

Tabletop Testbed for Attitude Determination and Control of Nanosatellites

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24 is also equipped with a Helmholtz cage whose purpose is to recreate the Earth magnetic field
25 conditions that spacecrafts encounter in orbit. The design and realization of this low-cost testbed
26 is presented in this paper. A simple and efficient automated balancing algorithm based on the
27 Least Squares Method (LSM) is proposed and validated by experiments. The performance of the
28 proposed simulator is evaluated and compared with previous works.

29 **INTRODUCTION**

30 In view of the great complexity and high budgets which usually concern spacecraft projects, it
31 is highly desirable to perform tests on ground-based platforms to reduce the implicated risks. The
32 effectiveness of taking tests on those platforms is closely related to their capacity to simulate the
33 peculiarities of the space environment. For instance, the absence of atmosphere, the presence of
34 microgravity and the magnetic field of Earth are characteristics that directly affect the design of
35 spacecrafts attitude determination and control systems.

36 Since the beginning of the space race, air bearing based platforms have been used as testbeds
37 for simulating spacecraft attitude motion. Depending on the number of degrees of freedom (DOF)
38 provided, these platforms can be classified as planar, rotational or combinational. Planar sys-
39 tems provide two translational degrees of freedom and, occasionally, a rotational degree of free-
40 dom (Schwartz et al. 2003).

41 The focus in this work is on the rotational systems, which aim to provide a frictionless rotational
42 movement with three degrees of freedom. The inherent difficulty of this type of platform is to achieve
43 this rotational freedom, leading the platform to some common build standards, such as those named
44 tabletop, umbrella and dumbbell (Schwartz et al. 2003). Since the tabletop design is used in this
45 work, special attention is given to this configuration, which is depicted in Fig. 1. In this type of
46 platform, the table is mounted directly on the air bearing. Although the rotational movement is
47 constrained by the mounting plate and the hemisphere design, this is the most common design
48 between the rotational systems as it is easier to balance when compared with umbrella and dumbbell
49 systems. Examples of tabletop designs are shown in Kim and Agrawal (2006) and Saulnier et al.
50 (2013).

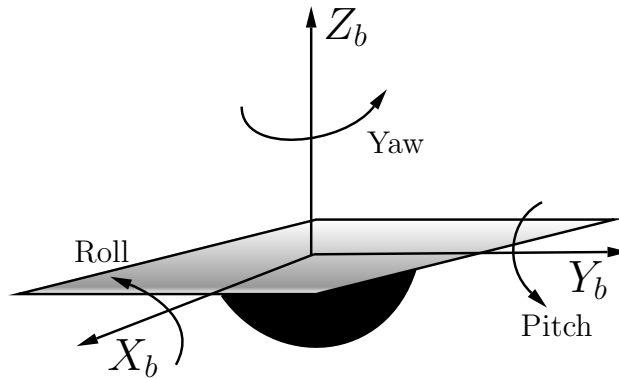


Fig. 1. Attitude angles in tabletop configuration.

51 Combinational systems are those which combine the features of both planar and rotational
 52 systems. For this reason, these platforms often provide 5 to 6 degrees of freedom. An example
 53 of combinational system is given in Gallardo and Bevilacqua (2011), which is a dynamic 6 DOF
 54 simulator. This platform is composed of two stages, one responsible for rotational motion and the
 55 other responsible for translational motion in a approximately $18m^2$ epoxy floor.

56 Common to all of these types of platform is the need of an efficient balancing procedure. The
 57 purpose of this balancing is to reduce the gravitational torque experienced by the platform. In order
 58 to accomplish this, the center of mass (CM) of the platform must be placed as close as possible
 59 to its center of rotation (CR), *i.e.* the unbalance vector magnitude must be as close as possible
 60 to zero. In Mittelsteadt and Mehiel (2007), it is reported the importance of distributing the masses
 61 as symmetrically as possible and it is reserved space in the initial project for implementation of
 62 an automatic mass balancing system. This problem is often solved manually, as shown in Romano
 63 and Agrawal (2003) and Peck et al. (2003), in which a minimum gravitational torque of $0.01 N \cdot m$,
 64 approximately, was achieved. In Carrara and Milani (2007), the need of balancing the system
 65 is mentioned, as well as the adopted procedure for accomplishing it manually. There are also
 66 numerical algorithms that search for the optimal placement for each equipment to be embedded in the
 67 platform (Xu et al. 2016). Another recent work addresses the necessity of implementing a balancing
 68 procedure (Carletta and Teofilatto 2017). In Thomas et al. (2018), the intent of implementing a
 69 CubeSat simulator, similar to the one described in this work, is presented. Moreover, the same

70 balancing method described in this work is set as the start point for solving the balancing problem
71 in Thomas et al. (2018), showing that this approach is being addressed nowadays in other facilities
72 over the world.

73 Manual balancing procedures may take hours to get appropriate results. For this reason, other
74 algorithms are based on automated processes, such as the algorithm presented in Kim and Agrawal
75 (2009), which is an adaptive control scheme developed using Lyapunov theory. Other studies on
76 adaptive control use, additionally, the Unscented Kalman Filter for tuning the vertical component
77 of the unbalance vector (Chesi et al. 2013).

78 In this work, aiming to provide a cost-effective solution for the balancing problem, the simple
79 and efficient well-known Least Squares Method (LSM) will be adapted and used to provide batch
80 estimations of the unbalance vector of the platform (Silva et al. 2016).

81 This paper presents an Attitude Determination and Control Systems (ADCS) testbed composed
82 of an air bearing table and a Helmholtz cage, being developed at the LAICA. This platform aims at
83 simulating two key conditions present in the in-orbit environment: the magnetic field of the Earth
84 and the frictionless conditions of rotations in space.

85 The air bearing table is installed inside the Helmholtz cage, a device used to induce a magnetic
86 field around the structure of the cage. In particular, inside the cage the induced magnetic field can
87 be adjusted in order to recreate the Earth magnetic field conditions that spacecrafts encounters in
88 orbit.

89 This paper is an extension of the work presented in Silva et al. (2016). More experiments were
90 run and further details were studied. This article is divided as follows. The second section shows a
91 description of the air bearing platform developed for testing nanosatellites, including an overview
92 of its physical parts and the hardware/software architecture. A description of the assembly aspects
93 involved with the project of the Helmholtz cage is also presented. The third section provides an
94 explanation of the balancing algorithm used and its theoretical foundations. The fourth section
95 shows some tests made to evaluate the performance of the algorithm used to make the air bearing
96 table balancing. Also, a comparative analysis with other balancing methods found in the literature

97 is made. Conclusions are given in the fifth section.

98 **SYSTEM CONFIGURATION**

99 This section describes the components of the proposed testbed in two separated subsections.
100 The first subsection addresses the constructive aspects of the hardware and the organization of the
101 software of the air bearing table. The second subsection presents the Helmholtz cage principle with
102 a description of its structure. The capability of magnetic field generation of the Helmholtz cage is
103 illustrated with a set of measurements.

104 **The air bearing table**

105 The air bearing table was conceived for testing attitude determination and control algorithms for
106 nanosatellites. The air bearing table developed at LAICA is an air bearing platform in the tabletop
107 configuration, as it is shown in Fig. 2 (Schwartz et al. 2003). In other words, the table is mounted
108 directly on the semisphere of the air bearing set. One major disadvantage of this configuration is the
109 limitation in the excursion of the roll and pitch angles, which will not exceed $\pm 45^\circ$. Nevertheless,
110 this excursion is sufficient for all the tests that will be carried and, as will be seen in the section
111 “BALANCING TECHNIQUES”, full range is not required for the balancing algorithm to provide
112 a consistent estimation of the unbalance vector.

113 The Movable Mass Units (MMU), Fig. 3, are responsible for adjusting the position of the
114 center of mass and have two degrees of freedom, even though only one is used in each of the three
115 MMUs. The two degrees of freedom of this device are accessible via a crank. In order to make
116 this movement automatic and controllable by the electronic system, a motor is mounted in place of
117 this crank for each MMU.

118 The electronic system that is embedded in the table contains:

- 119 1. Microcontroller: a complete USB-based microcontroller development system implemented
120 on the ATMEGA8 microcontroller is used. This platform, which is compatible with Arduino
121 software and libraries, controls all the electronic components embedded in the balancing
122 system of the air bearing table.

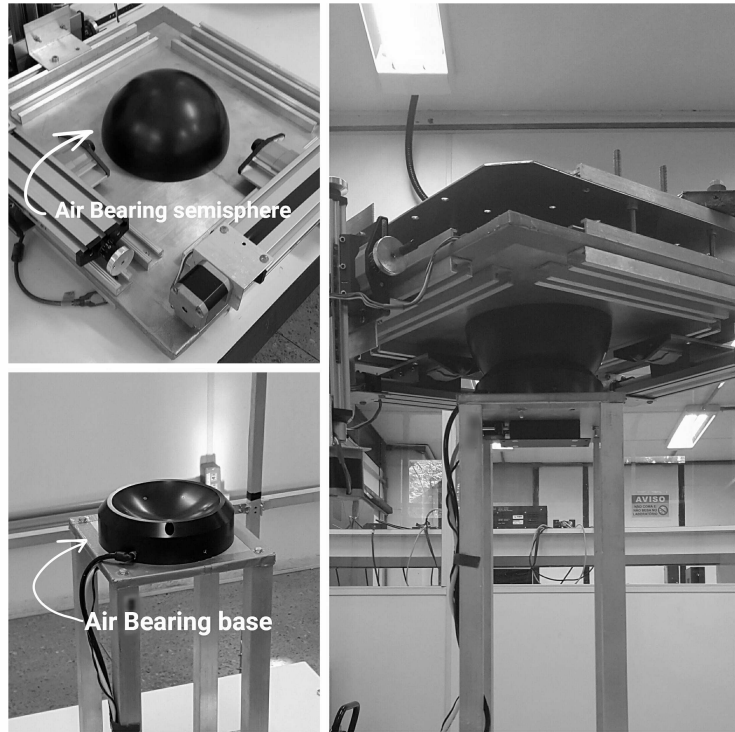


Fig. 2. The air bearing assembly.

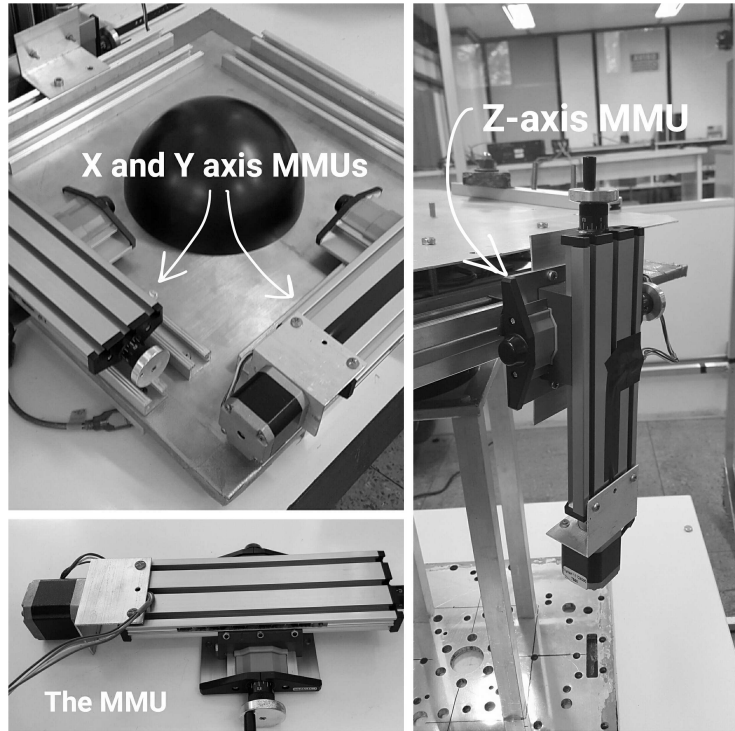


Fig. 3. The Movable Mass Units (MMUs).

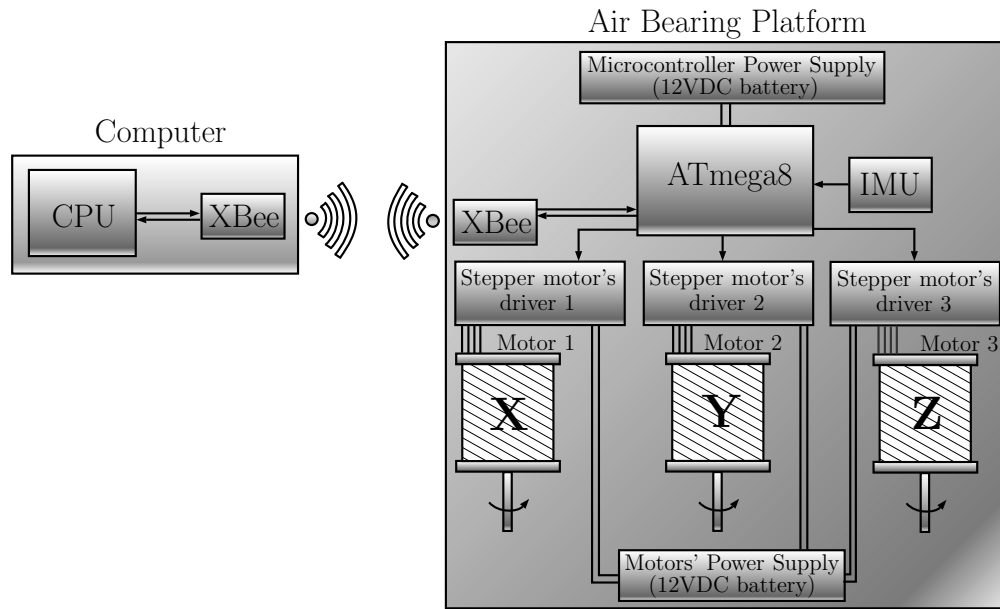


Fig. 4. Electronic components of the system.

- 123 2. Communication module: a XBee radio is used to make wireless communication with a
 124 computer that processes all the dynamic data collected.
- 125 3. Inertial Measurement Unit (IMU): an IMU with 9 Degrees of Freedom (DOF) is usedis used,
 126 specifically a magnetometer with 3 DOF, an accelerometer with 3 DOF and a gyroscope with
 127 3 DOF.
- 128 4. Motor drivers: there are 3 driver boards used to control each of the 3 motors mounted on the
 129 table.
- 130 5. Stepper motors: there are 3 motors mounted on the table. They make possible the translational
 131 movement of masses in three non-redundant degrees of freedom.
- 132 6. Batteries: two lithium polymer batteries power the system.

133 A schematic of the electronic system components is shown in Fig. 4.

134 **The Helmholtz cage**

135 For control schemes based on magnetorquers, there must be a way to control the magnetic field
 136 of the test environment. To provide this capability, a Helmholtz cage was built.

137 The Helmholtz cage consists of a set of coils in which electric current runs in order to gen-
138 erate a magnetic field, as predicted by the Biot-Savart law of electromagnetics. By controlling
139 the intensity and direction of this magnetic field, it is possible to simulate the orbital magnetic
140 environment (Brewer 2012).

141 To this end, the design has six square coils, two for each axis of the cage, used to generate an
142 homogeneous field according to the applied electric current. The magnitude of the generated field
143 is given, in each of the axes of the cage, by the following equation

$$144 \quad B = \frac{2\mu_0 Ni}{\pi a} \cdot \frac{2}{(1 + \gamma^2)\sqrt{2 + \gamma^2}} \quad (1)$$

145 where B is the generated field, μ_0 is the permeability of the environment, N is the number of wire
146 turns in the coil, i is the applied current, a is half the side of the coil and γ , the relation between the
147 distance within two coils in a pair and the side of a coil, is 0.5445. Further details on how Eq. (1) is
148 achieved may be found in Batista et al. (2017), which also clarifies the definition of γ , an optimal
149 construction parameter of the cage.

150 For the manufacturing of this equipment, it was decided to use “U” aluminum profiles, since
151 the material used cannot possess magnetic characteristics, with dimensions of 1 inch base, 1 inch
152 side and 3/32 inch thick ($1 \times 1 \times \frac{3}{32}$). The bars are attached using triangular aluminum side supports
153 and M5 stainless steel screws in order to build 2.5 meters side squares. The structure is covered
154 with enamelled copper wire, by means of constituting the coil.

155 Once the structure is assembled, the coils are connected to a direct electrical current supply
156 (DC), responsible for feeding the system and generating the magnetic field. The current supply is
157 automatically controlled through a software compatible with MATLAB that interprets the readings
158 from magnetometers mounted in the air bearing table and calculate the current to be applied in order
159 both to compensate the local magnetic field and establish the conditions suitable for the simulation
160 of the orbital field needed. Fig. 5 shows the air bearing platform surrounded by the Helmholtz cage.

161 Measurements taken with the maximum supplying current of 6A allowed to conclude that the

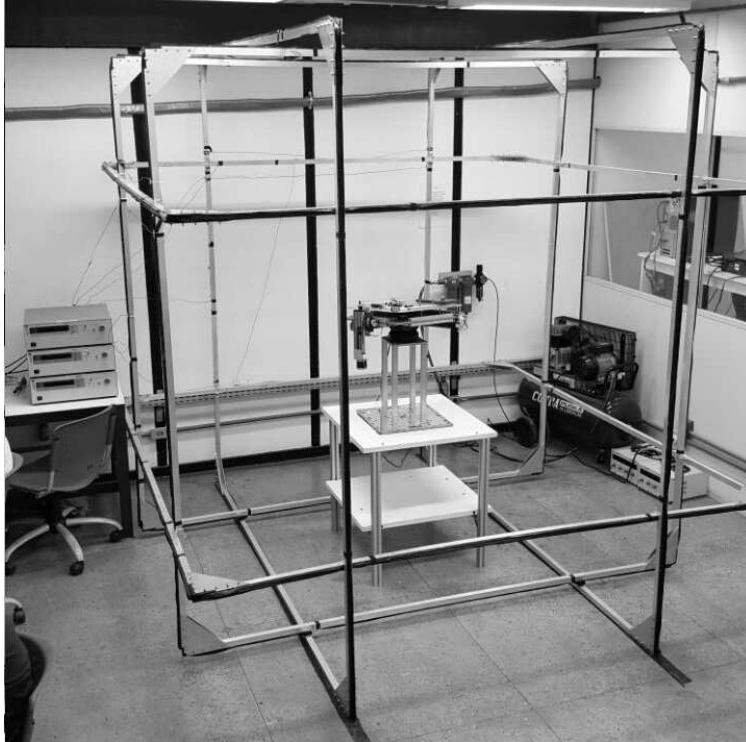


Fig. 5. The Helmholtz cage (image by authors).

162 cage is capable of generating approximately $180 \mu T$ in each of its axes, as can be seen in Fig. 6. It is
163 possible to see the magnetic field in the laboratory environment with the Helmholtz cage turned off
164 (initial portion of the graph) and turned on (final portion) in each axis. As can be seen in Fig. 6, the
165 environmental magnetic field is $-26 \mu T$, $1.3 \mu T$ and $18.05 \mu T$ in the X, Y and Z axes of the cage,
166 respectively. In other words, the cage is capable of nullifying the environmental magnetic field and
167 still provide around $150 \mu T$ generation capability in each axis, which is sufficient for simulating
168 most kinds of orbits. Another important aspect of the Helmholtz cage is the homogeneity of the
169 magnetic field in its interior. The desired behaviour of the generated magnetic field in a specific
170 instant is that it must remain constant, in direction and magnitude, in a volume which must cover the
171 air bearing table entirely. In de Loiola et al. (2018), different tests were made in order to quantify
172 the homogeneity of the field generated by the Helmholtz cage described in this work, reaching the
173 conclusion that the magnetic field remains constant, given some variation tolerance, along 100 cm
174 of each of the cage axes. Given that the air bearing table, when rotating, occupies a volume of

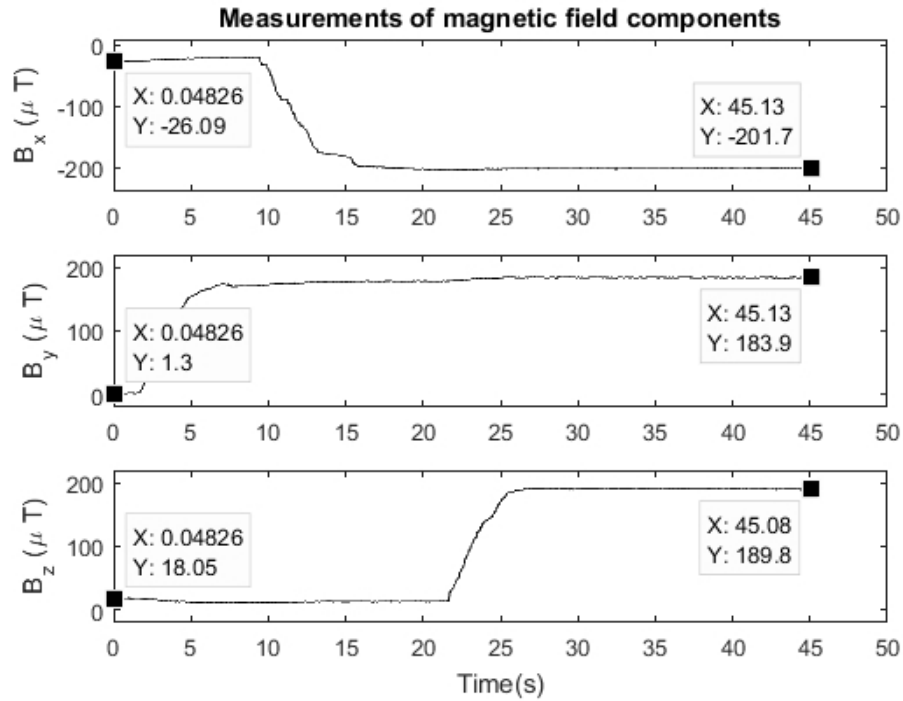


Fig. 6. Measurements of the environmental magnetic field taken at the center of the cage.

175 $44\text{ cm} \times 44\text{ cm} \times 44\text{ cm}$, the homogeneity requisite is guaranteed.

176 **BALANCING TECHNIQUES**

177 **Reference systems**

178 Two reference systems are established for the air bearing platform:

- 179 1. Inertial: the inertial frame, defined by the axes (X_i, Y_i, Z_i) is static and fixed in relation with
180 the laboratory. Its origin is located at the CR of the air bearing.
- 181 2. Body: the body frame, defined by the axes (X_b, Y_b, Z_b) is fixed in relation with the air bearing
182 table and moves with it. Its origin is coincident with the origin of the inertial frame.

183 Fig. 7 illustrates the relative position between the inertial and body frames when the table
184 performs a roll movement.

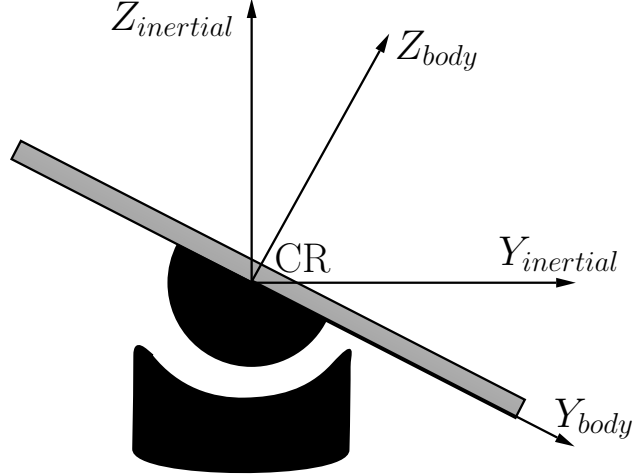


Fig. 7. Inertial and body frames during roll movement.

Platform dynamics

The platform can be modeled as a rigid body performing rotational movement and having its center of rotation fixed in both reference frames. For this system, the angular momentum taken at the center of rotation is given by Eq. (2) (Greenwood 1988),

$$\mathbf{H}_{CR} = \mathbf{r} \times M\mathbf{v}_G + \mathbf{H}_G, \quad (2)$$

where \mathbf{H}_G is the angular momentum taken at the center of mass of the platform, M is the total mass of the system, \mathbf{v}_G is the velocity vector at the center of mass and \mathbf{r} is the CM offset, which is a vector starting from the CR and pointing to the CM.

The system dynamics is obtained from the Newton second law, resulting that the torque applied to the system is equal to the time derivative of its angular momentum, that is,

$$\boldsymbol{\tau}_{CR} = \frac{d\mathbf{H}_{CR}}{dt}, \quad (3)$$

in which the resulting external torque, $\boldsymbol{\tau}_{CR}$, may take into account various torque effects, such as aerodynamic drag torque, actuation torque and, mainly, the gravitational torque.

Evaluating the right hand side of Eq. (3), with \mathbf{H}_{CR} given by Eq. (2), and taking into account the

199 rate of change of vectors in rotating frames (Young 1998), it follows

$$200 \quad \frac{d\mathbf{H}_{CR}}{dt} = (\mathbf{r} \times M\dot{\mathbf{r}}) + [\boldsymbol{\omega} \times (\mathbf{r} \times M\dot{\mathbf{r}})] + \dot{\mathbf{H}}_G + (\boldsymbol{\omega} \times \mathbf{H}_G) \quad (4)$$

201 in which $\boldsymbol{\omega}$ is the angular velocity of body frame when rotating around the inertial frame. Eq. (4)
202 can be written as

$$203 \quad \mathbf{A} \cdot \dot{\boldsymbol{\omega}} + \mathbf{B} = \boldsymbol{\tau}_{CR}, \quad (5)$$

204 in which the $\mathbf{A} = \mathbf{A}(M, \mathbf{r}, \mathbf{I})$ and $\mathbf{B} = \mathbf{B}(M, \mathbf{r}, \mathbf{I}, \boldsymbol{\omega})$ are 3×3 and 3×1 matrices and \mathbf{I} is the inertia
205 tensor of the system.

206 The acceleration vector can be obtained from Eq. (5) as

$$207 \quad \dot{\boldsymbol{\omega}} = (\mathbf{A})^{-1} \cdot (\boldsymbol{\tau}_{CR} - \mathbf{B}), \quad (6)$$

208 which can be solved simultaneously with the Euler angular rates to simulate the platform behavior.

209 **Dynamic model simplification**

210 As could be seen in the subsection “Platform dynamics”, the dynamics of the platform can be
211 described by Eq. (6). Although this equation takes into account all the dynamic effects experimented
212 by the platform, its implementation is not the most cost-effective since some simplifications can be
213 done without affecting the performance of the balancing algorithm. For instance, assuming that $\boldsymbol{\omega}$
214 and \mathbf{r} in Eq. (6) have small magnitudes compared to the other terms, it follows that,

$$215 \quad \begin{aligned} \dot{\boldsymbol{\omega}} &= (\mathbf{A})^{-1}(\boldsymbol{\tau}_{CR} - \mathbf{B}) \\ &\approx (\mathbf{I})^{-1} \cdot \boldsymbol{\tau}_{CR}. \end{aligned} \quad (7)$$

216 Additionally, assuming that the aerodynamic torque is negligible and considering null actuation
217 torque, the resultant torque is given solely by the gravitational torque, thus $\boldsymbol{\tau}_{CR} = \boldsymbol{\tau}_G$. This
218 gravitational torque $\boldsymbol{\tau}_G$ can be determined by the cross product $\boldsymbol{\tau}_G = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times M\mathbf{g}$, in which \mathbf{F}
219 is the moment force (weight) and \mathbf{g} is the local gravity vector.

220 Since all the vectors in the model must be referred to the same reference system, the reference
 221 system fixed to the table (body-frame) is chosen. The superscripts i and b are used to identify the
 222 quantities referred to the inertial and body frames, respectively. Consequently, the local gravity
 223 vector is given in the inertial frame as $(\mathbf{g})^i = g \cdot \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$, in which g is a scalar with magnitude
 224 equal to the local gravity.

225 Using the Euler rotation matrix in the ZYX sequence that relates these two reference frames,
 226 \mathbf{R}_i^b , the local gravity can be described in the body frame as

$$227 \quad (\mathbf{g})^b = \mathbf{R}_i^b \cdot (\mathbf{g})^i = \begin{bmatrix} g \cdot s\theta \\ -g \cdot c\theta s\phi \\ -g \cdot c\theta c\phi \end{bmatrix}, \quad (8)$$

228 in which the ϕ, θ, ψ notation is used for the roll, pitch and yaw angles.

229 Then, the gravitational torque may be calculated in the body frame as

$$230 \quad \boldsymbol{\tau}_G = \mathbf{r} \times M\mathbf{g} = Mg \begin{bmatrix} r_z c\theta s\phi - r_y c\theta c\phi \\ r_z s\theta + r_x c\theta c\phi \\ -r_y s\theta - r_x c\theta s\phi \end{bmatrix}, \quad (9)$$

231 where the r_x, r_y and r_z scalars are the components of the unbalance vector \mathbf{r} .

232 Knowing that the inertia products have negligible magnitude compared with the principal
 233 moments,

$$234 \quad \mathbf{I} \approx \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \Rightarrow \mathbf{I}^{-1} = \begin{bmatrix} \frac{1}{I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix}. \quad (10)$$

235 Finally, replacing Eq. (10) and Eq. (9) in Eq. (7) the dynamic model of the platform can be

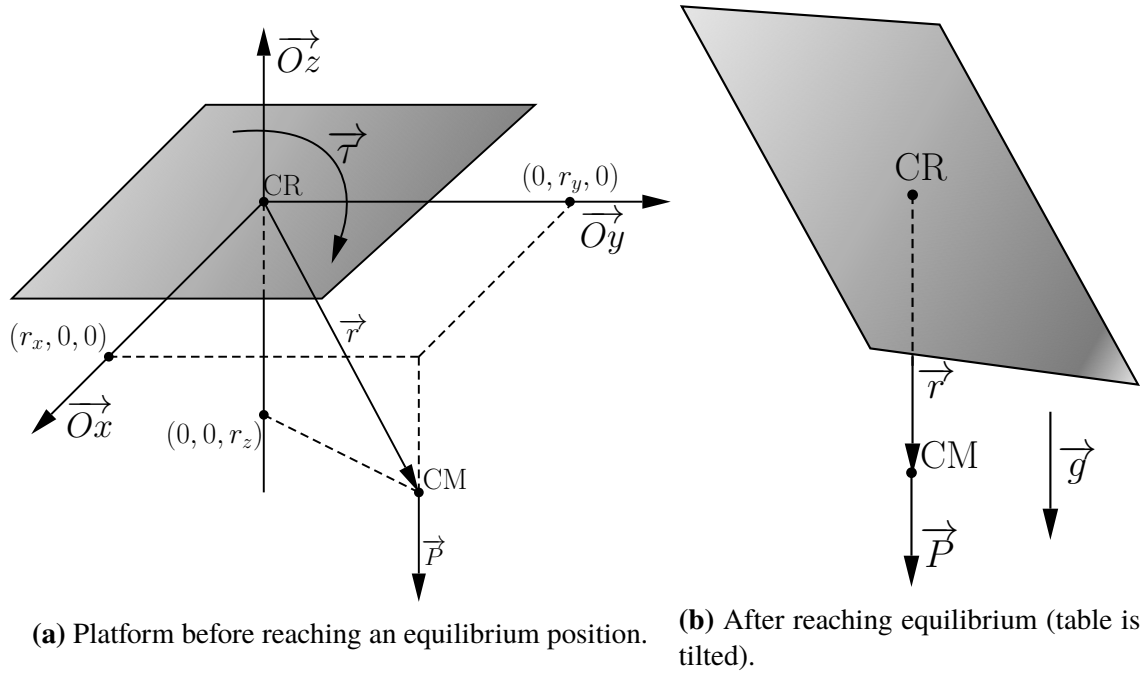


Fig. 8. Gravitational torque τ due to the presence of the gravitational force P .

236 represented in a simplified manner as Eq. (11)

$$237 \quad \dot{\omega} = \begin{bmatrix} \frac{Mg}{I_{xx}}(-r_y c_\phi c_\theta + r_z s_\phi c_\theta) \\ \frac{Mg}{I_{yy}}(r_x c_\phi c_\theta + r_z s_\theta) \\ \frac{Mg}{I_{zz}}(-r_x s_\phi c_\theta - r_y s_\theta) \end{bmatrix}. \quad (11)$$

238 **The batch estimation balancing algorithm**

239 After mounting all the components described in the section “SYSTEM CONFIGURATION”,
 240 it is expected that the table would tend to reach an unbalanced position or, in other words, the
 241 table would be tilted, as illustrated by Fig. 8. Being the center of mass (CM) in a position that is
 242 displaced from the center of rotation (CR) of the table, a gravitational torque is produced around
 243 the CR which tilts the platform.

244 The gravitational torque experienced by the table would interfere with the attitude control system
 245 of any nanosatellite placed on the table, therefore, it is necessary to minimize it. This gravitational
 246 torque is minimized by making the distance between the center of mass and the center of rotation

247 of the table as close as possible to zero.

248 A summary of the proposed algorithm is as follows:

- 249 1. First, dynamic data of the table are collected. This data are sent to the CPU through wireless
250 communication. These data consists of the roll and pitch angles and the angular velocities of
251 the table.
- 252 2. The CPU uses the data collected to make an estimation of the distance between the CM and
253 the CR of the table.
- 254 3. The CPU evaluates the required actuation, i.e. how much each motor will have to move, and
255 sends this command to the air bearing table through wireless communication.
- 256 4. After making the correction, dynamic data are collected again and the process is continued
257 iteratively until the measured distance between the CR and the CM of the table reaches a
258 predetermined threshold.

259 Considering the simplified version of the dynamic model of the testbed given in Eq. (11), the
260 Least Squares Method (LSM) is used in order to improve the estimation of the components of the
261 displacement vector (Young 1998). Firstly, Eq. (11) can be integrated over a short time period. In
262 this way, the gyroscope data can be used and the only three unknowns that remains in this equation
263 are the unbalance vector components. This is done under the assumption that the roll (ϕ) and pitch
264 (θ) angles are almost constant during a small time step. The result of this is given by Eq. (12)

$$\begin{aligned} (\Delta\omega_x)_{t_2-t_1} &= \frac{-Mg\Delta t}{2I_{xx}} \left\{ [(c_\phi c_\theta)_{t_2} + (c_\phi c_\theta)_{t_1}] r_y - [(s_\phi c_\theta)_{t_2} + (s_\phi c_\theta)_{t_1}] r_z \right\}, \\ (\Delta\omega_y)_{t_2-t_1} &= \frac{Mg\Delta t}{2I_{yy}} \left\{ [(c_\phi c_\theta)_{t_2} + (c_\phi c_\theta)_{t_1}] r_x + [(s_\theta)_{t_2} + (s_\theta)_{t_1}] r_z \right\}, \\ (\Delta\omega_z)_{t_2-t_1} &= \frac{-Mg\Delta t}{2I_{zz}} \left\{ [(s_\phi c_\theta)_{t_2} + (s_\phi c_\theta)_{t_1}] r_x + [(s_\theta)_{t_2} + (s_\theta)_{t_1}] r_y \right\}. \end{aligned} \quad (12)$$

266 that can be rewritten in the following manner

$$\underbrace{\begin{bmatrix} \Delta\omega_x \\ \Delta\omega_y \\ \Delta\omega_z \end{bmatrix}}_{\Delta\Omega} = \underbrace{\begin{bmatrix} 0 & \phi_{12} & \phi_{13} \\ \phi_{21} & 0 & \phi_{23} \\ \phi_{31} & \phi_{32} & 0 \end{bmatrix}}_{\phi} \cdot \underbrace{\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix}}_{\mathbf{r}}, \quad (13)$$

268 where the ϕ_{ij} terms are given as

$$\left\{ \begin{array}{l} \phi_{12} = -\frac{Mg\Delta t}{2I_{xx}} ((c_\phi c_\theta)_{t_2} + (c_\phi c_\theta)_{t_1}) , \\ \phi_{13} = \frac{Mg\Delta t}{2I_{xx}} ((s_\phi c_\theta)_{t_2} + (s_\phi c_\theta)_{t_1}) , \\ \phi_{21} = \frac{Mg\Delta t}{2I_{yy}} ((c_\phi c_\theta)_{t_2} + (c_\phi c_\theta)_{t_1}) , \\ \phi_{23} = \frac{Mg\Delta t}{2I_{yy}} ((s_\theta)_{t_2} + (s_\theta)_{t_1}) , \\ \phi_{31} = -\frac{Mg\Delta t}{2I_{zz}} ((s_\phi c_\theta)_{t_2} + (s_\phi c_\theta)_{t_1}) , \\ \phi_{32} = -\frac{Mg\Delta t}{2I_{zz}} ((s_\theta)_{t_2} + (s_\theta)_{t_1}) . \end{array} \right. \quad (14)$$

270 The LSM method is used because it finds a suitable estimation of the solution using all the data
 271 acquired from the sensors over time. It is also useful to prevent the occurrence of gross errors on
 272 the estimation caused by any kind of instantaneous sensor failure, since the estimation will not be

273 evaluated using data of only one instant. Oversampling Eq. (13) results in the following system

$$\begin{array}{c}
 \left[\begin{array}{c}
 (\Delta\omega_x)_{t_0} \\
 (\Delta\omega_y)_{t_0} \\
 (\Delta\omega_z)_{t_0} \\
 (\Delta\omega_x)_{t_1} \\
 (\Delta\omega_y)_{t_1} \\
 (\Delta\omega_z)_{t_1} \\
 \vdots
 \end{array} \right] = \underbrace{\left[\begin{array}{ccc}
 0 & (\phi_{12})_{t_0} & (\phi_{13})_{t_0} \\
 (\phi_{21})_{t_0} & 0 & (\phi_{23})_{t_0} \\
 (\phi_{31})_{t_0} & (\phi_{32})_{t_0} & 0 \\
 0 & (\phi_{12})_{t_1} & (\phi_{13})_{t_1} \\
 (\phi_{21})_{t_1} & 0 & (\phi_{23})_{t_1} \\
 (\phi_{31})_{t_1} & (\phi_{32})_{t_1} & 0 \\
 \vdots & \vdots & \vdots
 \end{array} \right]}_{\phi_L} \cdot \underbrace{\left[\begin{array}{c}
 r_x \\
 r_y \\
 r_z
 \end{array} \right]}_{\mathbf{r}}, \quad (15)
 \end{array}$$

274

275 that can be solved using the LSM providing the displacement vector shown in Eq. (16)

$$\mathbf{r} = [\phi_L^T \cdot \phi_L]^{-1} \cdot \phi_L^T \cdot \Delta\Omega_L. \quad (16)$$

276

277 After having a proper estimation of the distance between the CR and the CM, the actuation
 278 system is responsible for compensating the unbalanced vector components. Assuming that all
 279 MMUs displace the same amount of mass in each of the three non-redundant translational degrees
 280 of freedom of the table, the actuation parameters are given by

$$\Delta\mathbf{r}_{MMU} = -\frac{M}{m_{MMU}} \cdot \mathbf{r}_{CM}. \quad (17)$$

281

282 **Variation of the inertia tensor**

283 As one may notice, the inertia parameters of the platform are used in Eq. (16) in order to
 284 determine the unbalance vector components. These parameters are estimated in a CAD software
 285 and used to start the algorithm. For the current configuration of the platform, the inertia tensor is

286 given by

$$287 \mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 0.265 & -0.014 & -0.035 \\ -0.014 & 0.246 & -0.018 \\ -0.035 & -0.018 & 0.427 \end{bmatrix} [kg \cdot m^2], \quad (18)$$

288 and, as expected, the inertia products have much smaller magnitude than the principal moments of
289 inertia. After each movement of a MMU, this initial inertia tensor is changed. These changes may
290 be tracked in each iteration and incorporated to the algorithm to make corrections of the inertia
291 tensor (Kim and Agrawal 2009).

292 SIMULATION AND TESTS

293 The tests conducted in this work have illustrated the quality of the balancing procedure based
294 on batch estimation. By comparing the period of the platform oscillation with that of a simple
295 pendulum, it was possible to notice the improvement of the results after each interaction of the
296 balancing algorithm. Another way of verifying the balancing performance is to register the initial
297 and final positions of the platform. Starting from a tilted position, the platform shall conclude the
298 balancing procedure in an almost horizontal position, as the roll and pitch angles of the platform
299 became approximately null. However, a fact should be clarified: the roll and pitch angles are
300 expected to diminish to zero just in the case that the magnitude of the unbalance vector in the Z_b
301 axis stays much higher than the magnitude of the horizontal plane component of the unbalance
302 vector. Otherwise, the table could reach any other final inclination.

303 This fact points to a limitation concerning the balancing capability in the vertical axis (Z_b).
304 Although the X_b and Y_b components became well balanced, there is still a considerable unbalance
305 in the vertical axis of the platform since there is much more mass concentrated below its CR. There
306 are several ways to avoid this problem, as for instance increase the mass that each MMU can move
307 or simply add more weight above the CR of the platform. In this project, it was chosen to let
308 the Z_b component of the unbalance vector to reach larger values by implementing an interface for
309 mounting hardware above the initial configuration of the platform. The height of this plate related
310 to the table is adjustable with screws.

TABLE 1. Evolution of the oscillation period along the iterations of the balancing algorithm.

Air bearing table with mounting plate			
Iteration no.	$\ \mathbf{r}\ $ (μm)	Oscillation period	
		Calculated (s)	Measured (s)
Initial condition	4164.75	4.0827	4.3200
No. 1	3732.98	4.3124	4.7041
No. 2	3280.38	4.6003	4.7204
No. 3	2809.32	4.9710	5.4423
No. 4	2767.58	5.0084	5.5081
No. 5	2213.58	5.6002	5.8774
No. 6	2147.67	5.6855	6.2218
No. 7	1715.05	6.3622	6.9219
No. 8	1396.50	7.0506	7.7607
No. 9	1250.46	7.4510	8.5203
No. 10	910.12	8.7337	10.0773
No. 11	605.13	10.7108	12.9215
No. 12	289.42	15.4876	20.8118

311 This enabled the balancing process to position the CM even closer to the CR, as can be seen in
312 Fig. 9. Also, the final period of oscillation of the table in the roll and pitch axes increased to 22.83 s
313 and 20.81 s, respectively. In these new results, the MMUs were allowed to move a fixed maximum
314 in each iteration, in order to better track the evolution of the unbalance vector. Table 1 presents the
315 period of oscillation of the pitch axis starting from an arbitrary unbalance condition in which the
316 MMUs were positioned randomly. The estimates of the unbalance vector components at the end
317 of the balancing procedure were, for the X_b , Y_b and Z_b axes, $-14.1 \mu m$, $-9.0 \mu m$ and $-288.9 \mu m$,
318 respectively. The Z_b component did not achieve a magnitude similar to that obtained in the X_b and
319 Y_b axes, since the balancing procedure stopped in order to guarantee a stable position of the table,
320 avoiding the inverted pendulum behaviour. This behaviour is characterized by the positioning of
321 the CM above the CR, which makes the testbed move to the limit of the roll/pitch excursions.

322 Concerning the MMUs, each one has a total excursion of 134 mm. Each complete turn
323 performed by the stepper motor on the crank of the MMU displaces a mass of approximately 0.7 kg
324 for exactly 1 mm in the direction of the associate axis. Since the stepper motor driver is configured
325 in the 200-step mode, each turn corresponds to 200 voltage pulses sent to the driver. In other

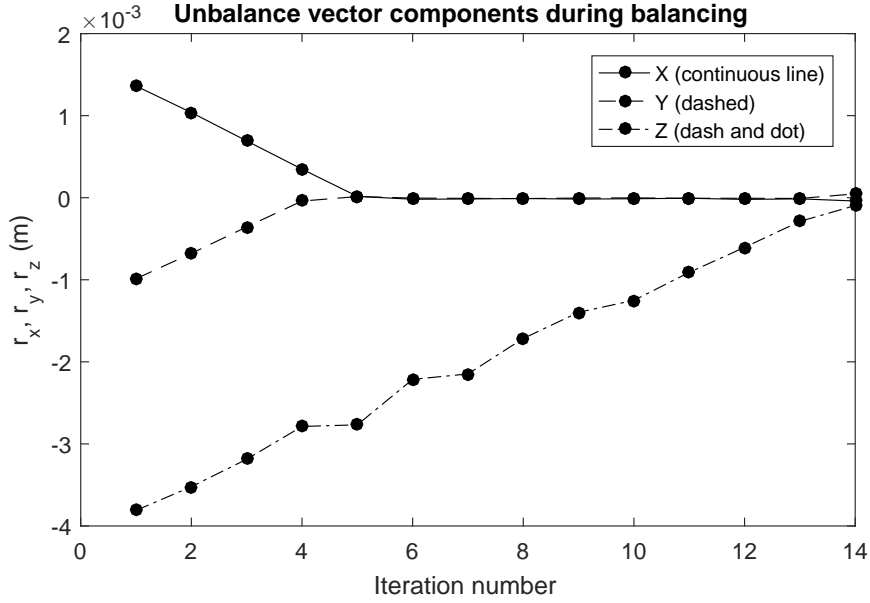


Fig. 9. Evolution of \mathbf{r} components after increasing mass above the CR.

326 words, it is possible to perform just $\frac{1}{200}$ of a turn by sending a single pulse to the motor, meaning a
 327 displacement of 5 thousandths of a millimetre of the movable mass. Eq. (19) shows the unbalance
 328 vector variation $\Delta \mathbf{r}$

$$329 \quad \Delta \mathbf{r} = \frac{m_{\text{MMU}}}{M} \begin{bmatrix} r_{mx} \\ r_{my} \\ r_{mz} \end{bmatrix}, \quad (19)$$

330 where r_{mi} , $i \in \{x, y, z\}$, are the displacements performed by each movable mass. Knowing that the
 331 testbed developed in this work weights 14 kg, Eq. (19) implies that the minimum change in any
 332 component of the unbalance vector \mathbf{r} is $0.25 \mu\text{m}$, whereas the maximum change is 6.7 mm . This
 333 range gives the maximum unbalance that can be compensated, as well as how close to the origin
 334 the MMUs can place the unbalance vector, what is in accordance with the initial and final values of
 335 $\|\mathbf{r}\|$ shown in Table 1. A $0.25 \mu\text{m}$ minimum step may indicate that a minimum of $3.5 \cdot 10^{-5} \text{ N} \cdot \text{m}$
 336 gravitational torque is reachable (see Eq. (9)). However, there are some obstacles, such as the noise
 337 level of the sensors measurements, that makes impossible the gravitational torque to reach this
 338 minimum gravitational torque level, as is mentioned posteriorly.

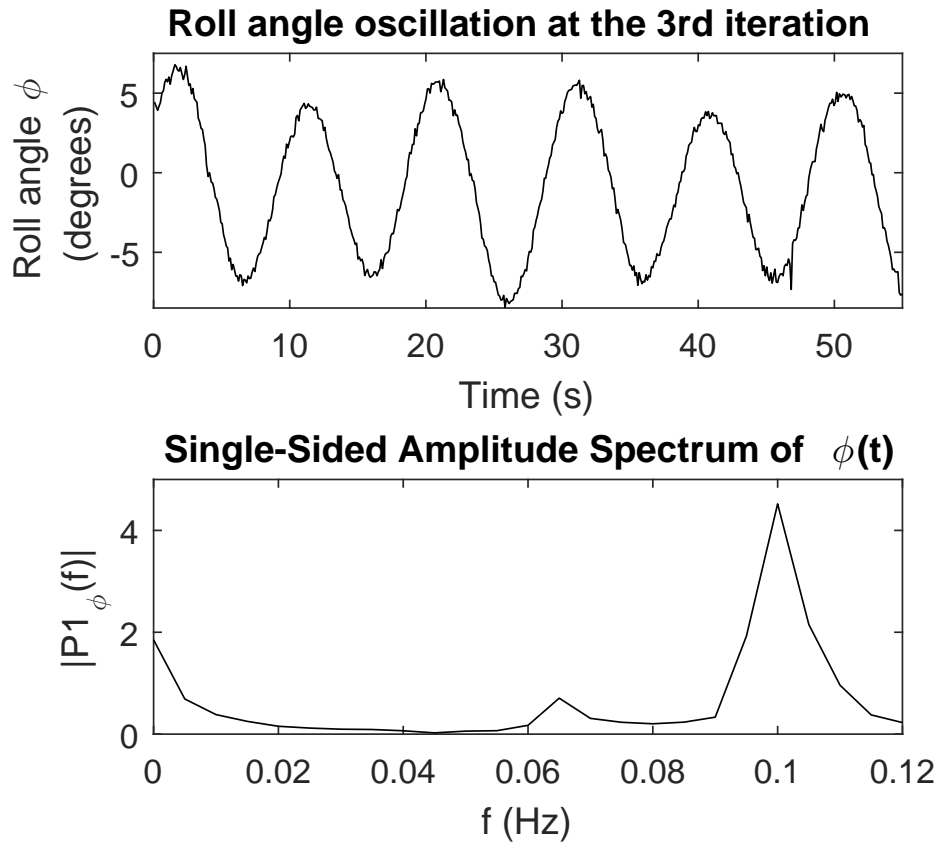


Fig. 10. Frequency spectrum using FFT.

339 Alternatively to the inspection method for estimating the oscillation period of the platform, in
 340 which the time difference between two peaks is measured, it is possible to analyze the frequency
 341 spectrum of these signals. In a second balancing test, the platform was first manually pre-balanced
 342 and 3 iterations of the balancing algorithm were executed. Applying the Fast Fourier Transform
 343 (FFT) to the oscillation signal obtained in the third iteration, it was possible to identify three main
 344 frequency components, as shown in Fig. 10. One is a constant component related to the steady-
 345 state equilibrium point of the platform and the other two components are related to the pendulum
 346 dynamics of the platform. The presence of two frequency components different from zero instead
 347 of one is related to the energy exchange between the roll and pitch axes. As can be seen in Fig. 10,
 348 the spectrum component of 0.1 Hz is dominant, what can also be noticed by checking the period
 349 of the signal in the time domain.

350 These results show that the proposed balancing method provided adequate balancing perfor-
351 mance. As a mean of comparison, the values obtained for the x and y components present the same
352 order of magnitude of similar works, as that shown in Liu et al. (2016), which reached a range of
353 $5\mu m$ for the unbalance vector magnitude.

354 Additionally, an analysis of the convergence of the unbalance vector was also performed,
355 similarly to that made by Young (1998). This analysis is important to define the minimum required
356 length in order to provide good estimations of the unbalance vector. For that, the platform was
357 excited with an initial angular momentum and sensor data were acquired during 5 minutes at a
358 sampling frequency of $10 Hz$. Then, the unbalance vector norm was calculated with various lengths
359 of data, as well as the correspondent standard deviation. Fig. 11 shows the initial 50 seconds of
360 this graph. The 5 initial samples of the estimated unbalance vector norm are not considered for
361 the determination of the standard deviation graph, since they introduce considerable bias. The
362 tests show that after 5 seconds there is already an expressive decay in the standard deviation of
363 the norm estimation and, after 40 seconds, changes in the unbalance vector norm are minimal.
364 In other words, 40 seconds of data acquisition at $10 Hz$ is proved to be enough for a reasonably
365 well estimation of the unbalance vector in the proposed tabletop testbed. One must also notice, in
366 Fig. 11, the value to which the standard deviation converges, about $0.2 \mu m$, which indicates the
367 minimum trustworthy estimate of the unbalance vector and is related to the noise level in the IMU
368 measurements.

369 Other balancing algorithms seen in the works published by Kim and Agrawal (2009) and Chesi
370 et al. (2013) could also be implemented in the proposed platform. However, some facts must be
371 mentioned: the results shown in Kim and Agrawal (2009) are based on an adaptive control method
372 that cannot be tested in a platform equipped with balancing masses only. The work developed
373 in Kim and Agrawal (2009) uses Control Moment Gyros (CMGs) which are responsible to track
374 a particular angular momentum trajectory and the error is used as feedback to the adaptive control
375 algorithm.

376 In this work, similarly to the work developed by Chesi et al. (2013), the only source of control

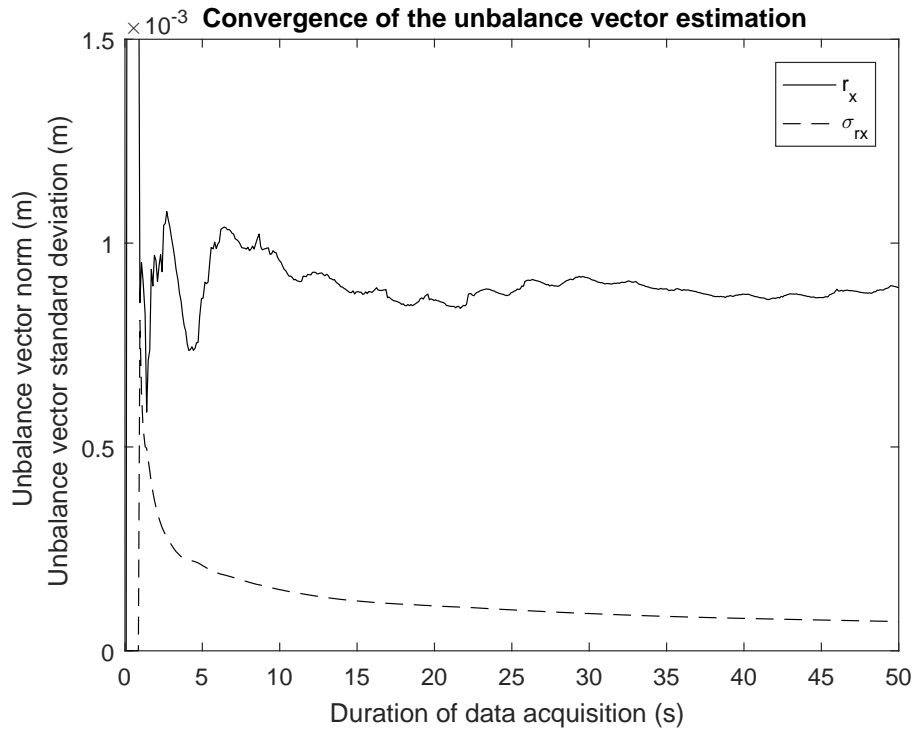


Fig. 11. Analysis of the LSM method convergence.

377 torque is that provided by the moving masses. Consequently, these torques are perpendicular to the
 378 gravity field. To avoid this restriction of generating torque in the vertical axis, a two-stage balancing
 379 algorithm was developed in which, in the first stage, only the unbalance vector components in the
 380 transverse plane are compensated using adaptive feedback control law. Then, in a second stage,
 381 an Unscented Kalman Filter is addressed in order to compensate for the last unbalance vector
 382 component, which is parallel to the gravity field.

383 In this work, the main limitation of implementing the adaptive control scheme developed
 384 by Kim and Agrawal (2009) is the absence of an alternative control torque source, whereas, for
 385 the two-stage scheme developed by Chesi et al. (2013), the bottleneck is the processing capacity
 386 of the adopted microcontroller. Both strategies utilize an onboard computer with high processing
 387 capacity, differently from the Commercial off-the-shelf (COTS) microcontroller used in this work
 388 which is already overwhelmed with the batch estimation implementation, even processing the LSM
 389 data in an external computer.

390 The sensor performance must be also analyzed. In Kim and Agrawal (2009), it is mentioned
391 that the tracking errors used as feedback tend to zero when there are no external torque disturbances.
392 However, it is emphasized that the momentum tracking errors are noisy when the angular measure-
393 ments are noisy. Thus, the mass balancing accuracy is highly sensitive to the quality of the sensor in
394 this balancing method. In fact, a comparison between the batch estimation and the adaptive control
395 made by Kim and Agrawal (2009) showed that, for some cases, the gravitational disturbances were
396 better diminished with batch estimation and, when the adaptive control presented better results,
397 the improvement was only about 46% at best. As shown in the section “SYSTEM CONFIG-
398 URATION”, the gyroscopes measurement resolution provided by the IMU is of 0.01 rad/s or,
399 equivalently, $0.5730^\circ/s$, much less than that provided by the IMU700 inertial measurement unit
400 used in that work, which is of less than $0.025^\circ/s$ (more than 23 times better). Consequently, it is
401 expected, *a priori*, that the implementation of Kim et al. method in this work would end in even
402 worse results. In Chesi et al. (2015) it is also mentioned the influence of unmodeled noise effects
403 in the measurements of the IMU. In this case, the IMU used - an ADIS16400 (Analog Devices) -
404 provided $0.05^\circ/s$ of resolution. The advantages and disadvantages of each balancing method are
405 summarized in Table 2.

406 In this context, the batch estimation method proved to be adequate in a low-cost system in which
407 the sensor data may not have the desired precision. The LSM method, when applied with enough
408 data, may suppress the noise influence.

409 Furthermore, as shown in Fig. 12, the platform oscillation decays with time, although it is
410 assumed that there is no friction in the air bearing. This occurs because the aerodynamic drag
411 torque, in fact, is present. As this effect is not predicted in the model simplification in Eq. (11),
412 it may cause deviations in the unbalance vector estimation provided by the LSM method in case
413 the data are collected for a long time. In other words, there is a trade off between the estimation
414 convergence and its precision. The aerodynamic drag problem is also addressed in Chesi et al.
415 (2013).

416 CONCLUSIONS

TABLE 2. Pros and Cons of each balancing method.

Silva et al.	
Pros	Satisfactory results are obtained using a low complexity algorithm. Does not require CMGs. Actuation is made using movable masses only. Less sensibility to sensor quality, since the noise effect is minimized by the Least Squares Method. Does not require an embedded controller, since calculations may be made in an external computer (consequently, there is no need of high processing capacity embedded in the onboard computer).
Cons	Does not solve the inverted pendulum problem. Correctness of the method is highly dependent on the accurate estimation of the inertia tensor. Method must be repeated several times until good balancing is achieved. Other methods may achieve better results.
Chesi et al.	
Pros	Does not require CMGs. Actuation is made using movable masses only. Capable of obtaining better results than those obtained with batch estimation.
Cons	Does not solve the inverted pendulum problem. Sensible to noise presence or low resolution in sensor measurements, in which case the batch estimation may obtain better results.
Kim et al.	
Pros	Solves the inverted pendulum problem. Capable of obtaining better results than those obtained with batch estimation.
Cons	Sensible to noise presence or low resolution in sensor measurements, in which case the batch estimation may obtain better results. Requires CMGs, <i>i.e.</i> , active torque actuators.

417 This article described a new platform developed at the University of Brasília for testing attitude
418 determination and control systems of nanosatellites. The platform simulates the attitude dynamics
419 of nanosatellites by using an air bearing table.

420 A LSM procedure has been proposed, based on the data from a COTS IMU, in order to reduce
421 the distance between the center of mass and the center of rotation of the air bearing table. A

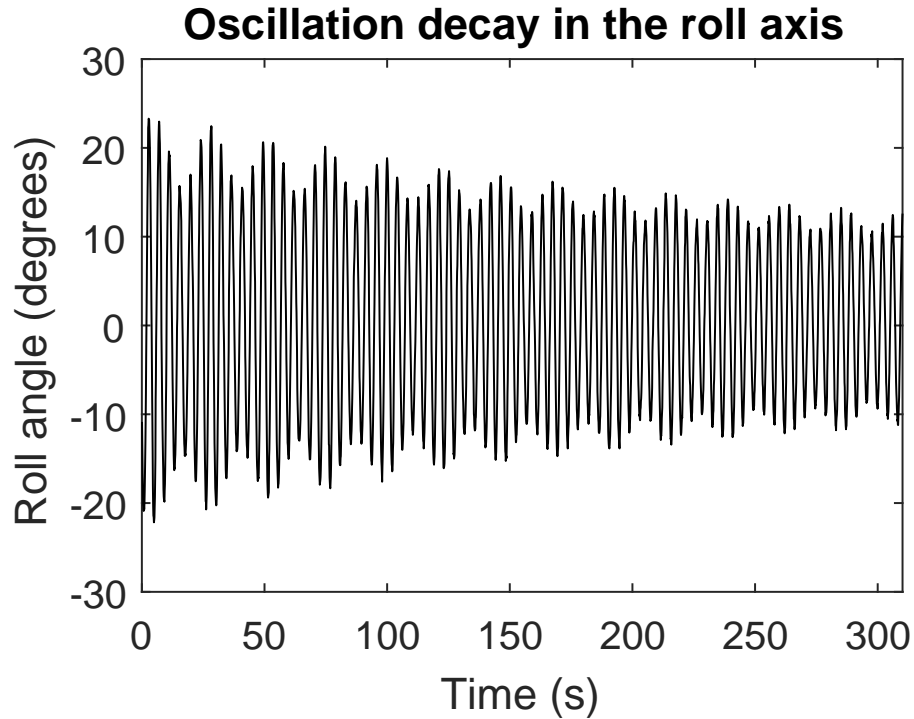


Fig. 12. Influence of the aerodynamic drag.

422 set of movable masses attached to the table are moved in accordance with the LSM algorithm
 423 and this allows the balancing of the platform. Results showed that, although only low-cost COTS
 424 electronic devices are used, the performance of the balancing system is satisfactory, since the
 425 achieved unbalance range is compatible with that shown in other works.

426 The testbed includes also an Helmholtz cage. The association between the testbed and the
 427 Helmholtz cage extends the range of simulation possibilities by making possible the simulation of
 428 the magnetic field of the Earth. Measurements taken during its operation showed that the cage is
 429 capable of generating enough magnetic field to run and test magnetic control algorithms, which
 430 will be done in future works.

431 **ACKNOWLEDGMENT**

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 434 (CAPES) and the National Council for Scientific and Technological Development (CNPq).

NOMENCLATURE

The following symbols are used in this paper:

s_{\bullet} = Sine of the angle variable denoted by \bullet ;

c_{\bullet} = Cosine of the angle variable denoted by \bullet ;

\times = Standard cross product for vectors in \mathbb{R}^3 ;

ω = Vector of angular velocities [rad/s];

M = Total mass of the platform [kg];

m = Mass [kg];

ϕ = Roll angle [rad];

θ = Pitch angle [rad];

ψ = Yaw angle [rad];

I_{ij} = Components of the inertia tensor, $i, j \in \{x, y, z\}$;

\mathbf{I} = Inertia tensor;

\mathbf{r} = Unbalance vector or CM vector;

i = Applied current in the coil [A];

a = Length of half the side of the coil [m];

L = Length of the pendulum rod [m];

T = Oscillation period [s];

τ = Torque [N.m];

γ = Aerodynamic coefficient of the platform;

\mathbf{H} = Vector of angular momentum;

\mathbf{v} = Vector of linear velocity;

\mathbf{R}_i^j = Rotation matrix relating the i, j reference frames;

g = Magnitude of the local gravity vector;

t_i = Subscript that denotes the variable taken at time t_i ;

G = Subscript related to the gravity field or vectors applied to the CM;

x, y, z = Subscripts used to denote scalar quantities related to the x, y or z axis;

b = Superscript of variables related to the body frame; and

i = Superscript of variables related to the inertial frame.

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