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Introducing the dynamic location-allocation problem.

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Introducing the Dynamic Customer Location-Allocation Problem

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***Abstract*—In this paper, we introduce a new stochastic Location-Allocation Problem which assumes the movement of customers over time. We call this new problem Dynamic Customer Location-Allocation Problem (DC-LAP). The problem is based on the idea that customers will change locations over a defined horizon and these changes have to be taken into account when establishing facilities to service customers demands. We generate 1440 problem instances by varying the problem parameters of movement rate which determines the possible number of times a customer will change locations over the defined period, the number of facilities and the number of customers. We propose to analyse the characteristics of the instances generated by testing a search algorithm using the stochastic *dynamic* evaluation (based on the replication of customer movement scenarios) and a deterministic *static* evaluation (based on the assumption that customer will not move over time). We show that the *dynamic* approach obtains globally better results, but the performances are highly related to the parameters of the problem. Moreover, the *dynamic* approach involves a significantly high computational overhead.**

***Index Terms*—Dynamic Customer Location-Allocation Problem, Static Approach, Dynamic Approach, Population-Based Incremental Learning Algorithm, Simulation Model**

I. Introduction

Location-Allocation Problem is a branch of location analysis concerned with determining optimal locations for a set of facilities to service a set of requirements to reduce the overall total costs [1].

Location-Allocation problems are formulated with a range of parameter values determining aspects such as the setup cost of facilities and levels of demand. However, this formulation can have limitations when tackling a real-world problem such as servicing customers demands over a defined time. The fundamental characteristics of Location-Allocation problems require that any rational model reflect some features of future uncertainty. Changes in the population growth and migration, market size, environmental factors and advancement in technology often drive the need of consumers which causes demand to be stochastic. For this reason, facilities are expected to be effective in servicing demand over an extended planning horizon. In cases where considerable capital and resource investment is required in establishing a facility, such as a case in the telecommunications industry, it is vital to take into consideration potential variations in demand over time [2].

In this paper, we introduce a new formulation of the Location-Allocation problem, which takes into account the actualised servicing costs and the movement of customers over a predefined period. We call this problem the Dynamic Customer Location-Allocation Problem (DC-LAP). The model generates random customer movements based on varying attractivity of locations. It is important to note that this problem does not fall in the domain of dynamic optimisation literature, where the fitness of solutions changes dynamically while a search algorithm is seeking an optimal solution. Here, "dynamic" refers to the movement of customers.

To study DC-LAP, we generate 1440 problem instances by varying three problem parameters: (1) movement rate, which determines the mobility of customers, (2) the number of facility locations and (3) the number of customers. We run the Population-Based Incremental Learning Algorithm (PBIL) [3] on these problems by using two evaluation methods. The first method referred to as, the *dynamic* approach, simulates customer movements to estimate the expected cost over time. The second method called, the *static* approach, assumes no customer movements and only evaluates the actualised servicing costs.

We compare the results of the two approaches concerning the different problem parameters to analyse the efficiency of each approach.

The paper is organised as follows. In section II we give a brief review of some prior work of dynamic models explored

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in the study of Location Analysis. In section III, we define the problem of Dynamic Customer Location-Allocation and describe the objective function and simulation model. In Section IV we describe the experimental setup and discuss result in section V. Finally, we conclude and present, future works in Section VI.

II. Problem Background

In this section, we briefly review some prior work of dynamic models of Location Analysis problems, in particular, the variations with location and demand. We will then introduce a new dynamic model of Location-Allocation problems which takes into account the movement of customers over a defined horizon.

Initial work in the area of studying the dynamic models of Location Analysis was conducted in [4], in this work Ballou located and relocated a single warehouse over a planning period. He aimed to maximise the net present value of costs based on the site of the warehouse. Ballou solved the problem using dynamic programming with backward recursion. In his approach, the collection of candidate locations is confined to the collection of locations which compare to solutions in the static problems for each period. The approach presented in this paper assumes demand and economic data to have a distinctive pattern.

In [5], a review of different heuristic methods for solving a deterministic dynamic model of Location problem was conducted. The model aims to minimise the total costs of all capacity additions and distribution costs. The cost is determined by ascertaining the time and place to affix capacity to meet all demands in all periods.

A Dynamic fixed charged facility location problem that requires facilities to stay shut once the choice is made to shut them down is examined in [6]. Such a deterministic model is suitable in the aspect of a shrinking market. The model is extended to permit already opened facilities to be shut down once and already closed facilities to be opened once. Branch and bound was employed to solve the models.

A dynamic variant of the P-median problem which consolidates fixed costs of opening and shutting down facilities in each period of the planning horizon is studied in [7]. (Also, the model may be viewed as a Dynamic fixed charge facility location problem with a restriction on the number of facilities that are required to be established in each period of the planning horizon). The model restricts the location moves allowed in each period of the planning horizon to a specified range. The model also limits the number of established facilities in each period to a range. By limiting the growth of the space, the authors can employ dynamic programming to solve the model. Small examples of the problem were solved with the use of a standard branch and bound integer programming package. The model presented in this work assumes a distinctive pattern of change in demand over the defined period.

Extensions of a deterministic Dynamic-set covering location problem and the Dynamic maximum covering location problems were considered in [8] and [9] respectively.

Recent work, such as the work presented in [10] exploited the use of the Benders decomposition for solving a class of the capacitated multi-period facility location problem. The approach proposed was targeted at the Benders subproblems associated with decomposition of large-scale capacitated multi-period discrete facility location problems. The paper argues that the approach can be extended to solving the stochastic single or multi-period discrete facility location problems. The model presented assumes changes in demands and transportation costs to be deterministic.

In [11] the authors proposed a new problem set for the discrete-time Robust optimisation over time (ROOT). In their work, the two types of problem sets were aimed at testing an algorithm's ROOT abilities in-terms of maximising average fitness and survival time. Four methods from the literature are investigated to test their ROOT abilities: A simple particle swarm optimiser with a restart strategy, An ideal tracking moving optimum algorithm, Jin's framework [12] and Fu's framework [13]. In both Jin's and Fu's framework a global radial basis function network is used as a surrogate model to approximate a solution's previous fitness and an auto-regression model for predicting a solution's future fitness. Experiments identified two major difficulties in solving ROOT problems, which to some extent, point out the requirement for a good ROOT algorithm, i.e. the difficulty to predict a solution's future fitness and the difficulty to evolve solutions based on inaccurate information.

Work presented in [14] explored a deterministic model of dynamic location problem arising from optimising the emergency service network of Police Special Forces in Serbia. They proposed a Variable Neighborhood Search method with a local search procedure and compared results with CPLEX 12.1. Although small problem instances were used to test the model, they argued that the model presented and solution approach can be applied to designing and managing other emergency service networks. In [15] a dynamic model is proposed with consideration to uncertainty demand over different periods from previously presented models of stochastic dynamic location problem. The stochastic model is converted to a deterministic one using stochastic constraint programming and solves the model using an industry solver.

Dynamic models of Location Analysis are categorised into Explicit and Implicit dynamic models [16]. In explicit dynamic models, facilities will be opened and possibly closed over a defined horizon also known as Multi-period. They include other factors such as relocation time, Number of re-locations and number of facilities to be relocated. Most works in the area of Dynamic Location problem have focused on explicit models [17]–[22].

Implicit dynamic models concern selecting profitable facilities locations to be opened once at the start and remain open over a defined time. Implicit dynamic models are dynamic because they recognise that problem parameters such as demand may vary across time and endeavour to anticipate these changes in the facility location scheme generated [23].

Since expected conditions are typically unpredictable, and projections are often inaccurate and subject to review, anticipating for future conditions can be challenging. Moreover, there is typically no predetermined exogenous time exceeding which conditions can be overlooked. Given this, and recognising that the first-period decision is the only one that must be implemented immediately [24], we hold that the goal of dynamic Location-Allocation planning should be to determine an optimal or near optimal first-period choice of facilities for the defined horizon. For this reason, we are interested in implicit models, and in particular, an enriched implicit model that captures characteristics of real-world scenarios.

Previous work has focused on opening and closing facilities over a defined horizon which incurs substantial costs, but in cases, where there is a large and mobile customer base, we are not aware of significant research work done in this area.

We, therefore, propose a dynamic variant of Location-Allocation problem called Dynamic Customer Location-Allocation problem (DC-LAP) where we model the movement of customers between different cities over a defined horizon. Customers are initially based in cities where facilities are located. The selected cities are each assigned an attraction rate, which is the probability of a city to attract a customer. We assume customers will move between cities at different times of the defined horizon. Therefore we simulate future events for each customer using a stochastic model that generates uncertainty in the movement of customers. At any point over the defined horizon, when customers have to move between cities, the attraction rate of the city influences the choice of the city a customer is likely to relocate.

III. Problem Formulation

In this section, we define the problem of Location-Allocation. We then introduce and define the new Dynamic variant of Location-Allocation problem called the Dynamic Customer Location-Allocation problem. We further describe the objective function for computing the fitness of a solution to the problem. Finally, we present a stochastic model for simulating customer movement over time.

A. Location-Allocation Problem

Location-Allocation Problems (LAP) are concerned with allocating locations to a set of facilities to service a set of customers in such a way as to optimise a cost function. One important LAP attribute is the ability of facilities to provide an unlimited amount of service, that is they can serve any number of customers. This model of LAP is known as Uncapacitated Location-Allocation problem. Our focus in this paper lies in the uncapacitated version of Location-Allocation problems. In this work we assume that each facility is located in the centre of a city; hence we use a city and facility interchangeably to mean the same thing.

Let a set of m cities $L = l_1, l_2, ..., l_m$ be a set of m potential locations, and $B = b_1, b_2, ...b_n$ be a set of n customers. Each $l_i \in [0, 1]^2$ and $b_i \in [0, 1]^2$ define the coordinates in a 2-dimensional plane.

Here, the cost d_{ij} of connecting customer b_j to location l_i is defined by the Euclidean distance between b_j to location l_i.

The decision variables are represented by a binary string $x = \{x_1, x_2, ..., x_m\} \in \{0, 1\}^m$ where 1 represents an opened facility and 0 represents a closed facility. Given c_i as opening cost of each facility. LAP is formulated as follows:

$$f(x) = \sum_{i=1}^{m} c_i x_i + C_0(x) \tag{1}$$

Where C_0 is the connection cost of each customer to an opened facility. Let $z_{ij} = 1$ if customer i is connected to facility j and $z_{ij} = 0$ if not.

$$C_0(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} z_{ij} \tag{2}$$

Subject to:

$$\sum_{i\ =1}^{m} \sum_{j\ =1}^{n} z_{ij} = 1 \tag{3}$$

i.e. each customer is connected to only one opened facility.

B. Dynamic Customer Location-Allocation Problem

The location of facilities in both private and public sector systems critically affects the ability of these systems to deliver the essential services [24]. Because facility location decisions are long-term strategic decisions, they impact on shorter-term decisions such as resource allocation. In the absence of substantial costs of opening a facility, the optimal option of locating facilities will be to site them optimally in each period to service changing demand. However, these costs are often substantial and therefore prevent repeated relocation of facilities. Hence, it imperative that location choices executed today consider expected future circumstances.

Thus Dynamic Customer Location-Allocation (DC-LAP) concerns finding the optimal locations for establishing facilities optimally in the first-period to service changing demands over a defined horizon. The idea of this new dynamic variant of Location-Allocation problem stems from the telecommunication industry where customers move between cities over time. Considering that telecommunication infrastructure involves considerable capital and resource investment, facilities need to be optimally positioned in order to service the changing customers' demands adequately. However, this model can be generalised to fit other location problems.

To help develop our dynamic model we make some assumptions about the parameters that define the model:

- Cause of Change: Changes can be assumed to have a distinctive pattern with deterministic and time-dependent parameters, or the pattern of change may be distinctive, but not deterministic; instead it is stochastic [25]. In this work, our dynamic model assumes the later.
- Location Type: Locations can be discrete where the locations to establish facilities are predetermined or continuous where the locations are not predetermined [23]. Our dynamic model assumes locations to be discrete.

- Types of facilities: Facilities can be exogenous, where the number of facilities to be located is predetermined or endogenous, where the optimal number of facilities are found by solving the problem [23]. We consider the later in our dynamic model.
- Type of Service: Services rendered by facilities can be homogeneous or heterogeneous [23]. Our dynamic model assumes facilities to offer similar services.
- Capacity Constraint: Facilities can be capacitated where they are limited in the number of customers they can optimally service or Uncapacitated where facilities are unconstrained in the number of customers they can optimally service. Our dynamic model is assumed to be uncapacitated.
- Time horizon: Time here can be finite or infinite. In this work, our dynamic model assumes a finite horizon.

In the following sections, we outline the approach to the Dynamic Customer Location-Allocation problem.

C. Problem formulation

Dynamic Customer Location Allocation Problem (DC-LAP) considers the potential movement of customers over a given time horizon t_{max}. This will influence the connection cost $C_t(x)$. The objective function defined in Equation 1 can then be formulated as:

$$f_{dynamic}(x) = \sum_{i=1}^{m} c_i x_i + C_0(x) + E\left[\sum_{t=1}^{t_{max}} C_t(x)(1+r)^{-t}\right] \tag{4}$$

Where the opening cost of facilities and the servicing cost $C_0(x)$ is a deterministic function.

The cost function C_t calculates the discounted total service costs of customers for times $\{t_1, t_2, ..., t_{max}\}$. r is a *discount rate*, typically applied to allow comparison of costs incurred at different times.

To generate an instance of this problem, we first uniformly generate cities location and their attraction rate randomly. Based on those locations, customers are then iteratively generated by randomly selecting a city (based on the attraction rates). The coordinates of the customer are then obtained by sampling its location from a normal distribution centred in the coordinates of the city. Hence, the coordinates b_i of customer i placed in city j is given by:

$$b_i = \{\mathcal{N}(x_{l_j}, 0.1), \mathcal{N}(y_{l_j}, 0.1)\} \tag{5}$$

An example of the set of facility locations and customers is presented in Figure 1.

D. Simulation

The simulation is based on the assumption that customers will move over time *i.e.* disappear from a location and reappear in another location. We also assume that the attraction rate of each city in the future is unknown.

Each simulation starts by generating new attraction rates for each city. For each customer, we then generate the times at which the customer is going to move over the next t_{max} years. For this purpose, we introduce a new parameter call movement rate mr ranging from 0 to 1 which regulates the mobility of customers. The movement times of each customer are sampled from a normal distribution centred in $mr \cdot t_{max}$. Hence, each customer will move on average $mr \cdot t_{max}$ times during the simulation. The process to generate movement times is shown in Algorithm 1.

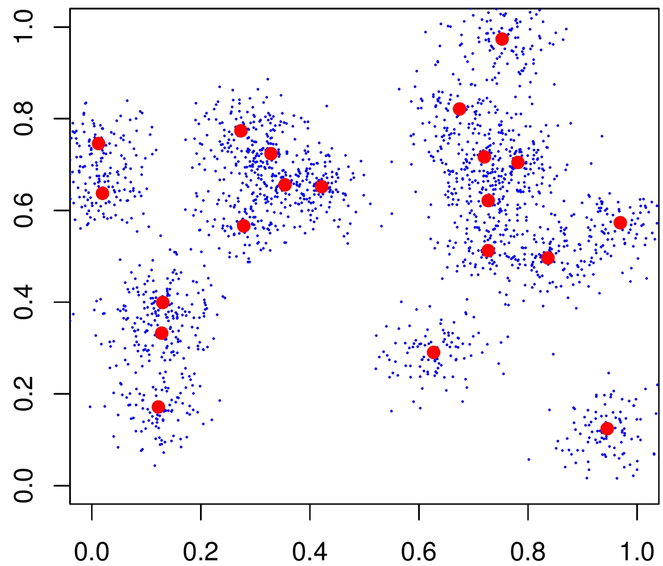


Fig. 1: Example of DC-LAP with $m = 20$ facilities (red dots) and $n = 2000$ customers (blue dots)

Algorithm 1 Customer movement dates generation

Require: Movement rate : mr
  Set of movement dates: $M = \emptyset$
  $t = 0$
  while $t < t_{max}$ **do**
    $t = t + \mathcal{N}(mr \cdot t_{max}, 0.1 \cdot t_{max})$
    $M = M \cup t$
  end while

Each simulation consists of generating customer movements and calculating their service costs over the t_{max}. The steps of a simulation are outlined in Algorithm 2.

Algorithm 2 Simulation Model

Require: L, B, t_{max}
  for Each Simulation **do**
    Generate attraction rates A' for each city
    for Each customer b_i in B **do**
      Generate movements of customer M from Algorithm 1
      for t_y in 1 to t_{max} **do**
        if $t_y \in M$ **then**
          Choose a new city for the customer based on A'
          Generate new location for customer in the new city
          Update cost for servicing customer based on open facilities
        end if
        Add cost of servicing customer b_i to total cost for year t_y
      end for
    end for
    Actualise costs obtained for t_{max} using discount rate
  end for

IV. Experimental Setup

For the purpose of this paper, we use the following parameters to generate DC-LAP instances. The choice of parameters are motivated by real-world telecommunication problems:

- The number of facilities $m = \{10, 20, 50, 100\}$.
- The number of customers $n = \{100, 500, 1000\}$.
- The movement rate $mr = \{0.25, 0.5, 0.75, 1\}$.
- Time horizon $t_{max} = 30$.

We set the discount rate $r = 0.05$. By varying the parameters *m, n,* and *mr* we generate 48 problem configurations. For each of the 48 configurations, we generate 30 instances creating a benchmark of 1440 problems using the method described in Section III-C.

To assess the necessity of simulating future movements of customers, we test the problem using two evaluation methods. The first one uses a stochastic approach as described in the previous section where each solution is evaluated over 100 simulations. We refer to this approach as *dynamic*. The second one consists in using a deterministic function which assumes that customers will not move over time. We refer to this approach as *static*. the static approach reformulates Equation 6 by:

$$f_{static}(x) = \sum_{i=1}^{m} c_i x_i + C_0(x) + \sum_{t=1}^{t_{max}} C_0(x)(1+r)^{-t} \quad (6)$$

To compare the two approaches we use Population-Based Incremental Learning Algorithm (PBIL) presented in [3]. This gives us the configurations: $PBIL_{static}$ and $PBIL_{dynamic}$. The Parameters used in this paper for $PBIL$ were the best parameters found in [3]. They include a population size of 50, the learning rate of 0.1 and a truncation size of 50% of the population for learning. Each run is allowed 10000 fitness evaluations. At the end of each run, the best solution is evaluated using the *dynamic* approach over 5000 simulations, and the expected cost of that solution is returned. We run each approach for each of the 1440 problems 20 times.

V. Results

In this section, We present the results obtained by the two approaches to the benchmark instances. We analyse in more details the effect of the problem parameters on the performances. Finally, we study the time complexity of the two approaches to the problem instances.

A. DC-LAP parameters influence on results

In order to better understand the problem characteristics, we analyse the performance of each approach according to the parameter values used in each configuration. To do so, we present a table summary of the number of wins, loss and ties of $PBIL_{dynamic}$ against $PBIL_{static}$ in Table I. A problem configuration is represented by a movement rate mr, the number of facilities m and the number of customers n. In Table I we show the number of wins, loss and ties achieved by $PBIL_{dynamic}$ on all 30 problems of a problem configuration. In Figure 2, we calculate the percentage difference between the $PBIL_{dynamic}$ and $PBIL_{static}$ for each of the 1440 problem instances in order to analyse the gain in cost of the solutions obtained by both approaches. Negative values indicate that the *dynamic* approach resulted in better performance. We analyse the results as follows:

Firstly, increasing the movement rate (see Figure 2a) appears to reduce the necessity of using the *dynamic* approach. Indeed, for a movement rate of 0.25, the average savings is of 0.61%, and it decreases to 0.002% for $mr = 1$. From Table I it appears that as mr increases the number of ties between both approaches also increases. Also, the number of wins obtained by $PBIL_{dynamic}$ on a higher mr appears to be relatively comparable to the number of wins obtained by the $PBIL_{static}$. The similarity can be explained by the fact that a higher mr indicates that customers will make little or no movement over the defined period, hence the higher the mr, the closer the problem is to a static problem. However, $PBIL_{dynamic}$ appears to achieve a higher number of wins on smaller mr. Because smaller mr indicates that customers will move more frequently, it creates highly different costs when simulating the movement of customers. Thus, it emphasises the necessity of simulating a large number of scenarios to obtain a robust fitness for solutions.

Secondly, the number of facilities corresponds to the dimension of the problem. According to Figure 2b, the more significant the number of facility locations the better the results obtained by the *dynamic* approach. The performance of $PBIL_{dynamic}$ highlights its scalability on significant problems.

Finally, for the number of customers (see Figure 2c), the *dynamic* approach favours the smaller number of customers. From Table I, we see that $PBIL_{dynamic}$ achieves a higher number of wins on problems with 100 and 500 customers even for a high mr of 0.75. The performance of $PBIL_{dynamic}$ is

TABLE I: Wins, Losses and Ties of $PBIL_{dynamic}$ when compared to $PBIL_{static}$ grouped by configurations of DC-LAP

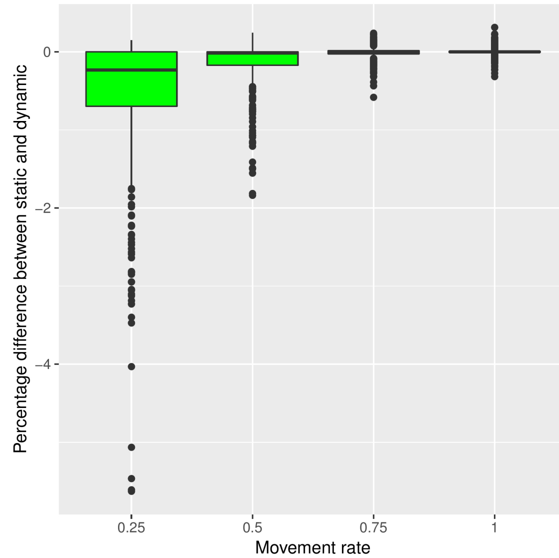
mr	m	n	Wins	Loss	Ties
0.25	10	100	20	0	10
0.25	10	500	12	3	15
0.25	10	1000	8	2	20
0.25	20	100	25	4	1
0.25	20	500	15	7	8
0.25	20	1000	18	9	3
0.25	50	100	30	0	0
0.25	50	500	29	1	0
0.25	50	1000	26	4	0
0.25	100	100	30	0	0
0.25	100	500	29	1	0
0.25	100	1000	28	2	0
0.5	10	100	10	5	15
0.5	10	500	6	6	18
0.5	10	1000	4	3	23
0.5	20	100	14	13	3
0.5	20	500	8	12	10
0.5	20	1000	15	9	6
0.5	50	100	29	1	0
0.5	50	500	24	6	0
0.5	50	1000	22	8	0
0.5	100	100	28	2	0
0.5	100	500	22	8	0
0.5	100	1000	19	11	0
0.75	10	100	7	2	21
0.75	10	500	3	1	26
0.75	10	1000	3	2	25
0.75	20	100	7	12	11
0.75	20	500	1	13	16
0.75	20	1000	6	9	15
0.75	50	100	23	7	0
0.75	50	500	15	15	0
0.75	50	1000	16	14	0
0.75	100	100	19	11	0
0.75	100	500	14	16	0
0.75	100	1000	13	17	0
1	10	100	0	0	30
1	10	500	1	1	28
1	10	1000	0	2	28
1	20	100	2	8	20
1	20	500	1	8	21
1	20	1000	2	7	21
1	50	100	14	16	0
1	50	500	12	18	0
1	50	1000	15	15	0
1	100	100	17	13	0
1	100	500	17	13	0
1	100	1000	17	13	0

explained by the fact that large customer base creates a more uniform distribution of the customers over an area. Thus, their movements are more likely to even out during simulations.

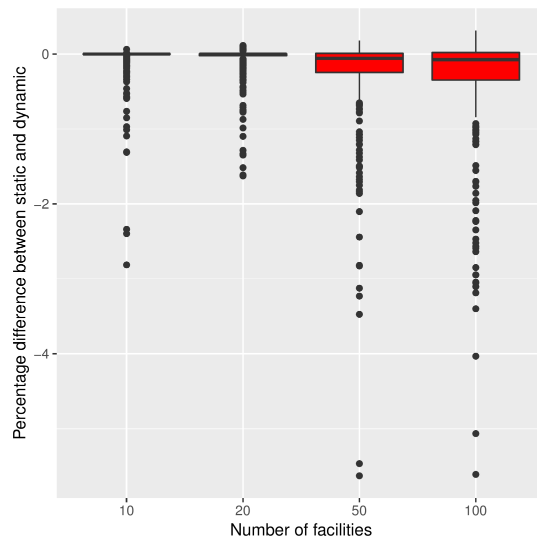
B. Computational time complexity

Table II shows the average computational time obtained by the two approaches for each of the 48 problem configurations. In the table $Static$ and $Dynamic$ represents $PBIL_{static}$ and $PBIL_{dynamic}$ respectively. $Dynamic/Static$ shows the difference between $PBIL_{dynamic}$ and $PBIL_{static}$.

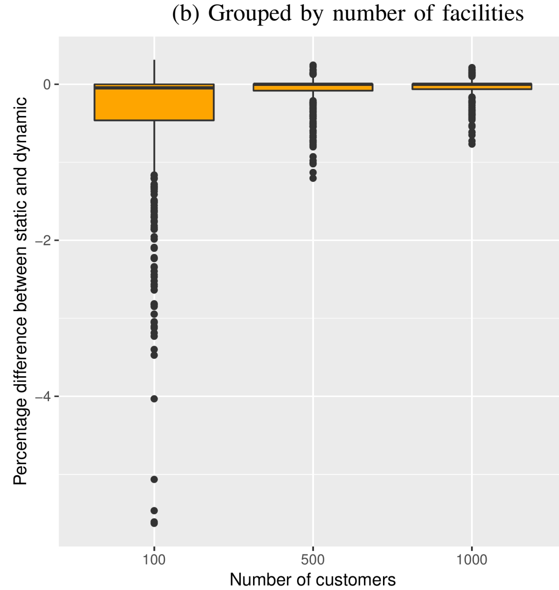
From the table, it is seen that for all problem configurations $PBIL_{dynamic}$ is computationally from 33 to 2451 times more expensive than $PBIL_{static}$. To understand why this is so, we



(a) Grouped by movement rate



(b) Grouped by number of facilities



(c) Grouped by number of customer

Fig. 2: Percentage difference between the *dynamic* and *static* approaches

need to analyse the impact each problem parameter has on the computational time of the $PBIL_{dynamic}$.

Firstly, we observe that the lower the moment rate, the more expensive the time recorded. A lower movement rate means that customers make more movements over time. Whenever a customer makes a move, new coordinates are computed for the customer and the distances calculated between the new location of the customer and the cities to obtain the least cost. Hence, the more movement a customer makes the more time it takes to generate new locations and compute the costs. On the other hand, a higher mr means that a customer will make little or no moves. Which means the simulation models generates fewer movement times and hence fewer new customer locations. From the table, the computational time of the $PBIL_{dynamic}$ is (depending on the configuration) 3 to 5 times higher between a movement rate of 0.25 and 1.

Secondly, the number of facilities also contribute to the computational time. To obtain the least cost of a new customer location, we calculate the cost of service between the customer location and the facilities. The computational cost of the $PBIL_{dynamic}$ increases by a factor of 2.5 to 3 between problems with 10 and 100 facilities.

Thirdly, the number of customers linearly increases the computational costs. Indeed, as each customer is subject to the simulation of its movements, the computational costs are directly proportional to the number of customers.

The sizeable computational cost difference between the two approaches emphasises the fact that the choice of methodology to tackle such problem is crucial.

VI. Conclusion

In this paper, we have introduced a new dynamic variant of the Location-Allocation Problem called the Dynamic Customer Location-Allocation Problem (DC-LAP) which considers the movement of customers over a defined horizon. The model used to simulate the movement of customers is based on the idea that the attractivity of different locations will vary over time. A DC-LAP is defined by an initial set of facility locations, a set of initial customers and the movement rate of the customers.

To analyse the influence of these parameters on the problems and its difficulty, we generated 1440 problems covering a range of 48 different configurations of numbers of facilities, numbers of customers and movement rates. We then compared the performance of the *dynamic* evaluation and *static* evaluation using a simple PBIL algorithm. For the *static* evaluation we assume that customers do not change their locations over the defined period whiles for the *dynamic* evaluation customers are assumed to make frequent moves between cities over the defined period. A combination of the evaluation approaches with $PBIL$ gives us the configurations $PBIL_{static}$ and $PBIL_{dynamic}$.

We observed that the performance of the $PBIL_{dynamic}$ obtained globally better results than the $PBIL_{static}$. However, we also noted that the significance in terms of wins and cost savings was highly dependent on the parameters of the problem.

TABLE II: Computational times in seconds of $PBIL_{static}$ and $PBIL_{dynamic}$ for each problem configurations

mr	m	n	*Static*(s)	*Dynamic*(s)	*Dynamic/Static*(s)
0.25	10	100	0.65	100.99	100.34
0.5	10	100	0.46	56.88	56.42
0.75	10	100	0.38	45.69	45.31
1	10	100	0.26	33.37	33.11
0.25	20	100	0.86	121.45	120.59
0.5	20	100	0.47	65.48	65.01
0.75	20	100	0.35	51.22	50.87
1	20	100	0.26	35.77	35.51
0.25	50	100	1.19	172.44	171.25
0.5	50	100	0.67	85.36	84.69
0.75	50	100	0.58	64.49	63.91
1	50	100	0.44	41.44	41
0.25	100	100	1.64	241.85	240.21
0.5	100	100	0.96	112.95	111.99
0.75	100	100	0.79	82.89	82.1
1	100	100	0.59	49.69	49.1
0.25	10	500	3.12	493.43	490.31
0.5	10	500	1.88	275.83	273.95
0.75	10	500	1.6	220.5	218.9
1	10	500	1.25	159.39	158.14
0.25	20	500	4.19	595.03	590.84
0.5	20	500	2.4	319.39	316.99
0.75	20	500	1.95	249.18	247.23
1	20	500	1.48	171.52	170.04
0.25	50	500	5.84	865.87	860.03
0.5	50	500	3.3	427.85	424.55
0.75	50	500	2.62	322.91	320.29
1	50	500	1.93	203.44	201.51
0.25	100	500	8.22	1225.31	1217.09
0.5	100	500	4.52	576.5	571.98
0.75	100	500	3.63	423.63	420
1	100	500	2.64	248.26	245.62
0.25	10	1000	6.84	999.69	992.85
0.5	10	1000	4.03	547.91	543.88
0.75	10	1000	3.33	441.84	438.51
1	10	1000	2.58	318.82	316.24
0.25	20	1000	8.23	1191.34	1183.11
0.5	20	1000	4.8	628.33	623.53
0.75	20	1000	4.03	498.67	494.64
1	20	1000	2.91	342.57	339.66
0.25	50	1000	12	1723	1711
0.5	50	1000	6.6	845.15	838.55
0.75	50	1000	5.34	645.87	640.53
1	50	1000	3.96	408.19	404.23
0.25	100	1000	16.71	2467.96	2451.25
0.5	100	1000	9.07	1159.61	1150.54
0.75	100	1000	7.27	857.84	850.57
1	100	1000	5.06	501.67	496.61

It is also important to note that the computational time of the $PBIL_{dynamic}$ can be extremely costly, especially in problems with a high frequency of customer movement, and a large number of facilities and customers. This computation overhead should be alleviated with the improvement in results obtained and should be taken into account when facing a given problem.

The homogeneity of the performances of the two approaches to the generated benchmark makes it a challenging problem to tackle. It raises the question of the computational effort one should dedicate to each solution evaluation in a stochastic environment concerning the global budget allocated to a search and the gain in performances. Future work will, therefore, focus on identifying better techniques to find the right balance between the number of simulations required and computational time complexity of the $PBIL_{dynamic}$.

We will then aim at relating this problem to real-world applications, using real city and customer data and applying, for instance, historical statistics to customer movement statistics.

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