivery VRP (Dethloff, 2001), the VRP with Time Windows (Agra et al., 2013), the VRP with Backhauls (Toth and Vigo, 1997). Several hybrid variants of the problem are also described in the literature, most of them inspired by real-life scenarios (see (Baldacci et al., 2007; Caceres-Cruz et al., 2014)).

As we could expect, different solution approaches for the VRP have been explored in the literature. Since the VRP is an NP-hard problem, heuristic and meta-heuristic approaches are more suitable for practical applications, often defined on large scale graphs. Despite the fact that they do not provide an approximation guarantee on the obtained solution, they have proved their efficiency empirically.

Exact methods for obtaining solutions with guar-

antee of optimality are often applied to small-sized instances and, sometimes, even to medium-sized ones (Fukasawa et al., 2006; Pecin et al., 2017). Among the exact approaches, we have solution methods based on integer (ILP) and mixed integer linear programming (MILP) formulations (Feillet, 2010; Simchi-Levi et al., 2005) and other enumerative algorithms (Fischetti et al., 1994). Exact approaches provide benchmark instances to the VRP problem studied and allow us to evaluate the performance of heuristics.

In the last years, technological advances in computer network architecture have added new features and applications to vehicle routing problems. Delay-Tolerant Networks and Wireless Sensor Networks are two examples.

Delay-Tolerant Networks (DTN) (Fall, 2003; Jain et al., 2004) are wireless networks designed to tolerate long delays or disruptions. One of the applications of DTN's that has gained popularity is to provide web service to remote locations. For example, Daknet (Pentland et al., 2004) is a network for rural connectivity that uses buses and local transport to carry messages and web connection to small and remote villages. Daknet establishes small kiosks in rural villages without web connectivity, allowing people to send email and information in an off-line manner. Another important application of DTN is to provide

connection for remote military stations which dispose of a set of vehicles to pick-up and deliver wireless information (Malowidzki et al., 2016).

Wireless Sensor Networks (WSN) (Akyildiz et al., 2002) are often used in critical applications such as habitat monitoring, war surveillance, submarine surveillance and monitoring, detection of biological, chemical and nuclear attack (Mainwaring et al., 2002; Winkler et al., 2008; Basagni et al., 2014; Vieira et al., 2015). In these applications, sensors are deployed in an area and are used to store information to be sent in the future to a control center via base stations. Mobile elements, such as unmanned aerial vehicles, have been incorporated in the WSN design (Teh et al., 2008).

The applications mentioned here from both technologies, DTN and WSN, need to provide vehicle routing strategies with wireless information transmission to the vehicles involved. Most of the work has been done considering the routing protocol (Bhoi et al., 2017; Celik and Modiano, 2010; Moghadam et al., 2011; Velásquez-Villada et al., 2014) while there is still a gap in the development of vehicle routing strategies. When developing efficient routing protocols, many authors consider that the vehicle route is already defined which is justified by applications taking advantage of an existing transportation infrastructure (Velásquez-Villada et al., 2014). In (Celik and Modiano, 2010), the vehicle route is assumed to exist but authors suppose vehicles can adjust their position (in order to receive information from some stations) and study the delay performance in the network. Other works, like (Kavitha and Altman, 2009), suppose an architecture is defined for the vehicle routing (cycle path or zig-zag path) and study the best placement for such architecture. To the best of our knowledge, the authors in (Basagni et al., 2014) are the only ones to investigate the vehicle routing problem from scratch together with the wireless transmission planning. They address an application of underwater wireless sensor networks for submarine monitoring. The authors considered a scenario with a set of S surfacing nodes and $|S|$ underwater nodes where they look for a routing to an autonomous underwater vehicle (AUV) during a time period T . The AUV must leave and return to a surface node while information generated by the set of underwater nodes is collected along a path that physically visits each station where information is collected. The information generated in a given underwater node i at a time point t_1 which arrives to a surface node at a time point t_2 has a given value v_{i,t_1,t_2} . The strategy adopted by the authors is the maximization of the value of the information collected. The authors in (Basagni et al., 2014) proposed an Integer Linear Programming (ILP) formula-

tion able to solve the problem with $|S| \in \{4, 5, 9, 12\}$ in a time that varies from a few hours to a few days.

In this work, we study a new version of the vehicle routing problem for information collection in wireless networks. The new characteristic added to this well-known problem is the possibility of picking up information via wireless transmission. In the context considered here, a vehicle is responsible for collecting data from a set of stations. The stations are equipped with technology capable of sending information via wireless connections to the vehicle when it is located in another sufficiently close station. Simultaneous transmissions are allowed. Time of transmission depends on the distance between stations, the amount of information transmitted, and other physical factors (e.g. obstacles along the way, installed equipment). Information to be sent outside of the network is generated at each station at a constant rate. Given a time horizon, the vehicle must visit the set of stations gathering information either when it is physically in a station or by wireless transmission. A unique base station is connected with the outside. The vehicle starts and ends its route at the base station. Any station can be visited several times but it can also never be visited.

Unlike in (Basagni et al., 2014), the value of information collected does not decrease with time. However, depending on the position of the base station, the vehicle can be conducted to return several times to the base in order to have access to some stations. The VRP treated in this work looks for a route to the vehicle and for an efficient planning on how to collect information from stations. Since only one uncapacitated vehicle is considered, the problem can also be seen as a version of the Traveling Salesman Problem (TSP) (Letchford et al., 2013). However, opposite to most versions of TSP, no constraint is imposed neither on the number nor on the frequency of visits to the stations. The focus of this work is the investigation of the strategies adopted in the vehicle routing problem and its exact solution by an ILP formulation.

The paper is organized as follows. Section 2 formally describes the problem being solved while notations and assumptions are presented. A MILP formulation is presented in Section 3 with three different objective functions being discussed at this section. Computational experiments are presented on Section 4. Periodicity on the remaining information at the stations after a sequence of vehicle routings is discussed in Section 4. Finally, some conclusions and research directions are presented at Section 5.

2 DESCRIPTION OF THE PROBLEM

The wireless network is modeled by a directed graph $D = (V, A)$. The node set $V = \{1, \dots, n\}$ represents the n stations of the network and the arc set A represents m directed paths connecting pairs of stations in V . A unique base station is considered and denoted as node 1. Weights t_{ij} and d_{ij} are associated to each arc (path) $(i, j) \in A$ representing, respectively, the time it takes to travel from station i to station j and the distance among these two stations. Let $T = \{1, 2, \dots, T\}$ be the time horizon considered. At the beginning of the time horizon, each station $j \in V \setminus \{1\}$ contains an amount C_j of information. For each station $j \in V \setminus \{1\}$, information is generated at a rate of r_j units per time point in T . Thus, the amount of information at station j at each time point $k \in T$, denoted by q_{jk} , is proportional to the elapsed time from the last extraction (either physically or through a wireless connection), i.e.,

$$q_{jk} = \begin{cases} C_j + kr_j, & \text{if station } j \text{ has not been visited} \\ & \text{before time point } k, \\ (k - t_{last})r_j, & \text{otherwise, where } t_{last} \text{ is the} \\ & \text{time of the last extraction.} \end{cases}$$

Only the base station is appropriately equipped for sending information outside the network. A unique vehicle is in charge of collecting data from all the stations in $V \setminus \{1\}$ and of transporting it to the base station. There is no capacity limit associated to the vehicle. At the beginning of the time horizon, the vehicle is located at the base station and at the end of the time horizon, it must return to the base station. Multiple visits are allowed to each node in V . Information can only be transmitted when the vehicle is located in one of the stations in V , i.e., no transmission is allowed while the vehicle is moving on an arc $(i, j) \in A$. We also assume that, once a station i starts a transmission to the vehicle, all the current information located in i at that moment must be transmitted.

Wireless transmission can be used to transfer data from a station $j \in V$ to the vehicle located in a station $i \in V \setminus \{j\}$. However, wireless transmission is only possible for close enough stations. Let r_{cov} be the maximum distance allowing wireless transmission. A station j can wireless transfer its data to (the vehicle located in) station i whenever $d_{ij} \leq r_{cov}$. We define a set $W = \{(i, j) \in V \times V \mid d_{ij} \leq r_{cov}\}$.

We assume that a transmission occurs with a fixed transmission power of P_t . The received power P_r is given by

$$P_r = \alpha P_t D^{-\eta}$$

where η is the pathloss parameter (which we shall take in this paper to be equal to two but our results carry on to arbitrary positive value of η) and where D is the distance between the receiver and transmitter. We shall assume that the vehicle has an antenna with an elevation of 1 unit. The coordinates of a sensor are given by $(x_s, y_s, 0)$ and those of the vehicle are $(x_b, y_b, 1)$, which means, the antenna on the vehicle is elevated by one unit with respect to the sensors. Thus, if $d = \sqrt{(x_s - x_b)^2 + (y_s - y_b)^2}$ then $D = \sqrt{1 + d^2}$. We use a linear approximation of the Shannon capacity (as a function of the power) for the data transmission rate (Tse and Viswanath, 2005) and write it as

$$Thp(d) = \log \left(1 + \frac{\alpha P_r}{\sigma} \right) \sim \frac{\beta P_r}{2\sigma} = \frac{\beta P_t}{2\sigma} (1 + d^2)^{-1}.$$

Here β is the antenna's gain and σ is the noise at the receiver (we assume independent channels and thus there are no interferences of other transmissions on the received signal from a sensor).

Thus, the time necessary for a wireless transmission of \bar{q} units of data between stations j and i is

$$\alpha_{ji}(1 + d_{ij}^2)\bar{q}, \quad (1)$$

with $\alpha_{ji} = \frac{\beta P_t}{2\sigma}$ depending on physical factors in stations i and j .

Simultaneous transmission is possible from a set of at most M stations to the vehicle located in a station $i \in V$. In this case, the simultaneous data transfer finishes only when each individual wireless transmission finishes. As a consequence, the time of a simultaneous transmission corresponds to the highest maximum individual wireless transmission.

The version of the VRP treated in this paper, denoted as Wireless Transmission VRP (WT-VRP), consists of finding a feasible routing for the vehicle together with an efficient planning for collecting information from stations $V \setminus \{1\}$. The criteria for measuring the efficiency of a collection planning will be discussed in the next section.

3 MATHEMATICAL FORMULATION

In this section, we introduce a mixed integer linear programming (MILP) formulation to the WT-VRP. Let us define an artificial arc set $A^0 = \{(i, i) \in V\}$. For each $(i, j) \in A \cup A^0$ and $k \in T$, we define the following decision variables.

$$x_{ijk} = \begin{cases} 1, & \text{if the vehicle crosses path } (i, j) \text{ and} \\ & \text{arrives to node } j \text{ at time point } k, \\ 0, & \text{otherwise.} \end{cases}$$

We observe that the action of crossing an artificial arc $(i, i) \in A^0$ models the following action: the vehicle is located at node i without neither moving nor transferring data. Let $t = (t_{ij})$ be the weight matrix associated with A . We also define $\hat{t} = t + I$ the weight matrix associated with $A \cup A^0$. For each $(j, i) \in W$, $k \in T$ and $l \in S_k = \{1, \dots, \bar{T} - k\}$, we also define,

$$w_{jikl} = \begin{cases} 1, & \text{if node } j \text{ is sending data to the vehicle} \\ & \text{while it is located in node } i, \text{ with} \\ & \text{transmission starting at time point } k \\ & \text{and lasting } l \text{ time units,} \\ 0, & \text{otherwise.} \end{cases}$$

Linear constraints defining the MILP formulation are presented next, divided in three sets according to their modeling purposes.

3.1 Problem constraints

The first set, the *Routing Constraints*, characterizes the way the vehicle moves around the set of stations.

$$\sum_{s:(1,s) \in A \cup A^0} x_{1s(1+t_{1s})} = 1, \quad (2)$$

$$\sum_{(s,1) \in A \cup A^0} x_{s1\bar{T}} = 1, \quad (3)$$

$$x_{ijk} \leq \sum_{(j,p) \in A \cup A^0 | k+t_{jp} \in T} x_{jp(k+t_{jp})} + \sum_{(u,j) \in W} \sum_{l \in S_k} w_{ujkl}, \\ \forall (i,j) \in A \cup A^0, \forall k \in T, \quad (4)$$

$$x_{ij(k+t_{ij})} \leq \sum_{(u,i) \in A \cup A^0} x_{uik} + \sum_{n|(n,i) \in W} \sum_{l=1}^{k-1} w_{ni(k-l)l}, \\ \forall (i,j) \in A \cup A^0, \forall k \text{ s.t. } k+t_{ij} \in T, \quad (5)$$

$$\sum_{l \in S_k} w_{jikl} \leq \sum_{(i,n) \in A \cup A^0} \sum_{s \in S_k | k+s+t_{in} \leq T} x_{in(k+s+t_{in})}, \\ \forall i \in V, \forall j \text{ s.t. } (j,i) \in W, \forall k \in T, \quad (6)$$

$$\sum_{(i,j) \in A} x_{ijk} \leq 1, \quad \forall k \in T, \quad (7)$$

$$\sum_{s=k+1}^{k+t_{ij}-1} \sum_{(m,n) \in A \cup A^0} x_{mns} \leq t_{ij}(1 - x_{ij(k+t_{ij})}), \\ \forall (i,j) \in A, \forall k \in T \text{ s.t. } k+t_{ij} \in \bar{T}, \quad (8)$$

$$x_{ijk} \in \{0, 1\}, \quad \forall (i,j) \in A, \forall k \in T, \quad (9)$$

$$w_{uvkl} \in \{0, 1\}, \forall (u,v) \in W, \forall k \in T, \forall l \in S_k. \quad (10)$$

Equations (2) and (3) ensure, respectively, that the vehicle starts and ends the routing at the base station. Inequalities (4) ensure that, if at a time point $k \in T$ the vehicle arrives at station $j \in V$ crossing arc $(i, j) \in A \cup A^0$, it either goes immediately to a neighboring

station (crossing an appropriate path $(j, p) \in A \cup A^0$), or it stays at node j for a data transfer from another station $u \in V$. Likewise, if at a time point $k+t_{ij} \in T$ the vehicle arrives to station $j \in V$ coming from station i , inequalities (5) impose that either the vehicle has arrived to station i at time point k , or a data transfer from at least one another station was occurring and it has finished at time point k . Once the vehicle has arrived to station $i \in V$, at a time point $k \in T$, if data transfer happens, inequalities (6) force the vehicle to leave i , in a future time point $k+s$, by crossing an arc $(i, n) \in A \cup A^0$. Inequalities (7) and (8) are responsible for the elimination of simultaneous paths. On one hand, inequalities (7) force the vehicle to leave a given station along a unique path. On the other hand, if the vehicle cross the arc (i, j) , leaving i at time point k , an inequality in (8) will prevent the vehicle from moving from time point $k+1$ to $k+t_{ij}-1$. Constraints (9) and (10) impose binary conditions on the variables defined.

Inequalities defining the second set of constraints are the *Data Transfer Constraints*.

$$\frac{1}{M} \sum_{(j,i) \in W} \sum_{l \in S_k} w_{jikl} \leq 1, \forall k \in T, \quad (11)$$

$$w_{jikl} \leq \sum_{(p,i) \in A} x_{pik}, \\ \forall (j,i) \in W, \forall k \in T, \forall l \in S_k, \quad (12)$$

$$\sum_{(i,n) \in A} \sum_{s=0}^{l-1} x_{in(k+s+t_{in})} \leq l(1 - w_{jikl}), \\ \forall (j,i) \in W, \forall k \in T, \forall l \in S_k. \quad (13)$$

Inequalities (11) define a bound of M simultaneous data transfers. Inequalities (12) impose that, at each time point $k \in T$, in order to start sending information to a station $i \in V$, the vehicle must previously arrive to this station. Inequalities (13) prevent the vehicle to leave station $i \in V$ while a data transfer is occurring: if w_{jikl} equals to 1, the vehicle cannot leave station i from time point k to time point $k+l-1$. When simultaneous data transfers occurs, this set of inequalities will be in charge of defining the total duration of the simultaneous transfer.

Before presenting the last set of constraints, we need to define the continuous variables of the MILP formulation. For each $j \in V$ and $k \in T$, let q_{jk} represent the amount of data accumulated at station j (waiting for transfer out of the network) at time point k . The last set of constraints is presented next and they

are the *Amount of Information Constraints*.

$$q_{j1} = C_j, \forall j \in V \setminus \{1\}, \quad (14)$$

$$q_{jk+1} = q_{jk} \left(1 - \sum_{i|(j,i) \in W} \sum_{l \in S_k} w_{jikl}\right) + r_j, \quad \forall k \in T \setminus \{\bar{T}\}, \forall j \in V, \quad (15)$$

$$\alpha_{ji}(1 + d_{ij}^2)q_{jk}w_{jikl} \leq l, \quad \forall (j,i) \in W, \forall k \in T, \forall l \in S_k. \quad (16)$$

Equations (14) set the initial load of each station $j \in V \setminus \{1\}$. Equations (15) are in charge of updating the load of stations along the time horizon. The amount of data accumulated in node j , at time point $k+1$, $q_{j(k+1)}$, is set to r_j in the case a data transfer started at time point k , otherwise it equals to $q_{jk} + r_j$. Finally, inequalities (16) define the time necessary for transferring q_{jk} data units (as defined by (1)) whenever $w_{jikl} = 1$.

Constraints (15) and (16) are quadratic ones and could be linearized by applying a classical change of variables. An alternative way of linearizing these constraints is by replacing them with the following big- M inequalities:

$$q_{j(k+1)} \geq q_{jk} + r_j - M_{jk} \sum_{j|(j,i) \in W} \sum_{l \in S_k} w_{jikl}, \quad \forall k \in T \setminus \{\bar{T}\}, \forall j \in V, \quad (17)$$

$$q_{jk} \geq r_j, \quad \forall j \in V, \forall k \in T \quad (18)$$

$$\alpha_{ji}(1 + d_{ij}^2)q_{jk} - N_{jik}(1 - w_{jikl}) \leq l, \quad \forall (j,i) \in W, \forall k \in T, \forall l \in S_k, \quad (19)$$

where

$$M_{jk} = r_j(k+1), \quad j \in V, k \in T,$$

$$N_{jik} = \alpha_{ji}(1 + d_{ij}^2)r_jk, \quad (j,i) \in W, \forall k \in T.$$

Consider a node $j \in V$ and a time point $k \in T$. If a transfer occurs from station j at time point k , the associated inequality in (17) becomes redundant and the associated inequality in (18) defines the valid lower bound $q_{jk} \geq r_j$. On the other hand, if no transfer occurs, the associated inequality in (18) becomes redundant and a valid lower bound $q_{j(k+1)} \geq q_{jk} + r_j$ is defined by (17). A minimization objective function of the WT-VRP (discussed in the next subsection) together with inequalities (17) and (18) will be in charge of appropriately setting the value of variables q_{jk} . In the same way, an inequality in (19) is either redundant ($w_{jikl} = 0$) or it becomes an inequality in (16) ($w_{jikl} = 1$).

3.2 Objective functions

As it has been described in Section 2, the WT-VRP looks for an efficient way of collecting data located

in remote stations. We discuss in this section three different criteria to measure the efficiency of a collection planning, each one giving birth to a different linear objective function. The first one maximizes the total amount of information extracted at the end of the time horizon T ; the second one maximizes the average of the information in the vehicle at each time point; while the third one maximizes the satisfaction of each station at the end of the time horizon T .

Let us analyze the first criteria. Consider that our goal is the extraction of as much information as possible from all stations, at the end of the finite time horizon T . Let η_j be the quantity of information extracted from a given station $j \in V \setminus \{1\}$, i.e.,

$$\eta_j = C_j + r_j(\bar{T} - 1) - q_{j\bar{T}}.$$

The amount of information extracted from all stations in $V \setminus \{1\}$ is

$$\sum_{j \in V \setminus \{1\}} \eta_j = \sum_{j \in V \setminus \{1\}} C_j + \sum_{j \in V \setminus \{1\}} r_j(\bar{T} - 1) - \sum_{j \in V \setminus \{1\}} q_{j\bar{T}}.$$

Since the only variables (from our MILP formulation) in this equation are the q_{jT} variables, for $j \in V$, we have that

$$\max \left\{ \sum_{j \in V \setminus \{1\}} \eta_j \right\} =$$

$$\sum_{j \in V \setminus \{1\}} C_j + \sum_{j \in V \setminus \{1\}} r_j(\bar{T} - 1) - \min \left\{ \sum_{j \in V \setminus \{1\}} q_{j\bar{T}} \right\}.$$

The first objective function, denoted **FO1**, minimizes the remaining amount of information at the end of the time period, over the set of stations, i.e.,

$$\text{Minimize } \sum_{j \in V \setminus \{1\}} q_{j\bar{T}}. \quad (20)$$

For the first criterion, no assumption is made about data location security: only the amount of information collected at the end of the time horizon matters. The second efficiency criterion is motivated by environments where information is safer once they leave the station to be stored at the vehicle (Basagni et al., 2014). Among the factors behind this supposition we have failures in station equipments, attack in case of military applications and energy resources limited. Let v_k be the amount of information in the vehicle at a time point $k \in T$,

$$v_k = \sum_{j \in V \setminus \{1\}} C_j + \sum_{j \in V \setminus \{1\}} r_j(k-1) - \sum_{j \in V \setminus \{1\}} q_{jk}.$$

We look for a solution, i.e., a routing and a collection planning, that maximizes the average over time of the amount of information in the vehicle, i.e.,

$$\max \left\{ \frac{1}{\bar{T}} \sum_{k \in T} v_k \right\} = \sum_{j \in V \setminus \{1\}} C_j +$$

$$\left(\sum_{j \in V \setminus \{1\}} r_j \right) \frac{(\bar{T}-1)}{2} - \frac{1}{\bar{T}} \min \left\{ \sum_{k \in T} \sum_{j \in V \setminus \{1\}} q_{jk} \right\}.$$

The second objective function, denoted **FO2**, minimizes the total amount of remaining information on the set of stations, over the whole time horizon, i.e.,

$$\text{Minimize } \sum_{k \in T} \sum_{j \in V \setminus \{1\}} q_{jk}. \quad (21)$$

In the third criterion, the satisfaction of each station will be taken into account. Let us define the satisfaction of a station $j \in V$ as the total amount of information extracted from j over the amount of information generated at j during the time horizon considered, i.e.,

$$s_j = \frac{C_j + r_j(\bar{T}-1) - q_j\bar{T}}{C_j + r_j(\bar{T}-1)} = 1 - \frac{q_j\bar{T}}{C_j + r_j(\bar{T}-1)}.$$

The maximization of the satisfaction over the whole set of stations is

$$\begin{aligned} & \max \left\{ \frac{1}{n-1} \sum_{j \in V \setminus \{1\}} s_j \right\} = \\ & = 1 - \frac{1}{n-1} \min \left\{ \sum_{j \in V \setminus \{1\}} \frac{q_j\bar{T}}{C_j + r_j(\bar{T}-1)} \right\}. \end{aligned}$$

For the particular case where $C_j = r_j$,

$$= 1 - \frac{1}{(n-1)\bar{T}} \min \left\{ \sum_{j \in V \setminus \{1\}} \frac{q_j\bar{T}}{r_j} \right\}.$$

Finally, the third objective function, denoted **FO3**, which maximizes the satisfaction of the stations, is defined as,

$$\text{Minimize } \sum_{j \in V \setminus \{1\}} \frac{q_j\bar{T}}{r_j}. \quad (22)$$

Consider the instance of the WT-VRP depicted in Figure 1 with five non-base stations. Figures 2–4 illustrate the solutions obtained by solving the MILP formulation, with the three different objective functions, assuming $\alpha_{ij} = 0.03$, for each $i, j \in V$, $r_{cov} = 2$, $\bar{T} = 20$ and $M = 3$. The movement of the vehicle is illustrated in each one of these figures by a time expanded network. Each set of nodes aligned horizontally is associated to a same station $i \in V$ in different time points. Each set of nodes aligned vertically represents the whole set of stations in a unique time point $k \in T$. Let us denote a node in the time expanded network by a pair ik , with $i \in V$ and $k \in T$. The scalar value inside a node ik gives us the amount of information in station i at time point k . A filled arrow from a node ik to a node jl represents a movement of the vehicle, leaving station i at time point k and arriving

$$d = \begin{bmatrix} 0 & 4 & 5 & 4 & 7 & 7 \\ 4 & 0 & 2 & 4 & 3 & 6 \\ 5 & 2 & 0 & 2 & 2 & 1 \\ 4 & 4 & 2 & 0 & 5 & 1 \\ 7 & 3 & 2 & 5 & 0 & 2 \\ 7 & 6 & 1 & 1 & 2 & 0 \end{bmatrix} \quad \bar{r} = \begin{bmatrix} 1 & 4 & 5 & 4 & 0 & 0 \\ 4 & 1 & 2 & 0 & 3 & 0 \\ 5 & 2 & 1 & 2 & 2 & 1 \\ 4 & 0 & 2 & 1 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 & 1 \end{bmatrix}$$

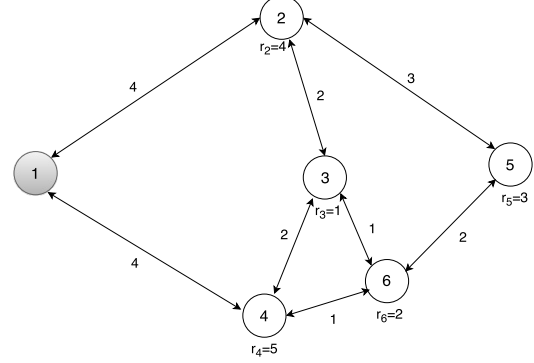


Figure 1: Directed (symmetric) graph describing an instance with 6 stations, data rates $r = (0, 4, 1, 5, 3, 2)$, time matrix \bar{r} and distance matrix d .

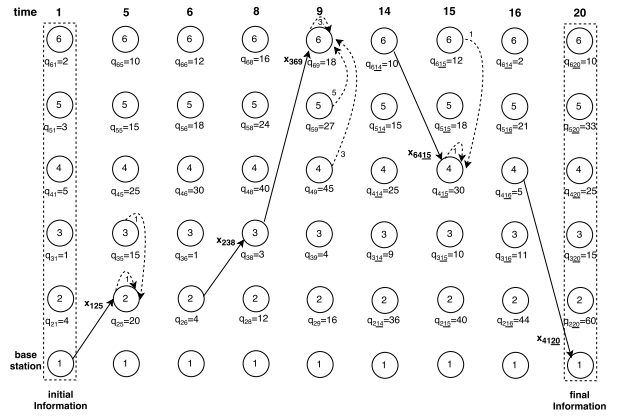


Figure 2: Solution obtained for the instance depicted in Figure 1 by the MILP formulation with objective function **FO1** defined in (20).

to station j at time point l . A dashed arrow from a node ik to a node jk indicates a data transfer occurs from station i to station j , starting in a time point after k . A weight associated with a dashed arc indicates the duration of the transfer.

The vehicle routing depicted in Figure 2 is $1 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 4 \rightarrow 1$. The vehicle leaves the base station 1 at time point 1 and arrives to station 2 at time point 5 with transmission from stations 2 and 3 starting immediately, each lasting 1 time unit. Then, the vehicle leaves station 2 at time point 6 arriving at station 6 at time point 9 for data transfer from stations 4, 5 and 6; data transfer at station 6 lasts $\max\{5, 3, 3\}$ time units. The vehicle leaves station 6 at time point 14 and continues its routing and data transfers until returning at time point 20 to the base station 1. The-

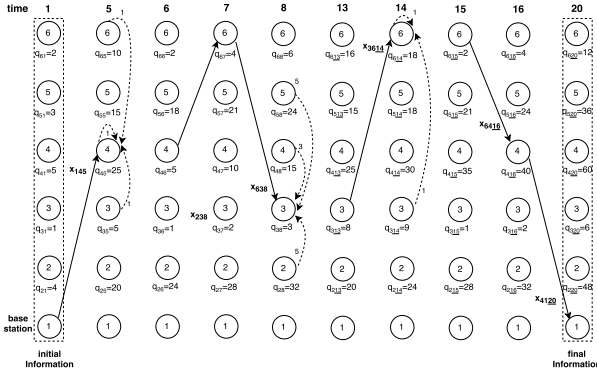


Figure 3: Solution obtained for the instance depicted in Figure 1 by the MILP formulation with objective function FO2 defined in (21).

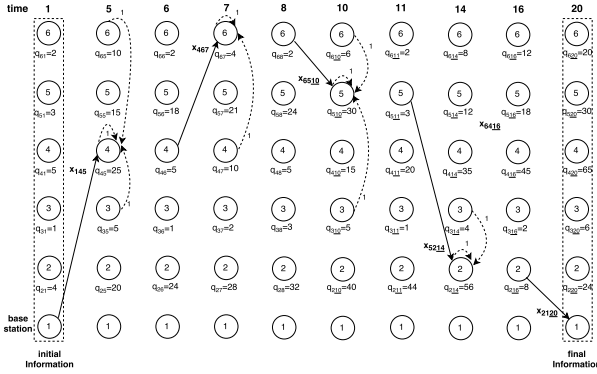


Figure 4: Solution obtained for the instance depicted in Figure 1 by the MILP formulation with objective function FO3 defined in (22).

total amount of information generated during the whole time horizon at each station, respectively from station 2 to 6, is 80, 20, 100, 60, 40. For the solution depicted in Figure 2, the remaining information at each station is, respectively, 60, 15, 25, 33, 10 which means, the total amount of information collected from each station is, respectively, 20, 5, 75, 26, 30. For this solution, the value of the three different objective functions are: 143 for **FO1**, 1575 for **FO2** and 51 for **FO3**.

The vehicle routing depicted in Figure 3 chooses for not visiting stations 2 and 5; wireless transmissions are used to collect information from these stations. For this solution, the value of the three different objective functions are: 162 for **FO1**, 1536 for **FO2** and 48 for **FO3**. The vehicle routing depicted in Figure 4 chooses for visiting each station in the network. This improves the satisfaction over the whole set of stations though it increases the amount of remaining information in the network. For this solution, the value of the three different objective functions are: 145 for **FO1**, 1598 for **FO2** and 45 for **FO3**.

4 COMPUTATIONAL EXPERIMENTS

In this section, we report computational experiments carried out with the formulation presented in Section 3 comparing the solutions obtained with the different objective functions proposed. The MILP problems were solved by IBM CPLEX Optimizer 12.6.1.0 (using 16 threads) on a server with a 16 processor Intel[®] Xeon[®] Processor E5640 12M Cache, 2.67 GHz and 32 GB of RAM memory. The CPU time limit was set to 1h for all instances. Before presenting the obtained results, we briefly describe the set of instances used in our experiments.

Instances We evaluate the MILP formulation on a set of 80 instances. Each instance is defined by a random graph $D = (V, A)$ and a given value \bar{T} . A set of 40 random graphs was generated as follows.

Let n and $\bar{\Delta}$ be, respectively, the total number of vertices and a border on the density of the graph to be generated. On a square of length B , the base station is located in the bottom-left vertex and the other $n - 1$ stations are placed randomly on the square of length $B - 2$ in the upper-right (as shown in Figure 5). We define A as the adjacency matrix associated with the complete graph defined by the n stations. Let $\Delta(D)$ denote the density of the graph $D = (V, A)$. We randomly select an arc $(i, j) \in A$ such that $D = (V, A \setminus (i, j))$ is a connected graph. We define $A = A \setminus (i, j)$. In case $\bar{\Delta} - \Delta(D) > 0.01$ we proceed with the random elimination of arcs; otherwise we stop and return the graph $D = (V, A)$. Finally, a rate r_j is randomly generated for each $j \in V \setminus \{1\}$. The distance matrix \mathbf{d} is defined by using the euclidean distance between each pair of vertices located in the square of side B . The time matrix \mathbf{t} is defined according to the adjacent matrix considering a vehicle of speed equal to 1 which means, $t_{ij} = d_{ij}$ if $(i, j) \in A$, $t_{ij} = 0$ otherwise. We generate graphs with number of vertices $n \in \{6, 8, 10, 12\}$ and upper bounds on the graph density $\bar{\Delta} \in \{0.5, 0.7\}$. For each combination of n and $\bar{\Delta}$, five random graphs are generated according to the procedure just described. In our experiments we considered the set of graphs just described together with $\bar{t} \in \{24, 28\}$.

Objective functions comparison Tables 2 and 3 exhibit the results obtained with the three objective functions on the set of 80 random instances. In both tables, the first multicolumn exhibits information about the instances: $|V|$ is the number of stations, d is the graph density, \bar{T} is the size of the time horizon considered. The fourth column informs

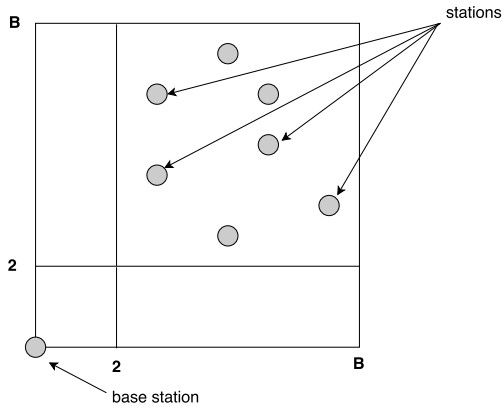


Figure 5: Schema used for random graph generation: the base station is located in the bottom-left vertex; the other stations are randomly placed in the square located in the upper-right.

the total amount of information generated by all stations in the network during the whole time horizon. The next five multicolumns display information about the solutions obtained with each different objective function: “*CpuTime*” is the time, in seconds, spent to solve the instance to optimality (“–” means the instance was not solved in the time limit); “*Gap*” is the MILP gap calculated between the best integer solution found and the final lower bound (a $Gap = 0$ means the solution was solved to optimality in the time limit); “*Inf.collected*” display the total amount of information collected by the vehicle during the whole time horizon; “*Av.Vehicle*” display the average amount of information in the vehicle over time; “*Av.Satisfaction*” display the average satisfaction over the set of stations.

Table 2 presents results obtained on instances with 6 and 8 stations and show, as we expected, the sensibility of the ILP formulation to the total number of stations in the network. In general, instances with more than eight stations cannot be solved in one hour of computation. Looking the optimal solutions obtained, data transmissions took no more than three time units. Thus, we solved a set of instances with 8, 10 and 12 stations by limiting data transmissions to four time units which reduces the number of binary variables in the formulation: variables w_{ijkl} are defined for $l \leq 4$. Table 3 presents the results obtained on instances with 8, 10 and 12 stations.

From results on these two tables, we conclude that the WT-VRP problem becomes more difficult as the number of stations and the time horizon increase. From Table 3, a total of 2 and 17 MILP problems were not solved to optimality, respectively, for $T = 24$ and $T = 28$ with gaps arriving to 30% when $T = 28$. From Table 2, a total of 7 and 29 MILP problems

were not solved to optimality, respectively, for $T = 24$ and $T = 28$ with gaps arriving to 69% when $T = 28$. The graphics in Figure 6 display the results obtained on the 47 instances solved to optimality by all three objective functions. In each graphic, the instances are ordered in increasing order of the average of the time spent to solve the three ILP problems. The three graphics compare the optimal solutions obtained according to the value of “*Inf.Collected*”, “*Av.Vehicle*” and “*Av.Satisfaction*”. We can observe that, in fact, the WT-VRP problem became more difficult as the total amount of information collected from the network increases.

From our results, we conclude the MILP formulation defined with objective function **FO2** is computationally easier: less instances not solved to optimality and, in average, less time spending and smaller gaps. From both tables, the total number of MILP not solved to optimality in the time limit is equal to 22, 11 and 22, respectively, for objective functions **FO1**, **FO2** and **FO3**. We can also observe from Figure 6 that, on one hand, the solutions maximizing the average satisfaction of the network impacts almost equally the average amount of information in the vehicle and the total amount of information collected. On the other hand, the solutions maximizing the average amount of information in the vehicle over time, impacts slightly more the average satisfaction of the network than the solutions maximizing the total amount of information collected.

Periodicity We compare now the optimal solutions obtained with each objective function with respect to the information left in the network at the end of a time period T . Let $C^0 = [C_0^0, C_1^0, \dots, C_n^0]$ be a vector with the amount of information stored in each station $i \in V$. By considering the vector C^0 as the initial conditions in the MILP formulation, we obtain a routing for the vehicle, a collection planning and the amount of remaining information in each station, denoted here as $C^1 = [C_0^1, C_1^1, \dots, C_n^1]$. Thus, the MILP formulation associated with an instance can be seen as a function $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ such that $F(C^0) = C^1$. In order to study the dynamics of function F , let us define $C^m = F^m(C^0)$ where $F^k(C) = F(F^{k-1}(C))$. We experimentally investigate the existence of values $k, \tau \in \mathbb{N}$ such that $F^\tau(C^k) = C^k$ starting with the initial conditions $C^0 = \mathbf{0}$. When such values exist, we say that the function F is periodic.

For this experiment, we use all the instances solved in less than 300s by our MILP formulation (with all three different objective functions). Consider an instance of the problem and a MILP formulation, i.e. a function $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$. Starting with $C^0 = \mathbf{0}$,

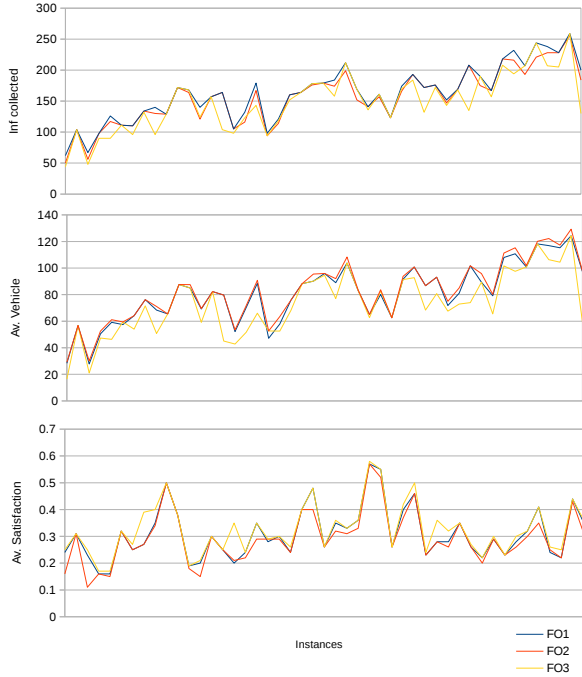


Figure 6: Results obtained on the set of 47 instances solved to optimality with all objective functions.

we solve this instance and obtain $F(C^0) = C^1$. Then, we continue to iterate until we discover that the function is periodic or we have solved the MILP formulation 30 times.

Table 1 displays the results obtained. The first multicolumn exhibits information about the instances used in this experiment as defined for the previous tables. The other three multicolumns inform us the results obtained for each objective function: the values obtained for k , τ and the number of isolated stations #isol. An isolated station i is a station such that, in the successive applications of function F , we arrive to a given \bar{k} , such that $C_i^{k'} > C_i^{\bar{k}}$, for each $k' > \bar{k}$. That means, the amount of information accumulated at station i at iteration \bar{k} arrives to a value that the optimal solution of the MILP formulation (due to the size of the time period \bar{T} and the assumption that once a transmission starts all the information in the station must be transmitted) is not able to reduce it. Thus, station i became isolated from outside of the network. The entry “–” for k and τ means the solution became periodic for a subset of stations but with the presence of isolated stations. Likewise, the entry “*” for k and τ means after the limit of 30 iterations no periodicity was achieved. From the results in Table 1, we can conclude that the formulation maximizing the average satisfaction of stations is more suitable to achieve periodicity and with smaller values of k .

Table 1: Results obtained when the periodicity of the solutions obtained by the ILP formulations is studied.

V	d	\bar{T}	FO1			FO2			FO3		
			k	τ	#isol	k	τ	#isol	k	τ	#isol
6	0.50	28	–	–	1	–	–	1	–	–	1
			–	–	1	–	–	1	–	–	1
			7	5	0	7	2	0	5	3	0
			–	–	1	–	–	1	–	–	1
6	0.50	30	4	2	0	3	2	0	3	1	0
			2	3	0	3	3	0	2	3	0
			–	–	1	–	–	1	–	–	1
			2	1	0	5	1	0	2	5	0
8	0.53	32	6	4	0	–	–	1	6	3	0
			4	1	0	3	2	0	5	1	0
			–	–	1	–	–	1	–	–	1
			*	*	0	*	*	0	5	3	1
			*	*	0	*	*	0	*	*	0
			3	2	0	7	4	0	5	2	0
			12	4	0	*	*	0	1	2	0

5 CONCLUSION

In this paper, we have introduced the WT-VRP, a version of the VRP defined on wireless networks where information can be delivered without physically visiting a node in the network. A MILP formulation was provided to model the WT-VRP. We discussed three different criteria to measure the efficiency of a solution which results in three different objective functions for the MILP formulation. Computational experiments were conducted on random instances. We conclude that the WT-VRP becomes more difficult as the total amount of information collected increases and, as we could expect, as the size of the time period increases. Our MILP formulation is computationally easier to solve when the average (over time) of the amount of information collected is maximized (FO2). However, we saw that the optimal solutions obtained with this criteria impacts the average satisfaction of the network (FO3). We have also studied the periodicity of the solutions obtained by each different efficiency criteria. In average, the solutions obtained with FO3 are more suitable to achieve periodicity. The results obtained in this work are important to access the hardness of the WT-VRP. In the future, it is important to strength the MILP formulation proposed here as well as to study heuristic approaches in order to be able to solve larger instances. Other characteristics could be considered in the future as the existence of multiple vehicles and the possibility of sending only part of the information located in a station. Another direction to continue this research in to enrich the version of the problem by considering a limit on the maximum traveling distance between consecutive visits to the base station. This limitation is imposed by the autonomy of the vehicle used in

applications described in (Vieira et al., 2015; Malowidzki et al., 2016).

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Table 2: Results obtained on the set of instances with $n = 6, 8$ and $T = 24, 28$.

V	d	\bar{T}	$\sum r_j T$	Cpu Time			Gap			Inf. collected			Av. Vehicle			Av. Satisfaction		
				FO1	FO2	FO3	FO1	FO2	FO3	FO1	FO2	FO3	FO1	FO2	FO3	FO1	FO2	FO3
6	0,5	24	312	1,64	1,77	1,84	0	0	0	62	50	44	28,33	29,16	16,41	0,24	0,16	0,25
			264	3,21	6,8	3,26	0	0	0	129	129	129	65,45	65,45	65,45	0,5	0,5	0,5
			408	2,83	4,14	2,89	0	0	0	111	111	111	57,45	59,45	52,58	0,32	0,32	0,32
			432	26,75	9,37	5,87	0	0	0	98	94	94	47,08	52,58	52,58	0,28	0,29	0,29
			312	3,39	3,91	3,9	0	0	0	140	130	96	68,33	71,08	50,75	0,35	0,34	0,4
			364	4,01	3,6	3,5	0	0	0	134	134	132	76,21	76,21	71,5	0,27	0,27	0,39
	0,72	24	308	34	67,48	49,51	0	0	0	161	157	161	80,14	83,64	82,71	0,55	0,52	0,55
			476	78,42	55,88	76,95	0	0	0	174	166	170	91,57	93,78	91,35	0,4	0,37	0,42
			504	187,75	77,63	125,07	0	0	0	176	176	172	93,14	93,14	80,71	0,28	0,28	0,36
			364	59,65	72,37	80,29	0	0	0	193	193	184	100,78	100,78	92,71	0,46	0,46	0,5
			384	3,55	3,18	3,49	0	0	0	110	110	96	64,16	64,16	54	0,25	0,25	0,27
			408	2,79	2,45	2,59	0	0	0	104	104	104	56,66	56,66	56,66	0,31	0,31	0,31
8	0,53	24	392	2,94	2,33	2,6	0	0	0	67	56	48	27,79	30	21	0,23	0,11	0,25
			240	57,61	19,91	68,77	0	0	0	141	141	136	65	65	62,7	0,57	0,57	0,58
			336	20,34	14,48	40,29	0	0	0	178	176	178	90,16	95,5	90,16	0,48	0,4	0,48
			448	51,46	10,92	11,23	0	0	0	164	164	164	88,28	88,28	88,28	0,4	0,4	0,4
			476	6,78	7,69	6,46	0	0	0	172	172	172	87,57	87,57	87,57	0,38	0,38	0,38
			364	13,22	11,14	11,8	0	0	0	105	105	98	52,17	53,46	42,78	0,2	0,21	0,35
	0,71	24	280	—	898	—	0	0	0	186	186	174	89,89	89,89	78,53	0,67	0,67	0,64
			392	2988,72	332,88	—	0	0	22,95	245	236	239	123,89	127,96	120,89	0,56	0,48	0,56
			672	186,79	80,30	27,80	4,25	0	19,97	142,5	139,5	135,1	72,70	74,19	68,31	0,39	0,36	0,41
			480	7,97	7,48	10,26	0	0	0	144	144	142	85,16	85,16	77,16	0,19	0,19	0,22
			528	28,63	21	60,62	0	0	0	157	157	142	82,29	82,29	70,79	0,3	0,3	0,3
			456	296,01	83,89	202,91	0	0	0	184	174	158	89	92	77,08	0,35	0,32	0,36
8	0,71	24	480	5,68	5,31	4,92	0	0	0	120	120	117	70	70	64,12	0,17	0,17	0,23
			784	32,85	20,98	27,27	0	0	0	179	167	179	88,54	90,79	88,54	0,35	0,29	0,35
			560	177,97	177,79	181,17	0	0	0	220	212	196	119	121,85	104,25	0,28	0,24	0,29
			616	1667,62	422,09	532,58	0	0	0	244	221	244	118,14	120,1	118,14	0,41	0,35	0,41
			532	139,86	85,39	305,35	23,42	15,27	13,39	236	236	212	118,67	118,67	105	0,45	0,45	0,42
			560	—	1160,55	2225,25	30,89	0	0	259	199	212	103,78	108,25	103,78	0,33	0,31	0,33
	0,71	24	528	—	1102,72	—	14,03	0	220	245	208	97,5	114,41	105,91	0,35	0,41	0,41	
			552	9,4	17,21	9,58	0	0	0	132	116	123	70,25	71,66	51,62	0,24	0,22	0,24
			504	44,75	42,79	50,36	0	0	0	169	152	169	83,04	83,33	83,04	0,36	0,33	0,36
			408	11,51	11,54	24,65	0	0	0	121	114	118	57,91	63,5	52,5	0,3	0,29	0,3
			480	137,78	91,97	339,78	0	0	0	169	169	168	81,04	85,04	72,91	0,35	0,35	0,35
			616	—	—	—	17,11	32,65	29,77	300	294	254	123	146,89	123,85	0,43	0,4	0,43
8	28	644	—	933	—	—	16,36	224	224	224	114,85	114,85	114,85	0,32	0,32	0,32		
		588	—	1889,39	—	14,13	0	213	199	213	105,89	108,5	105,89	0,39	0,33	0,39		
		476	—	602,05	1458,9	26,25	0	0	166	161	166	72,07	86,89	73,57	0,37	0,36	0,37	
		560	—	3124,41	—	17,47	0	217	205	215	98,96	107,32	93,17	0,39	0,36	0,38		
		551,20	213,34	544,42	388,11	20,40	23,96	18,11	194,3	186,15	185,95	95,17	100,09	90,52	0,34	0,32	0,35	

Table 3: Results obtained on the set of instances with $n = 8, 10, 12$ and $T = 24, 28$.

V	d	\bar{T}	$\sum r_j T$	Cpu Time			Gap			Inf. collected			Av. Vehicle			Av. Satisfaction		
				FO1	FO2	FO3	FO1	FO2	FO3	FO1	FO2	FO3	FO1	FO2	FO3	FO1	FO2	FO3
8	0,53	24	672	3,4	3,08	0	0	0	126	117	90	59,16	61,12	46,25	0,16	0,15	0,17	
			480	4,44	13,96	0	0	0	157	157	157	82,29	82,29	82,29	0,3	0,3	0,3	
			528	13,49	41,53	0	0	0	184	174	158	89	92	77,08	0,35	0,32	0,36	
			456	3,12	2,69	0	0	0	98	98	90	50,45	52,5	47,25	0,16	0,16	0,17	
			480	11,01	18,89	0	0	0	179	167	143	88,54	90,79	66,04	0,35	0,29	0,35	
			784	26,68	74	0	0	0	179	179	179	95,89	95,89	94,6	0,26	0,26	0,26	
			560	532,59	136,04	1128,2	0	0	244	221	244	118,14	120,1	118,14	0,41	0,35	0,41	
			616	—	432,9	—	8,33	0	17,09	232	232	210	120,57	120,57	102,28	0,38	0,38	0,4
			532	51,35	10,93	55,54	0	0	212	199	212	103,78	108,25	103,78	0,33	0,31	0,33	
			560	2233,91	170,94	1300,59	0	0	259	256	259	124,32	129,28	124,32	0,44	0,43	0,44	
10	0,52	24	566,8	325,80	79,93	293,16	8,33	0	17,09	187	180	174,20	93,21	95,28	86,20	0,31	0,30	0,32
			576	6,49	8,02	10,07	0	0	140	121	124	69,16	69,54	59,16	0,2	0,15	0,21	
			576	5,44	7,39	19,5	0	0	164	164	104	79,66	79,66	45,04	0,25	0,25	0,25	
			480	31,04	21,45	103,6	0	0	123	123	123	62,54	62,54	62,54	0,26	0,26	0,26	
			576	113,72	69,44	265,94	0	0	152	147	143	71,83	75	67,54	0,28	0,26	0,32	
			720	5,11	7,67	11,46	0	0	168	164	168	85	87,5	85	0,19	0,18	0,19	
			672	268,62	194,93	499,66	0	0	208	208	135	101,71	101,71	73,96	0,26	0,26	0,27	
			672	499,44	342,44	532,86	0	0	232	216	194	110,71	115,28	97,5	0,28	0,26	0,3	
			560	1607,21	766,31	—	0	0	220	218	220	107,28	111,28	107,28	0,38	0,36	0,38	
			672	3103,83	462,65	—	0	0	224	224	212	117	117	95,42	0,32	0,32	0,39	
10	0,66	24	840	1210,58	348,76	1393,03	0	0	228	228	205	115,28	117,28	104,39	0,22	0,22	0,25	
			696	414,41	147,61	841,84	0	0	207	193	207	100,75	101,87	100,75	0,32	0,3	0,32	
			768	15,88	19,67	28,5	0	0	160	160	152	75,83	75,83	68,16	0,24	0,24	0,26	
			648	596,51	207,01	285,6	0	0	172	172	132	86,66	86,66	68,33	0,23	0,23	0,24	
			696	82,49	32,37	186,65	0	0	200	184	130	97,33	97,83	59,58	0,35	0,31	0,36	
			576	1550,84	3583,52	1640,02	0	0	257	251	268	129,82	135,1	127,42	0,34	0,33	0,38	
			812	—	—	—	41,54	31,11	23,07	257	257	230	130,82	130,82	117,46	0,28	0,28	0,31
			896	—	651,33	2041,84	3,8	0	257	257	230	130,82	130,82	117,46	0,28	0,28	0,31	
			756	—	—	—	42,37	25,06	52,99	218	210	219	93,42	104,46	90,03	0,3	0,31	0,32
			812	—	2904,71	—	42,98	0	46,16	270	270	237	136,42	136,42	120,67	0,29	0,29	0,34
12	0,51	24	672	—	—	—	52,96	61,34	47,37	234	248	195	113,92	118,57	95,96	0,34	0,34	0,42
			683,8	634,11	575,02	561,47	36,73	39,17	36,02	200,05	196,25	177,75	98,21	100,23	85,58	0,28	0,27	0,30
			840	476,05	178,77	407,48	0	0	190	175	190	89,33	95,79	89,33	0,22	0,2	0,22	
			648	488,24	551,27	1742,87	0	0	238	228	207	116,83	122,25	106,29	0,24	0,25	0,26	
			960	266,89	189,44	738,49	0	0	218	218	208	107,91	111,25	101,66	0,23	0,23	0,23	
			936	—	3339,79	—	47,44	0	6,04	208	242	228	102,91	116,91	102,41	0,28	0,32	0,31
			744	—	—	—	54,07	32,36	24,69	235	245	269	108,2	126,41	116,2	0,32	0,34	0,38
			980	—	—	—	62,1	57,17	52,24	231	221	217	103	116,96	95,46	0,19	0,21	0,26
			756	—	—	—	51,36	58,88	57,01	275	274	240	131,6	140,25	108,14	0,24	0,24	0,25
			1120	—	—	—	47,3	19,98	56,54	323	310	263	159,32	159,14	127,21	0,26	0,25	0,25
1092	—	—	—	63,54	65,23	62,81	290	283	274	136,5	150,57	105,21	0,33	0,24	0,31			
868	—	—	—	64,99	69,61	59,71	276	302	296	124,32	147,96	126,21	0,33	0,35	0,38			
894,4	410,39	1064,82	962,95	55,83	50,54	45,58	248,40	249,80	239,20	117,99	128,75	107,81	0,26	0,26	0,29			