

# **A Fuzzy Model And Algorithm To Handle Subjectivity In Life Cycle Costing Based Decision-Making**

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## ABSTRACT

*A life cycle costing (LCC) algorithm that can effectively deal with judgmental assessments of input parameters is proposed. This algorithm is based on the fuzzy set theory and interval mathematics. The development of the algorithm is motivated by the need to handle in a systematic and a more objective way the imprecision in these subjective assessments. Three major issues were considered in the development of the algorithm. First, an appropriate mathematical framework for representing subjective imprecision was identified. Then, the original LCC closed-form equation was reformulated so that uncertainties in all input parameters can be modelled in an effective and convenient manner. Finally, the formulated model was implemented in the form of an efficient computational algorithm. The algorithm handles a number of alternatives with imprecise input data and ranks them automatically. The solution of a selected example problem is included to clarify the theory of the model.*

**Keywords:** Fuzzy Set Theory, Interval Analysis, Life Cycle Costing, Risk Assessment.

## 1. INTRODUCTION

In a typical LCC analysis, the analyst employs an explicit mathematical model based on the discounted cash flow (DCF) concept to calculate the present worth (PW) of all costs. Then, the techniques of risk analysis are usually used to add value to the quality of decision-making. In doing so, either the sensitivity analysis (SA) or a probabilistic technique, usually the Monte Carlo Simulation (MCS), is employed.

To carry out an MCS, it is required to determine a probability distribution function (PDF) for every uncertain variable. According to Edwards and Bowen (1998), it is unlikely that this could be achieved objectively because historic data for construction are too small. In the absence of historic data, subjective probabilities for the likely values of the variable under consideration have to be elicited from an expert. Even if historic data are available, it is common to adjust historic-based assessments with subjective opinions (Sobanjo, 1999). This seems to be inevitable in LCC analyses because historic data will never provide a precise solution and high quality judgment will always be required (Ashworth, 1996).

Some researchers claim that it is possible to produce meaningful PDFs using subjective opinions (e.g. Byrne, 1996). However, the authenticity of such assessments is still suspected as Byrne (1997) pointed out. Moreover, many researchers (Woodward, 1995; Chau, 1997; Byrne, 1997; Edwards and Bowen, 1998; among others) have criticized simulation techniques for their complexity and their expense in terms of computation time and expertise required to extracting the knowledge.

The confidence index (CI) approach (Kirk and Dell'Isola, 1995) is an approximate probabilistic approach to uncertainty assessment. The approach is based on the 0.67 probability rule of thumb for economic studies. Although, the CI approach has the advantage of being simple, it has two limitations. First, it is based on the assumption that uncertainties in cost data are normally distributed which is not always the case (Woodward, 1995). In addition, the approach is valid only when the differences between the best estimate of every cost and high and low estimates for that cost are within 25%.

On the other hand, the sensitivity analysis is a modeling technique that is used to identify the impact of a change in the value of a single uncertain parameter on the dependent variable; usually the PW in LCC analyses. However, the SA has two shortcomings. First, it does not aim to quantify risk but rather to identify factors that are risk sensitive. Thus, it does not provide a definitive method of making the decision. In addition, it is a univariate approach, i.e., only one parameter can be varied at a time (Flanagan and Norman, 1993). Thus, it is only effective when the uncertainty in one state variable is predominant.

These shortcomings relating to the sensitivity analysis and probabilistic techniques suggest that an alternative approach might be more appropriate. Recently, there has been a growing interest in many science domains in the idea of using the fuzzy set

theory (FST) to model uncertainty (Kaufmann and Gupta, 1988; Ross, 1995; Kosko, 1997, to mention a few). The fuzzy set theory seems to be the most appropriate in processes where human reasoning, human perception, or human decision making are inextricably involved (Ross, 1995; Kosko, 1997). In addition, it is easier to define fuzzy variables than random variables when no or limited information is available (Kaufmann and Gupta, 1985). Furthermore, mathematical concepts and operations within the framework of FST are much simpler than those within the probability theory especially when dealing with several variables (Ferrari and Savoia, 1998).

In the framework of the construction industry, some researchers (e.g. Ho and Lo, 1996; Spedding and Phillips, 1997) argue that fuzzy systems will have an increasing role to play in augmenting traditional methods of decision making in the design, control and management of the built environment. Ho and Lo (1996) carried out a survey to assess the potential use of fuzzy expert systems in general surveying practices. Results of the survey indicated that fuzzy expert systems would have potential application in valuation, investment appraisal, development consultancy, and project management.

Byrne (1995) pointed out the potential use of fuzzy logic as an alternative to probability-based techniques. In a subsequent paper (Byrne, 1997), he carried out a critical assessment of the fuzzy methodology as a potentially useful tool in discounted cash flow modeling. However, his work was mainly to investigate the fuzzy approach as a potential substitute for probabilistic simulation models. However, some researchers claim that probability may be viewed as a subset of the fuzzy set theory (e.g. Zadeh, 1995). In this sense, FST should not be treated as a replacement of the probability theory. Rather, it should be viewed as the source of additional tools that can enlarge the domain of problems that can be effectively solved.

Kaufmann and Gupta (1988) described how to manipulate fuzzy numbers in the discounting problem. They introduced an approximate method to simplify the mathematical calculations with fuzzy numbers. In this method, a function  $f(A)$ , where  $A$  is a triangular fuzzy number (TFN), can be approximated in general by another TFN. Sobanjo (1999) employed this simplified method to introduce a methodology for handling the subjective uncertainty in life cycle costing analyses. The model has the apparent advantage of being simple. However, it has the following limitations. First, both the interest rate, rehabilitation times, and the analysis period were assumed to be certain. Moreover, only TFNs were considered in representing decision variables. However, an expert should give his own estimates together with a choice of the most appropriate membership function for every state variable.

The rest of the paper is organized as follows. In the next section, an LCC model is formulated in a way that can handle uncertainty in all state variables. Then, basic concepts of FST are introduced with emphasis on operations on fuzzy numbers and intervals. This is followed by an investigation of issues necessary to implement the model in the form of an efficient algorithm on a microcomputer. Finally, conclusions and further future research are introduced. For convenience, principal symbols used in the development of the model are listed in an appendix.

## 2. FORMULATION OF THE MODEL

The life cycle cost for an alternative  $I$ , is the net present value,  $Nev_i$ , of all costs that emerge during the life cycle of the project and the alternative salvage value at the end of the analysis period,  $T$ , i.e.

$$NPV_i = C_{0i} + ARC_i^d + NRC_i^d - SAV_i^d \quad (1)$$

where  $C_{0i}$ , is the initial cost,  $SAV_i^d$  is the discounted salvage value, and  $ARC_i^d$ ,  $NRC_i^d$ , are the discounted values of annual and non-annual recurring costs, respectively. Discounting factors are easily derived and are available in most financial engineering texts (e.g. Kirk and Dell'Isola, 1995). The  $PWS$  factor used to discount a single future cost occurring at time  $t$ , is given by

$$PWS = (1+r)^{-t} \quad (2)$$

where  $r$ , is the discount rate. The  $PWA$  factor used to discount a series of  $T$  equal annual costs, is given by

$$PWA = \frac{1}{r} (1 - (1+r)^{-T}) \quad (3)$$

## 2.1 Discounting of Annual Costs

Assuming there are  $nar_i$  annual costs,  $A_{ij}$ ,  $ARC_i^d$  can be calculated as

$$ARC_i^d = \frac{1}{r} (1 - (1+r)^{-T}) \sum_{j=1}^{nar_i} A_{ij} \quad (4)$$

In almost all LCC models found in the literature, all annual costs were treated as a single figure,  $A$ . However, annual costs are assumed here to be the summation of  $nar_i$  components,  $A_{ij}$ , e.g. maintenance and operating costs. This was done to give experts the flexibility to assign different uncertainty levels to various annual costs depending on the nature of every cost.

## 2.2 Discounting of Non-Annual Costs

To discount non-annual recurring costs, assume there are  $nnr_i$  costs recurring every  $t_{ik}$  years. Then,  $NRC_i^d$  can be calculated by the  $PWS$  factor (Eq. 2) as

$$NRC_i^d = \sum_{k=1}^{nr_i} \sum_{n=1}^{n_{ik}} C_{ik} (1+r)^{-t_{ik}} \quad (5)$$

where  $C_{ik}$  are undiscounted non-annual costs and  $n_{ik}$  are their number of applications.

The inner summation in Eq. (5) represents a geometric series. Thus, Eq. (5) can be further simplified to

$$NRC_i^d = \sum_{k=1}^{nr_i} C_{ik} \frac{1 - (1+r)^{-n_{ik}t_{ik}}}{(1+r)^{t_{ik}} - 1} \quad (6)$$

where  $n_{ik}$  is calculated as follows

$$n_{ik} = \begin{cases} \text{int}\left(\frac{T}{t_{ik}}\right), & \text{provided that } \text{rem}\left(\frac{T}{t_{ik}}\right) \neq 0 \\ \frac{T}{t_{ik}} - 1, & \text{elsewhere} \end{cases} \quad (7)$$

The caveat applied to Eq. (7) accounts for the fact that non-annual costs recurring at the end of the last year of the analysis period are not taken into consideration.

### 2.3 Discounting of the Salvage Value

Discounting of the salvage value is attained in a straightforward way using the *PWS* factor (Eq. 2), as

$$SAV_i^d = (1+r)^{-T} S_i \quad (8)$$

where  $S_i$  is the undiscounted salvage and resale value of the alternative.

Substituting from Eqs. (4, 6 and 8) into Eq. (1) yields

$$NPV_i = C_{0i} + \frac{1}{r} \left(1 - (1+r)^{-T}\right) \sum_{j=1}^{nar_i} A_{ij} + \sum_{k=1}^{nr_i} C_{ik} \frac{1 - (1+r)^{-n_{ik}t_{ik}}}{(1+r)^{t_{ik}} - 1} - (1+r)^{-T} S_i \quad (9)$$

### 3. OVERVIEW OF THE FUZZY SET THEORY

#### 3.1 Basic Concepts and Definitions

Zadeh (1965) laid the foundation of fuzzy set theory (FST) as an extension of classical set theory. Fuzzy sets are sets of which the membership has grades in the real continuous interval  $[0, 1]$ , i.e.  $\mu_{\tilde{A}}(x) \in [0, 1]$ . As shown in Fig. (1), the end points of the interval  $[0, 1]$  conform to no membership ( $\mu(x)=0$ ) and full membership ( $\mu(x)=1$ ), respectively. However, the infinite number of points in between these end points can represent various degrees of membership. A number of commonly used features of membership functions are depicted in Figure (1).

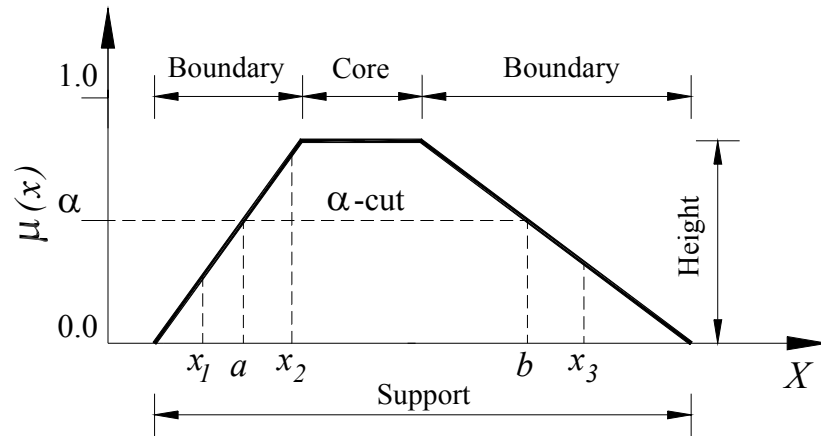


Fig. (1): Features of a fuzzy set.

If a fuzzy set has a height of 1, it is said to be normal. For any  $x_1 < x_2 < x_3$  (Fig. 1), a convex fuzzy set is one that satisfies the following relation

$$\mu_{\tilde{A}}(x_2) \geq \wedge (\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_3)) \quad (10)$$

An  $\alpha$ -cut of a fuzzy set is a crisp set,  $[a, b]$ , defined by

$${}^{\alpha}A = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\} \quad (11)$$



### 3.2 Fuzzy Numbers and Intervals

A fuzzy interval is a normal convex fuzzy set. If the core of such set is defined by one point only, it is called a fuzzy number. Fuzzy numbers and intervals represent approximate numeric quantities such as ‘about 4’ and ‘from about 3 to about 8’ (Fig. 2).

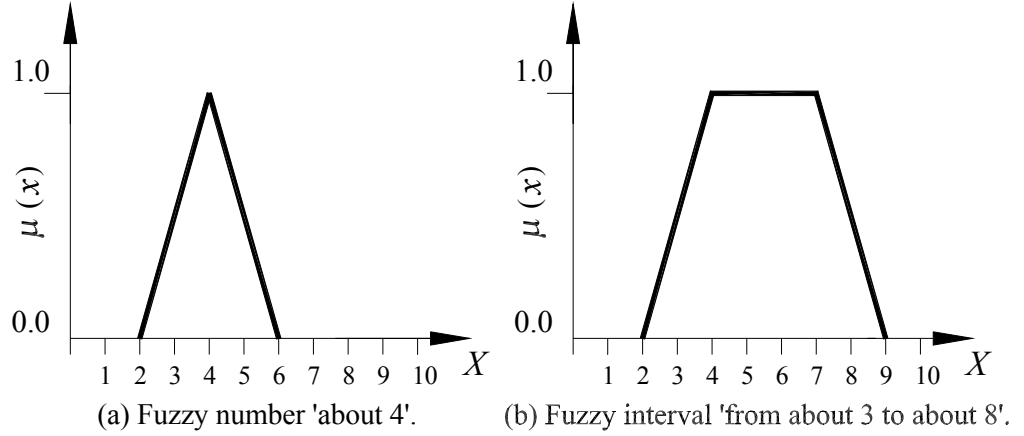


Fig. (2): Examples of fuzzy sets representing fuzzy quantities.

### 3.3 Arithmetic Operations On Fuzzy Quantities

The extension principle provides a general method for extending crisp mathematical concepts to deal with fuzzy quantities. The extension principle applied to a mapping  $y=f(x_1, x_2, \dots, x_n)$  is defined by:

$$\mu_{\tilde{B}}(y) = \bigvee_{y=f(x_1, \dots, x_n)} \bigwedge (\mu_{\tilde{A}_1}(x_1), \mu_{\tilde{A}_2}(x_2), \dots, \mu_{\tilde{A}_n}(x_n)) \quad (12)$$

The implementation of the extension principle is very complicated. This is because the solution of Eq. (12) is a non-linear programming problem (Dong et al., 1985). Several algorithms have been proposed to solve this problem. Dong et al. (1985) proposed a procedure known as the DSW algorithm. In this algorithm, membership functions are approximated with series of  $\alpha$ -cut intervals so that standard binary

operations of interval analysis can be utilised. These operations on two intervals  $[a, b]$  and  $[c, d]$  are as follows:

$$\lambda \cdot [a, b] = \begin{cases} [\lambda a, \lambda b] & \text{for } \lambda > 0 \\ [\lambda b, \lambda a] & \text{for } \lambda < 0 \end{cases} \quad (13)$$

$$[a, b] + [c, d] = [a + c, b + d] \quad (14)$$

$$[a, b] - [c, d] = [a - d, b - c] \quad (15)$$

$$[a, b] \times [c, d] = [\wedge(ac, ad, bc, bd), \vee(ac, ad, bc, bd)] \quad (16)$$

$$[a, b] \div [c, d] = [a, b] \cdot \left[ \frac{1}{d}, \frac{1}{c} \right] \quad \text{provided that } 0 \notin [c, d] \quad (17)$$

When  $a, b, c, d \geq 0$ , Givens and Tahani (1987) proposed a slight modification to the original DSW algorithm. In this case, Eq. (16 and 17) are simplified as follows

$$[a, b] \times [c, d] = [ac, bd] \quad (18)$$

$$[a, b] \div [c, d] = \left[ \frac{a}{d}, \frac{b}{c} \right] \quad (19)$$

These modifications require only one-fourth the number of multiplications (or divisions). Furthermore, the minimum or maximum operations are eliminated. For example, assume that a single future cost may vary from £1000 to £1100, and has to be discounted using a factor with possible values from 0.15 to 0.2. Equation (18) can be used to calculate the interval of the present worth as

$$[0.15, 0.2] \times [1000, 1100] = [150, 220]$$

Using Eq. (16), however, to do the same task would require ten operations as follows

$$[0.15, 0.20] \times [1000, 1100] = [\wedge(150, 165, 200, 220), \vee(150, 165, 200, 220)] = [150, 220]$$

Interval arithmetic does not follow the property of distributivity. For example, for three intervals,  $I, J$ , and  $K$ ,

$$I \cdot (J - K) \neq I \cdot J - I \cdot K \quad (20)$$

Dong and Shah (1987) attributed this to the treatment of two occurrences of identical interval numbers (i.e.,  $I$ ) as two independent interval numbers. Thus, they introduced a procedure known as the vertex method that effectively eliminates this problem. For the  $n$ -dimensional mapping,

$$y=f(x_1, x_2 \dots, x_n) \quad \text{where } x_i=[a_i, b_i], \quad i=1, 2, \dots, n \quad (21)$$

there are  $2^n$  combinations of boundaries of intervals, i.e.  $(a_1, a_2, a_3)$ ,  $(a_1, a_2, b_1)$ ,  $(a_1, a_2, b_2)$ , ... etc. Then, the extension principle becomes

$$f(x_1, x_2, \dots, x_n) = \left[ \bigwedge_j (f(c_j)), \bigvee_j (f(c_j)) \right], \quad j=1, 2, \dots, n \quad (22)$$

where  $c_j$  is the  $j$ th combination. This prevents the widening of the resulting function value set due to multiple occurrences of variables.

#### 4. MODEL IMPLEMENTATION

In this section, the model is implemented in the form of a computational algorithm. First, some important issues regarding the computing efficiency and ranking of fuzzy quantities are considered. Then, the algorithm is outlined.

##### 4.1 Computational Considerations

For fuzzy input, Eq. (9) can be rewritten as

$$N\tilde{P}V_i = \tilde{C}_{0i} + P\tilde{W}A \sum_{j=1}^{nar_i} \tilde{A}_{ij} + \sum_{k=1}^{nnr_i} \tilde{C}_{ik} P\tilde{W}N_{ik} - P\tilde{W}S \cdot \tilde{S}_i \quad (23)$$

where

$$P\tilde{W}A(\tilde{r}, \tilde{T}) = \frac{1}{\tilde{r}} \left( 1 - (1 + \tilde{r})^{-\tilde{T}} \right) \quad (24)$$

$$P\tilde{W}S(\tilde{r}, \tilde{T}) = (1 + \tilde{r})^{-\tilde{T}} \quad (25)$$

$$P\tilde{W}N_{ik}(\tilde{r}, \tilde{n}_{ik}, \tilde{t}_{ik}, \tilde{T}) = \frac{1 - (1 + \tilde{r})^{-\tilde{n}_{ik} \tilde{t}_{ik}}}{(1 + \tilde{r})^{\tilde{t}_{ik}} - 1} \quad (26)$$

As shown in Fig. (1), a fuzzy set may be considered as a crisp set with moving boundaries. In this sense, any convex fuzzy set can be described by the intervals associated with different levels of  $\alpha$ -cuts. This representation is computationally effective than other methods. In addition, it eliminates difficulties with other representation methods (Ross, 1995).

To guarantee computational efficiency and stability, the appropriate method for implementing the extension principle is identified based on the issues outlined on Sec. 3.3. A thorough investigation of Eqs. (23-26) reveals that

- Discounting factors are best computed by the vertex method.
- Because costs, salvage values, and discounting factors are non-zero and obviously positive, the restricted DSW method is the most appropriate in carrying out all multiplications.
- Finally, the net present value is determined by aggregating costs and subtracting the salvage value using the restricted DSW algorithm.

## 4.2 Ordering Of Alternatives

The primary objective of a life cycle costing analysis is to facilitate the effective choice between a number of competing alternatives. For analysis results that are deterministic, there is no ambiguity in ranking the alternatives. However, the choice may be ambiguous when results are associated with uncertainty, especially for a non-technical decision-maker. A standard way to extend the natural ordering of real numbers to fuzzy numbers was suggested as early as 1977 by Baas and Kwakernaak (1977). Since that

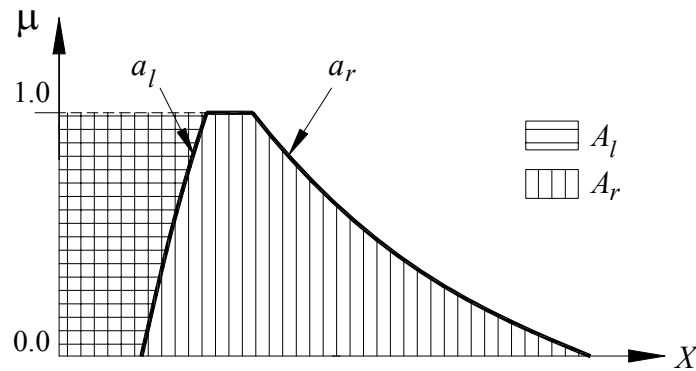
time, a large amount of literature has been developed in the area of fuzzy ranking. Some of these methods are reviewed by Chen and Hwang (1992).

Each ranking method seems to have some advantages and disadvantages. In selecting a ranking method to be employed in the algorithm, four factors were considered:

- The ability to deal with various shapes and types fuzzy quantities.
- The ease of interpretation.
- The efficiency of computations.
- The ease of computer implementation.

A simple ranking technique known as area compensation (Fortemps and Roubens, 1996) seems to satisfy these factors. The method is based on introducing a function  $R$  that maps the set of fuzzy quantities  $\tilde{F}$ , to the real line  $\Re$  and to use natural ordering. For a fuzzy quantity  $\tilde{A}$ ,  $R$  is given by

$$R_{\tilde{A}} = \frac{1}{2} \int_0^1 [\alpha a_l + \alpha a_r] d\alpha = \frac{1}{2} (A_l + A_r) \quad (27)$$



**Fig. (3): Variables used in computing the function  $R$ .**

Where  ${}^{\alpha}a_l$  and  ${}^{\alpha}a_r$  are the left and right slopes of the fuzzy set. As shown in Fig. (3), this function is the mean of the two areas  $A_l$  and  $A_r$ , bounded by the vertical axis and the left and right slopes, respectively. It is interesting that Kaufmann and Gupta (1988)

has proposed a criterion named the ‘removal’, which is given by Eq. (27).

To account for the fact that two fuzzy quantities may have the same removal, Kaufmann and Gupta (1988) proposed to order them according to their mode which is the mean value of the core. If the two fuzzy quantities still have the same mode, they are ordered according to their divergence.

### **4.3 The Algorithm**

Based on the foregoing arguments, the following algorithm may be proposed (Fig. 4).

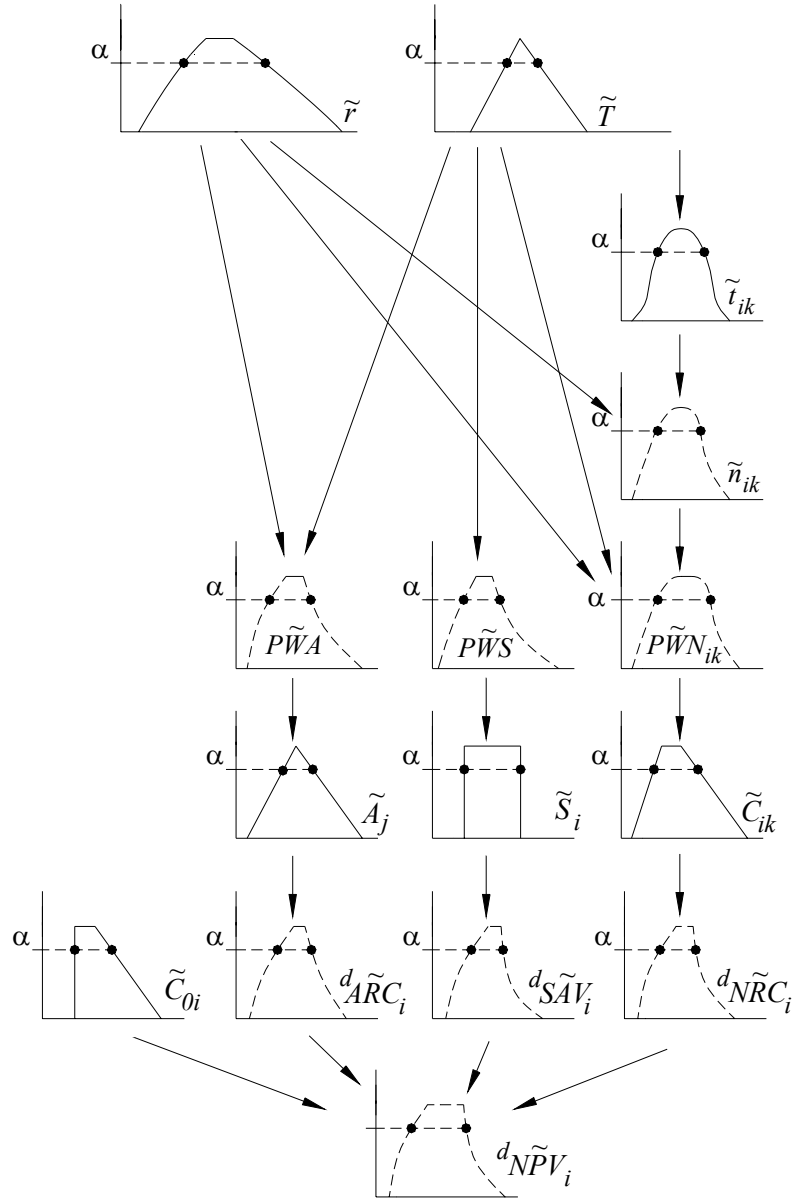


Fig. (4): Schematic representation of the Algorithm.

1. Experts express their assessments of uncertain state variables as fuzzy numbers and/or intervals. As shown in Fig. (4), these assessments (drawn with solid lines) may be conveniently expressed as any normal convex fuzzy set.
2. Select an  $\alpha$  value such that  $0 \leq \alpha \leq 1$ .
3. Find the intervals in the global input membership functions,  $\tilde{r}$  and  $\tilde{T}$ , that correspond to this  $\alpha$ .
4. Use the vertex method to calculate intervals of the discount factors  $P\tilde{W}A$  and

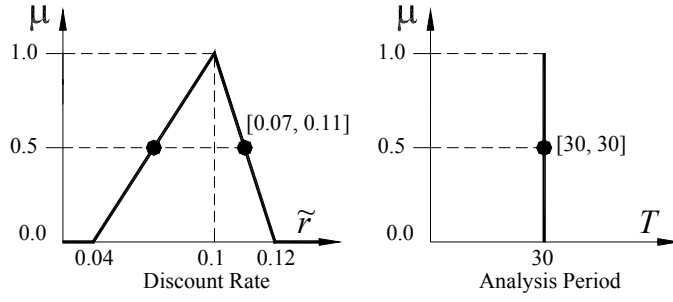
$P\tilde{W}S$  that correspond to this  $\alpha$  from Eqs. (24 and 25).

5. For alternative  $i$ , find the intervals in the alternative-specific input membership functions,  $\tilde{C}_{0i}$ ,  $\tilde{A}_{ij}$ ,  $\tilde{C}_{ik}$ , and  $\tilde{t}_{ik}$ , that correspond to this  $\alpha$ .
6. Use the restricted DSW algorithm to find the interval in the membership function  $n_{ik}$ , for the selected  $\alpha$ -cut level from Eq. (7).
7. Use the vertex method to find the interval in the  $PWN_{ik}$  factor for the selected  $\alpha$ -cut level using Eq. (26).
8. Repeat steps 6 and 7 for all non-recurring costs for alternative  $i$  ( $nnr_i$  times).
9. Using the restricted DSW algorithm, compute the interval for the output membership function for the selected  $\alpha$ -cut level using Eq. (23).
10. Repeat steps 2-9 for different values of  $\alpha$  to complete an  $\alpha$ -cut representation of the net present value for alternative  $i$ ,  $NPV_i$ .
11. Repeat steps 5 to 10 for all alternatives.
12. Alternatives are ranked according to their net present values using the ranking procedure outlined in Sec. 4.2.

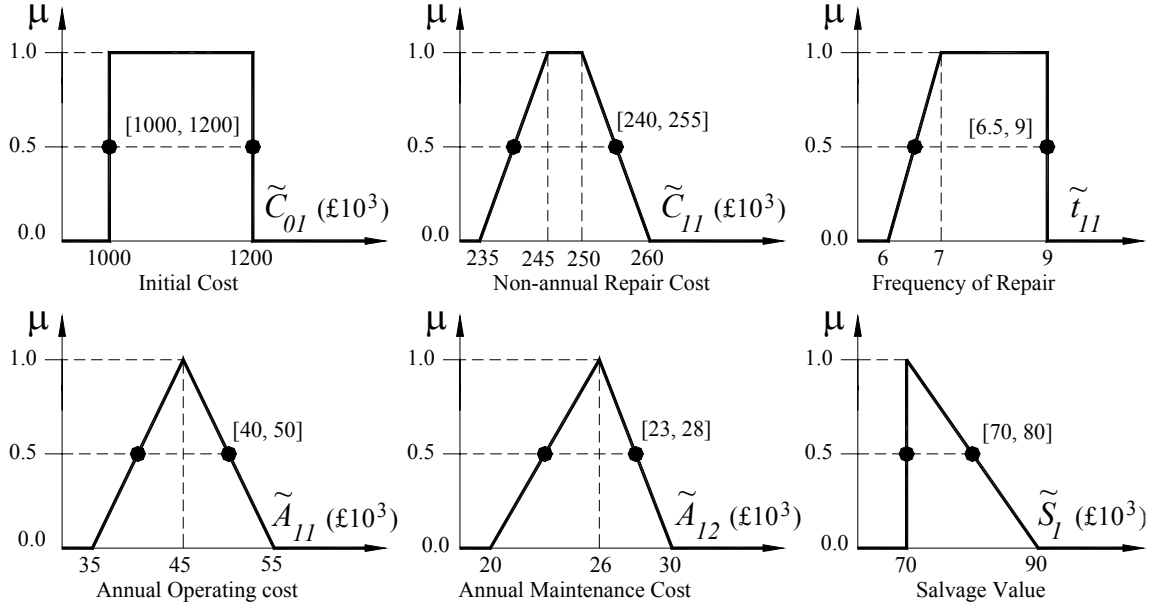
## 5. A NUMERICAL EXAMPLE

This section is devoted to the solution of an example problem to clarify the algorithm. Figure (5) shows the membership functions of various input parameters for two competing alternatives to be ordered according to their life cycle costs.

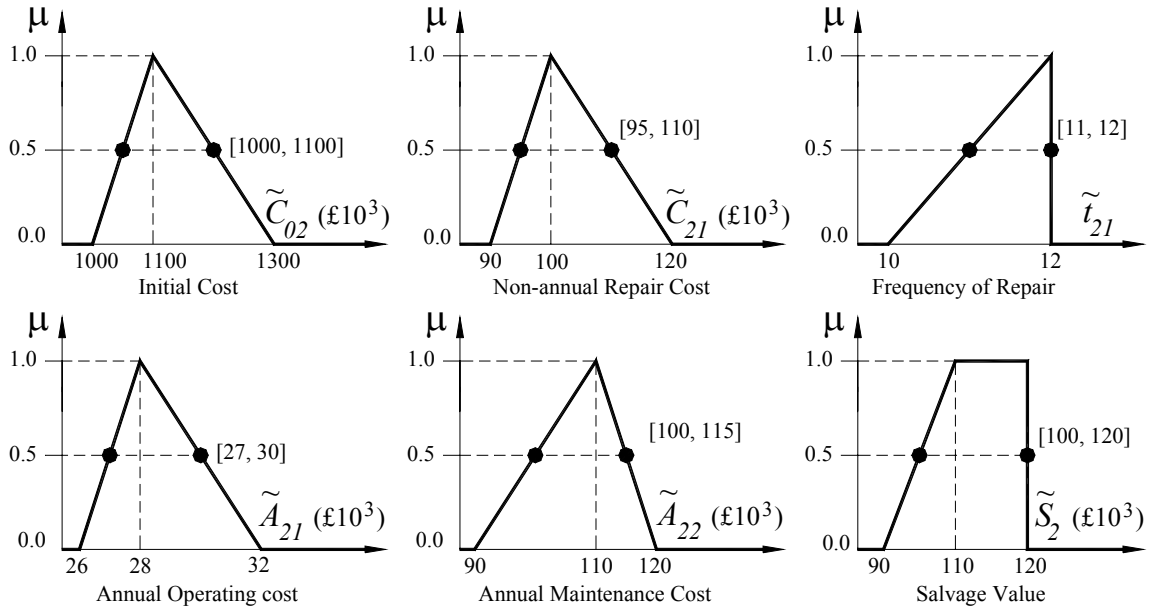




**a) Global variables.**



**b) Alternative A.**



**c) Alternative B.**

**Fig. (5): Membership functions for the example problem.**

Variables were assigned different membership functions (MFs) to reflect their uncertainty. For example, the initial cost of alternative A was assumed to vary from £1000,000 to £1200,000 without any preference given to any value in its interval. On the other hand, the initial cost of alternative B was assigned a triangular MF as a measure of the precision with which the cost is known to the expert.

The boundaries of all variables at  $\alpha=0.5$ , are shown also in Fig. (5). The calculation of the net present value of alternative A, is detailed in the following paragraphs for the chosen  $\alpha$ . For convenience, 0.5-cuts for input variables of Alternative A are given in Table (1).

**Table (1): 0.5-Cuts for Input Variables.**

Variable	Symbol	0.5-Cut
Initial cost (£1000's )	$^{0.5}C_{01}$	[1000, 1200]
Non-annual repair cost (£1000's )	$^{0.5}C_{11}$	[240, 255]
Frequency of repair cost (years)	$^{0.5}t_{11}$	[6, 8]
Annual operating cost (£1000's )	$^{0.5}A_{11}$	[40, 50]
Annual maintenance cost (£1000's )	$^{0.5}A_{12}$	[23, 28]
Salvage value (£1000's )	$^{0.5}S_1$	[70, 80]

First, the discounting factors are calculated using the vertex method. Both  $PWA$  (Eq. 24) and  $PWS$  (Eq. 25) are functions of the discount rate,  $r$ , and the analysis period,  $T$ . Because  $T$  is certain, only two combinations,  $(r = 0.07, T = 30)$  and  $(r = 0.11, T = 30)$ , are to be considered when using Eq. (22) as follows:

$$^{0.5}PWA(r, T) = [\wedge(PWA(0.07, 30), PWA(0.11, 30)), \vee(PWA(0.07, 30), PWA(0.11, 30))]$$

$$= [\wedge (8.6938, 12.40904), \vee (8.6938, 12.40904)] = [8.6938, 12.40904]$$

$${}^{0.5}PWS(r, T) = [\wedge (PWS(0.07, 30), PWS(0.11, 30)), \vee (PWS(0.07, 30), PWS(0.11, 30))]$$

$$= [\wedge (0.13137, 0.04368), \vee (0.13137, 0.04368)] = [0.04368, 0.13137]$$

Similarly, boundaries of the number of recurrences of the repair cost at  $\alpha=0.5$ ,  ${}^{0.5}n_{11}$ , can be calculated as follows

$$\begin{aligned} {}^{0.5}n_{11}({}^{0.5}t_{11}, {}^{0.5}T) &= \left[ \wedge \left( \left( \frac{30}{6} - 1 \right), \text{int} \left( \frac{30}{8} \right) \right), \vee \left( \left( \frac{30}{6} - 1 \right), \text{int} \left( \frac{30}{8} \right) \right) \right] \\ &= [\wedge (4, 3), \vee (4, 3)] = [3, 4] \end{aligned}$$

Then, boundaries of the discount factor,  ${}^{0.5}PWN_{11}$ , can be calculated in a similar way.

The  $PWN_{ik}$  factor is a function of  $r$ ,  $n_{ik}$ , and  $t_{ik}$ . Noting again that  $T$  is certain, only 8 ( $2^3$ ) combinations of boundaries of  $r$ ,  $n_{11}$ , and  $t_{11}$ , are to be considered. This is detailed in Table (2).

**Table (2): Calculation of the Possible Values of  $PWN$ .**

$(r, n_{11}, t_{11})$ Combinations	${}^{0.5}PWN_{11}(r, n_{11}, t_{11})$
(0.07, 3, 6)	1.4062
(0.07, 3, 8)	1.1952
(0.07, 4, 6)	1.6034
(0.07, 4, 8)	1.2326
(0.11, 3, 6)	0.9733
(0.11, 3, 8)	0.7039
(0.11, 4, 6)	1.0550
(0.11, 4, 8)	0.7397

These calculations reveal that the minimum and maximum are 0.7039 and 1.4062,

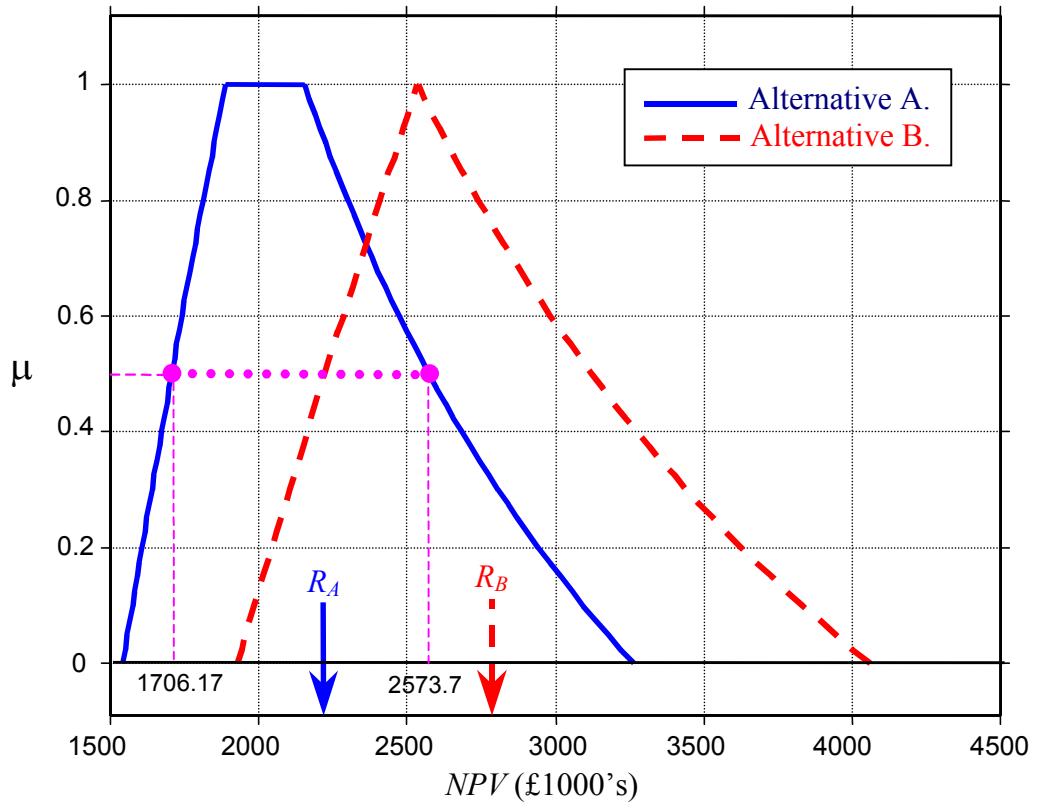
respectively. Thus,  ${}^{0.5}PWN_{11} = [0.7039, 1.4062]$ .

Since there are two annual costs, their sum is to be calculated using Eq. (14) as

$$\sum_{j=1}^2 {}^{0.5}A_{1j} = {}^{0.5}A_{11} + {}^{0.5}A_{12} = [40000, 50000] + [23000, 28000] = [63000, 78000].$$

Then, the 0.5-cut of the net present value,  ${}^{0.5}NPV_1$  is calculated as follows:

$$\begin{aligned} {}^{0.5}NPV_1 &= {}^{0.5}C_{01} + {}^{0.5}PWA \sum_{j=1}^2 A_{1j} + {}^{0.5}PWN_{11} C_{11} - {}^{0.5}PWS {}^{0.5}S_1 \\ &= [1000000, 1200000] + [8.694, 12.409] \times [63000, 78000] + \\ &\quad + [0.704, 1.603] \times [240000, 255000] - [0.0437, 0.1314] \times [70000, 80000] \\ &= [1000000, 1200000] + [547722, 967902] + [168960, 408858] - [3060, 10510] \\ &= [1716682, 2576760] - [3060, 10510] = [1706172, 2573700]. \end{aligned}$$



**Fig. (6): MFs for net present values of alternatives.**

This range in the output membership function is shown with a thick dotted line in Fig. (6). Using the same procedure for more values of  $\alpha$  in the interval  $[0, 1]$ , a complete representation of the output membership functions for both alternatives can be attained.

To ease the burden of calculations, the algorithm is implemented in a computer program. Figure (6) shows the output of the program for the given example. The first ranking criterion was sufficient to rank the alternatives. Removals for both alternatives,  $R_A$  and  $R_B$ , are plotted on Fig. (6). As shown, alternative A was found to have a clear advantage over alternative B. More details of the computer implementation of the algorithm and the solution of additional examples will be given in a future paper.

## **6. CONCLUSIONS AND FUTURE RESEARCH**

Traditional risk assessment techniques usually used in life cycle costing analyses were critically reviewed. The fuzzy set theory was found to be the most appropriate to handle imprecision that usually accompanies subjective assessments of input parameters.

A novel algorithm was designed around an explicit formulated analytical LCC model. The model is unique as it gives experts more flexibility and convenience in the assessment process. In addition, the model was introduced in a form that allows the handling of uncertainties in all state variables.

The algorithm has the apparent advantage of being transparent which allows more understanding of the decision-making process. This is mainly because it is theoretically based and the treatment of uncertainties is built in the model itself. The algorithm is also unique because alternatives are ordered automatically. This can put the decision-maker in a better position to make an informed decision.

To guarantee the computation efficiency, stability and robustness of the proposed algorithm, crucial issues in the computer implementation of the model were carefully investigated. These issues include the employment of the  $\alpha$ -cut concept, the optimization of fuzzy operations and the choice of an effective ranking procedure. The model is superior to that presented by Sobanjo (1999) due to its ability to deal with judgmental assessments of all state variables. In addition, it can manipulate various shapes of fuzzy quantities. Last but not least, it can be extended to deal with other uncertain quantitative and qualitative aspects of LCC (Kishk and Al-Hajj, 1999).

The model is the first in series being developed at the school of Construction, Property and Surveying, the Robert Gordon University. The objective of these models is to tackle some of the difficulties in the implementation of LCC in the industry. This will be achieved by integrating these models in a user-friendly decision support system. The theoretical framework for this system is outlined in Kishk and Al-Hajj (1999).

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## 8. APPENDIX: LIST OF SYMBOLS

$A_{ij}$	Annual recurring costs
$ARC_i^d$	Discounted annual recurring costs.
$[a, b]$	A crisp interval described by $a$ and $b$ ( $a \leq b$ ).



$\tilde{C}$	Fuzzy subset.
${}^{\alpha}C$	$\alpha$ -cut of the fuzzy set $\tilde{C}$ .
$C_o$	Initial costs.
$C_{ik}$	Non-annual recurring costs
$\tilde{F}$	Set of fuzzy quantities.
$int(a)$	Rounds $a$ to the nearest integer (towards zero).
$n_{ik}$	Number of recurrances of a non-annual recurring cost, $C_{ik}$ .
$nar$	Number of annual recurring costs.
$nnr_i$	Number of non-annual recurring costs, $C_{ik}$ .
$NRC_i^d$	Discounted non-annual recurring costs.
$PWA$	Present worth factor of annual recurring costs.
$PWN_{ik}$	Discounting factors for non-annual recurring costs, $C_{ik}$ .
$PWS$	Present worth factor for a single future cost.
$r$	Discount rate.
$R$	First ranking criterion (Removal).
$\Re$	Set of Real numbers.
$rem\left(\frac{a}{b}\right)$	Remainder after division of two numbers $a$ and $b$ .
$S_i$	The salvage value of the alternative at the end of the analysis period.
$SV_i^d$	Discounted salvage value.
$t_{ik}$	Frequencies of non-annual recurring costs, $C_{ik}$ .
$T$	Analysis period.
$\lambda$	Crisp real number.

$\mu_{\tilde{A}}(x)$  Membership function for the element  $x$  with respect to the fuzzy subset  $\tilde{A}$ .

$\mid$  condition Such that the condition is valid.

$\wedge$  Minimum.

$\vee$  Maximum.

$\in$  Inclusion.

$\notin$  Non-Inclusion.