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A Holistic Metric Approach to solving the Dynamic Location-Allocation Problem*

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Abstract. In this paper, we introduce a dynamic variant of the Location-Allocation problem: Dynamic Location-Allocation Problem (DULAP). DULAP involves the location of facilities to service a set of customer demands over a defined horizon. To evaluate a solution to DULAP, we propose two holistic metric approaches: Static and Dynamic Approach. In the static approach, a solution is evaluated with the assumption that customer locations and demand remain constant over a defined horizon. In the dynamic approach, the assumption is made that customer demand, and demographic pattern may change over the defined horizon. We introduce a stochastic model to simulate customer population and distribution over time. We use a Genetic Algorithm and Population-Based Incremental Learning algorithm used in previous work to find robust and satisfactory solutions to DULAP. Results show the dynamic approach of evaluating a solution finds good and robust solutions.

Keywords: Dynamic Uncapacitated Location-Allocation Problem · GA · PBIL · Holistic Metric · Stochastic model

1 Introduction

Location-Allocation Problem involves the location of a set of facilities to service a set of customers demands in such a way as to optimise a cost function, subject to a set of constraints [4]. LAP has been well-researched, and there are many formulations. LAP is, in general, a combinatorial problem and so the number of solutions increases exponentially with problem size, defined by the number of facilities and customers. Many metaheuristic methods have been proposed for LAP, including tabu search (TS) [2], simulated annealing (SA) [7], Variable neighborhood search.[3], Genetic algorithms (GA) [11],[5], Bees algorithm [9], Clustering search (CS) method using Simulated annealing (SA) [8], hybrid Particle swarm optimisation algorithm [10], hybrid intelligent algorithms [6]. LAP

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can be classified into capacitated problems (CLAP), where the capacity constraint of a facility applies, and uncapacitated problems (ULAP) where each facility is unconstrained in its service delivery. Our interest lies in ULAP, and in particular, a dynamic non-linear ULAP variant motivated by a real-world problem from the telecommunications industry.

ULAP is typically formulated with consideration to current parameters values. However, considering that customer distribution and growth will change over time, it becomes essential to plan the location of facilities with consideration to the time varying aspect of the problem. It is for this reason that we formulate the problem: Dynamic Uncapacitated Location-Allocation Problem (DULAP). DULAP involves the location of facilities to service a set of customer demands over a defined horizon, with the aim of reducing the overall total cost.

It is important to distinguish our approach in this paper from the dynamic optimisation literature. Classically, a dynamic optimisation is one where the value of solutions changes dynamically while a search algorithm is seeking an optimal solution. Here we explore the problem where the evaluation of a fixed solution incorporates dynamic or time-varying aspects of the problem.

To evaluate a solution to DULAP, we present two holistic metric approaches namely the Static and Dynamic approach. In the static approach, customers are assumed to remain the same over the defined period. In the Dynamic approach, customers are assumed to change over time. The change in customer movement is simulated using a stochastic model. To assess the benefits of simulating the movement of customers for decision making, we use a GA and a PBIL presented in [1]. Our objective is to study how a solution to DULAP can be assessed concerning robustness to forecast changes.

The paper is organised as follows: In Section 2 we present the problem formulation. In Section 3 we describe the experimental setup and results. Section 4 concludes the paper and highlights future work.

2 Problem Formulation

In this section, we discuss how the objective function is computed and how the movement of the customer population is simulated over time.

2.1 Objective Function

DULAP aims to minimise the overall total costs with regards to facilities and service costs over a defined horizon t expressed in years. Solutions to DULAP are evaluated using facilities and service costs. A solution x to the problem is defined by whether or not a facility is closed (0) or opened (1). We, therefore, select a binary representation, denoted by $x \in \{0,1\}^m$ where m is the number of possible facilities that can be opened. The objective function is expressed as:

$$f_{static}(x) = C_0(x) + \sum_{t=1}^{t_{max}} C_t(x)(1+r)^{-t} \tag{1}$$

where $r \in [0,1], \quad t_{max} = 25;$

The cost function C_0 involves the costs of opening and shutting down facilities, service costs of customers subject to main and backup connections, reassignment of customers and running costs of facilities at t_0. The cost function C_t calculates the discounted total costs for years $\{t_1, t_2, ..., t_{max}\}$. C_t involves service costs subject to main and backup connections and facility running costs. The discount rate r is set at 0.05. The two proposed ways of evaluating a solution both utilities the objective function but in two distinct ways. In the static and dynamic approach, the cost of C_0 remains constant.

For the static approach, we assume that customers will not change over the defined period. Hence, this makes the problem deterministic. This means that for time $\{t_1, t_2, ..., t_{max}\}$. The cost of C_t is the same for each year which is then discounted at a discount rate r.

For the dynamic approach, we assume that customers will change over time. This means that the problem is subject to changes in customer growth and distribution which are driven by a stochastic model. Hence, the problem becomes stochastic. Due to the stochastic nature of the problem, it becomes difficult to predict customer distribution in the future. The dynamic approach therefore proposes to simulate possible population movement for $\{t_1, t_2, ..., t_{max}\}$. The simulated customer movement gives varying costs for each year of the simulation. The cost for each year is then computed using C_t which are discounted at a discount rate r. While C_0, remains a deterministic function, future costs C_t are obtained from the expected costs over n simulations of customer growth. The dynamic approach then take the following form:

$$f_{dynamic}(x) = C_0(x) + E\left[\sum_{t=1}^{t_{max}} C_t(x)(1+r)^{-t}\right] \tag{2}$$

2.2 Simulation Model

Each facility is assumed to be located within a city. Parameters used in the model includes a growth rate, die-off rate, and a radius. For every year starting from time t_1, based on the growth rate of customers, new customers are generated randomly with a uniform probability of appearing anywhere within the defined radius of the city. Based on the die of rate, a percentage of customers are randomly removed from the city. Growth in customer sites within different regions varies in level of intensity depending on the demographics. The growth and die-off rates are based on the 2013 population statistics of the united states.

3 Experiments

In this section, we compare the two ways of evaluating a solution using the approaches described in section 2.1.

3.1 Algorithms

To compare the two approaches for evaluating a solution, we use two algorithms: GA and PBIL. The two algorithms were used in our previous work [1] to solve the static variant of DULAP. The parameters used in this paper for the algorithms were the best parameters found in [1]. For both GA and PBIL, population size is set to 50. The total number of evaluations is set to 10000 for both algorithms. For GA, we employ tournament selection with a selected size of 2 and a tournament size of 3, uniform crossover with a crossover rate of 0.9, bit-Flip mutation with a mutation rate of 0.2 and an elitism rate of 20% of the population. For PBIL a learning rate of 0.1 is used with a truncation size of 20% of the population.

3.2 Problem Instances

For our experiments, 30 different instances of the problem are generated with an initial set of 10000 customers. For all 30 problems,we used $m = 100$ facilities which remain the same for all problems. $n = 100$.

The two algorithms (*GA and PBIL*) when combined with the two ways of evaluating a solution gives us four configurations: GA-f_{static}, GA-$f_{dynamic}$, PBIL-f_{static} and PBIL-$f_{dynamic}$. Each configuration is executed 20 times for each problem. At the end of each run, the best solution obtained by an algorithm is evaluated over 5000 simulations using $f_{dynamic}$ to allow for comparison of results.

3.3 Experimental Results

Table 1 shows the results obtained by each configuration on the 30 instances of the problem averaged over 20 runs. For each problem, the best algorithm is highlighted in bold. Table 2 shows the average ranking of the four configurations for all 30 instances of the problem. The best-ranked configuration is highlighted in bold.

We apply the Friedman statistical test to the results presented in table 1 to find out if there is a statistical difference in the results obtained by each configuration. The *p-value* obtained by the Friedman test is 4.86E-11 which shows that there is a significant difference in results. We, therefore, apply the Holm's procedure to check the difference between the best ranking configuration and the other configurations in table 3.

Table 3 shows PBIL-$f_{dynamic}$ to be the best configuration. Although the dynamic approach finds better results, the time complexity is much higher on the average when compared with the static approach. The average time for a run of a configuration is 1800 seconds for the dynamic and 70 seconds for the static.

Table 1. Results of GA and PBIL configurations over 20 runs

Problem	GA-f_{static}	PBIL-f_{static}	GA-$f_{dynamic}$	PBIL-$f_{dynamic}$
P1	5.64E+07	5.50E+07	5.62E+07	**5.49E+07**
P2	5.59E+07	5.50E+07	5.63E+07	**5.50E+07**
P3	5.59E+07	5.51E+07	5.60E+07	**5.49E+07**
P4	5.56E+07	5.49E+07	5.62E+07	**5.48E+07**
P5	5.64E+07	5.50E+07	5.63E+07	**5.48E+07**
P6	5.59E+07	5.51E+07	5.65E+07	**5.49E+07**
P7	5.59E+07	5.51E+07	5.60E+07	**5.49E+07**
P8	5.63E+07	5.51E+07	5.66E+07	**5.49E+07**
P9	5.62E+07	5.50E+07	5.60E+07	**5.48E+07**
P10	5.58E+07	5.51E+07	5.73E+07	**5.50E+07**
P11	5.57E+07	5.50E+07	5.62E+07	**5.48E+07**
P12	5.62E+07	5.51E+07	5.58E+07	**5.49E+07**
P13	5.62E+07	5.50E+07	5.58E+07	**5.48E+07**
P14	5.64E+07	5.50E+07	5.62E+07	**5.48E+07**
P15	5.59E+07	5.50E+07	5.57E+07	**5.49E+07**
P16	5.65E+07	5.50E+07	5.67E+07	**5.48E+07**
P17	5.56E+07	5.51E+07	5.59E+07	**5.49E+07**
P18	5.63E+07	5.50E+07	5.62E+07	**5.48E+07**
P19	5.67E+07	5.50E+07	5.60E+07	**5.48E+07**
P20	5.58E+07	5.50E+07	5.69E+07	**5.48E+07**
P21	5.67E+07	5.50E+07	5.65E+07	**5.48E+07**
P22	5.60E+07	5.51E+07	5.64E+07	**5.49E+07**
P23	5.65E+07	5.51E+07	5.61E+07	**5.48E+07**
P24	5.73E+07	5.49E+07	5.58E+07	**5.48E+07**
P25	5.68E+07	5.50E+07	5.56E+07	**5.48E+07**
P26	5.62E+07	5.50E+07	5.67E+07	**5.48E+07**
P27	5.72E+07	5.51E+07	5.64E+07	**5.49E+07**
P28	5.63E+07	5.50E+07	5.64E+07	**5.48E+07**
P29	5.67E+07	5.51E+07	5.70E+07	**5.49E+07**
P30	5.66E+07	5.50E+07	5.64E+07	**5.48E+07**

Table 2. Average Rankings of the algorithms

Algorithm	Ranking
GA-f_{static}	3.50
PBIL-f_{static}	2.00
GA-$f_{dynamic}$	3.50
PBIL-$f_{dynamic}$	**1.00**

Table 3. Holm Table for $\alpha = 0.05$

i	algorithm	p	Holm
3	GA-$f_{dynamic}$	6.38E-14	0.016
2	GA-f_{static}	6.38E-14	0.025
1	PBIL-f_{static}	2.70E-03	0.050

4 Conclusion

In this paper, we introduced a dynamic variant of ULAP. We proposed two ways of evaluating a solution to DULAP. We used two algorithms from previous paper to find satisfactory and robust solutions to the new problem. The results show that PBIL-$f_{dynamic}$ produced the best results in all problem instances.

A comparison of the two approaches shows that when making important decision to establish facilities which are expected to service demand over a defined period. It becomes essential to consider how users demand and distribution might evolve. By simulating possible alternatives of how users might change, one can generate a solution that is robust enough to ensure that facilities can perform optimally for a defined time.

This approach of evaluating a solution can be extended to other fields of operational research where user demand and demographic patterns over time are often stochastic. Although the dynamic approach explored in this paper produces satisfactory solutions, the number of simulations needed to reflect real-world scenarios can be difficult to determine. Too many simulations may be computationally expensive in the time it takes to evaluate a solution. Too little simulations might not be reflective enough of the problem. Future work will, therefore, focus on finding a measure of balance between the number of simulations required and the time complexity of evaluating a solution to the problem.

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